High resolution (p, p') on ^{207}Pb and ^{209}Bi†

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The inelastic scattering of protons from ^{207}Pb and ^{209}Bi has been measured with energy resolution on the order of 1/5000. Many states were observed and a number of weak coupling multiplets were identified. Use of the collective model and weak coupling theory enabled spin and parity assignments to be made. Calculations for the observed single particle states with noncentral forces and with core polarization are presented.

[NUCLEAR REACTIONS ^{207}Pb(p,p'), ^{209}Bi(p,p'), $E=35$ MeV; measured $\sigma(\theta)$, $\theta=10$–$100°$. Deduced L, β_L; microscopic DWBA analysis.]

I. INTRODUCTION

Nuclei that are only one or two particles away from a shell closure permit the valence nucleon-core interaction to be investigated. The lead mass region is well suited for such investigation due to the purity of the double shell closure and the knowledge of many states in ^{208}Pb. This paper reports the (p,p') study of ^{207}Pb and ^{209}Bi which can be considered as a ^{208}Pb core with a valence neutron hole or proton particle. Inelastic proton scattering was used to excite a variety of states in these nuclei. Collective, single particle, and apparently complex excitations have all been observed and angular distributions recorded.

Experimentally, ^{207}Pb and ^{209}Bi are difficult to study because of the high level density and fractionation of inelastic transition strength. In ^{208}Pb many levels are well separated. In ^{207}Pb or ^{209}Bi, however, weak coupling to core excitations produces a spread of inelastic transition strength among many levels. Often, members of the multiplet are separated from one another or other states by only a few keV of excitation energy. For example, the 3.1 MeV multiplet in ^{209}Bi, apparently arising from the $h_{9/2}$ valence proton weak coupled[1] to the 3.2 MeV 5^- vibration in ^{208}Pb, has doublet members separated by less than 5 keV, spans an excitation energy region of only 225 keV, and lies within 15 keV of other states. Such problems necessitate the use of ultra-high resolution techniques for separation and identification of multiplet members from other levels. With data of high quality spin-parity assignments for multiplet constituents and searches for weak coupled states built on high ($E_x \approx 5$ MeV) excitation energy collective core states are possible.

Aside from the weak coupling excitations, inelastic proton scattering from these nuclei allows study of the single particle and single hole states and of the extent of core polarization in their excitation. Core polarization effects in transitions to the most well known single hole states in ^{207}Pb[2-4] and to the $i_{13/2}$ proton state in ^{209}Bi[5] have been examined previously. Here it was hoped to determine the importance of both the ^{208}Pb core and the noncentral forces in the excitation of some of the ^{209}Bi states.

Section II discusses the experimental setup and procedure. The reduction of the data, angular momentum transfer identification, and comparison with previous work are discussed in Sec. III. Calculations involving the weak coupling theory and the microscopic distorted wave Born approximation (DWBA) are shown in Secs. IV and V.

II. EXPERIMENTAL PROCEDURE

The experiment used 35 MeV protons extracted from the Michigan State University cyclotron with beams on target ranging between $\frac{1}{2}$ and 1 μA, the smaller current being used on the lower melting point bismuth. Protons scattered from targets of ^{209}Bi and ^{207}Pb were observed using both a wire proportional counter and photographic emulsions in the focal plane of the Enge split-pole spectrometer. The high resolution cyclotron-spectrograph system were used to obtain typical plate data resolution of 5–10 keV full width at half maximum (FWHM). The plate data spanned the region of excitation energy between about 0.5 and 8.0 MeV. The counter data had a resolution which was detector limited to about 50 keV FWHM and examined the lowest 5 MeV of excitation.

Initially, angular distributions were measured using thick lead and bismuth targets and the wire counter-scintillator setup.[6] Protons exciting the low-lying states were generally well resolved with good statistics. Measurement of the elastic angular distribution was also made. Comparing the elastic cross sections with the optical model calculations using Becchetti-Greenlees best-fit parameters[7] determined the absolute normalization to about 5%. Comparing the completely resolved inelastic states in both plate and counter data gave the absolute normalization of the plate data to about 10%. Whenever possible the better statistics counter data is displayed.

The high resolution data was recorded on Kodak 25 μm NTB emulsion with a piece of 0.051 cm stainless steel shim stock before the plate to enhance track brightness and to absorb heavier mass particles. Spectra were recorded from 10 to 100°. Fifteen angles were recorded for the plate data. Most plate data was taken with a $1\times1°$ spectrometer entrance slit, but some ^{209}Bi spectra were taken with a $2\times2°$ slit as a reasonable compromise of resolution and count rate. Before beginning a run, the resolution was optimized using the on-line focal plane line width determination system and dispersion matching.[8]

Typical spectra of ^{207}Pb and ^{209}Bi are shown in Fig. 1. Also shown is a spectrum of ^{208}Pb to allow comparison. The fragmentation of ^{208}Pb collective states into multiplets is apparent. Many single particle states were resolved and are also indicated. The increase in level density from ^{208}Pb to ^{207}Pb to ^{209}Bi is striking. Discrete structure can be seen up to 6 MeV in the two lead spectra but the bismuth spectrum is essentially a continuum above 5.5 MeV of excitation.

Since Bi is monisotopic, few contaminants were found in the bismuth data, the major ones being oxygen and carbon from the thin carbon foil-Formvar backing. The ^{207}Pb targets were made from an isotopically enriched lead sample obtained from the Oak Ridge National Laboratory and was 99.81% ^{207}Pb, 0.13% ^{208}Pb, and had small amounts of other lead isotopes. The lead targets also had backings. Target thickness was about 100 μg/cm^2 and 3 mg/cm^2 for the plate and counter studies, respectively.

III. DATA

A. Excitation energies

The excitation energies of the 170 levels observed in ^{207}Pb and the 80 levels seen in ^{209}Bi are listed in Tables I and II along with the results of recent *Nuclear Data Sheets*.[9,10] The energy calibration for each plate exposure was made using lead or bismuth levels whose focal plane positions were unambiguously known and whose excitation energies were well determined in previous high resolution studies. Levels used for calibration are noted in the tables. The calibration involved using the best experimentally determined excitation energies initially, predicting average energies with all plate spectrum, and iterating until the average energies were consistently obtained. The well-known levels of ^{12}C, ^{13}C, and ^{16}O were also used in

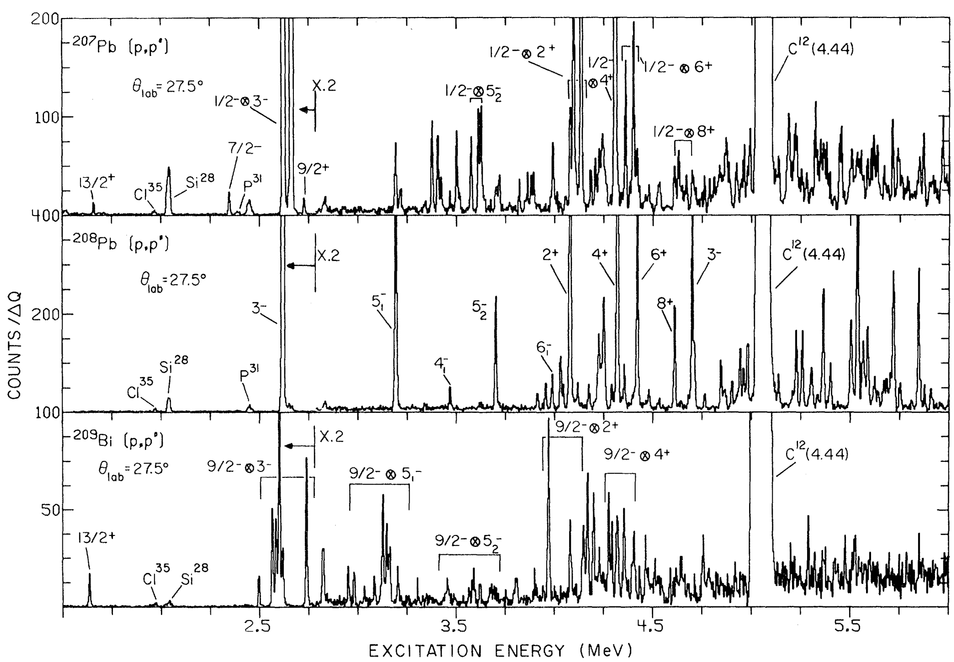

FIG. 1. Typical spectra of ^{207}Pb, ^{208}Pb, and ^{209}Bi. Multiplets built on strong levels in ^{208}Pb are apparent in other spectra.

TABLE I. Energy levels, l transfers, and deformation parameters for ^{207}Pb.

Present work			Compilation [e]		
E_x [a] $\pm\Delta E_x$	L	β_L	E_x [a]	J	L
0.5709 [b]	2	0.026	0.569 67	$\frac{5}{2}^-$	
0.8986 [b]	2	0.025	0.8976	$\frac{3}{2}^-$	
1.6337 [b]	6, 7	0.019, 0.019	1.633 29	$\frac{13}{2}^+$	
2.3398 [b]	4	0.024	2.339 89	$\frac{7}{2}^-$	4
			2.368?		
			2.563?		
2.6230 [b]	3	0.076	2.6241	$\frac{5}{2}^+$	3
2.6626 [b]	3	0.087	2.6619	$\frac{7}{2}^+$	3
2.702 ± 0.005			2.705?		
2.7276 [b]	5	0.024	2.726	$\frac{9}{2}^+$	6
			2.840?		
			2.902?		
			2.909?		
			3.004?		
			3.057?		
			3.180?		
3.200 ± 0.003			3.202		5
3.223 ± 0.002	5	0.013	3.222		5
			3.267?		
			3.298	$\frac{1}{2}^-$	
			3.319?		
			3.335?		
			3.344?		
3.384 ± 0.002	(5)	0.027	3.382	$(\frac{9}{2}^+, \frac{11}{2}^+)$	5
3.413 ± 0.002	4	0.021	3.409	$\frac{9}{2}^-$	4
3.429 ± 0.002	(5)	0.016	3.426		6
3.476 ± 0.003	(5)	0.013			
3.509 ± 0.002	(5)	0.025	3.499		
3.583 ± 0.002	5	0.023	3.580		5
3.620 ± 0.002	5	0.028	3.620		5
3.634 ± 0.002	3	0.019	3.632		
3.650 ± 0.003	≈9	≈0.020			
3.672 ± 0.003					
3.709 ± 0.004					
3.726 ± 0.003					
3.829 ± 0.003	5	0.014			
3.857 ± 0.004	7, 8	0.014, 0.015			
3.869 ± 0.002	(5)	0.016			
3.887 ± 0.003					
3.901 ± 0.002	7, 8	0.023, 0.023			
3.925 ± 0.005 [d]					
3.986 ± 0.002					
3.999 ± 0.003					
4.017 ± 0.003					
4.034 ± 0.005	(3)	0.007			
4.062 ± 0.004					
4.088 ± 0.004			4.089		2
4.103 ± 0.003	2	0.036	4.113		
4.140 ± 0.003	2	0.045	4.127		2
4.190 ± 0.003	7, 6	0.026, 0.024			
4.213 ± 0.003	5	0.022			
4.232 ± 0.005					
4.250 ± 0.004					
4.270 ± 0.004	(6)	0.010			
4.287 ± 0.006	4	0.008	4.288		4
4.313 ± 0.004	4	0.067	4.318		
4.342 ± 0.006	(3)	0.009	4.339		6
4.364 ± 0.003	6	0.042	4.380		6
4.387 ± 0.004			4.387		
4.404 ± 0.003	6	0.047			
4.422 ± 0.003	(2)	0.017			
4.465 ± 0.005 [d]					
4.479 ± 0.004					
4.494 ± 0.005	(8)	0.009			
4.514 ± 0.004			4.513		
4.527 ± 0.004	(2)	0.010			
4.538 ± 0.004			4.541		
4.558 ± 0.003			4.546		
4.592 ± 0.006					
4.612 ± 0.003					
4.630 ± 0.003	(8)	0.028	4.629		
4.656 ± 0.005	2	0.013			
4.671 ± 0.003	(8)	0.025			
4.700 –4.730 [c]					
4.733 ± 0.003					
4.745 ± 0.003	(8)	0.016			
4.761 ± 0.004					
4.785 ± 0.004	≥7				

