Pion-nucleus scattering in a cluster model*

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The forward amplitude for pion-nucleus scattering in the N^* region is studied using a Glauber formalism with a cluster model of the nucleus. It is found that the major features of the data for light nuclei (^{6}Li, ^{7}Li, ^{9}Be, ^{12}C, ^{16}O) are reasonably well reproduced by using a simple parametrization of the cluster structure. These results are in sharp contrast to predictions of a nucleon model at a comparable level of simplicity. We argue that a Glauber series representation of the amplitude is viable in a cluster model but not reliable in a nucleon model essentially because the interlocking effects of multiple scatterings and strong particle correlations inside the nucleus are easy to approximate in the former model and are difficult to account for in the latter model. Several modifications of the basic cluster model are considered and estimated to be small. More studies, experimental and theoretical, on angular distributions are urged.

[NUCLEAR REACTION ^{2}H, ^{3}He, ^{4}He, ^{6}Li, ^{7}Li, ^{9}Be, ^{12}C, ^{16}O, ^{27}Al, ^{32}S; forward amplitude in the N^* region calculated with a cluster model using a Glauber approximation and compared to data.]

I. INTRODUCTION

Different methods are generally used to analyze the pion-nucleus interaction.[1] For low energies (below 100 MeV pion laboratory kinetic energy), optical potentials[2] are usually used. For very high energies (above 1 GeV), the Glauber multiple scattering series[3] is used almost exclusively. Both methods have been extended into the intermediate energy region, especially the so-called N^* region around 170 MeV where the pion nucleus total cross section appears resonating, but with mixed success.[1] There are also other approaches[1] to calculating the pion-nucleus interaction, particularly in the awkward N^* region. All these methods may be viewed as different means to render tractable the same underlying Watson multiple scattering series[4] which is exact, formal, but not amenable to calculations. A common feature of most of these calculations is the use of the pion-nucleon scattering amplitudes as the basic input. This is, of course, rooted in our general acceptance of nucleons as basic ingredients of all nuclei. In this note, we offer a variation.[5] We consider certain target nuclei (^{6}Li, ^{7}Li, ^{9}Be, ^{12}C, ^{16}O, ...) to be composed of four basic kinds of clusters (nucleons, deuterons, and α particles) and use the pion-cluster scattering amplitudes as the basic input. The Glauber formalism is still used to carry out the calculation. Our aim is twofold: first, to show that the Glauber series is viable in the N^* region if a cluster model is used, and second, to show that clusters in light target nuclei offer an attractive alternative to nucleons as the basic nuclear substructure. We confine our attention in this note to the forward part of the pion-nucleus amplitude. This is motivated in part by the wealth of experimental information on total pion-nuclear cross sections[6-10] in the last few years and in part by the paucity of angular distributions available which hampers our effort either to parametrize the basic pion-cluster amplitudes or to compare our results to data.

II. BASIC MODEL

There are several variants of the Glauber amplitude[3, 11-13] for scattering off a target which contains a finite number of scattering centers. But one essential feature remains the same. A closed form expression of the scattering amplitude is obtained which can be written as a finite series of multiple scattering terms. To test our contentions about clusters and the Glauber series in the N^* region, we use a model which is a slight generalization of a simple formula given by Glauber.[14] Complications will be discussed later in Sec. IV.

Following Glauber, we make the following major assumptions: (a) all correlations among the clusters are ignored; (b) the scattering on each cluster is predominantly forward with no rescattering on the same cluster; (c) the clusters remain at rest throughout the scattering process; and (d) dependences on certain momentum transfer variables, as detailed below, are of the Gaussian type. For notation, let the pion be incident with laboratory momentum q, kinetic energy T, and in the process of scattering, transfer momentum $\vec{\Delta}$ to the target nucleus. Further, let there be k kinds of clusters with N_1 clusters of the 1st kind, N_2 clusters of the 2nd kind $\cdots$, etc., for a total of

$$N = N_1 + N_2 + \cdots + N_k \tag{1}$$

clusters in the target nucleus. Masses are M_π, M_i, and M_T for the pion, ith kind cluster, and target nucleus, respectively. The pion-nucleus amplitude F_T can be expressed as a finite series with N multiple scattering terms:

$$F_T(\vec{\Delta}; q) = \sum_{n=1}^{N} F^{(n)}(\vec{\Delta}; q). \tag{2}$$

Each order $F^{(n)}$ of multiple scattering is itself a sum of all possible scatterings on n clusters with no rescattering on the same cluster:

$$F^{(n)}(\vec{\Delta}; q) = \sum_{n_1=0}^{N_1} \sum_{n_2=0}^{N_2} \cdots \sum_{n_k=0}^{N_k} \binom{N_1}{n_1}\binom{N_2}{n_2} \cdots \binom{N_k}{n_k} F^{(n)}_{n_1 n_2 \cdots n_k}(\vec{\Delta}; q). \tag{3a}$$

Here, n_i is the actual number of scattering on clusters of the ith kind and is restricted by

$$n = n_1 + n_2 + \cdots + n_k, \tag{3b}$$

and $\binom{N_i}{n_i}$ is the binomial coefficient representing the number of ways in which n_i scatterings on N_i centers can occur. For our no-correlation Gaussian model

$$F^{(n)}_{n_1 n_2 \cdots n_k}(\vec{\Delta}; q) = \left(\frac{i}{2q}\right)^{n-1} \frac{[h_1(q)]^{n_1}[h_2(q)]^{n_2} \cdots [h_k(q)]^{n_k}}{\gamma_1^{n_1}\gamma_2^{n_2}\cdots\gamma_k^{n_k}(n_1/\gamma_1 + n_2/\gamma_2 + \cdots n_k/\gamma_k)} e^{-\Delta^2/(n_1/\gamma_1 + n_2/\gamma_2 + \cdots n_k/\gamma_k)}, \tag{4}$$

where

$$\gamma_i \equiv \tfrac{1}{2}\beta_i + \lambda_i .$$

The function $h_i(q)$ and the parameter β_i are derived from an assumed Gaussian form for the scattering amplitude f_i for pions on a cluster of the ith kind:

