

Three-body force in nuclear matter

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We report calculations on the contribution of the three-body potential W to the energy of nuclear matter which include realistic correlations in all internucleon distances which are consistent with realistic two-body potentials (V) in the calculation of the dominant second order [$\langle V(Q/e)W \rangle$] contribution to the binding energy. For the Reid soft core potential we get a binding energy contribution of 6 MeV/nucleon. This is to be compared with the binding energy of 1.7 MeV/nucleon obtained using hard core potentials. The latter result is consistent with the calculations of Loiseau, Nogami, and Ross. The physical reason for the large difference between the results obtained from hard core and soft core two-body potentials is clarified.

I. INTRODUCTION

As calculations of the binding energy of nuclear matter using two-body forces have become more and more refined, it has become increasingly important to know how large the effects of three-body and other internucleon forces are, so that they can be included if necessary. Considerable work has already been done on binding energy calculations for three-body forces. In the early work, correlations introduced into the wave function of nuclear matter by two-body forces were treated in an *ad hoc* manner. The calculations were performed in configuration space^{1,2} and correlations were treated by introducing a cutoff into the wave function at small distances between nucleons 1 and 2 in Fig. 1.

Brown and Green³ (referred to as BG) also considered the problem, including the same correlations but calculating in momentum space. The relation between the momentum space and configuration space calculations was discussed by Bhaduri, Nogami, and Ross,⁴ who showed that differences arose due to "contact interactions" in the nucleon pairs 1, 3 and 2, 3. The unphysical contact interactions are removed by taking proper account of correlations in these nucleon pairs which are then prevented from coming into contact. Hence, cutoffs were put into all three internucleon distances by Loiseau, Nogami, and Ross⁵ (we will refer to this paper as LNR).

The role of correlations in the calculation of binding energy contributions was later clarified by McKellar and Rajaraman,⁶ in a Bethe-Faddeev-style summation of all the correlation-producing three nucleon diagrams. This justified the approach of LNR, provided that realistic correlations derived from solutions of the Bethe-Gold-

stone equation are used in the computations.

The important contribution of BG and LNR was the recognition that most of the binding from the three-body force W comes from the second order cross term $\langle V(Q/e)W \rangle$ between W and the two-body force V . This is a consequence of the strong tensor components in both W and V . It therefore seems appropriate to use a realistic two-body potential for V in the computation of this important term.

In this paper we perform several calculations of the binding energy contribution of the three-body force using the effective potential method of LNR. We use correlated wave functions for the three nucleons. These wave functions are derived from solutions of the Bethe-Goldstone equation for the particular two-body potential V considered.

Bethe⁷ and Day⁸ have shown how to calculate correlated wave functions for three nucleons from the Bethe-Goldstone wave function for two nucleons. The particular case they considered was the Reid-hard-core potential. We have also used the results of Negele⁹ and Siemens¹⁰ for the solution of the Bethe-Goldstone equation with the Reid-soft-core potential to construct three-body correlated wave functions appropriate to this potential.

Realistic $NN\pi$ form factors are used throughout and, as in LNR, results are also given for the case of no form factors for comparison purposes only.

There has been some recent controversy^{11,12} about the use of correlated wave functions in all three internucleon distances. In an earlier paper¹³ we discussed this point, and for the reasons outlined there, continue to correlate all three nucleons in the same way. This earlier paper also briefly reported the soft-core results presented here.

Strictly, of course, the binding energy contributions of $W_{2\text{PE}}$ are, in an obvious notation, the first order contribution

$$\Delta E^{(1)} = \langle W_{2\text{PE}} \rangle / N$$

and the second order contribution

$$\Delta E^{(2)} = \Delta E_1^{(2)} + \Delta E_2^{(2)},$$

where

$$\Delta E_1^{(2)} = 2 \langle V^{nn} \frac{Q}{e} W_{2\text{PE}} \rangle / N$$

and

$$\Delta E_2^{(2)} = \langle W_{2\text{PE}} \frac{Q}{e} W_{2\text{PE}} \rangle / N.$$

