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^{40,48}Ca(d, p) reactions near the Coulomb barrier: Comparison of coupled-channel and distorted-wave Born-approximation calculations

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The (d, p) reactions on ^{40}Ca and ^{48}Ca targets for incident deuteron energies near the Coulomb barrier (2–5.5 MeV) are studied by a coupled-channel (CC) method in which the $\Delta l = 1$ stripping channel has been coupled to the incident deuteron channel. The method employs a set of energy independent optical potential parameters and predicts differential cross sections and polarizations in both the incident deuteron channel and the stripping channel. The results are compared with the measured values and those obtained from the distorted-wave Born-approximation calculations. It is found that the effect of the coupling is reasonably large even in this low energy region. There is an indication that the CC predictions for ^{40}Ca targets will improve with the inclusion of the coupling of the $\Delta l = 3$ stripping channel. It is also suggested that some portion of the j dependence in (d, p) angular distributions may be simulated by the coupling of the stripping channel.

[NUCLEAR REACTIONS ^{40,48}Ca(d,d), (d, p), E = 2.0–5.5 MeV; calculated $\sigma(E, \theta)$, $T_{11}(\theta)$, coupled-channel model, DWBA.]

I. INTRODUCTION

The distorted-wave Born-approximation (DWBA) has had considerable success in describing the deuteron stripping (d,p) cross sections and polarizations. However, in recent years some limitations of the DWBA theory, as applied to (d,p) reactions, have become more apparent. One of them is that the elastic deuteron wave function used in evaluating stripping overlap integral is obtained on the assumption that the deuteron structure remains undisturbed in the vicinity of the nucleus, the whole structure being deflected by a force acting on its center of mass (c.m.). In view of the small binding energy of the deuteron, it is unlikely that the deuteron will survive as such in the nuclear interior. There are two important channels which are excited with relatively large probability—the stripping channels and the deuteron breakup channels. The theoretical analysis of the (d,p) reactions has now reached a degree of sophistication which does not ignore the coupling of the reaction channels, mentioned above, with the incident deuteron channels. These new developments have been nicely summarized by several authors.[1]

Stated in another way, the DWBA method ignores entirely the internal degrees of freedom of the deuteron despite the fact that large distortions are expected to occur in the internal deuteron wave functions when the deuteron c.m. approaches the surface of the target nucleus. A previous attempt by Rawitscher,[2] in incorporating the internal degrees of freedom of the deuteron into a stripping calculation, consists of introducing a set of coupled equations which describes the dissociation (and recombination) of the incident deuteron into (and from) the stripping channel. These equations have recently been reformulated by Rawitscher and Mukherjee[3] in order to include the spins of the proton, the deuteron, and the final nucleus involved in the reaction, so that the question of the j dependence in the stripping cross section and also the polarization both in the elastic channel and the stripping channel could be investigated. In the simple form of these equations as they are presently employed, the only stripping transition, explicitly included, is one with $\Delta l = 1$. The calculations have been carried out so far for 11 MeV deuterons with ^{40}Ca as the target.[3] However, the coupled channel (CC) model mentioned above is yet to be tested for different energies and different nuclei.

The nucleus of calcium is well suited for this type of calculation for the following reasons. The stripping cross sections have a large value so that the CC calculation may be more suitable than the DWBA calculation, since the former method automatically describes the multiple transitions which take place between the incident channel and several of the strong reaction channels.

In the present investigation the CC model is applied to a more appropriate physical situation, namely, the study of ^{48}Ca(d,p)^{49}Ca reaction. The ^{48}Ca target is better suited for testing the model than ^{40}Ca, since the stripping to the $f_{7/2}$ state, ignored in the calculation, does not occur in ^{48}Ca, and there are fewer p states. We have singled out the low deuteron energies because the applicability of the CC model when the incident deuteron energies are lowered also remains to be seen.