$$f_i(\vec{\Delta}; q) = h_i(q) e^{-\beta_i \Delta^2/2}. \tag{5}$$

We identify $h_i(q)$ as the forward amplitude and relate it to the ratio α_i of its real and imaginary parts and the total cross section σ_i by

$$h_i(q) = f_i(0; q) = \frac{q}{4\pi}\sigma_i(T)[i + \alpha_i(T)]. \tag{6}$$

The parameter β_i is the diffraction slope, to be related to the shape of the angular distribution near forward angles. The parameter λ_i is derived from an assumed Gaussian dependence for the form factor S_i for an uncorrelated cluster of the ith kind:

$$S_i(\vec{\Delta}) \equiv g_i(\vec{\Delta}) e^{-\lambda_i \Delta^2}, \tag{7}$$

where $g_i(\vec{\Delta})$ is a slowly varying function of $\vec{\Delta}$. We note that since S_i is defined in terms of the position space wave function $\psi_i(\vec{R}_i)$ by

$$S_i(\vec{\Delta}) = \int d^3\vec{R}_i \, |\psi_i(\vec{R}_i)|^2 e^{-i\vec{\Delta}\cdot\vec{R}_i}, \tag{8}$$

we have, generally,

$$S_i(0) = g_i(0) = 1, \tag{9}$$

so that λ_i, the form factor slope, can be related to some property of the wave function such as the mean squared position

$$\langle \vec{R}_i^{\,2} \rangle \equiv \int d^3\vec{R}_i \, |\psi_i(\vec{R}_i)|^2 \vec{R}_i^{\,2}. \tag{10}$$

In particular, we note that a Gaussian dependence for S_i follows from a Gaussian type wave function. For example, if

$$\psi_i(\vec{R}_i) = \text{constant} \times (a_0 - a_1 R_i^{\,2}/\lambda_i) e^{-R_i^{\,2}/8\lambda_i}, \tag{11}$$

then for $a_1 = 0$ we have

$$\begin{aligned} &\langle \vec{R}_i^{\,2} \rangle = 6\lambda_i, \\ &S_i(\vec{\Delta}) = e^{-\lambda_i \Delta^2}, \end{aligned} \tag{12}$$

and for $a_0 = 0$, we have

$$\begin{aligned} &\langle \vec{R}_i^{\,2} \rangle = 14\lambda_i, \\ &S_i(\vec{\Delta}) = (1 - \tfrac{4}{3}y^2 + \tfrac{4}{15}y^4) e^{-y^2}, \\ &y \equiv \sqrt{\lambda_i}\,\Delta. \end{aligned} \tag{13}$$

Near the N^* region, the parametrization in Eq. (5) is not expected to be valid for target nucleons but is admissible for target deuteron and α particle since these nuclei have diffraction peaks that set in at much lower energies. Unfortunately, the scarcity of angular distribution information means that not much is known about β_i and λ_i. To minimize the uncertainty, we confine ourselves to the forward part of F_T only. Then β_i and λ_i will not contribute to the single scattering term $F^{(1)}(0; q)$, which turns out to be the dominant contribution to $F_T(0; q)$. We define, analogous to Eq. (6),

$$F_T(0; q) = (q/4\pi)\sigma_T(T)[i + \alpha_T(T)]. \tag{14}$$

TABLE I. Parameter values for Eq. (16). Units are as follows: σ_r and σ_∞ are in mb; T_r, Γ_r in GeV; B_α in $(\text{GeV}/c)^{-1}$; C_α in $(\text{GeV}/c)^{-2}$; and B_σ, μ, ν, A_α are dimensionless. The expressions for σ_i and α_i are fitted over the range of kinetic energies as shown.

				σ_i					α_i	
	σ_r	σ_∞	T_r	Γ_r	B_σ	μ	ν	A_α	B_α	C_α
π^+p, π^-n	192	12	0.18	0.116	0.075	3	0	4.394	−31.358	39.989
π^-p, π^+n	66	13	0.18	0.09	0	0	0.5	5.397	−41.970	72.445
$\pi^\pm\,{}^2$D	235	37	0.18	0.13	0.1	3	0	3.790	−27.718	38.854
$\pi^\pm\,{}^4$He	330	88	0.145	0.19	0.14	4	0	1.561	−8.237	−2.088
Range			$0.05 \leq T \leq 0.45$						$0.10 \leq T \leq 0.34$	

In short, by inputting σ_i, α_i, β_i, and λ_i on the clusters, we obtain σ_T and α_T for the target nucleus, which can then be compared to experiment over a range of energies and target nuclei.

Explicit expressions for σ_T and α_T are simple if there are only two clusters present. In this case,

$$\begin{aligned}
&\sigma_T = \sigma^{(1)} + \sigma^{(2)} , \\
&\sigma^{(1)} = \sigma_1 + \sigma_2 , \\
&\sigma^{(2)} = -(1 - \alpha_1\alpha_2)\frac{\sigma_1\sigma_2}{8\pi(\gamma_1+\gamma_2)} , \\
&\alpha_T = \alpha^{(1)} + \alpha^{(2)} , \\
&\alpha^{(1)} = \frac{1}{\sigma_T}(\alpha_1\sigma_1 + \alpha_2\sigma_2) , \\
&\alpha^{(2)} = -\frac{1}{\sigma_T}(\alpha_1 + \alpha_2)\frac{\sigma_1\sigma_2}{8\pi(\gamma_1+\gamma_2)} .
\end{aligned} \tag{15}$$

We make several rather obvious observations: (a) no information about the cluster structure of the target appears in the single scattering contributions $\sigma^{(1)}$ and $\alpha^{(1)}$—a fact generally true of no-recoil models; (b) the minus sign for the double scattering contributions can be interpreted as a unitarity effect, the so called "shadow correction"[3]; (c) the factor $(\gamma_1+\gamma_2)$ plays the role of an intercluster separation that determines how probable double scattering is; and (d) the energy dependence of α_1 and α_2 will tend to suppress the contribution $\sigma^{(2)}$ for energies away from the resonance region where $\alpha_1 \approx 0 \approx \alpha_2$.