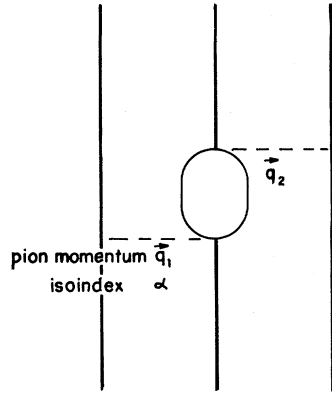
by⁵

$$\begin{aligned} W_{2\text{PE}}(\vec{r}_1 \vec{r}_2 \vec{r}_3) = & -\frac{2}{9} C_p \vec{\tau}_1 \cdot \vec{\tau}_2 [\vec{\sigma}_1 \cdot \vec{\sigma}_2 \bar{U} \bar{X} + S_{12}(\hat{u}) \bar{U} \bar{X} + S_{12}(\hat{x}) \bar{U} \bar{X}] \\ & + [9\hat{u} \cdot \hat{x} \vec{\sigma}_1 \cdot \hat{u} \vec{\sigma}_2 \cdot \hat{x} - S_{12}(\hat{u}) - S_{12}(\hat{x}) - \vec{\sigma}_1 \cdot \vec{\sigma}_2] \bar{U} \bar{X} \} Y_u Y_x, \end{aligned} \quad (2.1)$$

where $\hat{u} = \vec{r}_3 - \vec{r}_1$, $\hat{x} = \vec{r}_2 - \vec{r}_1$, $\hat{x} = \vec{x}/|\vec{x}|$, and \bar{U} , \bar{X} , \bar{X} are functions of $|\hat{u}|$ and $|\hat{x}|$, respectively, and are given by

$$\bar{X} = 1 - \xi \frac{\eta^2}{\mu^2} e^{-(\eta-\mu)x}, \quad (2.2)$$

$$\begin{aligned} \bar{X} = & \left(1 + \frac{3}{\mu x} + \frac{3}{\mu^2 x^2} \right) \\ & - \xi \frac{\eta^2}{\mu^2} \left(1 + \frac{3}{\eta x} + \frac{3}{\eta^2 x^2} \right) e^{-(\eta-\mu)x}. \end{aligned} \quad (2.3)$$



	Space	\vec{r}_1	\vec{r}_3	\vec{r}_2
Nucleon coordinates	Spin	$\vec{\sigma}_1$	$\vec{\sigma}_3$	$\vec{\sigma}_2$
	Isospin	$\vec{\tau}_1$	$\vec{\tau}_3$	$\vec{\tau}_2$

FIG. 1. The Feynman diagram for $W(3)$ with nucleon and pion coordinates shown.

However, calculation of $\Delta E^{(2)}$ in this form is almost intractable, and for this reason we follow LNR and replace $W_{2\text{PE}}$ by an effective two-body potential $V^{2\text{PE}}$ defined so that $\langle W_{2\text{PE}} \rangle = \langle V^{2\text{PE}} \rangle$. The validity of this approximation for the second order terms has yet to be tested in detail.

First, in Sec. II we define the effective two-body force and then in Sec. III use it as calculations of the contribution of three-body forces to the binding energy of nuclear matter.

II. EFFECTIVE TWO-BODY FORCE

The two pion exchange three-body potential $W_{2\text{PE}}$ derived from the process shown in Fig. 1 is given

The coefficient C_p is given by

$$C_p = (f\mu^2/3\pi)^2 \int_0^\infty dp \sigma_{33}(p) (p^2 + \mu^2)^{-1} = 0.61 \text{ MeV}, \quad (2.4)$$

where $f^2 = 0.08$ is the πN coupling constant, μ is the pion mass, and $\sigma_{33}(p)$ is the total cross section of the p -wave πN scattering in the $I=J=\frac{3}{2}$ state. Other notations used in (2.1) are

$$S_{12}(\hat{x}) = 3\vec{\sigma}_1 \cdot \hat{x} \vec{\sigma}_2 \cdot \hat{x} - \vec{\sigma}_1 \cdot \vec{\sigma}_2, \quad (2.5)$$

$$Y_x = e^{-\mu x} / \mu x. \quad (2.6)$$

ξ and η are constants which depend on the choice of $NN\pi$ form factor and are tabulated in Table I.

We now define an effective two-body potential:

$$V_{2\text{PE}}(r_{12}) = \int W_{2\text{PE}}(\vec{r}_1 \vec{r}_2 \vec{r}_3) \rho(\vec{r}_1 \vec{r}_2 \vec{r}_3) d\vec{r}_3, \quad (2.7)$$

where $\rho(\vec{r}_1 \vec{r}_2 \vec{r}_3)$ is the density distribution of the three nucleons. Calculating this effective potential is simply a trivial intermediate step if we are only interested in the first order term of the binding energy. For the second order term it is an essential step in obtaining a tractable calculation.