TABLE I *(Continued)*

Present work			Compilation [e]		
E_x [a] $\pm\Delta E_x$	L	β_L	E_x [a]	J	L
4.806 ± 0.005					
4.835 ± 0.006 [d]					
4.870 ± 0.004					
4.884 ± 0.003					
4.921 ± 0.004					
4.943 ± 0.004					
4.957 ± 0.004					
4.975 ± 0.006					
4.987 ± 0.005 [d]	3	0.022			
5.018 ± 0.005	≈9	≈0.021			
5.039 ± 0.005	≈9	≈0.021			
5.053 ± 0.005					
5.081 ± 0.004	3	0.029			
5.117 ± 0.006 [d]					
5.129 ± 0.005 [d]			5.129		
5.156 ± 0.006	(3)	0.010			
5.177 ± 0.006 [d]	3	0.029	5.178		
5.193 ± 0.005					
5.217 ± 0.005			5.219		
5.245 ± 0.008	3	0.017	5.252		
5.267 ± 0.005	3	0.016			
5.290 ± 0.005					
5.310 ± 0.005					
5.321 ± 0.005	3	0.020			
5.336 ± 0.005	3	0.023			
5.352 ± 0.005	3	0.026			
5.369 ± 0.005	(6)	0.027			
5.383 ± 0.005					
5.402 ± 0.006					
5.428 ± 0.005	3	0.018	5.417		
5.440 ± 0.005	3	0.022			
5.454 ± 0.005					
5.474 ± 0.004					
5.487 ± 0.006					
5.501 ± 0.005	4	0.017			
5.526 ± 0.004 [d]	(7)	0.027			
5.537 ± 0.005					
5.548 ± 0.006					
5.569 ± 0.005	3	0.020			
5.584 ± 0.004					
5.598 ± 0.005					
5.614 ± 0.006	(3)	0.019			
5.648 ± 0.006					
5.668 ± 0.005					
5.689 ± 0.005					
5.720 ± 0.006					
5.735 ± 0.006					
5.765 ± 0.007					
5.803 ± 0.006					
5.822 ± 0.006					
5.840 ± 0.006					
5.868 ± 0.006					
5.897 ± 0.007					
5.915 ± 0.008					
5.934 ± 0.007					
5.952 ± 0.005					
5.959 ± 0.006					
5.998 ± 0.006					
6.010 ± 0.005					
6.031 ± 0.006					
6.041 ± 0.007					
6.064 ± 0.007					
6.073 ± 0.006					
6.090 ± 0.007					
6.105 ± 0.006	(3)	0.015			
6.146 ± 0.005 [d]					
6.170 ± 0.008	(7)	0.017			
6.188 ± 0.007 [d]	(7)	0.024			
6.228 ± 0.007					
6.251 ± 0.006					
6.262 ± 0.006					
6.276 ± 0.006	(3)	0.016			
6.310 ± 0.007					
6.332 ± 0.007					
6.360 ± 0.005					
6.381 ± 0.007					
6.402 ± 0.006					
6.449 ± 0.007					

TABLE I *(Continued)*

Present work E_x [a] $\pm \Delta E_x$	L	β_L	Compilation [e] E_x [a]	J	L	Present work E_x [a] $\pm \Delta E_x$	L	β_L	Compilation [e] E_x [a]	J	L
6.483 ± 0.008	(3)	0.013				6.788 ± 0.009					
6.547 ± 0.008						6.864 ± 0.008					
6.627 ± 0.008 [d]						6.912 ± 0.008 [d]					
6.654 ± 0.008						6.939 ± 0.009					
6.670 ± 0.008						6.955 ± 0.009					
6.716 ± 0.007 [d]						7.048 ± 0.009					
6.762 ± 0.007 [d]											

[a] All energies in MeV.
[b] State used in energy calibration.
[c] Spectral region with unresolved multiplet structure.
[d] Level with probable multiplet structure.
[e] Reference 9.

the calibration whenever possible.

The large level population at even low excitation energies prevented the use of many levels in the calibration. In ^{207}Pb and ^{209}Bi the excitation energies above 2.730 and 3.155 MeV, respectively, were determined by extrapolation. Therefore, below these energies the error is simply the standard deviation. Above these energies the error is the standard deviation plus an additional 1 keV/MeV of extrapolated energy. This systematic error is an estimate of both the interpolation error and the uncertainties caused by the high level density.

B. Inelastic angular distributions

Figures 3 through 5 display angular distributions which are reasonably well fitted by collective model calculations for these isotopes. The latter are discussed in Sec. III D and the former are discussed here. In all cases only the levels apparent in six or more plate exposures are displayed in the figures. For both isotopes, as indicated in Tables I and II, some regions of the spectra have level densities too great for states to be resolved. The error bars drawn indicate only statistical errors and are shown only when larger than the size of the symbol. Gaps in the angular distributions occur when the peak of interest was obscured by a contaminant.

In some cases multiplet structure is suggested in the data by a larger-than-average peak width or by resolution of a peak into a doublet at a few angles. States having such features have been noted in the tables as possible multiplets.

In ^{207}Pb many of states observed have been previously reported. There are some states reported in (d, d') and (d, t)[11] or other[9] studies which were not observed here. An upper limit of about 40 μb/sr can be set for the peak differential cross section for excitation of these states by (p,p'). A large number of previously unidentified levels have been observed, especially in the excitation energy region above 4.5 MeV. Because of the large level density, identification of levels at high excitation which correspond to those seen in other reactions is quite uncertain.

In both ^{207}Pb and ^{209}Bi, (p,p') excites states which have been populated in a variety of single nucleon transfer reactions.[11-17] In ^{207}Pb the $p_{1/2}$, $f_{5/2}$, $p_{3/2}$, $i_{13/2}$, and $f_{7/2}$ neutron hole configurations were seen and in ^{209}Bi the $h_{9/2}$, $f_{7/2}$, $i_{13/2}$, $f_{5/2}$, $p_{3/2}$, and $p_{1/2}$ proton particle levels were excited. Thus these nuclei permit the study of hole and particle states in the same major oscillator shell. The angular distributions for these levels have fairly characteristic shapes and many of them are discussed in Sec. V.

In ^{209}Bi, inelastic proton scattering apparently excites most states seen in direct reactions but is most sensitive to collective nuclear motion. Some of the states populated in the (p,p') study of Cleary[10,25] were not seen here. That study with 14.95 and 16.1 MeV protons examined ^{209}Bi both on and off the $g_{9/2}$ isobaric resonance. At those energies many of the configurations formed are sensitive to the compound nucleus energy and are not expected to be strongly populated at 35 MeV.

Another study of ^{209}Bi using the ^{207}Pb(α, d)[19] reaction excited levels not observed here. This particular transfer reaction is expected to excite configurations involving $(\pi\nu\nu^{-1})$ as the final state, where the neutron hole is in the $p_{1/2}$ orbital. Because (p,p') can be described by a one body operator in the quantum space of the nucleus it is ex-

TABLE II. Energy levels, l transfers, and deformation parameters for ^{209}Bi.

Present work			Compilation (Ref. 10)		
E_x [a] $\pm\Delta E_x$	L	β_L	E_x [a]	J	L
0.8959 [b]	(2)	0.013	0.8966	$\frac{7}{2}^{-}$	2
1.6081 [b]	3	0.027	1.6085	$\frac{13}{2}^{+}$	3
2.492 ± 0.001	3	0.026	2.492	$\frac{3}{2}^{+}$	3
2.564 ± 0.001	3	0.047	2.563	$\frac{9}{2}^{+}$	3
2.581 ± 0.002	3	0.041	2.582	$\frac{7}{2}^{+}$	3
2.599 ± 0.001	3	0.074	2.599	$\frac{11}{2}^{+}$	3
			2.601	$\frac{13}{2}^{+}$	3
2.617 ± 0.002	3	0.035	2.616	$\frac{5}{2}^{+}$	3
2.7404 [b]	3	0.057	2.741	$\frac{15}{2}^{+}$	3
2.766 ± 0.002	4	0.013	2.762		
2.8251 [b]			2.822	$\frac{5}{2}^{-}$	
			2.827		
			2.91		
2.956 ± 0.003	(4)	0.014	2.957		
2.986 ± 0.001	5	0.021	2.987	$\frac{13}{2}^{+}$	5
3.038 ± 0.002	5	0.013	3.038	$\frac{3}{2}^{+}$	5
3.091 ± 0.003	5	0.014	3.091	$\frac{5}{2}^{+}$	5
3.118 ± 0.002			3.116	$\frac{3}{2}^{-}$	
3.1339 [b]	5	0.036	3.135	$\frac{11}{2}^{+}$, $\frac{19}{2}^{+}$	5
3.1534 [b]	5	0.032	3.154	$\frac{17}{2}^{+}$, $\frac{7}{2}^{+}$ or $\frac{17}{2}^{+}$, $\frac{9}{2}^{+}$	5
3.168 ± 0.002	5	0.026	3.170	$\frac{15}{2}^{+}$	5
			3.197		
3.211 ± 0.001	5	0.020	3.212	$\frac{9}{2}^{+}$ or $\frac{1}{2}^{+}$, $\frac{7}{2}^{+}$	5
			3.222		
3.309 ± 0.003	(3)	0.009	3.311		
3.358 ± 0.002			3.363		
			3.379		
			3.393		
			3.406		
			3.433 ?		
			3.450		
3.466 ± 0.002 [d]	5	0.019	3.465		
			3.476		
			3.489		
3.501 ± 0.005			3.503		
3.579 ± 0.003	5	0.012	3.579		$L=5$ Group
3.597 ± 0.002 [d]	5	0.020	3.597		
3.633 ± 0.004			3.640	$\frac{1}{2}^{-}$	
			3.670		
3.685 ± 0.003	5	0.015	3.683		
3.703 ± 0.004	5	0.015	3.692		
3.710 –3.750 [c]			3.719		
			3.735		
			3.753		
3.765 ± 0.003 [d]			3.763		
3.803 ± 0.004	(3)	0.013	3.802		
3.815 ± 0.002	(7, 8)	0.031, 0.026	3.818		
3.839 ± 0.004 [d]			3.839		
3.855 ± 0.003			3.855		
			3.880		
3.892 ± 0.003			3.893		$L=2$ Group
			3.909		
3.924 ± 0.005 [d]	(3)	0.013	3.919		
			3.937		
3.950 ± 0.005	(3)	0.012	3.950		
			3.962		
3.981 ± 0.003	2	0.033	3.981		
			3.994		
4.013 ± 0.005			4.015		
			4.038		
4.047 ± 0.005			4.050		
			4.079		
4.092 ± 0.004	2	0.027	4.096		
4.116 ± 0.004	(7)	0.022	4.121		
			4.133		
4.157 ± 0.004	2	0.027			
4.177 ± 0.004	3	0.033	4.178		
4.210 ± 0.004	3	0.029			$L=4$ Group
4.235 ± 0.004					
4.257 ± 0.004					
4.286 ± 0.003	4	0.034	4.276		
4.301 ± 0.003	≈7	≈0.033			
4.326 ± 0.003			4.32 ?		
4.362 ± 0.003	4	0.032			
4.397 ± 0.003			4.397		
4.411 ± 0.003	≈8	≈0.035	4.421	$\frac{1}{2}^{-}$	