To interpolate between experimental data points[15–19] for values of σ_i and α_i, we use the following parametrized form for the energy dependence:

$$\begin{aligned}
&\sigma_i(T) = \left\{[(1+B_\sigma)\sigma_r - \sigma_\infty]\frac{Z^2}{Z^2+(X+1)^2} + \sigma_\infty\right\}\frac{X^{\mu+\nu}}{B_\sigma + X^\mu} , \\
&\alpha_i(T) = A_\alpha + B_\alpha T + C_\alpha T^2 , \\
&X \equiv T/T_r , \\
&Z \equiv [M_i^2 + M_\pi^2 + 2M_i(T_r + M_\pi)]^{1/2}\Gamma_r/(2M_i\Gamma_r) .
\end{aligned} \tag{16}$$

The parameters σ_r, σ_∞, T_r, Γ_r, B_σ, μ, ν, A_α, B_α, and C_α are given in Table I for the cluster of interest: neutron (n), proton (p), deuteron (d), and helium-4 (α). We remark that no data exist for α_d. We use our model with nucleons as input clusters to generate values of α_d for the parabolic fit. So there is some uncertainty here, but we believe that the effect is small.[20] Little is known about β_i and λ_i. Qualitatively, they are related to the structure of the cluster and the target. For simplicity, we choose the wave functions for all clusters in the form defined by Eqs. (11) and (12) and further assume that

$$\langle \vec{R}_i^{\,2}\rangle = \langle \vec{r}_T^{\,2}\rangle , \tag{17a}$$

where $\langle \vec{r}_T^{\,2}\rangle$ is the mean squared radius of the target nucleus[21–23] as given in Table II. The value of β_i is assigned:

$$\beta_i = 10\ (\text{GeV}/c)^{-2} \approx 0.4\ \text{fm}^2 , \tag{17b}$$

TABLE II. Values for $\langle r_T^2\rangle^{1/2}$, the rms radius of the target nucleus, in fm. In addition to the experimental values, we also show "good fit" values as discussed in the text which generally yield fits to better than 5% at the peak.

	^{2}D	^{3}He	^{4}He	^{6}Li	^{7}Li	^{9}Be	^{12}C	^{16}O	^{27}Al	^{32}S
Experiment	1.96 [a]	1.87 [b]	1.68 [c]	2.54 [d]	2.39 [d]	2.46 [e]	2.40 [e]	2.61 [e]	3.03 [e]	3.24 [e]
Good fit	···	···	···	2.78	2.71	2.95	3.02	3.10	3.10	3.70

[a] R. Hofstadter, Ref. 21.
[b] H. Collard *et al.*, Ref. 21.
[c] H. Frank *et al.*, Ref. 21.
[d] Reference 22.
[e] Reference 23.

as suggested by π-nucleon data[24] at very high energies. We may also take the point of view that λ_i and β_i are free parameters of the model to be adjusted for a good fit to the energy dependence of the total cross section in the N^* region. We note such "good fit" values in Table II.

III. DISCUSSION

We present in Fig. 1 the result of calculations for the scattering of pions on ^{2}D, ^{3}He, and ^{4}He using the usual nucleon model, i.e., $^2\mathrm{D}=(n,p)$, $^3\mathrm{He}=(n,p)$, $^4\mathrm{He}=(2n,2p)$. It is readily seen that although the nucleon model is quite successful for a loose structure like ^{2}D, it fails for a smaller and more tightly bound structure like ^{4}He. Although the magnitude is roughly correct, the calculated peak is not satisfactory with regard to position and shape. The fault may lie with the fact that the Glauber (i.e., small angle and no rescattering) approximation is not valid in the known p-wave resonance region, or that the assumption of no nucleon correlation is inadequate. In a tightly bound nucleus like ^{4}He, both effects are interlocked. At present, however, multiple scattering theories beyond the Glauber series and many-particle correlations beyond that for two particles present enormous calculational difficulties. So it seems that improvements on the nucleon model in the N^* region along these lines can proceed only at great cost. If one further extends this nucleon model to other light nuclei (^{6}Li, ^{7}Li, ^{9}Bi, ^{12}C), one reaches essentially the same conclusion. The nucleon model, without improvements, fails to reproduce well the peaking total cross section in magnitude, shape, and especially, position; the disagreement grows progressively worse as more and more nucleons are included.

In Figs. 2–6, we present the calculated σ_T and α_T for a range of nuclei using the cluster model

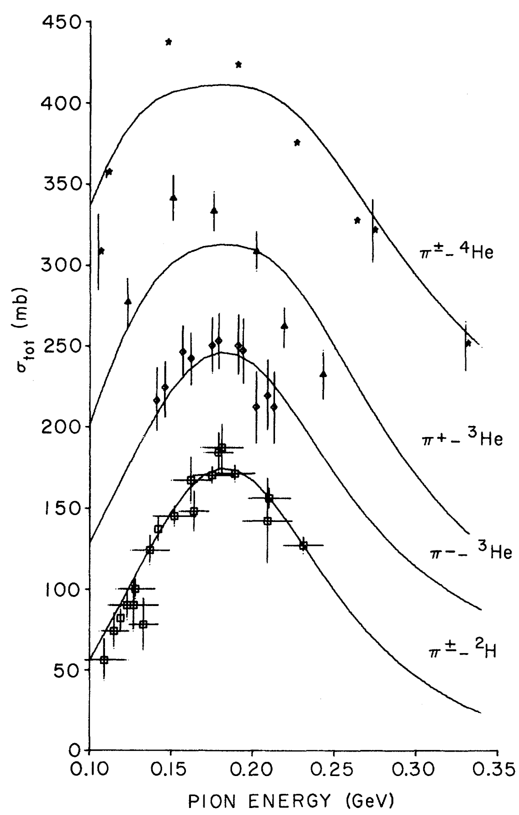

FIG. 1. Total cross section for $\pi^{\pm}$-^{2}D, π^+-^{3}He, π^--^{3}He, and $\pi^{\pm}$-^{4}He. For clarity in the display, we have, before plotting, (a) subtracted 50 mb for $\pi^{\pm}$-^{2}D; (b) subtracted 10 mb for π^--^{3}He; and (c) added 100 mb for $\pi^{\pm}$-^{4}He. References to the experimental data are: (a) for $\pi^{\pm}$-^{2}D, Ref. 17 (the π^+ and π^- data have not been averaged); (b) for $\pi^{\pm}$-^{3}He, Ref. 10; and (c) for $\pi^{\pm}$-^{4}He, Ref. 19. Predictions of a nucleon model are shown (see Table II).

