Comment on well matching in distorted-wave–Born-approximation predictions for two-nucleon transfers*

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The sensitivity of many calculations in the distorted-wave–Born-approximation (DWBA) to optical model parameter ambiguities has led to the introduction of a well-matching condition which was discussed in a recent paper. The claim for the merit of this condition was based primarily on its success and consistent usefulness in standard DWBA calculations. A theoretical argument given for its plausibility is explained in greater detail. It is argued that the absence of restraints on optical model potentials may be expected to lead to poor results with the common DWBA approximations whereas the well-matching condition helps to insure the smallness of some generally neglected terms.

In studies of one- and two-nucleon transfer reactions of the type $A(a,b)B$, where $b=a+x$, authors have frequently noted a strong sensitivity of distorted-wave–Born-approximation (DWBA) calculations to the choice of the optical model potentials U_a and U_b. In Ref. 1 we discussed new experimental evidence and possible reasons for the consistent usefulness of a well-matching condition for the real parts of the potentials of the form

$$\mathrm{Re}(U_a + V_x - U_b) \approx 0 \tag{1}$$

which puts constraints on the magnitude and geometry of the potentials to be used in DWBA calculations. We observed that there is an apparent need for this or a very similar condition in cases where significant contributions to the conventional DWBA transition amplitude arise from the nuclear interior.

Although the claim for the importance of such a relation between U_a, U_b, and the "bound state" potential V_x (which is chosen to generate the center of mass motion of x, i.e. the transfered nucleon or cluster of nucleons) was based primarily on the empirical evidence presented, we contended in Sec. II A and B of Ref. 1 that the usefulness, if not the necessity, of a well-matching condition might be understood from the common approximations to the distorted-wave theory (DW) which lead to the common (zero range) DWBA model. The plausibility argument given for well matching was kept very brief because it had been discussed previously for the case of single particle transfer reactions.[2]

However, Werby's comment[3] shows that our arguments can be misunderstood or misinterpreted; hence we present them here in more explicit terms: The exact transition amplitude in the distorted-wave theory T_f^{DW} for two-particle initial and final states [see Eq. (2) in Ref. 1, or Eq. (1) in the preceding comment] can be written as the sum of the terms $T_f^{\mathrm{DW}} = T_f^{\mathrm{DWBA}} + T_f^C + T_f^E$, where

$$T_f^{\mathrm{DWBA}} = \langle \chi_b^{(-)} \phi_b \Phi_B \mid V_{ax} \mid \phi_a \Phi_A \chi_a^{(+)} \rangle , \tag{2a}$$

$$T_f^C = \langle \chi_b^{(-)} \phi_b \Phi_B \mid V_{aB} - U_a \mid \phi_a \Phi_A \chi_a^{(+)} \rangle , \tag{2b}$$

$$T_f^E = \langle \chi_b^{(-)} \phi_b \Phi_B \mid (V_{aB} + V_{xB} - U_b) \frac{1}{E^+ - H} (V_{aB} + V_{ax} - U_a) \mid \phi_a \Phi_A \chi_a^{(+)} \rangle . \tag{2c}$$

The operators $V_{\alpha\beta}$ in these equations must be understood as sums of physically well defined two-body interaction potentials between the nucleons in the clusters labeled α and β, but the potentials U_a, U_b in the DW theory are model potentials which are introduced to cancel the "dominant" (diagonal) terms of V_{aA} and V_{bB}. They are arbitrary except that they must not lead to the rearranged state in the exit channel.[4]

The situation changes considerably if the common DWBA approximations are made, for in typical DWBA calculations[5] only T_f^{DWBA} of (2a) is retained. T_f^E and T_f^C would give account of multistep and core excitation processes; but even where such effects are unimportant T_f^E and T_f^C are not necessarily small. The well known discrete and continuous ambiguities in U_a and U_b[6] influence T_f^{DWBA} by introducing different shapes of the scattered waves in the nuclear surface and interior. Only in the exact DW theory are such effects compen-

sated by corresponding changes in T_f^C and T_f^E. In the commonly used calculational approximations to the DW theory [Eq. (2a)] we should try to minimize these contributions of the neglected terms T_f^C and T_f^E to the extent possible. We argued[1] that this seems to be achieved by choosing U_a and U_b so that the average value of the sums of potential operators in T_f^C and T_f^E is made very small. Formally this was expressed by the equations

$$\| V_{aB} + V_{xB} - U_b \| \equiv \| V_{bB} - U_b \| \approx 0, \tag{3a}$$

$$\| V_{aB} - U_a \| \approx 0, \tag{3b}$$

where the symbol $\| \ldots \|$ stands for an integration over all but the relative coordinates of the centers of mass of b and B, or a and A, respectively. Equation (3a) amounts to the relation $U_b \approx \| V_{bB} \|$, i.e. the postulate that the potential U_b have the depth and geometry of a scattering potential obtained from a folding procedure. The physical idea implicit in the use of (3a) is that for the nuclear surface, which is the source of most transfer contributions, the *diagonal* terms from V_{bB} are most nearly canceled by a folding potential. Condition (3b) is introduced in order to " minimize" T_f^C. We realize, of course, that these conditions are not fulfilled exactly by any conventional optical potentials, but we can see that if U_a and U_b are chosen according to (3a) and (3b) one is led to the relation

$$U_a + \| V_{xB} \| - U_b \approx 0 \tag{4}$$

which is very similar to the empirical relation (1).

Since Werby has not taken the arguments of Ref. 1 in their intended meaning, it appears a bit futile to answer his comment on a point by point basis. The plausibility argument of Sec. II A was not offered as a "derivation" of the well-matching condition, and was specifically restricted to the common, simplified calculations with expression (2a), which because of the physical asymmetry of light projectiles and heavy targets is most useful in the final state form. Conditions (3a) and (3b) were written as folding integrals, not as matrix elements. [Incidentally, no restrictions placed on U_a and U_b of the form discussed in Ref. 1 can violate post-prior equality for the full DW theory or that form of the DWBA where only the (post-prior symmetric) term T^E is neglected but the sum of T_f^C and T_f^{DWBA} is carried without approximations.]

It remains an interesting and important question to ask if the sensitivity to optical parameters and the usefulness of well matching would persist if the sum $(T_f^{\mathrm{DWBA}} + T_f^C)$ were calculated with good accuracy. On the basis of our present understanding we are inclined to think that the multistep term T^E is probably quite sensitive to the optical potentials, and so would be $(T_f^{\mathrm{DWBA}} + T_f^C)$, its post-prior equality notwithstanding.

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