TABLE II *(Continued)*

Present work			Compilation (Ref. 10)			Present work			Compilation (Ref. 10)		
$E_x{}^{a} \pm \Delta E_x$	L	β_L	$E_x{}^{a}$	J	L	$E_x{}^{a} \pm \Delta E_x$	L	β_L	$E_x{}^{a}$	J	L
4.441 ± 0.004^{d}	4	0.017	4.447	$\frac{7}{2}^-$		5.241 ± 0.007					
4.469 ± 0.003^{d}						5.282 ± 0.005					
4.485 ± 0.004						5.312 ± 0.005^{d}			5.304		
4.512 ± 0.005			4.519			5.333 ± 0.005					
4.532 ± 0.004^{d}	≈ 8	≈ 0.021				5.360 ± 0.006^{d}					
4.592 ± 0.006			4.601			5.423 ± 0.006^{d}			5.43		
4.613 ± 0.005^{d}						5.463 ± 0.005					
$4.630 - 4.745^{c}$			4.650			5.509 ± 0.006^{d}					
			4.745			5.569 ± 0.010^{d}			5.57		
4.760 ± 0.004^{d}						5.769 ± 0.005			5.77		
4.791 ± 0.006^{d}						5.795 ± 0.007^{d}					
4.828 ± 0.005						5.835 ± 0.008^{d}					
4.853 ± 0.005									6.394		
4.949 ± 0.004^{d}									7.169		
4.965 ± 0.005									7.176		
4.998 ± 0.006									7.416		
5.056 ± 0.005^{d}									7.637		
5.131 ± 0.006^{d}	≈ 7	≈ 0.022									

[a] All energies in MeV.
[b] State used in energy calibration.
[c] Spectral region with unresolved multiplet structure.
[d] Level with probable multiplet structure.

pected that states excited in both (p,p') and (α, d) should involve $(\pi_{h_{9/2}}\nu\nu^{-1}_{p_{1/2}})$. The states not excited in both reactions are the 2.91, 2.979, and 4.133 MeV levels suggesting that these are 2p-1h states not involving the $h_{9/2}$ proton orbital.

Many levels in the region above 4.6 MeV of excitation were resolved in this study of ^{209}Bi that were not reported before. In this excitation region the extremely high population of levels and the fractionation of strength makes resolving states very difficult.

C. Discussion of the collective model

The primary information required for a conventional collective model (CM) calculation is the nucleon-nucleus elastic scattering optical model potential. This potential is usually obtained with search codes that vary the model parameters until the best χ^2 fit to elastic scattering data and, often, to polarization data is achieved. Figure 2 compares the ^{207}Pb and ^{209}Bi elastic data with optical model calculations using the best-fit Becchetti-Greenlees[7] (BG) parameters and those obtained with the optical model search program GIBELUMP.[20] Since there is no polarization data for 35 MeV protons the search was performed with fixed BG spin-orbit geometry. Spin-orbit sets adopted from other studies at other bombarding energies gave similar good agreement between the measured and calculated elastic cross sections.

However, collective model calculations with the BG and fitted sets yield different cross section predictions. With the BG and fitted optical models for either nucleus, the CM gives about a 40% difference between results for the same l transfer. However, the ratio of l transfers within a set is identical. This ambiguity in normalization of the DWBA cross section has been noticed previously.[21] For this reason and because of the lack of polarization and large angle data the BG optical model parameters have been used in all macroscopic and microscopic model calculations in this paper. Use of these parameters gives excellent agreement with previous CM analysis of these nuclei using the (p,p') reaction.

In the collective model calculations, the code DWUCK[22] was used with 40 partial waves, integration limit of 20 fm, and integration step of 0.1 fm. Both the real and imaginary parts of the optical model were deformed and, since there is little sensitivity to the reaction Q value, all calculations were for transitions to states lying at 5 MeV of excitation energy. Coulomb excitation was included in the $L=2$ and 3 cases but only in the former excitation is the contribution significant. The deformation parameter β_L was calculated as the square root of the ratio of the experimental and predicted CM cross sections.

For L greater than about 5 the forward angle CM fits to the data are not good. The data consistently shows forward angle strength not predicted by the CM. This fact and the rather similar angular distributions for $L \geq 6$ makes large L assignments quite tentative.

The results for the CM fits are discussed below. The actual fits are shown in Figs. 3 through 5 and the deformation parameters and L assignments are listed in Tables I and II.

D. l transfers and deformation parameters for ^{207}Pb

The results of CM fits to the ^{207}Pb data are displayed in Figs. 3 and 4. Whenever possible comparison of the data with angular distributions for levels of known l transfer was made. In ^{207}Pb, considerable fractionation of the strength seen in ^{208}Pb generally occurs. States are discussed in the following sections according to L. The deformation parameters for the single particle states differ from those given elsewhere.[4] The values given here probably represent the inelastic strength of these states better.

1. *Quadrupole excitations*

The states at 0.571 and 0.899 MeV of excitation, previously identified[11-15,18] as single particle states, are excited predominantly with $L=2$. The rate of fall of the angular distributions are well reproduced and the phase of the data is also reason-

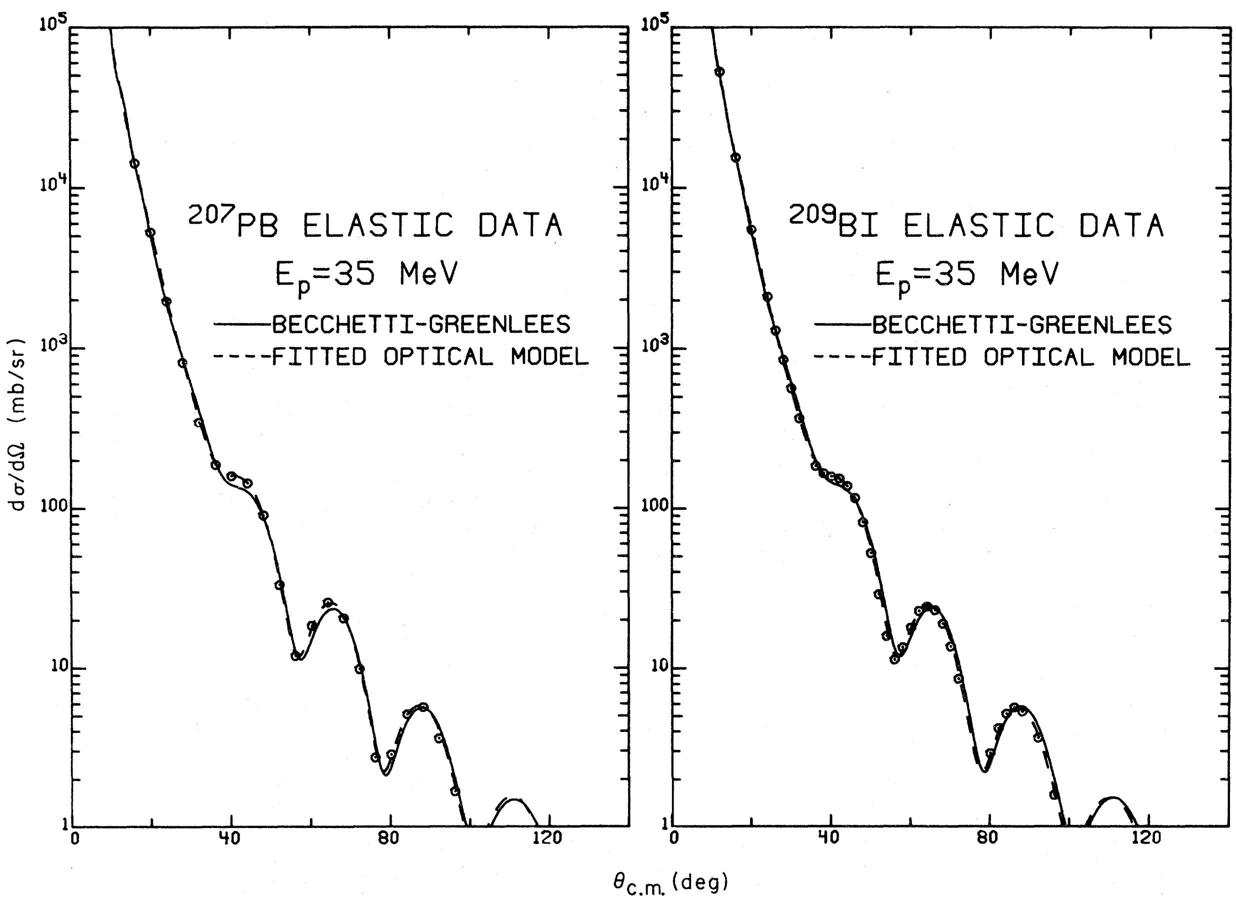

FIG. 2. Comparison of the measured elastic angular distributions with calculations described in the text.

ably well given. These $f_{5/2}$ and $p_{3/2}$ neutron hole states have been shown[2-4] to have significant contributions from quadrupole core polarization excitations.

The ^{208}Pb 2^+ state at 4.086 MeV is apparently split into a doublet with members at 4.103 and 4.140 MeV. Vallois *et al.*[23] have identified strong quadrupole excitations at 4.090 and 4.125 MeV. Within experimental error, the excitation energies and deformation parameters from that study agree with our values. However, a 4.115 MeV state seen[11] in (d,p) suggests that a doublet may lie near this excitation energy. Our data do reveal a weakly excited state at about 4.112 MeV which is unfortunately seen at only a few angles. When resolved its cross section is less than 5% of that of the 4.103 MeV state.

Tentative identifications of states involving $L=2$ transitions have been made for the states at 4.422 and 4.527 MeV. Identification of the 4.656 MeV level as $L=2$ is fairly certain.

2. Octupole excitations

There were many transitions involving $L=3$ angular momentum transfer. This is to be expected since there are many 1p-2h configurations that can arise from the large number of 1p-1h octupole configurations in the ^{208}Pb core. The well known doublet with members at 2.623 and 2.663 MeV dominates any inelastic spectrum and was so intense at some angles as to be unscannable. Both members of the doublet have characteristic $L=3$ shapes.