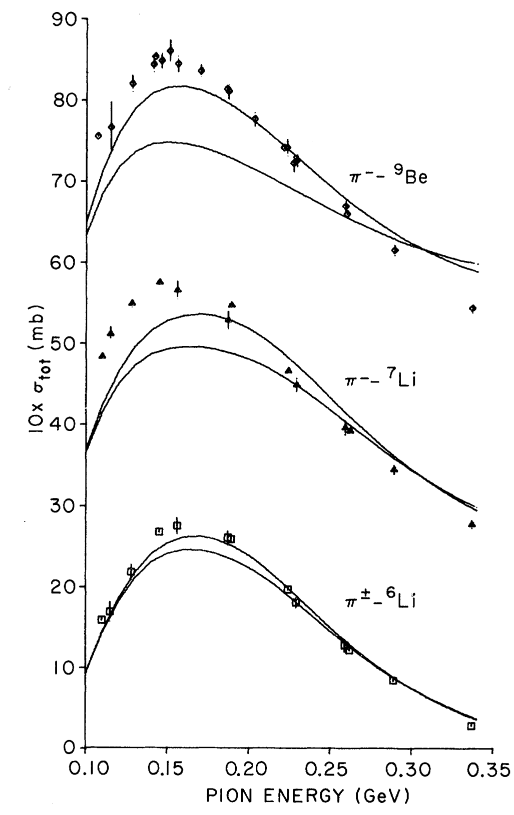

FIG. 2. Total cross section for $\pi^{\pm}$-^{6}Li, π^--^{7}Li, and π^--^{9}Be. For clarity in the display, we have, before plotting, (a) subtracted 200 mb for $\pi^{\pm}$-^{6}Li; and (b) added 200 mb for π^--^{9}Be. References to the experimental data are: (a) for $\pi^{\pm}$-^{6}Li, Refs. 7 and 9 (the average over $\pi^+$and π^- is done); (b) for π^--^{7}Li, Refs. 7 and 9; and (c) for π^--^{9}Be, Refs. 7, 9, and 10. Predictions of the cluster model using two values of $\langle \vec{r}_T^{\,2} \rangle$ are shown (see Table II).

and compare them to available experimental data.[6-10] The nuclei can be divided into three groups: (a) those requiring only α clusters, viz., $^{12}C = (3\alpha)$, $^{16}O = (4\alpha)$, $^{32}S = (8\alpha)$; (b) those requiring two kinds of clusters, viz., $^{6}Li = (d, \alpha)$, $^{9}Be = (n, 2\alpha)$; and (c) those requiring three kinds of clusters, viz., $^{7}Li = (n, d, \alpha)$, $^{27}Al = (n, d, 6\alpha)$. Considering the crudeness of the cluster model employed, the agreement with data (to better than 10% at the peak) for nuclei below ^{16}O is gratifying. For nuclei that require a large number of clusters, e.g. 8 for ^{27}Al and ^{32}S, the failure of the model is not surprising. In some cases, better agreement with data can be obtained by adjusting $\langle \vec{r}_T^{\,2} \rangle$. These values are shown in Table II. Although these values for $\langle \vec{r}_T^{\,2} \rangle$ are still reasonable, we do not attach too much significance to a better fit other than the fact that it is within easy reach in the cluster model. The tentative value of the model and its insensitivity to changes in β_i and λ_i preclude a definitive set of values for β_i and λ_i.

A remarkable feature common to these calculations is the appearance of the downshifted peak in σ_T. Typically, the peak position shifts from 190 MeV for a target nucleon to approximately 150 MeV for target ^{12}C. The mechanism for such a 40 MeV shift has been attributed to many factors.[1] In our model, we can identify two: many-nucleon correlations and unitarity. First, the dominant single scattering term has a peak that is already downshifted by about 30 MeV because the input α cluster (as four correlated nucleons) contains such a downshift. Second, interference between single and higher order multiple scattering terms (a unitarity effect) further contributes a downshift of about 10 MeV. Of course, the downshift in ^{4}He itself is due solely to a similar unitarity effect

