Comments

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Comment on a systematic distorted-wave Born-approximation prediction for two-nucleon transfer: Application to (d, α) experiments*

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In a recent study of (d, α) reactions on medium and heavy nuclei, DelVecchio and Daehnick employ a well-matching condition which limits the choice of the optical model potentials. It is shown that in the authors' derivation of the well-matching condition, they violate the post-prior equality necessary for distorted-wave transition amplitudes.

In the recent paper by DelVecchio and Daehnick[1] the authors employ an approximation or an assumption that creates an inconsistency in the distorted-wave (DW) theory of direct reactions. For the reaction $a+A\to b+B$, with the transferred cluster or nucleon being x and

$$b=a+x ,$$

$$B=A-x ,$$

the DW amplitude is, in the final state formalism,[2]

$$T_f^{\mathrm{DW}}=N\left\langle \chi_b^{(-)}\phi_b\,\phi_B\right|\left(1+(V_{aB}+V_{Bx}-U_b)\frac{1}{E^{(+)}-H}\right)(V_{aB}+V_{ax}-U_a)\left|\phi_a\,\phi_A\chi_a^{(+)}\right\rangle , \tag{1}$$

where the same designations are used here as in Ref. 1. The authors then impose the following conditions:

$$\| V_{aB}+V_{Bx}-U_b\| =0 , \tag{2a}$$

$$\| V_{aB}-U_a\| =0 , \tag{2b}$$

in order to reduce Eq. (1) to the usual distorted-wave Born approximation (DWBA) frequently employed in the literature for the case of single nucleon transfer reactions:

$$T_f^{\mathrm{DWBA}}=N\langle \chi_b^{(-)}\phi_b\,\phi_B|\,V_{ax}\,|\,\phi_a\,\phi_A\chi_a^{(+)}\rangle . \tag{3}$$

Our objective is to show that if one assumes condition (2a) to be an essential condition in reducing Eq. (1) to Eq. (3), then by the same reasoning one can show that such a condition leads one to a serious discrepancy when one compares the post and prior forms of the DWBA amplitudes. In addition, condition (2a) is not equivalent to making the Born approximation, as we discuss below, nor does the Born approximation suggest condition (2a).

We would also like to point out that it is not clear from Ref. 1 what Eqs. (2a) and (2b) mean. Moreover, though the potential terms V_{xy} should designate the sum of two-body interactions between particles in x with those in y, the authors at one point treat the interaction V_{aB} as if it were an averaged potential when they use (2a) and (2b) to lead to their well-matching condition $V_a+V_{Bx}-U_b\approx 0$. Nonetheless, in what follows, Eqs. (2a) and (2b) will be employed in the same manner as it was in the paper in question.

The initial state formalism for the DW amplitude which must be equal to the final state transition amplitude is[2]

$$T_i^{\mathrm{DW}}=\left\langle \chi_b^{(-)}\,\phi_b\,\phi_B\right|(V_{aB}+V_{Bx}-U_b)\left[1+\frac{1}{E^{(+)}-H}(V_{aB}+V_{ax}-U_a)\right]\left|\phi_a\,\phi_A\,\chi_a^{(+)}\right\rangle . \tag{4}$$

Any approximation or assumption employed in the DW theory must, in principle, preserve the initial-state–final-state or post-prior equality. However, it is apparent by examining Eq. (4) that if condition (2a) is used in exactly the same way that it was used to derive Eq. (3) then one has

$$T_i^{\mathrm{DW}}=0 \ . \qquad (5)$$

This not only destroys the post-prior equality, but it causes the cross section derived in an alternate formalism to be zero. It therefore does not seem reasonable to employ condition (2a) as part of a theoretical basis for deriving the well-matching condition employed in Ref. 1.

We would also like to point out that Eqs. (1) and (4) both contain the term $(V_{aB}+V_{Bx}-U_b)[1(E^{(+)}-H)] \times(V_{aB}+V_{ax}-U_a)$. As pointed out by Goldberger and Watson,[3] it is the above term, and *not the term* $(V_{aB}+V_{Bx}-U_b)$ that is assumed negligible when one makes the Born approximation. Thus, upon making such an approximation, one arrives at the DWBA transition amplitudes in the post and prior form, which clearly still maintain the post-prior equality

$$T_f^{\mathrm{DWBA}}=N\langle \chi_b^{(-)}\phi_b\phi_B| V_{aB}+V_{ax}-U_a | \phi_a\phi_A\chi_a^{(+)}\rangle \ , \quad (6)$$

$$T_i^{\mathrm{DWBA}}=N\langle \chi_b^{(-)}\phi_b\phi_B| V_{aB}+V_{Bx}-U_b | \phi_a\phi_A\chi_a^{(+)}\rangle \ . \quad (7)$$

Equation (2b), which is frequently used in the literature (if we interpret it to mean

$$\langle \chi_b^{(-)}\phi_b\phi_B| V_{ab}-U_a | \phi_a\phi_A\chi_a^{(+)}\rangle$$

is negligible compared to

$$\langle \chi_b^{(-)}\phi_b\phi_B| V_{ax} | \chi_a^{(+)}\phi_b\phi_B\rangle \ ,$$

then reduces Eq. (6) to Eq. (3). Note that this need not imply $V_{aB}=U_a$, since both terms are only expected to be large in the nuclear interior, and it is possible for absorption and phase averaging effects due to the distorted waves to render the term $V_{aB}-U_a$ small in such a region. One can interpret this result to mean that the matrix element of V_{ax} is equal to or approximately equal to that of $V_{aB}+V_{Bx}-U_b$. This in fact is usually done and is why one uses the initial rather than the final state form for the transition amplitude.

Finally, it may be pointed out that, though it is possible for a term such as $V_{aB}-U_a$ to be negligible in some average sense, i.e., the transition matrix element, it is difficult to see how condition (3c) of Ref. 1 and a similar equation in an earlier article[4] can be satisfied, namely that $U_a+V_{Bx}-U_b$ sums to zero when the three terms are functions of three different coordinates.

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