In the preliminary study of the CC model without spin consideration, it was noticed that the effective coupling between the stripping channel and the deuteron channel is particularly large near 5 MeV. This is seen from the fact that the iterations carried out in solving the coupled equation converge poorly, and can be attributed to the fact that the overlap between the two channels is particularly large in this energy region. However, one can raise a question: What are the potential parameters for deuterons when the deuteron energies are below the Coulomb barrier (4.75 and 6.30 MeV for ^{48}Ca and ^{40}Ca targets, respectively)? These parameters, in fact, cannot be obtained from elastic scattering, since this is predominantly Rutherford scattering, thus essentially insensitive to nuclear distortions. But in the present investigation, due to the large Q value of the (d,p) reaction, the kinetic energy available to the proton is much greater than the Coulomb barrier in spite of the low deuteron kinetic energy. Hence the stripping matrix element can be sensitive to the nuclear distortion, and the distinctive features of the (d,p) angular distributions allow the extraction of nuclear structure information. Information concerning energy dependence of the optical potential parameters and the j dependence can also be extracted from such studies.

II. FORMALISM

The method of calculation and the approximations used are the same as in Refs. 2 and 3. A short summary is given below. The stripping channel is phenomenologically coupled to the incident deuteron channel. The effect of the coupling is to produce a nonlocal deuteron nucleus interaction which has a spin-orbit and tensor character and which also gives the right type of energy dependence for the equivalent local potential. The coupling also causes a damping of the deuteron wave function in the interior of the nucleus. This reduction has an effect upon the stripping cross section which is reminiscent of a cutoff in the DWBA calculations but does not appreciably affect the main stripping peak. In the entire formulation of the CC model the nonorthogonality between the stripping and the deuteron channel is ignored to a large extent. The effect due to the deuteron breakup channel is also left out.

The coupled equations of the radial waves have the following form:

$$[(H_D)_{\mathcal{L},I} - E_D]F^{\mathcal{L}''}_{\mathcal{L},I}(r) = N \sum_{LJj} B(\mathcal{L}I;\, LJj)u_j(r)F^{\mathcal{L}''}_{LJjI}(r), \tag{1a}$$

$$[(H_p)_{L,J} - E_j]F^{\mathcal{L}'}_{LJjI}(r) = \sum_{\mathcal{L}} B(\mathcal{L}I;\, LJj)u_j(r)F^{\mathcal{L}'}_{\mathcal{L},I}(r), \tag{1b}$$

where

$$B(\mathcal{L}I;\, LJj) = (-)^{\mathcal{L}}\frac{D_0}{\sqrt{4\pi}}[1][l][L][j][J][\mathcal{L}]\begin{pmatrix} L & l & \mathcal{L} \\ 0 & 0 & 0\end{pmatrix}\begin{Bmatrix} L & \frac{1}{2} & J \\ l & \frac{1}{2} & j \\ \mathcal{L} & 1 & I\end{Bmatrix} \tag{1c}$$

with

$$[n] = (2n+1)^{1/2},$$

$$(H_D)_{\mathcal{L},I} = \langle(\mathcal{L}1)IM_I \mid K_D + U_D \mid (\mathcal{L}1)IM_I\rangle, \tag{1d}$$

and

$$(H_p)_{L,J} = \langle(L\tfrac{1}{2})JM_J \mid K_p + U_{A+1} \mid (L\tfrac{1}{2})JM_J\rangle.$$

Here $F^{\mathcal{L}''}_{\mathcal{L},I}(r)$ and $F^{\mathcal{L}'}_{LJjI}(r)$ are the distorted deuteron and proton waves, respectively, and u_j is the bound neutron wave. The symbols in round parentheses and curly brackets in Eq. (1c) are, respectively, "$3j$" and "$6j$" symbols. $(\mathcal{L}1I)$, $(L\frac{1}{2}J)$, and $(l\frac{1}{2}j)$ are set of quantum numbers for the incident deuteron, the outgoing proton, and the captured neutron, respectively. $\mathcal{L}''$ and $\mathcal{L}'(=\mathcal{L}+1, \mathcal{L}, \mathcal{L}-1)$ depend on the initial spin projection of the deuteron denoted by μ_0. The constant $D_0 = (1.5)^{1/2}\times 10^2 F^{3/2}$ MeV appears for the zero range approximation.