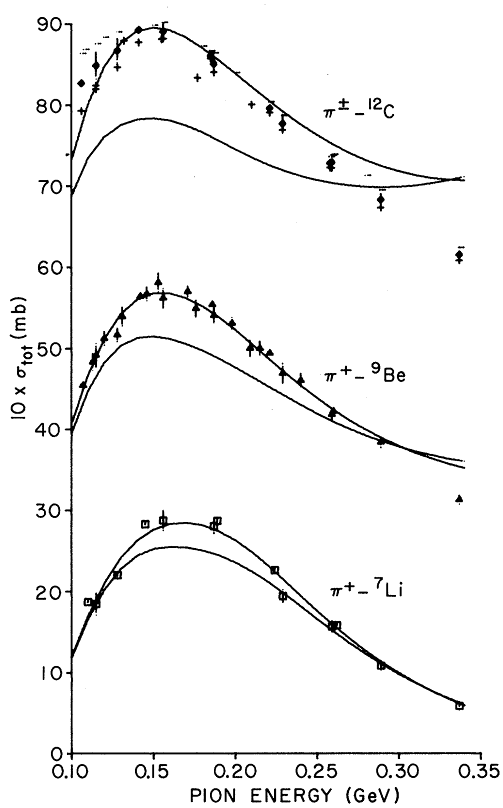

FIG. 3. Total cross section for π^+-^{7}Li, π^+-^{9}Be, and $\pi^{\pm}$-^{12}C. For clarity in the display, we have, before plotting, (a) subtracted 200 mb for π^+-^{7}Li; and (b) added 200 mb for $\pi^{\pm}$-^{12}C. References to the experimental data are: (a) for π^+-^{7}Li, Refs. 7 and 9; (b) for π^+-^{9}Be, Refs. 7, 8, 9, and 10; and (c) for $\pi^{\pm}$-^{12}C, Refs. 6, 7, 8, and 9 (the π^+ and π^- total cross sections are shown separately and without error bars as + and −; the averaged total cross section, where available, is shown as ◇). Predictions of the cluster model using two values of $\langle \vec{r}_T^{\,2} \rangle$ are shown (see Table II).

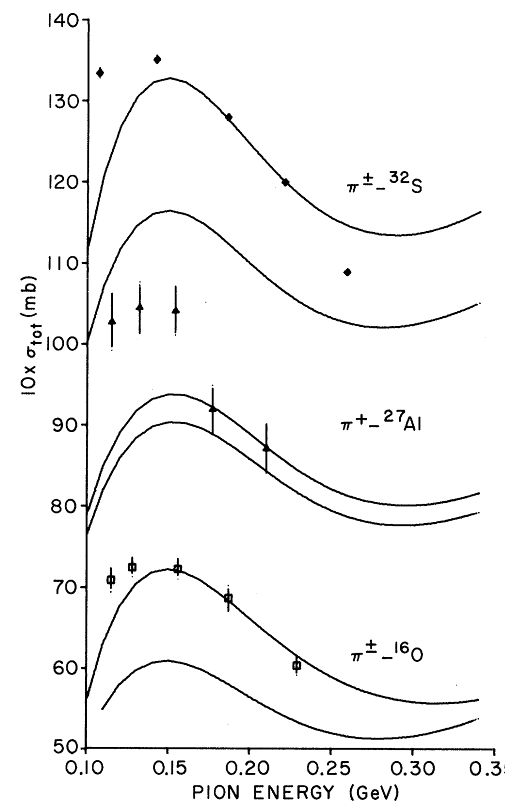

FIG. 4. Total cross section for $\pi^{\pm}$-^{16}O, π^+-^{27}Al, and $\pi^{\pm}$-^{32}S. For clarity in the display, we have, before plotting, (a) subtracted 100 mb for $\pi^{\pm}$-^{16}O; and (b) subtracted 100 for π^+-^{27}Al. References to the experimental data are: (a) for $\pi^{\pm}$-^{16}O, Ref. 9; (b) for π^+-^{27}Al, Ref. 8; and (c) $\pi^{\pm}$-^{32}S, Ref. 7. Predictions of the cluster model using two values of $\langle \vec{r}_T^{\,2} \rangle$ are shown (see Table II).

operating among its correlated nucleon constituents. But, as we pointed out before, we really do not know how to do a good calculation for a small system like ^{4}He.

Another common feature of these calculations is that, in the cluster model, the Glauber series is actually perturbative in nature when the total number of clusters involved is not too high. For example, for ^{12}C, three scatterings on the three α clusters are involved. The first, second, and third order multiple scattering contributions to the total cross section in the peak region are in the ratio 9 : 3 : 1. At higher energies, the single scattering term is even more dominant, as is expected. In contrast, a nucleon model for ^{12}C, or even our α-cluster model for ^{32}S, actually generates in the N^* region multiple scattering contributions that are as big as, if not bigger than, the resultant sum.[25] With such large cancellations [the successive terms in the Glauber series, Eq. (4), alternate in over-all sign] occurring, one may well question if a finite series like the Glauber series can represent at all the actual multiple scatterings.

There is an obvious theoretical reason why the Glauber series works better in the cluster model than in the nucleon model. The point is that by using experimental data in the cluster model, one is, in effect, doing an *exact* sum of the underlying and presumably infinite Watson multiple scattering series over a subset of nucleons in the target nucleus. At the same time, major nucleon correlations in the target nucleus are as well accounted for as clusters are major components of the tar-

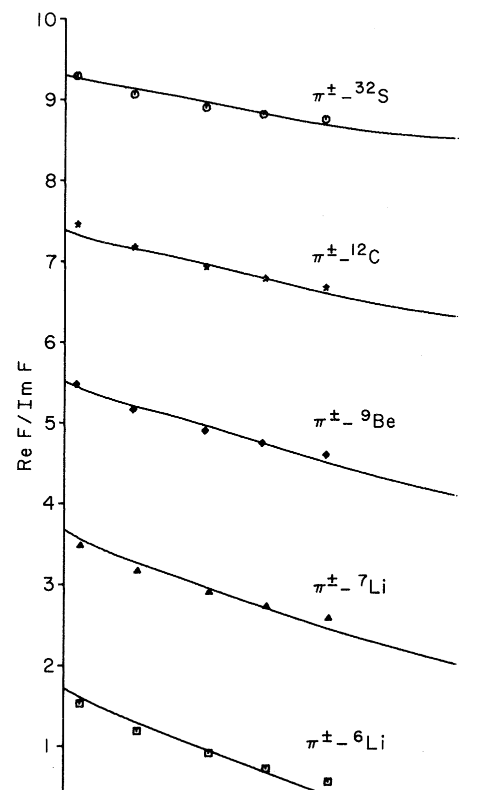

FIG. 5. Ratio of real to imaginary parts of the forward amplitude for $\pi^{\pm}$-^{6}Li, $\pi^{\pm}$-^{7}Li, $\pi^{\pm}$-^{9}Be, $\pi^{\pm}$-^{12}C, and $\pi^{\pm}$-^{32}S. For clarity in the display we have, before plotting, added to α_T the following values: 1, 3, 5, 7, and 9 for ^{6}Li, ^{7}Li, ^{9}Be, ^{12}C, and ^{32}S, respectively. The experimental data are taken from Ref. 7. The prediction of the model, averaged over π^+ and π^-, is shown for experimental values of $\langle \vec{r}_T{}^2 \rangle$ as listed in Table II.