TABLE I. $NN\pi$ form factor parameters.

Form factor	ξ	η^2/μ^2
I	0.00	...
II	0.72	5.73
III	1.00	10.0

In general, $\rho(\vec{r}_1\vec{r}_2\vec{r}_3)$ is the square of a three-body correlated wave function and depends on spin and isospin as well as the three internucleon distances.

If we assume that $\rho(\vec{r}_1\vec{r}_2\vec{r}_3)$ is separable, and independent of the spin and isospin variables we can write

$$\rho(\vec{r}_1\vec{r}_2\vec{r}_3) = \bar{\rho}\xi(r_{12})\xi(u)\xi(x), \quad (2.8)$$

where $\bar{\rho}$ is the (constant) density of nuclear matter. Notice that while we have neglected spin-isospin dependence in the three-body wave function, the spin-isospin structure of the three-body potential is still included. This has the consequence that the effective potential (2.7) becomes like a one-pion-exchange potential (OPEP), splitting into a central and tensor part containing spin-isospin structure for nucleons 1 and 2 only:

$$V_{2\text{PE}}(r_{12}) = \vec{\tau}_1 \cdot \vec{\tau}_2 [\vec{\sigma}_1 \cdot \vec{\sigma}_2 V_c^{2\text{PE}}(r_{12}) + S_{12}(\hat{r}_{12}) V_t^{2\text{PE}}(r_{12})]. \quad (2.9)$$

LNR, as they had no underlying theory for the choice of correlation functions ξ , admitted the possibility of using different correlation functions for $\xi(r_{12})$ and $\xi(x)$ and $\xi(u)$. They chose for $\xi(x)$ and $\xi(u)$ the cutoff $\theta(r-c)$, and for $\xi(r_{12})$ the cutoff $\theta(r-d)$ with c not necessarily equal to d .

More recently, Green *et al.*^{11,12} used smooth curves instead of step functions to describe the

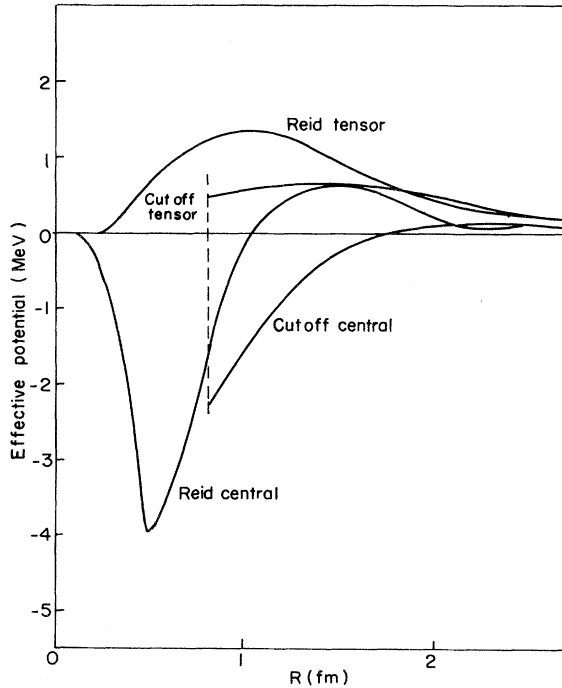


FIG. 2. The two-pion-exchange effective potentials with cutoff and Reid-soft-core correlations (form II).

correlation. The curves were derived from exact solutions to the two-body Bethe-Goldstone equation in nuclear matter with a Reid-hard-core potential. Unfortunately, this treatment used smooth correlations in only one internucleon coordinate, leaving unphysical contact interactions in the other two. In this paper we follow McKellar and Rajaraman, and our earlier paper¹³ and adopt the same correlation function for each internucleon distance. We believe, for the reasons given there, that this is the physically correct approach.