K and U represent the kinetic energy operator and the optical potential, respectively. The constant N simulates the presence of other coupled channels not included explicitly in the calculation, such as (d,n) channels. It is treated as a free parameter and has the value of either 2 or 3.

The asymptotic form of the deuteron wave $F^{\mathcal{L}''}_{\mathcal{L},I}(r)$ is given by

$$F^{\mathcal{L}''}_{\mathcal{L},I} \sim \delta_{\mathcal{L}\mathcal{L}''}F^C_{\mathcal{L}} + X^{\mathcal{L}''}_{\mathcal{L},I}H^C_{\mathcal{L}}, \tag{2}$$

where $F^C_{\mathcal{L}}$ and $H^C_{\mathcal{L}}$ are incident and outgoing Coulomb waves such as $F^C_{\mathcal{L}} \sim \sin(\varphi_{\mathcal{L}})$:

$$\varphi_{\mathcal{L}} = \kappa_D r - \tfrac{1}{2}\mathcal{L}\pi + \sigma_{\mathcal{L}} - \eta_{\mathcal{L}}\ln(2\kappa_D r), \tag{2a}$$

$\sigma_{\mathcal{L}}$ and $\eta_{\mathcal{L}}$ being the usual Coulomb phase shifts and Coulomb parameter, respectively. The quantity X defined in Eq. (2) is composed of two parts. One, called $X^{(u)}$, is the contribution which arises in the absence of coupling, and the other, $Z^{\mathcal{L}''}_{\mathcal{L},I}$, is due to the presence of coupling:

$$X^{\mathcal{L}''}_{\mathcal{L},I} = \delta_{\mathcal{L}\mathcal{L}''} X^{(u)}_{\mathcal{L},I} + Z^{\mathcal{L}''}_{\mathcal{L},I} . \tag{3}$$

The $X^{(u)}$ is related to the uncoupled deuteron phase shifts $K^{(u)}_{\mathcal{L},I}$ according to

$$X^{(u)}_{\mathcal{L},I} = [\exp(2i K^{(u)}_{\mathcal{L},I}) - 1]/2i , \tag{4}$$

and contribution due to coupling can be expressed by

$$Z^{\mathcal{L}''}_{\mathcal{L},I} = \left(\frac{-2\mu_D}{\hbar^2 \kappa_D}\right) N \int_0^\infty F^{(u)}_{\mathcal{L},I} \sum_{LJj} B(\mathcal{L}I; LJj) u_j F^{\mathcal{L}''}_{LJjI}(r)\, dr . \tag{5}$$

The normalization of $F^{(u)}_{\mathcal{L},I}$ is such that

$$F^{(u)}_{\mathcal{L},I}(r) \sim \exp(i K^{(u)}_{\mathcal{L},I}) \sin(\varphi_{\mathcal{L}} + K^{(u)}_{\mathcal{L},I}) , \tag{6}$$

where $\varphi_{\mathcal{L}}$ defined in Eq. (2a) contains the Coulomb phases. The regular solutions of the homogeneous part of Eq. (1b) are also needed further on. They are denoted by

$$F^{(u)}_{LJj}(r) \sim \exp(i K^{(u)}_{L,J}) \sin(\varphi_L + K^{(u)}_{L,J}) , \tag{7}$$

where φ_L is equivalent to $\varphi_{\mathcal{L}}$ with L and κ_p replacing $\mathcal{L}$ and κ_D in Eq. (2a).