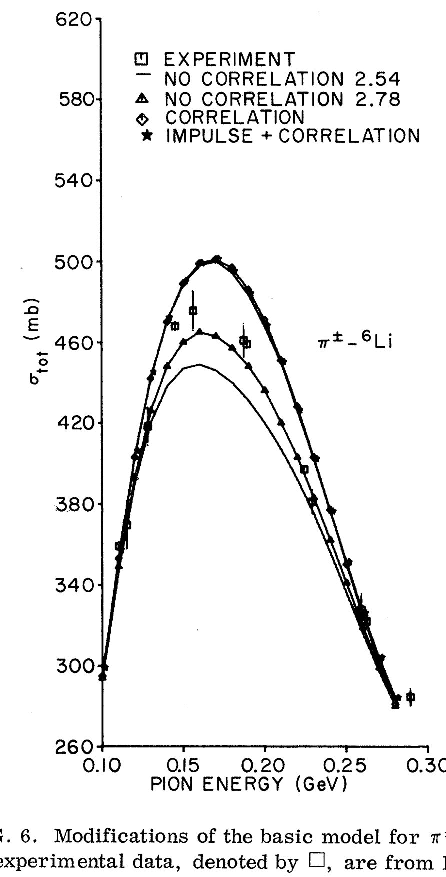

FIG. 6. Modifications of the basic model for $\pi^{\pm}$-^{6}Li. The experimental data, denoted by □, are from Refs. 7 and 9. The theoretical calculations are as follows: (a) the basic model, as defined by Eqs. (4), (11), (12), and (17) and $\langle \vec{r}_T{}^2 \rangle^{1/2} = 2.54$ fm, is shown as a line without symbols; (b) the basic model with $\langle \vec{r}_T{}^2 \rangle^{1/2} = 2.78$ fm is shown as a line with △; (c) a model with correlations, as defined by Eqs. (18) and (19), $a_0 = 0$, and $\langle \vec{r}_T{}^2 \rangle^{1/2} = 2.54$ fm, is shown as a line with ◇; and (d) a model with $\sigma^{(1)}$ as calculated in the impulse approximation [Eq. (20)] and $\sigma^{(2)}$ as calculated in the correlated model is shown as a line with ✶.

get nucleus. This exact subsum cannot be duplicated by a finite Glauber series with nucleons in the resonance region[26] even if the same correlations were—but in reality seldom are—put in. This is, from a calculational point of view, the most appealing feature of the cluster model. We are bypassing a difficult calculation involving infinitely many scatterings on strongly correlated nucleons by using the scattering data on the clusters which act as approximates of the correlated nucleons. It is why the single scattering term in the cluster model can approximate the data while it is way off in a nucleon model. Our qualitative success here can be viewed as evidence from strong interaction in support of clustering in light nuclei.[27]

IV. MODIFICATIONS

We next test the sensitivity of the cluster model to changes in its parameters and underlying assumptions. For this, we shall fix our attention on ^{6}Li, although the general behavior discussed below should hold for other light nuclei as well. There is much evidence[28,29] that points to a (d, α) structure for ^{6}Li. In fact, electromagnetic studies have provided information about the two-cluster correlation in the form of an intercluster harmonic oscillator type wave function[28] of the form of Eq. (11). Furthermore, with only two clusters the kinematics is simple. The Glauber series contains two terms, as given by Eq. (15). We test our cluster model for ^{6}Li with regard to the following modifications (the results are plotted in Fig. 6).

1. Parameter variations. If we change the rms radius of ^{6}Li from 2.54 fm to 2.78 fm, we increase the value of $\gamma_\alpha = \gamma_d$ by 20% [see Eqs. (4), (12), and (17)]. The result happens to be a better fit at the peak for σ_T. The peak value is raised by about 5%, as expected. Since the effective cluster separation, as given by $(\gamma_d + \gamma_\alpha)$, is increased, we expect a smaller $\sigma^{(2)}$ and hence a larger σ_T [see Eq. (15)]. The insensitivity to fairly large changes in γ_i is indicative of the perturbative nature of the Glauber series in the cluster model.[25] Reasonable changes in b_i produce hardly any change in σ_T since $\sigma^{(1)}$ and $\langle \vec{R}_i^2 \rangle$ are generally much larger than $\sigma^{(2)}$ and b_i, respectively. We note that for other light nuclei, similar effects of the changes in peak value and insensitivity to changes in γ_i are observed.

2. Correlation effects. We introduce into our model for ^{6}Li two-cluster correlation by assuming that the d and α clusters have an intercluster separation $\vec{R}$ with a wave function of the form[28]

$$\psi(\vec{R}) = \text{constant} \times (a_0 + a_1 R^2/\lambda) e^{-R^2/8\lambda}, \tag{18}$$

with $\langle \vec{R}^2 \rangle = 6\lambda$ if $a_1 = 0$ and $\langle \vec{R}^2 \rangle = 14\lambda$ if $a_0 = 0$. The forward Glauber amplitude analogous to Eq. (4) is

$$F_T(0; q) = f_\alpha(0; q) + f_d(0; q) + \frac{i}{2\pi q} \int d^2\vec{\Delta}_r f_\alpha(\vec{\Delta}_r; q) f_d(\vec{\Delta}_r; q) S(\vec{\Delta}_r), \tag{19a}$$

$$S(\vec{\Delta}_r) = \int d^3\vec{R} \, |\psi(\vec{R})|^2 e^{-i\vec{\Delta}_r \cdot \vec{R}}.$$

The mean squared separation distance may be related to the mean square radius of ^{6}Li with corrections for cluster sizes by

$$\langle \vec{R}^2 \rangle = \tfrac{3}{4}[6\langle \vec{r}_T^{\,2} \rangle - 4\langle \vec{r}_\alpha^{\,2} \rangle - 2\langle \vec{r}_d^{\,2} \rangle]. \tag{19b}$$

The result, as compared to the no-correlation model, is to raise the total cross section by about 5% at the peak and less elsewhere. The reason, again, is that the two clusters are, on the average, farther apart if correlated than if not correlated. The effect of changing the form of the wave function from $a_1 \neq 0$ and $a_0 = 0$ to $a_0 \neq 0$ and $a_1 = 0$ is also about 5%.