A fully correlated three-body wave function has been derived by Day.⁸ An approximation to this wave function reduces it to the separable form proposed originally by Moszkowski.¹⁴ This gives the correlation function

$$\rho_M = \bar{\rho} [1 - \chi(u)]^2 [1 - \chi(x)]^2 [1 - \chi(r_{12})]^2, \quad (2.10)$$

where χ is the on-shell defect wave function for the two nucleon system. χ is found by solving the Bethe-Goldstone equation in nuclear matter; this has been done by Negele⁹ and Siemens¹⁰ using the Reid-soft-core potential. For the Reid-hard-core-derived correlations, we use the quadratic approximation for χ calculated by Bethe⁷:

$$\chi(r) = \begin{cases} 1; & r < 0.5 \text{ fm} \\ \left(\frac{1.1-r}{1.1-0.5}\right)^2; & 0.5 \text{ fm} \leq r \leq 1.1 \text{ fm} \\ 0; & r > 1.1 \text{ fm} \end{cases} \quad (2.11)$$

In Fig. 2 we compare the cutoff-derived effective potential ($c = 0.8$ fm) to the effective potential computed using Reid-soft-core correlations. Both curves were calculated with form factor II.

In the next section, we will evaluate binding energy contributions of the three-body force using the various correlation functions discussed above.

III. BINDING ENERGY CALCULATIONS

Having calculated an effective two-body potential, we can now compute its binding energy contributions to nuclear matter in first and second order. In the effective potential approximation the contributions are

$$\begin{aligned} \Delta E^{(1)} &= \langle V_c^{2\text{PE}} \rangle / N, \\ \Delta E^{(2)} &= \left[2 \left\langle V^{nn} \frac{Q}{e} V^{2\text{PE}} \right\rangle + \left\langle V^{2\text{PE}} \frac{Q}{e} V^{2\text{PE}} \right\rangle \right] / N \\ &= \Delta E_1^{(2)} + \Delta E_2^{(2)}. \end{aligned} \quad (3.1)$$

(N is the nucleon number, Q is the Pauli operator, and e is an energy denominator.) V^{nn} is the two nucleon potential needed to compute the second order cross term $\Delta E_1^{(2)}$. LNR use OPEP, which

is given by

$$V^{\text{OPEP}}(r) = \frac{f^2 \vec{\tau}_1 \cdot \vec{\tau}_2}{2\pi^2 \mu^2} \int d\vec{q} \frac{\vec{\sigma}_1 \cdot \vec{q} \vec{\sigma}_2 \cdot \vec{q}}{q^2 + \mu^2} K^2(q^2) K'(q^2) e^{i\vec{q} \cdot \vec{r}} . \quad (3.2)$$

[Note the error in sign in Eqs. (10) and (15) of BNR³.] Only the central part of the effective potential contributes to the binding energy in first order. In second order, both central and tensor parts contribute. We denote these parts separately by

$$\Delta E_i^{(2)} = \Delta E_{i,c}^{(2)} + \Delta E_{i,t}^{(2)} \quad (i = 1, 2) . \quad (3.3)$$

The integrals implicit in (3.1) are over configuration space and over momentum space. Different insights into the calculation are obtained depending on which integral is done first, so we describe both methods.

A. Momentum space method

This is the classic method of Euler¹⁵ in which the configuration space integrals are done first. The formulas for the ΔE are then

$$\Delta E^{(1)} = a_1 \int_0^{2k_F} \tilde{V}_c^{2\text{PI}}(q) \left[2 - 3 \frac{q}{2k_F} + \left(\frac{q}{2k_F} \right)^3 \right] q^2 dq , \quad (3.4)$$

$$\Delta E_{1c,t}^{(2)} = a_2^{c,t} \int_0^\infty U \left(\frac{q}{2k_F} \right) \tilde{V}_{c,t}^{2\text{PI}}(q) \tilde{V}_{c,t}^{nn}(q) dq , \quad (3.5)$$

$$\Delta E_{2c,t}^{(2)} = a_3^{c,t} \int_0^\infty U \left(\frac{q}{2k_F} \right) [\tilde{V}_{c,t}^{2\text{PI}}(q)]^2 dq . \quad (3.6)$$