The theoretical expressions for the cross sections and the polarizations, both in the deuteron channel and the stripping channel in terms of the phase shifts defined above, have been evaluated by Rawitscher and Mukherjee and are given in Ref. 3. Once the phase shifts are known by solving the coupled equations, the calculation of the observables becomes straightforward.

III. POTENTIAL PARAMETERS

The formalism discussed in the previous section is applied to the example of deuterons incident on the ^{40}Ca and ^{48}Ca targets with incident energy in the range 2 to 5.5 MeV. The optical potential employed for U_D and U_{A+1} in the coupled equations has the standard form

$$U_D = V_0(1+e^x)^{-1} + i W_0(1+e^{x'})^{-1} - (\hbar/m_\pi c)^2 V_{so}(\tfrac{1}{2})[I(I+1) - \mathcal{L}(\mathcal{L}+1) - 2] r^{-1} \frac{d}{dr}[e^{x''}+1]^{-1} + U_c , \tag{8a}$$

$$U_{A+1} = V_0(1+e^x)^{-1} + 4i W_D \frac{d}{dx}(1+e^{x_D})^{-1} - \left(\frac{\hbar}{m_\pi c}\right)^2 V_{so}[J(J+1) - L(L+1) - \tfrac{3}{4}] r^{-1} \frac{d}{dr}[e^x + 1]^{-1} + U_c , \tag{8b}$$

where

$$x = (r - r_0 A^{1/3})/a_0; \quad x' = (r - r_w A^{1/3})/a_w ,$$
$$x_D = (r - r_D A^{1/3})/a_D; \quad x'' = (r - r_{so} A^{1/3})/a_{so} , \tag{8c}$$

and where U_c is a Coulomb potential due to a uniform charge distribution which extends out to a radius R_c. The value of R_c adopted for the numerical calculation is 4.4 fm. There are nine parameters for the deuteron potential U_D, three each for the real and imaginary parts of the central potential, and three for the real spin-orbit potential. The proton potential U_{A+1} also has nine parameters.

TABLE I. Potential parameters. All quantities are either in MeV or in fm.

Nucleus	Energy	V_0	r_0	a_0	W	r_W	a_W	V_{so}	r_{so}	a_{so}
				Deuteron optical potential						
^{48}Ca	2.5	95.0	1.11	0.74	6.47	1.54	0.66	10.0	1.11	0.60
	5.0	95.0	1.22	0.64	13.8	1.26	0.65	10.0	1.22	0.60
	5.5	95.0	1.14	0.81	10.4	1.57	0.66	10.0	1.14	0.60
^{40}Ca	5.0	112.0	1.05	0.85	8.5	1.66	0.515	9.0	0.90	0.60
	2.0	110.0	1.20	0.90	20.0	1.55	0.47	0.0		
^{48}Ca, ^{40}Ca	CC	120.7	0.906	0.846	60 [a]	1.40	0.90	7.5	0.85	0.60
				Proton optical potential						
^{48}Ca	2.5–5.5	51.4	1.24	0.63	8.6	1.19	0.64	7.5	1.24	0.63
^{40}Ca	5.0	53.5	1.29	0.70	6.0	1.26	0.70	6.0	1.20	0.70
	2.0	56.2	1.25	0.65	7.5	1.25	0.65	0.0		
^{48}Ca, ^{40}Ca	CC	60–0.5E	1.2	0.65	11	1.25	0.47	6.0	1.20	0.70

[a] Volume type.

In the DWBA calculation, deuteron optical potential is obtained by fitting the d-^{40}Ca or d-^{48}Ca data at various energies. As a result, these parameters are energy dependent. The imaginary part of U_D is surface derivative type. The numerical values of these parameters are given in Table I.

