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Regge-type representations of the partial wave amplitudes for spin-zero–spin-one systems

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Regge-type representations are developed for the scattering amplitude of spin-one particles elastically scattered from nuclei. This work forms an addition to a recent paper by Shanta and Satpathy on the Regge-type representations.

I. INTRODUCTION

Recently the Regge-pole theory[1] has been widely used for nuclear scattering. Knowing the analytic properties of the S matrix in the complex λ $(=l+\frac{1}{2})$ plane, the Regge-type representations of the partial wave amplitude have been derived in terms of a few pole parameters in the right-half λ plane for spin-zero[2] and spin-half[3] particles. The analyses done with these representations have tested the validity of the Regge-pole approach to nuclear scattering and have given information about the spin and parity of some nuclear levels.

The purpose of this communication, which adds to the work described in Ref. 3, is to extend the complex angular momentum method to the study of scattering of spin-one particles. In Sec. II, the Regge-type representations have been given in a form suitable for the calculation of differential cross sections.

II. REGGE-TYPE REPRESENTATIONS FOR SPIN-ZERO–SPIN-ONE SYSTEM

The elastic scattering of particles with spin one from spin-zero nuclei is generally described in terms of the five amplitudes given below[6]:

$$A(k,\cos\theta)=A_C(k,\cos\theta)+\sum_{l=0}^{\infty}[(l+1)a^+(l,k)+la^-(l,k)]e^{2i\sigma_l}P_l(\cos\theta),$$

$$B(k,\cos\theta)=A_C(k,\cos\theta)+\sum_{l=0}^{\infty}\tfrac{1}{2}[(l+2)a^+(l,k)+(2l+1)a^0(l,k)+(l-1)a^-(l,k)]e^{2i\sigma_l}P_l(\cos\theta),$$

$$C(k,\cos\theta)=\sum_{l=0}^{\infty}\frac{1}{\sqrt{2}}[a^+(l,k)-a^-(l,k)]e^{2i\sigma_l}P_l^1(\cos\theta), \tag{2.1}$$

$$D(k,\cos\theta)=\sum_{l=0}^{\infty}\frac{1}{\sqrt{2}}\frac{1}{l(l+1)}[l(l+2)a^+(l,k)-(2l+1)a^0(l,k)-(l-1)(l+1)a^-(l,k)]e^{2i\sigma_l}P_l^1(\cos\theta),$$

$$E(k,\cos\theta)=\sum_{l=0}^{\infty}\frac{1}{2}\frac{1}{l(l+1)}[la^+(l,k)-(2l+1)a^0(l,k)+(l+1)a^-(l,k)]e^{2i\sigma_l}P_l^2(\cos\theta),$$

where $A_C(k,\cos\theta)$ is the Coulomb amplitude, k is the wave number, and σ_l is the Coulomb phase shift. The partial wave amplitudes $a^{\pm,0}(l,k)$ are related to the nuclear S matrix by the relation $a^{\pm,0}(l,k)=(\bar{S}^{\pm,0}-1)/2ik$, corresponding to total angular momentum $j=l\pm 1$ and $j=l$.

The Regge and Khuri representations of the various partial wave amplitudes are obtained following the method described in Ref. 3, and hence details of the derivations are not given. It is found that the Regge representations are given by

$$\frac{1}{2\lambda}[(\lambda\pm\tfrac{1}{2})^3\pm 2]a^{\pm}(\lambda,k)=\sum_{n=1}^{N^{\pm}}\beta_n^{\pm}\frac{(\lambda_n^{\pm}\pm\frac{1}{2})^3\pm 2}{\lambda^2-(\lambda_n^{\pm})^2},$$
$$(\lambda^2-\tfrac{1}{4})a^0(\lambda,k)=\sum_{n=1}^{N^0}\beta_n^0\frac{(\lambda_n^0)^2-\frac{1}{4}}{\lambda^2-(\lambda_n^0)^2}, \tag{2.2}$$

where $\lambda_n^{\pm,0}$ and $\beta_n^{\pm,0}$ are the poles and residues, respectively, and $N^{\pm,0}$ is the number of poles in the right-half λ plane.