Here k_F is the Fermi momentum (1.36 fm⁻¹). U is related to the Euler function $P(x)$ ^{15,16} by

$$U(x) = \frac{3}{10} x P(x) . \quad (3.7)$$

U is a smooth function of x rising from 0 at $x=0$ to 1 for x slightly greater than 1, where it heals. The tildes on $V_{c,t}$ denote the central and tensor momentum transforms:

$$\tilde{V}_c(q) = \int_0^\infty V_c(r) j_0(qr) \xi(r) r^2 dr , \quad (3.8)$$

$$\tilde{V}_t(q) = \int_0^\infty V_t(r) j_2(qr) \xi(r) r^2 dr , \quad (3.9)$$

where $\xi(r)$ is the square of an appropriate correlated wave function defined above. We use the same correlations in (3.8) and (3.9) as we have already used in deriving the effective potentials $V_{c,t}(r)$ [see Sec. II, cc. Eq. (2.8)]. The constants

a_1 , a_2 , and a_3 in (3.4)–(3.6) have the values

$$a_1 = \frac{-27\rho\pi}{4k_F^3} = -1.433 ,$$

$$a_3^c = \frac{-24 M k_F^3}{\pi} = -0.1475 \text{ MeV}^{-1} \text{ fm}^{-5} ,$$

$$a_3^t = 2a_3^c ,$$

$$a_2^{c,t} = 2a_3^{c,t} . \quad (3.10)$$

Dimensions for the above constants are chosen so that the potentials $V^{2\text{PI}}$ and V^{nn} can be used in MeV in Eqs. (3.8) and (3.9) and all distances can be measured in femtometers. The conversion factor for $\hbar = c = 1$ units is given by

$$197.32 \text{ MeV} = 1 \text{ fm}^{-1} . \quad (3.11)$$

By evaluating the momentum transforms of the effective potentials we have discussed in Sec. II, we are able to calculate their binding energy contributions. The results of these calculations are summarized in Table II.

Note that we have used form factors in OPEP, which are given by (3.2). The effect of these form factors is much the same as that of correlations ρ —both reduce the potential at short distances. Form factors were therefore ignored in OPEP, but not in $W_{2\text{PI}}$ by LNR. We believe that this is inconsistent. Moreover, although the form factor and the short range correlations have the same effect on the potential, they represent physically different effects and both should be included. Their results for the case where the cutoffs c and d in (2.9) are both equal to 0.8 fm are given in column 1 of Table II for each of the three form factors included in (3.2) are shown in column 2; we have found that their inclusion modified $\Delta E_1^{(2)}$ and hence, the total by 0.3 MeV repulsion for both form factors II and III. The net result, with form factors in (3.2) is about 1.5 MeV attraction for form factors II and III.

The two-body correlations used in (2.10) also affect the results. Column 3 shows the results obtained using form factors in (3.2) and Reid-hard-core correlations. The contributions are only modified slightly for this two-body correlation function, giving a result of 1.7 MeV attraction for both form factors II and III. We would conclude this section with a fairly stable result of 1.7 MeV attraction were it not for the set of results in column 4 of Table II. These last results were obtained using the Reid-soft-core potential both as V^{nn} in Eq. (3.1) and in the Bethe-Goldstone equation when calculating the correlation function ξ in (2.9).^{9,10} The total binding energy contributions (column 4, Table II) are just over 6 MeV attraction. This large result is somewhat puzzling.

TABLE II. Binding energy contributions of the three-body force to nuclear matter (MeV).

Form factor		LNR (no form in OPEP) ^a $c=d=0.8$ fm	LNR+form in OPEP $c=d=0.8$ fm	Reid-hard-core correlations	Reid-soft-core correlations
I	$\Delta E^{(1)}$	1.3	1.3	1.3	-3.2
	$\Delta E_1^{(2)}$	-6.0	-6.0	-8.2	-17.2
	$\Delta E_2^{(2)}$	-0.5	-0.5	-0.6	-1.7
	(no form) Total	-5.2	-5.2	-7.5	-22.1
II	$\Delta E^{(1)}$	1.0	1.0	0.9	-0.4
	$\Delta E_1^{(2)}$	-2.5	-2.2	-2.5	-5.7
	$\Delta E_2^{(2)}$	-0.1	-0.1	-0.1	-0.3
	Total	-1.6	-1.3	-1.7	-6.4
III	$\Delta E^{(1)}$	1.3	1.3	1.1	0.3
	$\Delta E_1^{(2)}$	-3.0	-2.7	-2.6	-6.0
	$\Delta E_2^{(2)}$	-0.2	-0.2	-0.2	-0.4
	Total	-1.9	-1.6	-1.7	-6.1

^a These are calculated independently to LNR and agree with LNR errata except for $\Delta E^{(1)}$, $I=1.3$ which they give as -1.3 .