In the CC calculation, the deuteron optical potential parameters used are those which were obtained previously[2] by fitting the elastic d-^{40}Ca scattering data over the energy range 7–22 MeV. The parameters are energy independent. The imaginary part of U_D is volume type. Its depth of 60 MeV is chosen large enough so as to absorb the deuteron channel wave functions which penetrate into the nuclear interior. The choice of the parameters of the spin-orbit potential in both the deuteron and the proton channels are arbitrary. However, we are guided by the corresponding DWBA analysis. It has been observed that the stripping peaks are not sensitive to the parameters of the spin-orbit potential.

The proton optical potential parameters used in the DWBA calculations are obtained from the analysis of elastic proton scattering data on ^{40}Ca and ^{48}Ca targets. Because the proton elastic scattering data at the appropriate energies were not available, the proton optical potential used in the analysis of these reactions was obtained from experiments performed at higher energies or on different nuclei or by adjusting their parameters to give the best fit to the stripping angular distributions. Neither of these approaches can be considered satisfactory in view of the unknown variation with energy of these parameters. Recently Liers[4] found that the elastic proton scattering on ^{48}Ca between 6 and 12.7 MeV shows considerable fluctuations.

In the CC calculation, the choice of proton optical potential poses a dilemma. There is probably no good way of resolving this dilemma other than solving, for example, a set of coupled equations for p-^{40}Ca scattering in which d-^{39}K is an intermediary state. Then adjust proton parameters until the experimental proton cross section is reproduced and finally use these parameters for (d,p) calculation. If the nonorthogonality terms are included this procedure would be very interesting to follow. To maintain consistency with the previous CC results[2] at higher energies, the proton optical potential parameters are kept same for both the isotopes of calcium. We find stripping differential cross section are best fitted with this choice.

The bound neutron wave function is obtained separately by solving the Schrödinger equation for a particle moving in a real central potential of Woods-Saxon form and a spin-orbit potential of Thomas form. The depth of the central potential is adjusted by the numerical code so as to give the correct neutron binding energy. The depth of the spin-orbit potential is kept fixed. The parameters of the neutron shell potential are given in Table II.

The coupled equations are solved by employing our numerical code DPDJ4 which was employed previously to study the (d,p) reaction for 11 MeV deuterons on ^{40}Ca targets.[3] The calculating machine employed is the CDC-3600 located at the Tata Institute of Fundamental Research, Bombay, India. The running time to solve the coupled equations and to obtain the cross sections and polarizations in deuteron and two stripping channels lies between 20 to 40 min central processing unit time. The total memory required is 45 000 words location, and access to this is done by splitting the code into two parts. The inhomogeneous equations are solved by the use of Green's functions. The number of iterations required to achieve an accuracy of 10^{-3} in the coefficients of the asymptotic waves lies between 5 and 30.

IV. CROSS SECTIONS AND POLARIZATIONS

We now proceed to calculate the cross sections and polarizations both in the incident deuteron channel and the stripping channel by the CC and the DWBA methods and compare them with experiments. The potential parameters used for the present calculation have been described already in the text. A careful study of the expected shapes of $l=1$, (d,p) angular distributions is quite important because of the purity of the $2p_{3/2}$ and $2p_{1/2}$ single particle states in ^{49}Ca. This is an important ex-

TABLE II. Bound state parameters. All quantities are either in MeV or in fm.

					Neutron shell potential					
					Real central			Spin-orbit		
Nucleus	State	Binding energy	Excitation energy	Q value	V_{0n}	r_{0n}	a_{0n}	V_{son}	r_{son}	a_{son}
^{41}Ca	$2p_{1/2}$	4.42	3.95	2.2	58.82	1.21	0.65	8.05	1.21	0.65
	$2p_{3/2}$	6.42	1.95	4.2	58.22	1.21	0.65	8.05	1.21	0.65
^{49}Ca	$2p_{1/2}$	3.06	2.02	0.84	55.57	1.14	0.65	8.05	1.14	0.65
	$2p_{3/2}$	5.19	0	2.97	55.62	1.14	0.65	8.05	1.14	0.65