3. Fermi motion. We relax the requirement of static clusters somewhat to allow relative motion between the pion and each cluster. To estimate the effect of this, we use the observation that the single scattering term is dominant and compare it to an impulse approximation[30]:

$$F_{\text{imp}}(0; q) = \sum_{i=1}^{2} \int d^3\vec{q}_i \, |\varphi_i(\vec{q}_i)|^2 \left(\frac{M_T W_i}{M_i W_T} \right) f_i(0; Q_i). \tag{20}$$

$\varphi_i(\vec{q}_i)$ is the momentum space wave function for the ith cluster having a laboratory momentum $\vec{q}_i$. The variables Q_i and w_i are, respectively, the pion momentum and the total pion-cluster energy in the pion-cluster center of mass frame; w_T is the total pion-target nucleus energy in the pion-target nucleus center of mass frame. Note that w_T is a function of q while Q_i and w_i are functions of q and $\vec{q}_i$. This should be an important correction for low energies, especially in a nucleon model. For a cluster model, however, the massive d and α clusters plus the usually sharply peaked $\varphi_i(\vec{q}_i)$ near $\vec{q}_i = 0$ renders this correction a small effect; not more than 2% at 100 MeV and less than that at higher energies for ^{6}Li.

There are other corrections to a Glauber type model that can be considered. Among them are: (a) optical analogs like Fresnel diffraction[31] and multiple internal reflections[32]; (b) excitation of pionic and nuclear inelastic intermediate states[33]; and (c) spin and isospin effects.[34] These are expected to be small, at least as far as σ_T is concerned. However, detailed calculation may reveal surprises. In any case, they are probably easier to observe in angular distributions than in total cross sections.

V. CONCLUSION

We have presented calculations for the forward pion nuclear amplitude that support the following observations. The Glauber series, when used in a simple cluster model, (a) is able to reproduce the data on total cross sections σ_T to within 10% at the peak for light nuclei, and the data on α_T at least qualitatively; (b) accounts for the downshift in peak position; and (c) is a perturbative series with the single scattering term dominating. However, the point to emphasize is that we have the bulk of the data described by using only the gross features (in terms of σ_i, α_i, β_i, λ_i) of the pion-nucleus system. This is both good and bad. Because σ_T is not sensitive to changes in the parameters, one cannot hope to extract from $F_T(0;q)$ much definitive information about the pion-nucleus interaction, a fact generally acknowledged for any model. This is the bad part. The good part is that, unlike a nucleon model, the corrections needed to be applied to the basic model are indeed perturbative as far as $F_T(0;q)$ is concerned. One can then use other measurements, such as angular distributions, as a discriminator among the variants of the basic model. One can thus accept or reject different hypotheses about the pion-nucleus system with confidence since one knows that the forward point in elastic scattering $F_T(0;q)$ is relatively stable to changes once the basic model is correct. Our calculations of $F_T(0;q)$ tends to support, albeit not very strongly, the concept of clustering in light nuclei. Angular information, in the form of elastic, quasielastic, and inelastic pion-nuclear processes, which are more sensitive to correlations, higher order multiple scatterings, and other corrections, will be more definitive in this respect. Hopefully, this is where experiments and calculations will proceed to next.

As for immediate improvements on our model, two comments can be made. First, the consistently larger values of $\langle \bar{r}_T^2 \rangle$ needed in our no-correlation model to obtain a good fit (see Table II and Figs. 2–4) is an indication that putting in intercluster correlations should be a major consideration.[5] Correlations keep the clusters farther apart, decrease the double scattering contribution, and increase the resultant peak value. Second, we note that the prediction of the model at the peak for π^+-^{3}He, π^--^{7}Li, and π^--^{9}Be is consistently worse than that for π^--^{3}He, π^+-^{7}Li, and π^+-^{9}Be. This, of course, reflects the fact that the resonance cross section is much larger in the π^+-p or π^--n channel than in the π^--p or π^+-n channel. It calls for a more careful treatment of the extra nucleon in terms of a more accurate parametrization of the pion-nucleon amplitude, a more realistic wave function for the extra nucleon, and more importantly, some accounting for the p-wave resonance nature of the pion-nucleon interaction in the multiple scattering series—say, in the form of a perturbative backward scattering correction.[11] In addition, better data on σ and α for $\pi^{\pm}$-D and $\pi^{\pm}$-^{4}He are certainly needed. But beyond these, improvements on the model may not warrant the effort until one has enough confidence in the concept of nuclear clusters through corroborating evidence from angular distributions as previously described.

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[1] For a review on the pion nucleus interaction, see Lectures from the LAMPF Summer School on Pion-Nucleus Scattering, July, 1973, edited by W. R. Gibbs and B. F. Gibson, V, Los Alamos Report No. LA-5443-C, 1973 (unpublished); see also Carl B. Dover, BNL Report No. BNL-18747, 1974 (unpublished).

[2] L. S. Kisslinger, Phys. Rev. 98, 761 (1955).

[3] R. J. Glauber, in *High Energy Physics and Nuclear Structure*, edited by G. Alexander (North-Holland, Amsterdam, 1967), p. 311.

[4] K. M. Watson, Phys. Rev. 89, 575 (1953).

[5] Other similar cluster model calculations of interest are: P. Osland, Nucl. Phys. B56, 308 (1973) for π-^{9}Be, and R. B. Raphael and M. Rosen, Particles and Nuclei 2, 29 (1971) for p-^{12}C.