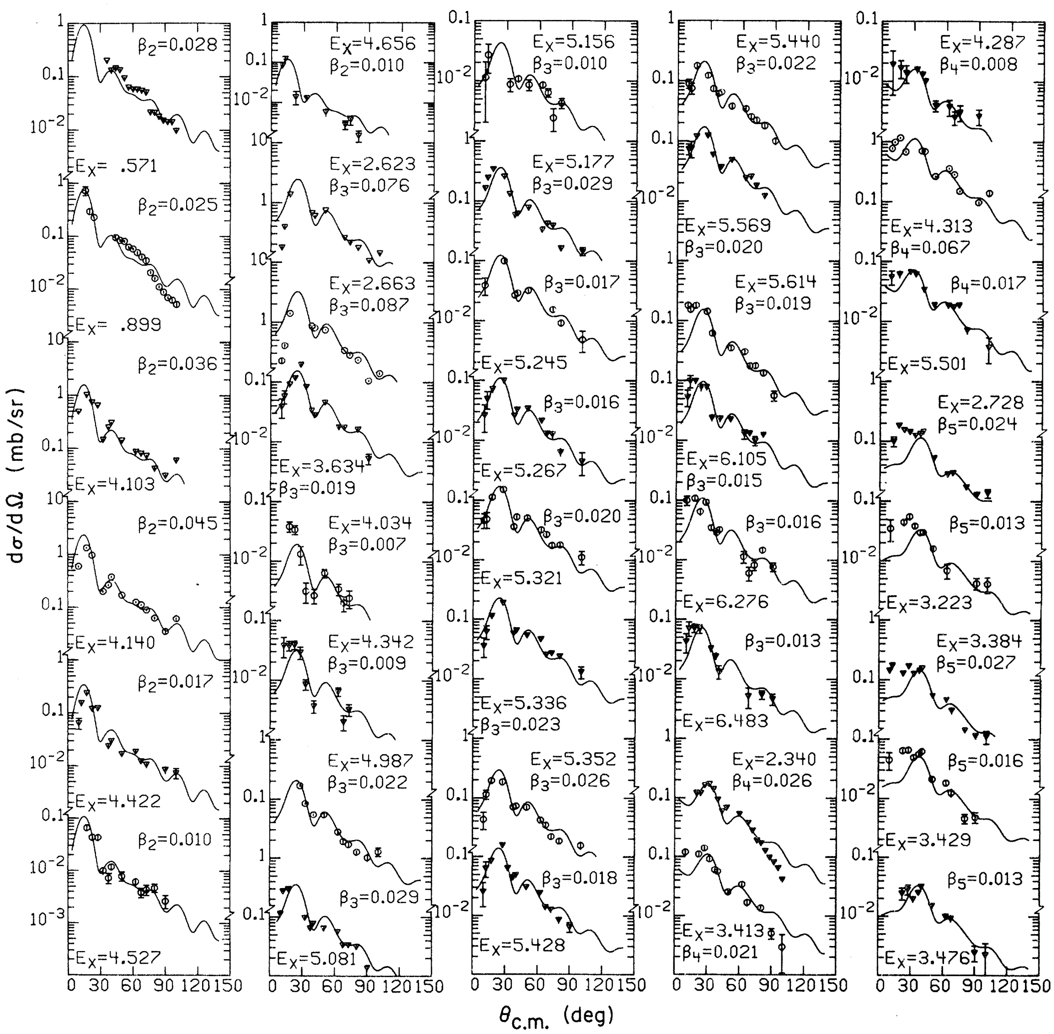

FIG. 3. Collective model fits for all identified states in ^{207}Pb. Displayed with the fit is the excitation energy of the state and the deformation parameter, β_L, corresponding to orbital angular momentum transfer L.

The 4.342 MeV level is believed to be an octupole excitation. Vallois *et al.*[23] identified a state at 4.340 ± 0.015 MeV as being an $L=6$ level. As there is no state observed in our data within 15 keV of the 4.342 MeV state we conclude that either a doublet is present at that energy or the initial L assignment is incorrect.

The level at 5.177 MeV appears to be a multiplet. An $L=3$ assignment has been made for the strongest member.

3. States involving L=4

Collective model calculations for states involving $L=4$ transitions were similar to only a few experimental angular distributions. The $L=4$ strength appears concentrated in only a few levels. The 2.340 MeV state, which has been identified as the $f_{7/2}$ neutron hole state, has an angular distribution with characteristic $L=4$ shape. According to direct DWBA theory this state can be reached only by $L=2$ or 4.

The state at 4.313 MeV has been assumed to be the unresolved weak coupling doublet built on the 4.323 MeV 4^+ vibration in ^{208}Pb. This ^{207}Pb state has a small satellite at 4.287 MeV which is weakly excited but has an identifiable $L=4$ shape.

4. States with L=5

As in ^{208}Pb there are many states that involve $L=5$ transitions. In particular, the region from

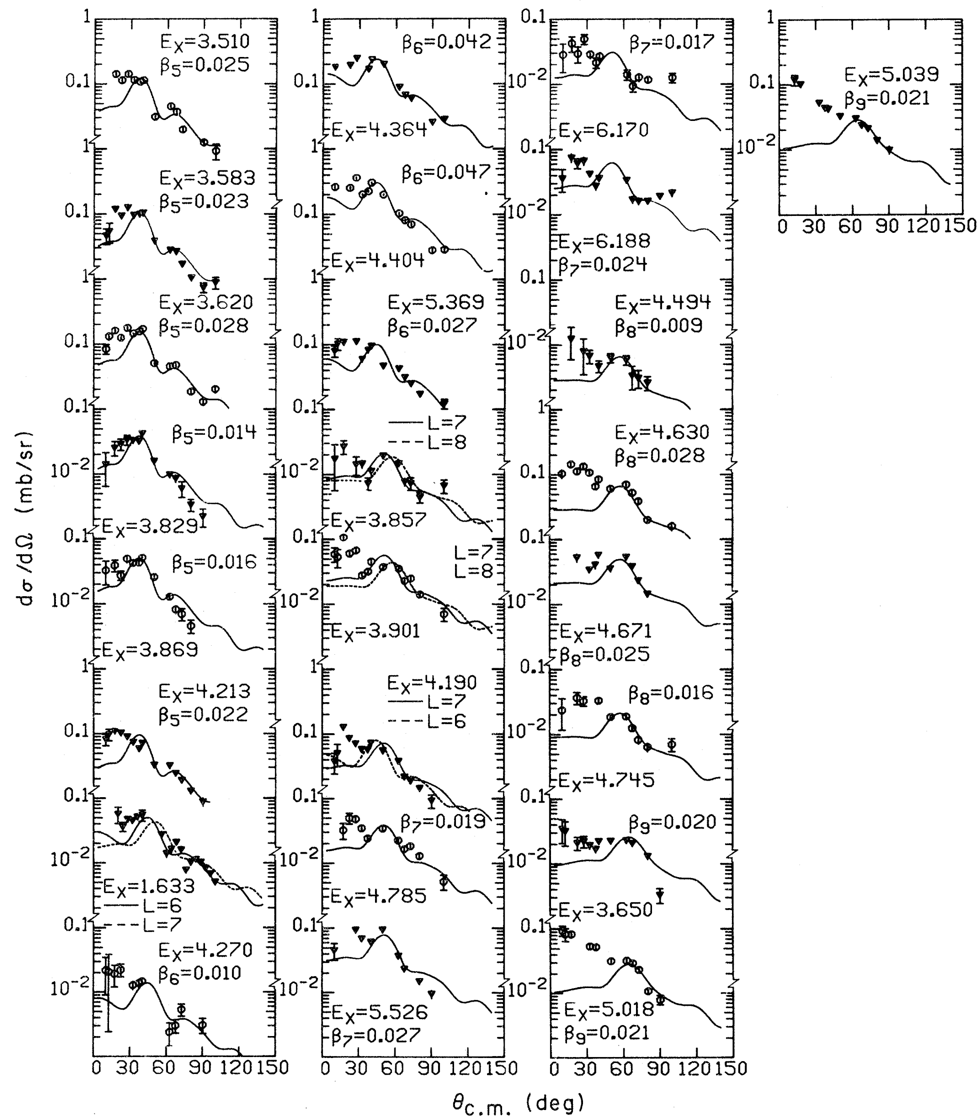

FIG. 4. Collective model fits for all identified states in ^{207}Pb. Displayed with the fit is the excitation energy of the state and the deformation parameter, β_L, corresponding to orbital angular momentum transfer L.

3.20 to 3.62 MeV of excitation has many weakly excited levels that have l transfers of 5. The states at 3.583 and 3.620 MeV both have $L=5$ shapes. The 2.728 and 3.429 MeV levels were previously assigned[23] $L=6$. We have assigned $L=5$ for both states. The 2.728 MeV level is seen[11] in (d,p) with $l_n=4$. The neutron configuration of $g_{9/2}$ coupled to ^{206}Pb(0.00 MeV) which has been suggested[11,14,15,18] for this state is consistent with both identifications.

A significant fraction of the $L=5$ strength seen in ^{208}Pb is not seen in ^{207}Pb. Probably most interesting is the lack of $L=5$ strength in the excitation region of ^{207}Pb corresponding to the first excited 5^- state in ^{208}Pb. This missing strength is discussed in Sec. IV B, below.

Higher excitation $L=5$ strength seen in ^{208}Pb has not been observed in ^{207}Pb. This may be due to configuration mixing or masking of the strength by other ^{207}Pb levels. We also noted a similar lack of octupole strength corresponding to that seen in high-lying levels in ^{208}Pb. This probably has the same explanation.

5. *States with $L\geqslant 6$*

A few states apparently involve $L=6$ transfers. The weak coupling doublet with parentage in the 6^+ state in ^{208}Pb apparently lies at 4.364 and 4.404 MeV. The 1.634 MeV state which is highly excited in single particle transfers[11-17] is excited in (p,p') by $L=6$ or 7. Direct DWBA theory allows the transition to proceed through only $L=5$ or 7.

All angular momentum transfer assignments for $L=7$ or larger are quite tentative. As noted above, this is due to the generally similar shapes of these high l transfers. States which possibly involve high spin transfer are found at 3.650, 3.857, 3.901,