We can see from Fig. 2 that the effective two-body potential using Reid-soft-core correlations differs appreciably from the hard-core-derived potentials. However, the above derivation of bind-

ing energies does not make clear which feature of these curves is primarily responsible for the differences. We turn to the other method of evaluating (3.1) to clarify this point.

B. Configuration space method

The integrations over r and then q in Eqs. (3.8) and (3.4) can be done in the other order, giving

$$\Delta E^{(1)} = \int_0^\infty \Omega_1(r) \xi(r) V_c^{2Pi}(r) dr, \quad (3.12)$$

where

$$\Omega_1(r) = \frac{12a_1}{k_F r^2} \left(\cos k_F r - 4 \frac{\sin k_F r}{k_F r} - 4 \frac{\cos k_F r}{k_F^2 r^2} + 1 + \frac{4}{k_F^2 r^2} \right). \quad (3.13)$$

$\Omega_1 \xi$ can then be thought of as a "weight function" which selects the relevant parts of the effective potential V_c^{2Pi} in Eq. (3.12). In Fig. 3 we have drawn a graph of $\Omega_1(r)$. When $\xi(r)$ is a cutoff, the weight function $\Omega_1 \xi$ is merely the part of $\Omega_1(r)$ outside the cutoff distance (0.8 fm cutoff is shown dotted). For the Reid-derived correlations, $\xi(r)$ varies smoothly up to a healing distance of 2.1 fm and the weight function $\Omega_1 \xi$ is also shown in Fig. 3 for this case. Now comparing Figs. 2 and 3 we can quickly verify, using Eq. (3.12), that the Reid-derived and cutoff-derived effective potentials should give oppositely signed first order energy

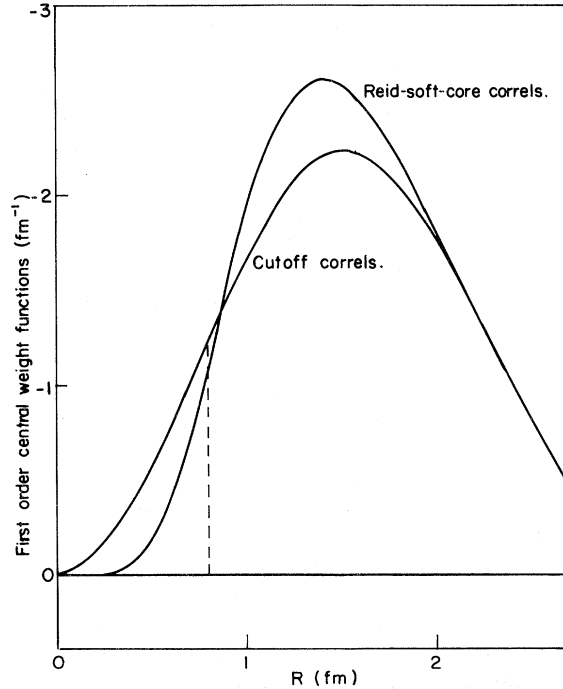


FIG. 3. Central weight functions for calculation of $\Delta E^{(1)}$.

contributions.

The second order terms can be handled in a similar fashion. As the function U in Eq. (3.5) and (3.6) heals quickly to 1, the q integrations can be done analytically except in the small q region which we treat as a wound. The second order binding energy contributions can then be written in the form

$$\Delta E_{1c,t}^{(2)} = \int_0^{\infty} \int_0^{\infty} \Omega_2^{c,t}(r, r') V_{c,t}^{2PI}(r) V_{c,t}^{nn}(r') \times \xi(r) \xi(r') dr dr' , \quad (3.14)$$

$$\Delta E_{2c,t}^{(2)} = \int_0^{\infty} \int_0^{\infty} \Omega_3^{c,t}(r, r') V_{c,t}^{2PI}(r) V_{c,t}^{2PI}(r') \times \xi(r) \xi(r') dr dr' , \quad (3.15)$$

where the four weight functions Ω_2^c , Ω_2^t , Ω_3^c , and Ω_3^t are defined by

$$\Omega_{2,3}^c(r, r') = a_{2,3}^c r^2 r'^2 \int_0^{\infty} U \left(\frac{9}{2k_F} \right) j_0(qr) j_0(qr') dq , \quad (3.16)$$

$$\Omega_{2,3}^t(r, r') = a_{2,3}^t r^2 r'^2 \int_0^{\infty} U \left(\frac{9}{2k_F} \right) j_2(qr) j_2(qr') dq . \quad (3.17)$$

If U were equal to one, we could use the analytic result

$$\int_0^{\infty} j_l(qr) j_l(qr') dq = \frac{\pi}{2(2l+1)} \frac{[\min(r, r')]^l}{[\max(r, r')]^{l+1}} \quad (3.18)$$

which can be derived from the normalization integral for the spherical Bessel functions. This result, together with a short numerical integration in the small q region, gives the two-dimensional weight functions $\Omega_{2,3}^{c,t}$.