[6] CERN-ORSAY collaboration, Nucl. Phys. B17, 168 (1970).

[7] C. Wilkin *et al.*, Nucl. Phys. B62, 61 (1973); note that the uncertainties quoted here are statistical only and are, therefore, smaller than those quoted in Refs. 6, 8, 9, and 10.

[8] N. D. Gabitzsch, Ph.D. thesis, Rice University, 1973 (unpublished).

[9] Birmingham-Rutherford-Surrey collaboration, Rutherford High-Energy Laboratory Report No. RL-74-013, 1974 (unpublished).

[10] C. B. Spence, Ph.D. thesis, College of William and Mary, 1974 (unpublished).

[11] I. Dadic, M. Martinis, and K. Pisk, Ann. Phys. (N.Y.) 64, 647 (1971).

[12] T. A. Osborn, Nucl. Phys. B46, 538 (1972) and I. Manning, Phys. Rev. D 5, 1472 (1972).

[13] A. Baker, Phys. Rev. D 6, 3462 (1972); S. J. Wallace, *ibid.* 8, 1846 (1973).

[14]See Eq. (25) in Ref. 3.

[15]Data on σ_i for pion-nucleon scattering are taken from G. Giacomelli, P. Pini, and S. Stagni, A Compilation of Pion-Nucleon Scattering Data, CERN Report No. CERN-HERA-69-1, 1969 (unpublished).

[16]Data on α_i for pion-nucleon scattering are obtained by using the $l=0$, 1 partial wave phase shifts (CERN experimental solution) from π-N Partial Wave Amplitudes, a Compilation, Particle Data Group UCRL Report No. UCRL-20030, 1973 (unpublished).

[17]Data on σ_i for pion-deuteron scattering are obtained from: P. J. Issacs *et al.*, Phys. Rev. 85, 803 (1952); H. L. Anderson *et al.*, *ibid.* 86, 413 (1952); J. Ashkin *et al.*, *ibid.* 96, 1104 (1954); D. E. Nagle, *ibid.* 97, 480 (1955); K. C. Rodgers and L. M. Lederman, *ibid.* 105, 247 (1957); E. G. Pewitt *et al.*, *ibid.* 131, 1862 (1963); and A. A. Carter *et al.*, *ibid.* 168, 1457 (1968).

[18]Data on α_i for pion-deuteron scattering are not available. See text for details.

[19]Data on α_i and σ_i for pion–helium-4 are obtained from Ref. 7; additional data on σ_i are from E. C. Fowler *et al.*, Phys. Rev. 91, 135 (1953) and Y. A. Budagov *et al.*, Zh. Eksp. Teor. Fiz. 42, 1191 (1962) [transl.: Sov. Phys.–JETP 15, 824 (1962)].

[20]Our own calculation for σ_T (see Fig. 1) confirms the fact that because of the loose deuteron structure, π-^{2}D scattering in the N^* region is well represented (to within 10%) by a simple Glauber amplitude. So we expect the same for the calculated α_D. We note further that α_D does not contribute to $\sigma^{(1)}$ [see for example Eq. (15)], which is the dominant term in σ_T. So a 10% error on α_D is certainly tolerable in a simple model like ours.

[21]R. Hofstadter, Rev. Mod. Phys. 28, 214 (1956); H. Collard *et al.*, Phys. Rev. 138, B57 (1965); and H. Frank *et al.*, Phys. Lett. 19, 719 (1965).

[22]L. R. Suelzle, M. R. Yearian, and H. Crannell, Phys. Rev. 162, 992 (1967).

[23]C. S. Wu and L. Willets, Annu. Rev. Nucl. Sci. 19, 527 (1969).

[24]The diffraction slopes for hadrones on target nucleons are all of the order of 10 $(\mathrm{GeV}/c)^{-2}$ [see the table in R. Serber, Phys. Rev. Lett. 13, 32 (1964)] for incident momenta above 5 GeV/c. Assuming that b_i has a geometric origin, one can presumably scale by an $A^{2/3}$ factor to get a value of $b_i=25$ $(\mathrm{GeV}/c)^{-2}$ for ^{4}He. However, since b_i has little effect on σ_T, we have arbitrarily set $b_i=10$ $(\mathrm{GeV}/c)^{-2}$ for all clusters considered.

[25]In the nucleon mode, $\sigma^{(2)}$ exceeds σ_T at the peak beginning with ^{6}Li. In the cluster model, this occurs at ^{16}O.

[26]See, however, J. M. Eisenberg [Ann. Phys. (N.Y.) 71, 542 (1972)] for a discussion where, for static targets, the Glauber series is a better than expected approximation to the Watson series due to cancellations among off-shell and higher multiple scattering contributions in the latter series.

[27]For references on clustering in nuclei, see Y. G. Neudatchin and Yu. F. Smirnov, in *Progress of Nuclear Physics*, edited by D. M. Brink and J. H. Mulvey (Pergamon Press, New York, 1969), Vol. 10, p. 273; and H. Faraggi, Comments Nucl. Part. Phys. 5, 79 (1972).

[28]See Yu. A. Kudeyarov *et al.*, Nucl. Phys. A163, 316 (1971); B. K. Jain, *ibid.*, A194, 651 (1972) and references cited therein.

[29]Z. Gromadzski and J. V. Noble, Phys. Lett. 51B, 9 (1974).

[30]For a detailed discussion of the intricacies in the impulse approximation see K. L. Kowalski and D. Feldman, Phys. Rev. 130, 276 (1963).

[31]K. Gotlfried, Ann. Phys. (N.Y.) 66, 868 (1971); and G. Fäldt, Nucl. Phys. B46, 460 (1972).

[32]K. Gabathuler and C. Wilkin, CERN Report No. CERN TH-1756, 1973 (unpublished).

[33]See J. Pumplin and Marc Ross, Phys. Rev. Lett. 21, 1778 (1968), and references cited therein.

[34]A. T. Hess and J. M. Eisenberg, Phys. Lett. 47B, 311 (1973).