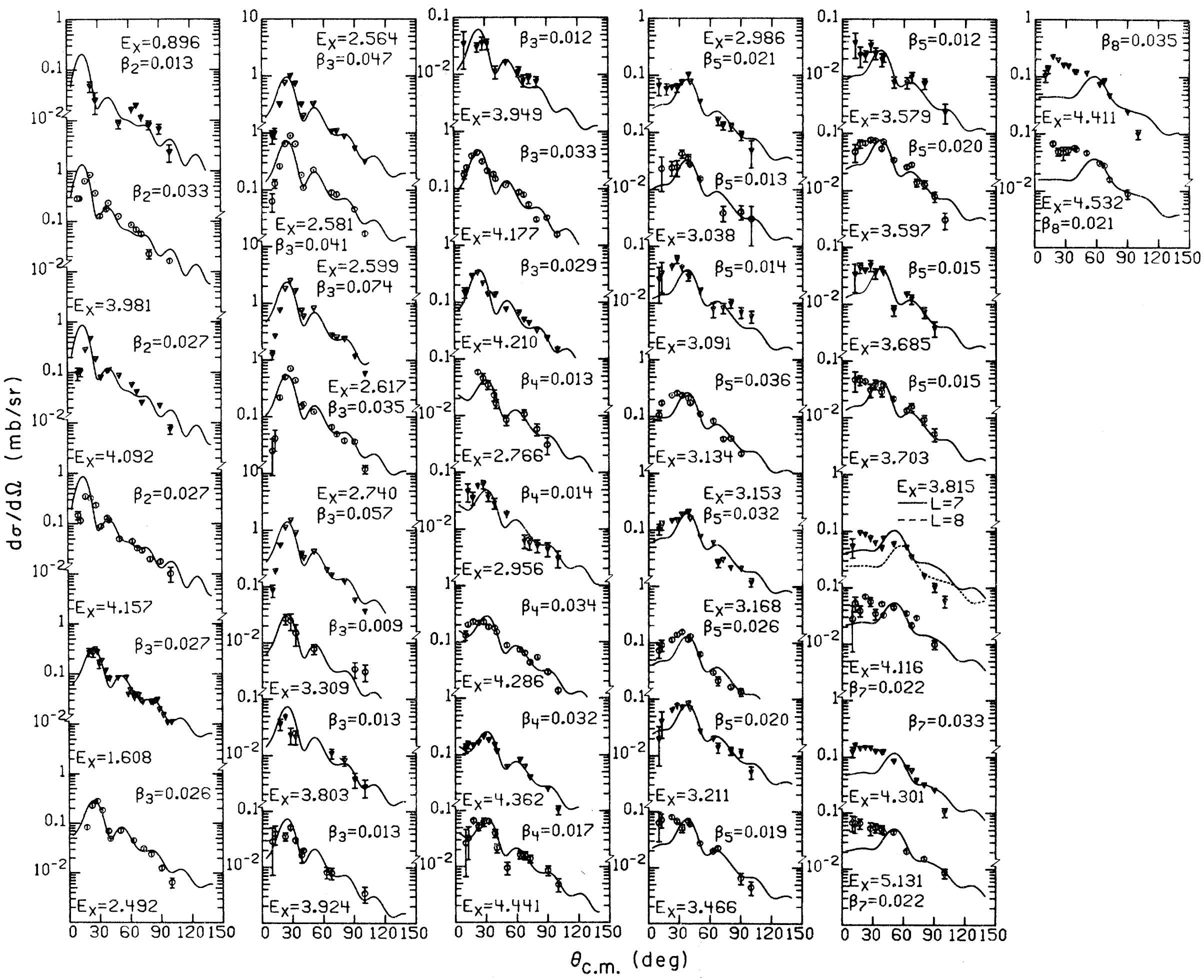

FIG. 5. Collective model fits for all identified states in ^{209}Bi. Displayed with the fit is the excitation energy of the state and the deformation parameter, β_L, corresponding to orbital angular momentum transfer L.

4.494, 4.630, 4.671, 4.745, 4.785, 5.018, 5.039, 5.526, 6.170, and 6.188 MeV. Most of these levels have tentative assignments and many appear to have multiplet structure.

E. l transfers and deformation parameters for ^{209}Bi

The ^{209}Bi(p,p') data displayed in Fig. 5 has been compared with CM characteristic shapes. The large level density and the apparent extreme splitting of the strength of ^{208}Pb core excitations made L assignment difficult. In general, the bismuth angular distributions were similar to those of ^{207}Pb.

1. Quadrupole excitations

The single particle state at 0.896 MeV $(J^{\pi}=\frac{7}{2}^-)$ seen in the $(^3\text{He}, d)$[16] and (α, t)[17] reactions is populated primarily by an $L=2$ transition although $L=0$, 2, 4, 6, 8 are allowed for transitions to this state.

Three distinct quadrupole excitations at 3.981, 4.092, and 4.157 MeV have been resolved. The total deformation parameter for this triplet is about 0.050. Bertrand and Lewis,[24] in an inelastic proton study of ^{209}Bi, reported an $L=2$ group centered at about 3.96 MeV of excitation and with a $\beta_2=0.049$. These two measurements are in good agreement. The 3.981 MeV level has been suggested[10] to be an unresolved doublet.

2. Octupole excitations

There were a number of states seen with characteristic $L=3$ angular distributions. Most interesting is the dominant six-member group centered at about 2.6 MeV. This is the well-known multiplet resulting from the $h_{9/2}$ proton coupling to the octupole vibration at 2.615 MeV in ^{208}Pb.

The $i_{13/2}$ single particle level at 1.608 MeV also has an angular distribution well fitted by an $L=3$ CM calculation. This state has been shown[5] to have a large admixture of the $\frac{13}{2}^+$ member of the 2.6 MeV multiplet.

Other states with $L=3$ were found at 3.309, 3.803, 3.924, 3.950, 4.177, and 4.210 MeV. The last two levels show a fairly large concentration of octupole strength in an excitation region where no comparable strength is found in ^{208}Pb.

3. Levels with $L=4$

Two low-lying states at 2.766 and 2.956 MeV were observed that previously were seen[25] in (p,p') work near 15 MeV bombarding energy. There, the lower state was concluded to be a mem-

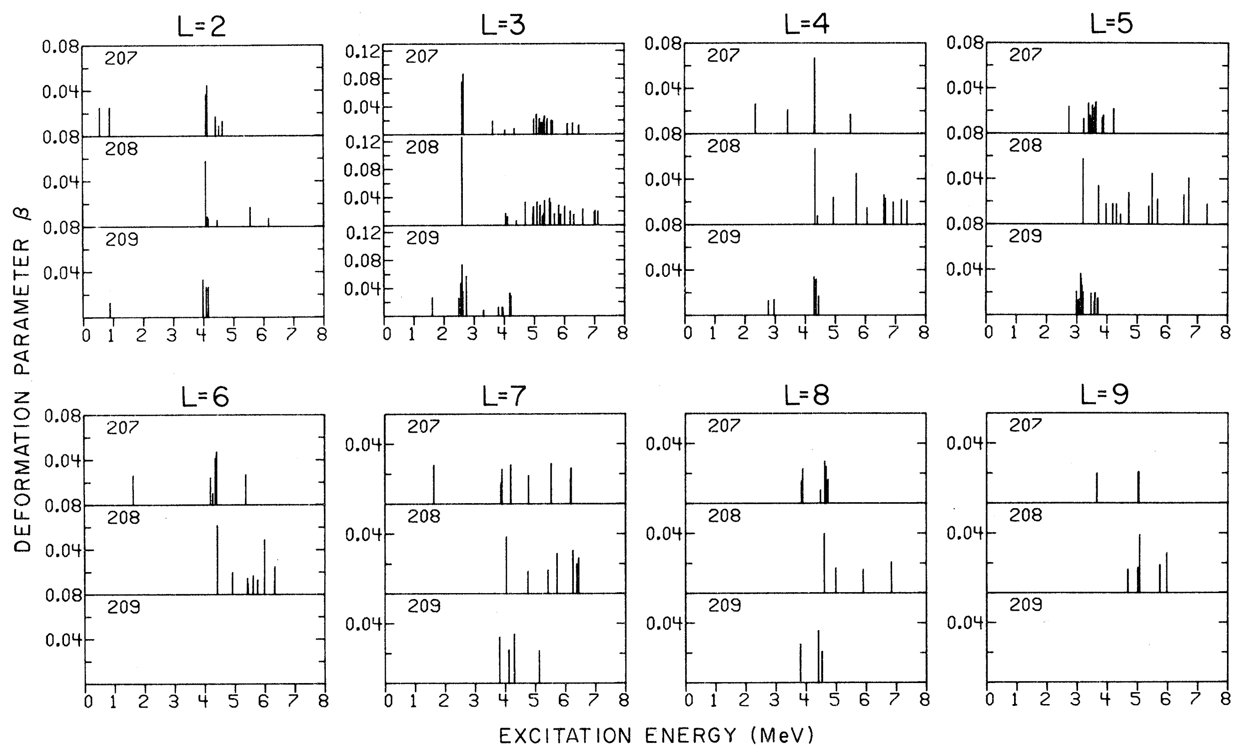

FIG. 6. Summary of the collective model results for the three nuclei. The deformation parameter β_L is plotted against excitation energy for a number of l transfers.

ber of the multiplet built on the lowest 5^- level in ^{208}Pb. The upper state was assigned a spin of $\frac{3}{2}^+$. Our data indicate that the cross sections for these states are fitted only by the $L=4$ shape. This is in disagreement with the previous conclusions.

A strongly populated group near 4.3 MeV was also observed to have two members with $L=4$ shapes. The members lie at 4.286 and 4.362 MeV. The 4.441 MeV excitation, which is probably a multiplet, has been assigned $L=4$. The level at 5.509 MeV may correspond to the $L=4$ level at 5.20 ± 0.5 MeV observed[24] at 62 MeV, but we could not make an assignment of L.

4. *Transitions with L=5*

The bismuth spectra have two dominant groups at about 3.13 and 3.56 MeV and with characteristic $L=5$ shapes. These groups have been identified in other (p,p') studies as $L=5$ excitations. The 62 MeV work[24] extracted deformation parameters of 0.050 and 0.029 for the lower and higher states, respectively. We have obtained total deformations of 0.065 and 0.037 for these transitions. The disagreement may arise from problems in background subtraction in the higher energy data. We are in good agreement with the results given in Ref. 25.

No other $L=5$ identifications could be made.

5. *States with L≥6*

A few states were identified as having angular momentum transfer greater than 5. As in the case of the ^{207}Pb data, the experimental angular distributions for high l transfers are difficult to distinguish because of the similar shapes. States at 4.116, 4.301, and 5.131 MeV showed possible $L=7$ strength. States at 4.411 and 4.532 MeV revealed possible $L=8$ strength. No $L=6$, 9, or 10 transitions were found in the data. This might result from the fractionation of core strength by weak coupling.

F. Summary of the collective model results

The results of the CM fits are presented in Fig. 6. There, the strength for each l transfer ranging from 2 to 9 has been displayed according to excitation energy for each of the three nuclei, ^{207}Pb, ^{208}Pb, and ^{209}Bi. It is clear that the distribution of inelastic strength is quite similar in each nucleus. This similarity will be discussed in the next section.

IV. WEAK COUPLING MODEL

A. Discussion

It is evident from the spectral plots of Fig. 1 and from the deformation parameter versus excitation energy display of Fig. 6 that the strong excitations in ^{208}Pb split into multiplets in ^{207}Pb and ^{209}Bi. The characteristics of these multiplets are that they are centered about the energy of the core excitation, their total strength is about equal to that of the core level, and the ratio of members' cross sections is roughly constant. The cross sections for ^{208}Pb states, which apparently are the bases on which these multiplets are built, are compared with the angular distributions for corresponding multiplet members in Figs. 7 and 8. The similarity between angular distributions of the core state and states in the odd-A nuclei is quite striking.

These properties are given by the weak coupling model[27] which assumes that a valence particle or

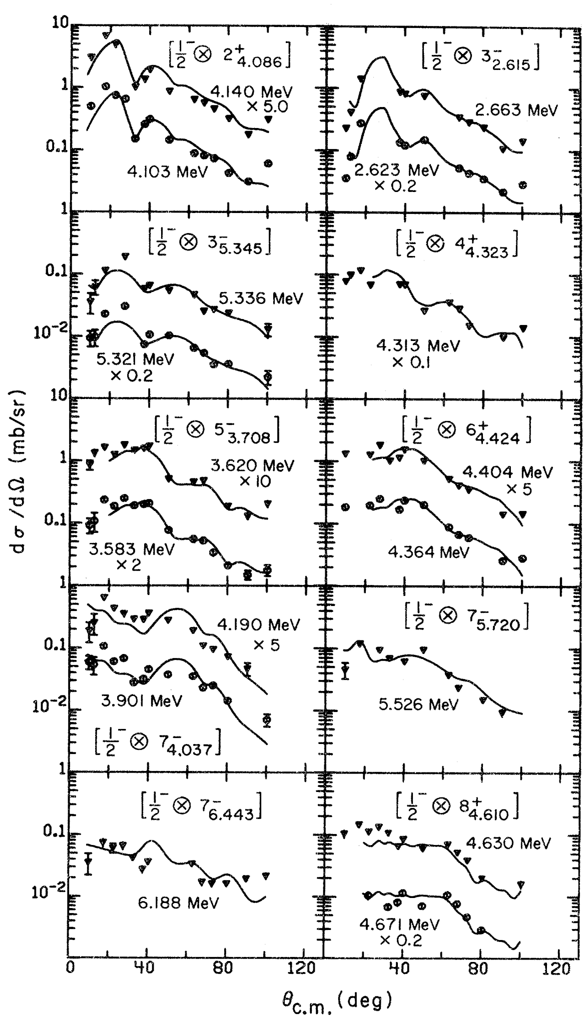

FIG. 7. Comparison of ^{208}Pb angular distributions with cross sections for weak coupling multiplets in ^{207}Pb built on the indicated ^{208}Pb excitation. The curves result from smooth interpolation through the ^{208}Pb data for the indicated level.

hole nucleon interacts only weakly with a collective core excitation. This assumption leads to a number of predictions which have been applied to the data assuming a ^{208}Pb core and which are summarized in Tables III and IV. These predictions are also discussed below. It should be noted that Cleary *et al.*[28] have found that weak coupling to excitations in a ^{210}Bi core can equally well describe levels in ^{209}Bi in the excitation energy region between 2.98 and 3.65 MeV. There is, however, no evidence to decide the more proper alternative.