For the second order contribution $\Delta E_2^{(2)}$ no further reduction is possible and we use the weight functions $\Omega_3^{c,t}$ with Eq. (3.15). However, for the dominant cross term $\Delta E_1^{(2)}$, we can now do the r' integration in Eq. (3.14) provided we know the particular two-body potential V^{nn} which we intend to use. We can then do the integration

$$\tilde{\Omega}_2^{c,t}(r) = \int_0^{\infty} \Omega_2^{c,t}(r, r') V_{c,t}^{nn}(r') \xi(r') dr , \quad (3.19)$$

leaving $\Delta E_1^{(2)}$ in the form

$$\Delta E_{1c,t}^{(2)} = \int_0^{\infty} \tilde{\Omega}_2^{c,t}(r) \xi(r) V_{c,t}^{2PI}(r) dr . \quad (3.20)$$

This is a one-dimensional integral similar to (3.12) so that $\tilde{\Omega}_2^t \xi$ is a useful weight function for graphical evaluation of the contribution $\Delta E_1^{(2)}$.

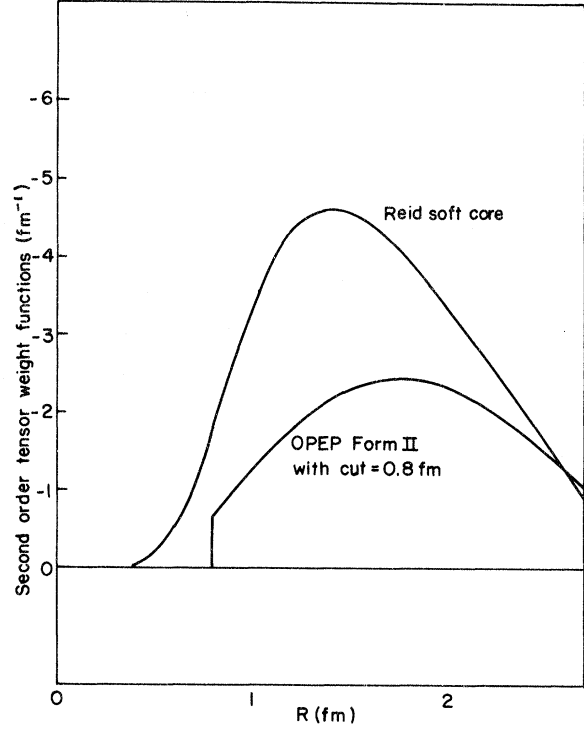


FIG. 4. Tensor weight functions for calculation of $\Delta E_1^{(2)}$.

As $\Delta E_{1c}^{(2)}$ is negligible compared to the tensor part $\Delta E_{1t}^{(2)}$, we only show the tensor weight function $\tilde{\Omega}_2^t \xi$. This is plotted in Fig. 4 for both the Reid-soft-core potential and OPEP in Eq. (3.19). Combining Fig. 4 with the tensor curves in Fig. 2, we can roughly verify the results for $\Delta E_1^{(2)}$ by eye.

IV. CONCLUSION

It is clear that the differences between the results are due to the nature of the effective two-body potential and not to a cancellation effect caused by any oscillation in the central and tensor weighting functions; the latter are smooth functions peaking at an internucleon distance of about 1.5 fm. Both the central and tensor part of the Reid-soft-core-derived effective two-body potential lead to increased binding. The tensor component has the more significant influence.

It is clear that consistency requires that the same two nucleon potential should be used in calculating the two-body force contribution to the binding energy, in the cross term with the three-body potential, and in the correlations used in calculating the three-body force contribution. Our results show that the Reid-soft-core potential,

used consistently in this way, leads to a considerable overbinding of nuclear matter.

On the other hand, the Reid-hard-core potential would appear to give more reasonable results, in

that it does not lead to overbinding.

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