Similarly, the Khuri representations are given by

$$\frac{a^{\pm}(\lambda,k)}{\lambda^2}=\sum_{n=1}^{N^{\pm}}\frac{\beta_n^{\pm}}{(\lambda_n^{\pm})^2}\frac{e^{(\lambda_n^{\pm}-\lambda)\xi}}{\lambda-\lambda_n^{\pm}},$$
$$\frac{a^{0}(\lambda,k)}{\lambda(\lambda+\frac{1}{2})}=\sum_{n=1}^{N^{0}}\beta_n^{0}\frac{1}{\lambda_n^{0}(\lambda_n^{0}+\frac{1}{2})}\frac{e^{(\lambda_n^{0}-\lambda)\xi}}{\lambda-\lambda_n^{0}}. \tag{2.3}$$

The modified Regge representations of Ref. 4 are obtained from (2.3) by incorporating the appropriate factors, and we get

$$\frac{a^{\pm}(\lambda,k)}{\lambda^2}=\sum_{n=1}^{N^{\pm}}\frac{1}{(\lambda_n^{\pm})^2}\frac{\beta_n^{\pm}}{\lambda-\lambda_n^{\pm}}\frac{\exp(ie^{-\lambda\xi})-1}{\exp(ie^{-\lambda_n^{\pm}\xi})-1}\left(\frac{2\lambda}{\lambda+\lambda_n^{\pm}}\right)^m,$$
$$\frac{a^{0}(\lambda,k)}{\lambda(\lambda+\frac{1}{2})}=\sum_{n=1}^{N^{0}}\frac{\beta_n^{0}}{\lambda-\lambda_n^{0}}\frac{1}{\lambda_n^{0}(\lambda_n^{0}+\frac{1}{2})}\frac{\exp(ie^{-\lambda\xi})-1}{\exp(ie^{-\lambda_n^{0}\xi})-1}\left(\frac{2\lambda}{\lambda+\lambda_n^{0}}\right)^m, \tag{2.4}$$

where m is a suitable positive integer, chosen to render the background terms small.

For charged particle scattering, one has to incorporate the Coulomb threshold factor[5]

$$\xi_{\lambda-1/2}{}^2=e^{-\pi\eta}\frac{\Gamma(\lambda+\frac{1}{2}+i\eta)\Gamma(\lambda+\frac{1}{2}-i\eta)}{[\Gamma(\lambda+\frac{1}{2})]^2} \tag{2.5}$$

by multiplying each of the Eqs. (2.1) to (2.3) by the factor

$$\xi_{\lambda-1/2}{}^2/\xi_{\lambda_n^{\pm,0}-1/2}{}^2\,. \tag{2.6}$$

Further, one has to modify the Khuri and modified Regge representations by incorporating the following form factor:

$$\frac{\lambda(e^{-\lambda_n^{\pm,0}\bar{\xi}}+e^{-R/a_0})}{\lambda_n^{\pm,0}(e^{-\lambda\bar{\xi}}+e^{-R/a_0})}, \tag{2.7}$$

which originates from the Woods-Saxon form of the optical potential:

$$V(r)=\frac{e^{-r/a_0}}{(e^{-r/a_0}+e^{-R/a_0})},$$

where a_0 is the diffuseness parameter, R is a parameter of the order of nuclear radius, and

$$\bar{\xi}=\cosh^{-1}[1+1/(2a_0{}^2k^2)].$$

The derivation of the form factor is carried out in Ref. 2.

Methods of determination of pole parameters of the representations given in (2.2) to (2.4) are the same as described in Ref. 3 for spin-$\frac{1}{2}$ particles. The pole parameters so determined can be used for calculating the partial wave amplitudes $a^{\pm,0}(\lambda,k)$ and hence the differential cross sections for elastic scattering of spin-one particles from spin-zero nuclei according to the formula[6]

$$\frac{d\sigma}{d\Omega}=\tfrac{1}{3}[|A|^2+2(|B|^2+|C|^2+|D|^2+|E|^2)]. \tag{2.8}$$

Thus, the Regge-pole formulation presented here is suitable for the analysis of deuteron-nucleus scattering.

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