According to the weak coupling model, members of a multiplet have cross sections, σ_m, which are related to the core cross section, σ_c, by a simple spin-statistics factor. If J_c, J_w, and j_m are the spins of the core excitation, the weak coupled particle, and a particular multiplet member, respectively, then

$$\sigma_m = \frac{(2j_m+1)}{(2J_c+1)(2J_w+1)}\,\sigma_c .$$

Summing this expression over possible multiplet spins predicts that the total multiplet strength should equal that of the core. Due to experimental difficulties, the data for these nuclei were taken at slightly different scattering angles. Since the cross sections vary fairly rapidly with angle, a direct comparison of the total strength with that of the core could not be done reliably. However, by using the CM deformation parameters the angle averaged strengths of the data can be compared. For this reason the total effective deformation parameters for the multiplet is compared with the corresponding core deformation in the tables.

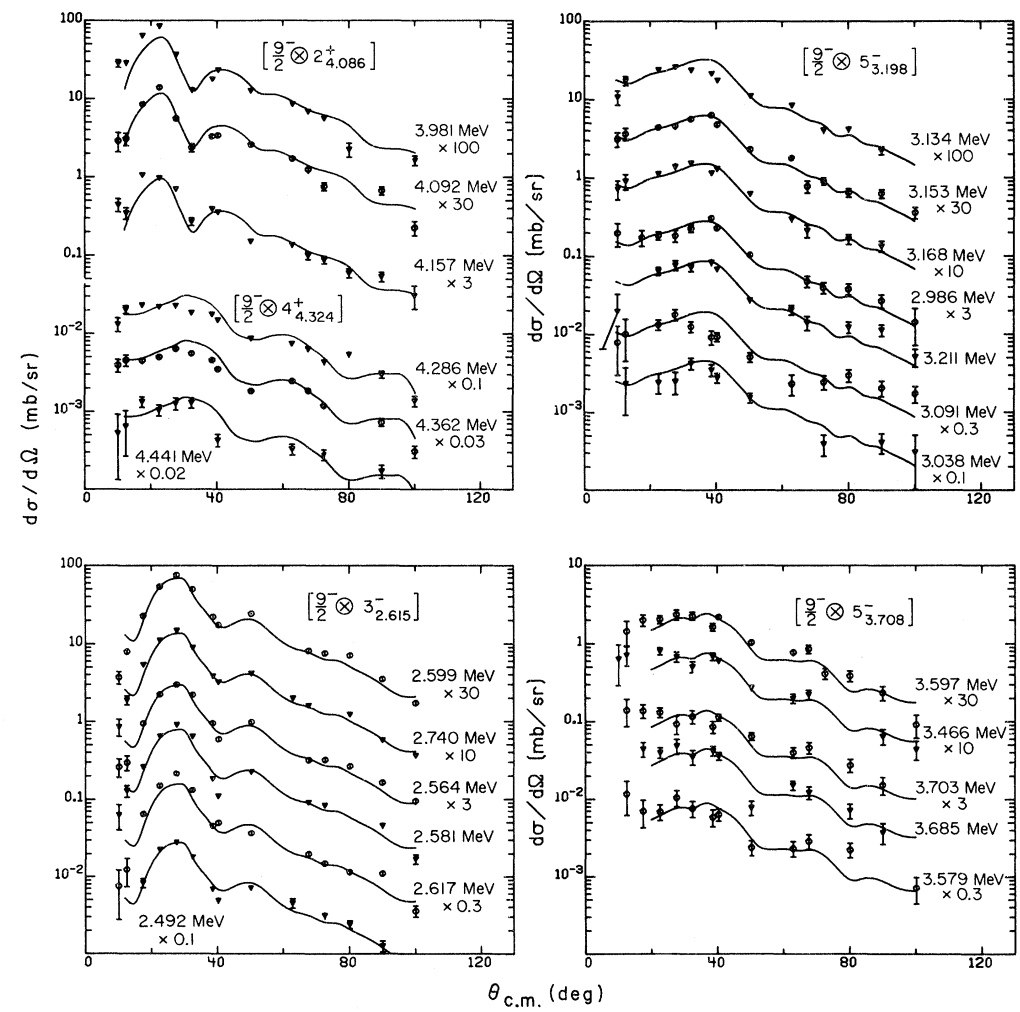

FIG. 8. Comparison of ^{209}Bi angular distributions with cross sections for weak coupling multiplets in ^{209}Bi built on the indicated ^{209}Bi excitation. The curves result from smooth interpolation through the ^{209}Bi data for the indicated level.

TABLE III. Weak coupling results for ^{207}Pb.

L	E_{core} (MeV)	SWES [a] (MeV)	β_L (core)	$[\sum \beta_L{}^2(E_i)]^{1/2}$	E_i (MeV)	J_i^π	Ratio [b]
3	2.615	2.646	0.126	0.120	2.623	$\frac{5}{2}^+$	1.06 ± 0.02
					2.663	$\frac{7}{2}^+$	0.94 ± 0.02
5	3.708	3.603	0.034	0.036	3.583	$\frac{9}{2}^+$	0.85 ± 0.03
					3.620	$\frac{11}{2}^+$	1.12 ± 0.04
7	4.037	4.055	0.038	0.035	3.901	$\frac{13}{2}^+$	0.89 ± 0.05
					4.190	$\frac{15}{2}^+$	1.04 ± 0.05
2	4.086	4.125	0.058	0.058	4.103	$\frac{3}{2}^-$	1.10 ± 0.02
					4.140	$\frac{5}{2}^-$	0.93 ± 0.02
4	4.323	4.313	0.067	0.067	4.313	$\frac{7}{2}^-, \frac{9}{2}^-$	1.00 ± 0.02
6	4.424	4.386	0.062	0.063	4.364	$\frac{1}{2}^-$	0.97 ± 0.02
					4.404	$\frac{13}{2}^-$	1.01 ± 0.02
8	4.610	4.649	0.040	0.037	4.630	$\frac{17}{2}^-$	1.16 ± 0.05
					4.671	$\frac{15}{2}^-$	0.76 ± 0.04
3	5.345	5.330	0.035	0.030	5.321	$\frac{5}{2}^+$	1.03 ± 0.05
					5.336	$\frac{7}{2}^+$	0.96 ± 0.04
7	5.720	5.526	0.027	0.027	5.526	$\frac{13}{2}^+, \frac{15}{2}^+$	1.00 ± 0.05
7	6.443	6.188	0.024	0.024	6.188	$\frac{13}{2}^+, \frac{15}{2}^+$	1.00 ± 0.06

[a] Spin weighted energy sum: $\sum E_i(2J_i+1)/\sum(2J_i+1)$.
[b] For degenerate levels the ratio is identically one.

In the case of a single multiplet member the above equation predicts that the expression

$$\frac{(2J_c+1)(2J_w+1)}{(2j+1)} \frac{\sigma_m}{\sigma_c} = 1$$

only for the choice of $j=j_m$. If members of the group are degenerate then a similar expression, involving a sum of $(2j+1)$ factors in the denominator, will be one only for the proper choice of spins. This method of checking spin assignments has been used and the results for each multiplet level is displayed in the ratio column of Tables III and IV. Again, since the core and multiplet data were not measured at exactly the same angles, the core cross section was taken to be the sum of multiplet member cross sections. If the complete strength of the multiplet has been identified this is a safe procedure. For most cases it appears, from comparison with the core strength, that the total strength has been found. The ratio listed is the weighted average of the values determined at each possible angle. The given error corresponds to the mean deviation in the ratio. The tables also give the intensity weighted energy average for each multiplet. Of course, in the limit of no particle-core interaction, this energy is expected to be identical to that of the core excitation.

B. ^{207}Pb results

1. Coupling to the 3^- core state

The doublet arising from the coupling of the $p_{1/2}$ neutron hole to the lowest octupole level in ^{208}Pb has members at 2.623 and 2.663 MeV. Calculations for these states have been performed by Hamamoto[29] and indicate that the intensity ratio should conform to the expected weak coupling prescription. However, those results[29] also suggest that the states should absorb only 94% of the observed core strength. We have found that the total strength of this doublet is about 95% that of the ^{208}Pb 3^- vibration and that the intensity of each member is fairly consistent with the assigned spins of $\frac{5}{2}^+$ and $\frac{7}{2}^+$. A study,[30] involving inelastic proton excitation functions of the 5^- and 4^- analog resonances in ^{208}Bi, supports these spin assignments and is independent of assumptions about the weak coupling model.

The 2.623 and 2.663 MeV states have been examined in a variety of inelastic scattering experiments.[23,26,31-33] Two of these studies[32,33] were un-

TABLE IV. Weak coupling results for ^{209}Bi.

L	E_{core} (MeV)	SWES [a] (MeV)	β_L (core)	$[\sum \beta_L{}^2(E_i)]^{1/2}$	E_i (MeV)	J_i^π	Ratio [b]
3	2.615	2.620	0.126	0.120	2.492	$\frac{3}{2}^+$	0.68 ± 0.02
					2.563	$\frac{9}{2}^+$	1.08 ± 0.03
					2.581	$\frac{7}{2}^+$	0.98 ± 0.03
					2.599	$\frac{11}{2}^+, \frac{13}{2}^+$	1.02 ± 0.02
					2.617	$\frac{5}{2}^+$	0.97 ± 0.03
					2.740	$\frac{15}{2}^+$	0.93 ± 0.02
5	3.198	3.072	0.058	0.065	2.986	$\frac{11}{2}^+$	0.92 ± 0.07
					3.038	$\frac{3}{2}^+$	1.14 ± 0.12
					3.091	$\frac{5}{2}^+$	0.92 ± 0.09
					3.134	$\frac{13}{2}^+, \frac{19}{2}^+$	0.93 ± 0.05
					3.153	$\frac{7}{2}^+, \frac{15}{2}^+$	1.06 ± 0.06
					3.169	$\frac{17}{2}^+$	1.05 ± 0.06
					3.211	$\frac{9}{2}^+$	1.04 ± 0.07
5	3.708	3.591	0.034	0.037	3.466	$\frac{13}{2}^+, \frac{15}{2}^+$	0.98 ± 0.09
					3.579	$\frac{11}{2}^+$	0.87 ± 0.11
					3.597	$\frac{1}{2}^+, \frac{3}{2}^+, \frac{5}{2}^+, \frac{19}{2}^+$	1.04 ± 0.09
					3.685	$\frac{7}{2}^+, \frac{9}{2}^+$	0.83 ± 0.09
					3.703	$\frac{17}{2}^+$	1.00 ± 0.10
2 [b]	4.086	4.061	0.058	0.050	3.981	$\frac{9}{2}^-, \frac{11}{2}^-$	1.11 ± 0.04
					4.092	$\frac{13}{2}^-$	0.93 ± 0.04
					4.157	$\frac{5}{2}^-, \frac{7}{2}^-$	0.82 ± 0.03
4 [b]	4.324	4.335	0.067	0.050	4.286	$\frac{7}{2}^-, \frac{15}{2}^-, \frac{17}{2}^-$	1.02 ± 0.04
					4.362	$\frac{1}{2}^-, \frac{9}{2}^-, \frac{11}{2}^-, \frac{13}{2}^-$	0.96 ± 0.04
					4.441	$\frac{5}{2}^-$	0.91 ± 0.07

[a] Spin weighted energy sum: $\sum E_i(2J_i+1)/\sum(2J_i+1)$.

[b] For these l transfers the total strength in ^{209}Bi probably has not been found. Spin assignments are very tentative.

able to resolve the doublet but did detect strength about equal to the strength of the core state. The remaining studies[23,26,31] found the relative strengths generally consistent with the predictions of a weak coupling model. Reference 23 observed about 91% of the total strength, however. A (d, d') experiment[26] reported only 87% of the core strength. The deuteron experiment was performed at 13 MeV so that compound nucleus processes may be important. Therefore, it seems that inelastic experiments generally support the weak coupling model for this doublet.

Since the ^{208}Pb 2.615 MeV octupole vibration is very collective it might be expected that neutron holes other than the $p_{1/2}$ could couple to this vibration as well. The work of Grosse *et al.*[34] has suggested that levels observed at about 3.210 and 3.580 MeV could correspond to configurations with the $f_{5/2}$ and $p_{3/2}$ holes coupling to the octupole, respectively. We were unable to assign an l-transfer value to the 3.200 MeV level observed in our data. The 3.223 MeV excitation has an $L=5$ assignment. The state seen at 3.583 MeV of excitation energy has a definite $L=5$ assignment and seems to be a member of the weak coupling doublet built on the second 5^- core state. The only state in this excitation region having an $L=3$ identification is the 3.634 MeV level which has a transition rate only 2% of the core octupole. Thus, there appear to be no multiplets in ^{207}Pb arising from coupling of the ^{208}Pb 3^- level to other neutron holes.

2. Coupling to the ^{208}Pb first 5^- state

The level at 3.198 MeV excitation in ^{208}Pb is a strong collective state with about 10 single particle units strength. However, there are apparently no $L=5$ states in ^{207}Pb in the region about 3.2 MeV that exhaust more than 5% of the inelastic transition strength of the core state. A strong level at 2.728 MeV does involve an angular momentum transfer of 5 but this level has been shown[14,15,18] to have the configuration of the $g_{9/2}$ neutron coupled to the ground state and first excited 2^+ state of ^{206}Pb. Thus it seems that the weak coupling model breaks down here. This has been noted[23,26] before. The missing strength probably can be explained by the fact that the ^{208}Pb 5^- wave function[35] has a large (>0.6) amplitude neutron $(g_{9/2}\text{-}p_{1/2}{}^{-1})$ component. Thus, the inelastic strength for excitation of the core 5^- level is severely hindered by the missing $p_{1/2}$ strength.

Using the Green's function method for the random phase approximation,[41] Bertsch, Schlomo, and Tsai have performed[42] calculations for the lowest-lying octupole and the first and second $L=5$ excitations in ^{207}Pb, ^{208}Pb, and ^{209}Bi. The results are shown in Table V. A slight decrease of the ^{208}Pb octupole transition strength is predicted for both ^{207}Pb and ^{209}Bi. (The ^{207}Pb result agrees well with the octupole calculations of Hamamoto.[29]) The table also shows a predicted 40% decrease (increase) in the inelastic strengths of the first (second) $L=5$ group in ^{207}Pb. Our data for ^{207}Pb indicate that more than 90% of the lower, $L=5$ state's strength is missing and that essentially all of the upper, $L=5$ excitation strength in ^{208}Pb is seen in ^{207}Pb. Further, these calculations predict that the strength for both $L=5$ transitions in ^{209}Bi should nearly equal that of the corresponding core excitations.

TABLE V. Random-phase-approximation inelastic transition strengths (single particle units) for the lowest $L=3$ and two lowest $L=5$ excitations in ^{207}Pb, ^{208}Pb, and ^{209}Bi.

	^{207}Pb		^{208}Pb		^{209}Bi	
L	E_x (MeV)	G_L	E_x (MeV)	G_L	E_x (MeV)	G_L
3	2.81	38	2.70	40.9	2.75	37.1
5	3.29	6.8	3.07	11.0	3.10	10.4
5	3.70	4.1	3.70	2.9	3.70	2.9

3. Coupling to the ^{208}Pb second 5^- state

We observe states at 3.583 and 3.620 MeV which are definitely $L=5$ excitations and whose summed strength agrees fairly well with the core-particle model. However, the relative intensities are not in good agreement with the predictions.

4. Coupling to the ^{208}Pb quadrupole excitation

The levels at 4.103 and 4.140 MeV are excellent candidates for weak coupling members of a multiplet with parentage in the 4.085 MeV 2^+ excitation in ^{208}Pb. The inelastic transition rates are in good agreement and the intensities agree fairly well with the weak coupling model prescription. Alster[32] detected 100% of the core cross section in his (α, α') study and Vallois *et al.*[23] reported an intensity identical to that of the core but observed relative population of the levels not in agreement with theory.

5. Unresolved multiplet at 4.313 MeV

Although there were no experimental indications of multiplet structure for this level we conclude that the 4.313 MeV state corresponds to a weak coupling doublet built on the 4^+ level in ^{208}Pb. Other studies[1,10,19] reported a single level at this energy and observed a cross section equal to that of the core state. We also observe the same strength as that of the core vibration so that a doublet assignment for this level seems fairly certain.

6. Other possible weak coupling levels

The 6^+(4.424 MeV) and the 8^+(4.610 MeV) in ^{208}Pb are both fairly strongly excited in (p,p') and could be expected to lead to multiplets in ^{207}Pb. The ^{207}Pb levels at 4.364 and 4.404 MeV, with $L=6$, and the levels at 4.630 and 4.671 MeV, with $L=8$, have relative intensities and summed cross sections in agreement with the weak coupling model predictions.

Although $L=7$ levels observed in ^{207}Pb and ^{208}Pb have tentative spin identification it seems that multiplets with parent 7^- core states have been found. The total strength and location near the core excitation energies suggests identification of these levels as weak coupling states. The lowest 7^- state in ^{208}Pb, at 4.037 MeV, leads to two levels at 3.901 and 4.190 MeV in ^{207}Pb. The summed strength is slightly less than that observed in the core and the relative intensities are in fair agreement with the model.

The 7^- levels in ^{208}Pb at 5.720 and 6.443 MeV may correspond to degenerate doublets in ^{207}Pb at 5.526 and 6.188 MeV. However, the ^{207}Pb states are separated from the excitation energies of the corresponding core states by a much larger energy gap than the other levels discussed above. The strength of these levels is essentially equal to that of the core states. The large energy separation and the uncertain L assignments, however, makes identification of these levels as weak coupling multiplets quite tentative.

Lastly, the doublet with constituents at 5.321 and 5.336 MeV, apparently $\frac{5}{2}^+$ and $\frac{7}{2}^+$ states, respectively, may have parentage in the 5.345 MeV octupole excitation in ^{208}Pb. The total core strength is nearly reproduced and the relative intensities are about in the ratio given by the model. Again, a possible explanation of the missing strength lies in mixing with nearby octupole levels.

C. ^{209}Bi results

1. Coupling to the 3^- core state

In the particle-core coupling model, coupling of the $h_{9/2}$ proton to the ^{208}Pb octupole vibration can lead to a septuplet of states. Our study and a number of other charged particle studies[25,26,31] of this multiplet have only resolved six members. However, assuming a $(2J+1)$ cross section dependence, the strength of the 2.599 MeV state suggests that this level is a degenerate $\frac{11}{2}^+$, $\frac{13}{2}^+$ doublet. Coulomb excitation[36] has shown that this level is a doublet with members separated by only about 2 keV, the larger spin state lying higher. We have found the total strength of this multiplet nearly equal to that of the core excitation. The assigned spins are in agreement with those given in Refs. 26, 25, 31, and 36 and the relative intensities agree quite well with the weak coupling model predictions.

2. Coupling to the ^{208}Pb first 5^- state

Spin assignments for this multiplet have been made and compared with the weak coupling theory in Table IV. A total strength greater than that of the core excitation was observed. The intensities follow a $(2J+1)$ rule quite well and, as shown in Table VI, the agreement with previous spin assignments is good. In all assignments but that of Francillon, Terrien, and Vallois[1] the $\frac{1}{2}^+$ level of the multiplet has not been located. Since this $\frac{1}{2}^+$ state is expected to have a very small cross section, identification of this level is expected to be difficult. The 3.309 MeV level, identified by Francillon *et al.* as the $\frac{1}{2}^+$ state, has a distinct $L=3$ shape in our data. Unless a doublet lies at this energy it appears that the 3.309 MeV state can not be a member of the multiplet. Cleary[25] suggests that a very weak state seen at 2.847 MeV may be the $\frac{1}{2}^+$ level but the cross section was so small that an angular distribution could not be measured. Our spectra show no states near 2.847 MeV.

TABLE VI. Spin and parity assignments for the $\frac{9}{2}^- \times 5_1^-$ multiplet in ^{209}Bi.

E_x (MeV)	Present work	Ref. 1	Ref. 25	Ref. 26
2.766	...	...	$\frac{3}{2}^+$	...
2.986	$\frac{11}{2}^+$	$\frac{13}{2}^+$	$\frac{19}{2}^+$	$\frac{13}{2}^+$
3.038	$\frac{3}{2}^+$	$\frac{3}{2}^+$	$\frac{5}{2}^+$	$\frac{3}{2}^+$
3.091	$\frac{5}{2}^+$	$\frac{7}{2}^+$	$\frac{7}{2}^+$	$\frac{5}{2}^+$
3.134	$\frac{13}{2}^+$, $\frac{19}{2}^+$	$\frac{11}{2}^+$, $\frac{19}{2}^+$	$\frac{11}{2}^+$, $\frac{15}{2}^+$	$\frac{11}{2}^+$, $\frac{19}{2}^+$
3.153	$\frac{7}{2}^+$, $\frac{15}{2}^+$	$\frac{5}{2}^+$, $\frac{17}{2}^+$	$\frac{9}{2}^+$, $\frac{17}{2}^+$	$\frac{7}{2}^+$, $\frac{17}{2}^+$
3.169	$\frac{17}{2}^+$	$\frac{15}{2}^+$	$\frac{13}{2}^+$	$\frac{15}{2}^+$
3.211	$\frac{9}{2}^+$	$\frac{9}{2}^+$	...	$\frac{9}{2}^+$
3.315	...	$\frac{1}{2}^+$	...	...

Using techniques independent of any weak coupling assumptions, Cleary also identified the 2.986 MeV level as having spin $\frac{19}{2}^+$. However, the strength he measured for this level was much less than that predicted by the weak coupling model assuming $J^\pi=\frac{19}{2}^+$. The strength that was measured is consistent with our spin assignment and the weak coupling picture. Being very sure of the spin assignment, however, Cleary attributed the difference between the weak coupling model and experiment to mixing of this level with the higher-lying $\frac{19}{2}^+$ state associated with the decouplet built on the ^{208}Pb second excited 5^-. Our analysis of the two $L=5$ multiplets, however, indicates that mixing of these two states is not required if one assumes that the 2.986 MeV level has spin $\frac{11}{2}$. It should also be noted that Cleary concluded that the 3.211 MeV level had a microscopic configuration based on coupling of the $h_{9/2}$ particle to the unnatural parity 4^- level in ^{208}Pb. Our bismuth data indicate that the 3.211 MeV level is the $\frac{9}{2}^+$ weak coupling member of the 5_1^- multiplet.

Our assignment of doublet spins to the 3.153 MeV level is consistent with the results of Ref. 26 which found two members at about this energy and with separation of about 4 keV. That work also suggested possible doublet structure and spin assignment for the 3.134 MeV level and concluded that its members are separated by at most 3 keV.

3. Coupling to the ^{208}Pb second 5^- state

Five members of a multiplet near 3.6 MeV have angular distributions similar to those of the second 5^- level in ^{208}Pb. The total cross section is slightly greater than that seen in the core. It also seems that many of the levels are degenerate since coupling of the valence proton to the core excitation is expected to result in 10 states. This apparent degeneracy makes the spin assignments quite uncertain.

4. Other possible weak coupling levels

Excitations involving angular momentum transfers of 2 and 4 were identified in ^{209}Bi that lie near the excitation energies of the first 2^+ and 4^+ levels in ^{208}Pb. In both cases the total strength of the core was not observed and it seems that some fragmented strength has not been resolved. However, spins have been assigned assuming that all possible strength was observed and that the relative intensities are given by the $(2J+1)$ rule. Therefore, the spins given in Table IV for the $L=2$ and $L=4$ multiplets are quite tentative.

About 75% of the core quadrupole strength was found. Cleary[25] reported an additional $L=2$ excitation at 4.213 MeV and observed about 72% of the expected strength. Reference 10 has suggested that the 3.981 MeV level is really a doublet. A γ-ray resonance experiment[37] on ^{209}Bi identified $L=2$ transitions to levels at 3.977, 4.083, 4.156, 4.176, and 4.206 MeV, the lower three levels corresponding to our identified $L=2$ states. The levels seen here at 4.177 and 4.210 MeV have definite assignments of $L=3$, although doublet structure is possible. It seems that complete identification of the 2^+ and 4^+ weak coupling multiplets in ^{209}Bi requires higher resolution than currently possible.

V. SINGLE PARTICLE STATES AND A MICROSCOPIC MODEL

Both ^{207}Pb and ^{209}Bi have states strongly populated in single particle transfer reactions and thus identified as single particle levels. Most of these states have been observed in the present (p,p') study. It is expected from electromagnetic measurements[38] and other inelastic scattering experiments[2,4,5] that the inelastic transitions to these states involve strength greater than that given by a model involving a single valence nucleon.

Studies of ^{207}Pb(p,p') at both 20 and 35 MeV bombarding energy have been made[2-4] before. These studies showed that core polarization effects were important in excitation of the neutron single particle states. It is our intention to analyze the scattering to the proton single particle levels in ^{209}Bi using techniques identical to those of Ref. 4.

The single particle orbits seen in ^{209}Bi lie at 0.896, 1.608, 2.825, 3.118, and 3.633 MeV of excitation energy and have spins and parities of $\frac{7}{2}^-$, $\frac{13}{2}^+$, $\frac{5}{2}^-$, $\frac{3}{2}^-$, and $\frac{1}{2}^-$, respectively. Following Ref 4, inelastic scattering to these states was first calculated using a simple valence proton model. The calculations used an effective bound state interaction (the bare G matrix derived from the Hamada-Johnston potential) for the projectile-target interaction. The BG optical model was used for the distorted waves and the effects of knock-on exchange were included via the approximation of Petrovich.[39] The calculations were done with the code DWUCK.[22] All possible LSJ triads were included. For the $\frac{13}{2}^+$ level twenty such triads are possible. For each state, the cross sections for each LSJ transition were summed to give the total cross section. For the $\frac{13}{2}^+$ level, the $L1J$ transitions were comparable in strength to the usually dominant $L0L$ transitions. The results of these central force and valence particle calculations are given in Fig. 9 by the short dashed curves. In all instances the calculations fall at least a factor of 10 below the data.

The effects of the noncentral nucleon-nucleon forces were investigated using the code DWBA70.[40] Because of numerical limitations only the cross sections for the $\frac{1}{2}^-$ and $\frac{3}{2}^-$ states could be calculated. The Serber exchange mixture was used for the central interaction and the spin-orbit and tensor forces were identical to those used previously.[4] The long dashed curves for these two states shown in Fig. 9 show the results of these noncentral and central forces with valence particle calculations. Apparently, noncentral forces cannot sufficiently enhance the theoretical cross sections to match the strength of the data.

Finally, microscopic core polarization calculations were done. The 2p-1h admixtures in the wave functions for these levels were calculated using first order perturbation theory. For those states whose quadrupole transition rates have been measured,[38] the core polarization wave functions give $B(E2)$ values in fair agreement with experiment. Values of 22 and 572 $e^2\text{fm}^4$ were calculated without effective charge for the $\frac{7}{2}^-$ and $\frac{5}{2}^-$ transitions, respectively. The measured values[38] are 24 and 288 $e^2\text{fm}^4$. Transition densities obtained with the resulting wave functions were folded with the effective bound state interaction used above. The zero range approximation was again used to account for knock-on exchange and the code DWUCK[22] was again utilized. The results of these calculations are given by the solid lines in Fig. 9. In all cases but the $\frac{13}{2}^+$ transition the agreement with the data has greatly improved. In the case of the $\frac{3}{2}^-$ level the calculated strength falls only about a factor of 2 below the data. For the $\frac{5}{2}^-$ cross section the calculation gives a good fit to the data.

The worst case is the $\frac{13}{2}^+$ calculation where the core polarization results essentially reproduce the valence calculations. In the core polarization results the $L0L$ transitions have become more dominant while the $L1J$ transitions have lost much of the strength possessed in the valence model. The net result is that the cross section remains about the same as it was in the valence calculation. This

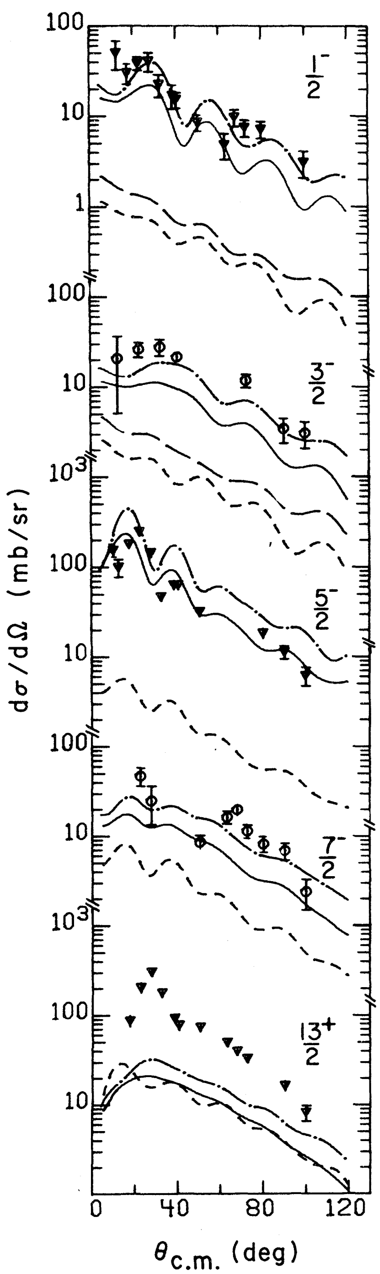

FIG. 9. Calculations for the single particle states in ^{209}Bi. The meaning of the curves is given in the text.

state has been shown[5,16] to have a large admixture of the weak coupled $\frac{13}{2}^+$ state. The effect of this admixture has been studied[5] in a (p,p') experiment at 39.5 MeV where good agreement with the experimental cross section was obtained only when the weak coupled admixture was included. Since the perturbation prescription used here cannot produce the coherent 2p-1h components found in the admixture, the present results for the $i_{13/2}$ single particle state are to be expected.

Finally, complex coupling was included. An imaginary collective model form factor was added to the approximate exchange, microscopic core polarization form factor for each $L0L$ transition. The strength of the CM contribution was given by an effective deformation parameter obtained by fitting CM results, calculated with only the real portion of the CM form factor, to the approximate exchange, microscopic core polarization results. The $L0L$, complex coupling cross sections were summed with $L1J$ cross sections to give the dash-dot curves in the figure. For the $\frac{1}{2}^-$, $\frac{3}{2}^-$, and $\frac{7}{2}^-$ excitations, these calculations give good fits to the data. Improvement when complex coupling has been noted before.[3,4,43] For the $\frac{5}{2}^-$ level, the complex coupling results overestimate the data. This is consistent with the overestimate of the $B(E2)$ by the same microscopic wave function. The complex coupling cross section for the $\frac{13}{2}^+$ level fails to reproduce the strength of the data. Again, the weak coupling admixture must be treated explicitly if a proper estimate of the inelastic strength is to be given.

To summarize, it seems that the single particle states can only be explained when core polarization effects are treated. The microscopic calculation involving a simple 2p-1h model for the single particle states in ^{209}Bi apparently can account for much of the observed core polarization strength in transitions not involving contributions from coherent excitations of the core.

VI. CONCLUSION

The (p,p') reaction has allowed an intensive study of the macroscopic behavior to be made. In both ^{207}Pb and ^{209}Bi collective model fits to states enabled the transferred angular momentum to be identified. A large number of states in both nuclei had features corresponding to the weak coupling of the valence hole or particle to core excitations. In ^{209}Bi the extremely high level density and fractionation of strength permitted only a few multiplets to be studied. Of these, the weak coupling groups corresponding to the first 3^- and the first and second 5^- levels in ^{208}Pb had most of their strength identified and were found to conform to a weak coupling prescription. Spins and parities were assigned using this fact and were found in good agreement with previous studies. The less dense level structure in ^{207}Pb apparently permitted more weak coupled states to be identified. Most of the states expected to be built on the very strong ^{208}Pb core excitations were observed and a few high-lying ^{207}Pb states were found corresponding to high-lying core states. Most interesting was the absence of a weak coupling multiplet with parentage in the lowest 5^- level in ^{208}Pb. This missing strength may possibly be explained by examining the ph structure of the core state.

The single particle states in ^{207}Pb and ^{209}Bi were excited in this (p,p') study and examined using microscopic models. As expected from electromagnetic measurements, transitions to these states were found to be greatly enhanced by the core polarization effects. Calculations with the single valence nucleon, exchange effects, and noncentral forces apparently cannot reproduce the observed cross sections. A first order perturbation theory calculation using a large number of neutron and proton shell model orbitals gave a core transition density comparable to that of the valence particle. The DWBA calculations with the core polarization density and purely central forces gave reasonable reproductions of the data in all cases but that of the ^{209}Bi $\frac{13}{2}^+$ state which has been shown to have significant mixing with the weak coupling $\frac{13}{2}^+$ lying at higher excitation. It is concluded that these single particle states are properly described only in models which properly account for core polarization.

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