perimental fact which distinguishes ^{48}Ca from ^{40}Ca as a good example of a closed shell nucleus. The (d,p) reactions studied here are as follows: (i) ^{40}Ca$(d,p)^{41}$Ca, leading to the $2p_{1/2}$ and $2p_{3/2}$ states of ^{41}Ca having excitation energy 3.95 and 1.95 MeV, respectively; (ii) ^{48}Ca$(d,p)^{49}$Ca, leading to the $2p_{3/2}$ ground state and $2p_{1/2}$ (2.02 MeV) first excited state of ^{49}Ca. The incident deuteron energy varies from 2 to 5.5 MeV.

In comparison with ^{40}Ca, much less experimental studies have been possible in the region of ^{48}Ca, because hardly any nearby nuclide is long lived enough to be used as targets in scattering experiments.[6] Even ^{48}Ca, though stable, is extremely scarce because of its low natural abundance. However, recently Bockelman and his group at Yale[7] have studied (d,p) reaction on a ^{48}Ca target for incident deuteron energy above the Coulomb barrier (13 to 19 MeV). The analysis of these measurements by the CC model will appear in a future publication. For incident deuteron energy across the Coulomb barrier, the measurements on ^{48}Ca-$(d,p)^{49}$Ca reactions are taken from the work of Roy and Bogaards.[8] The experiments on ^{40}Ca$(d,p)^{41}$Ca reactions are numerous. At 2 MeV deuteron energy the data are taken from the experiment of Fodor, Szentpetery, and Zimamje and Gómez del Campo, Richards, and Rapaport,[9] while at 5 MeV deuteron energy, the cross section measurements of Schwandt and Haeberli,[10] Kocher and Haeberli,[11] and that of Leighton *et al.*[12] have been averaged out.

For the incident deuteron energy far below the Coulomb barrier (~2 MeV) we note from Fig. 1 that the deuteron elastic scattering cross section is predominantly Rutherford and (d,p) angular distributions do not show any fine structure. However, as we increase the incident deuteron energy, the maxima and minima in the cross sections gradually show up. Figure 2 shows the fit to the deuteron elastic scattering cross sections and the vector polarizations for ^{48}Ca and ^{40}Ca targets for incident deuteron energies of 5.5 and 5.0 MeV, respectively. The corresponding stripping angular distributions and the vector analyzing powers are shown in Fig. 3. In defining the vector analyzing power which describes the asymmetry of the proton angular distribution obtained from the stripping reaction of an incident vector polarized deuteron beam on spin zero target, we have followed the convention of Haeberli's group.[10] A great advancement in polarized beam experiment has been made by this group.[10] However, the polarization measurements

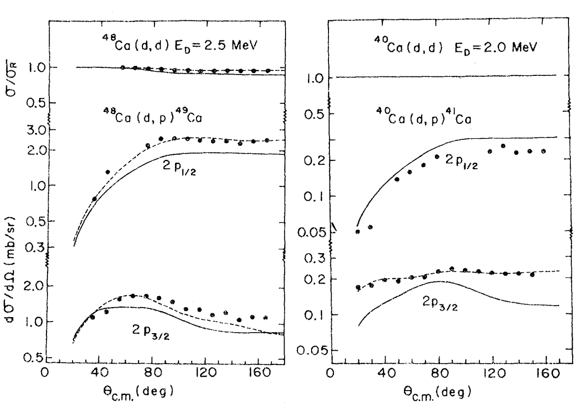

FIG. 1. Deuteron elastic scattering cross sections (as compared to Rutherford) and p-stripping differential cross sections for ^{48}Ca and ^{40}Ca targets at deuteron energies far below the Coulomb barrier. The measurements (solid circles) on the ^{48}Ca target are taken from Ref. 8 and those on the ^{40}Ca target are taken from Ref. 9. The solid and dashed curve represent the CC model and the DWBA predictions, respectively.

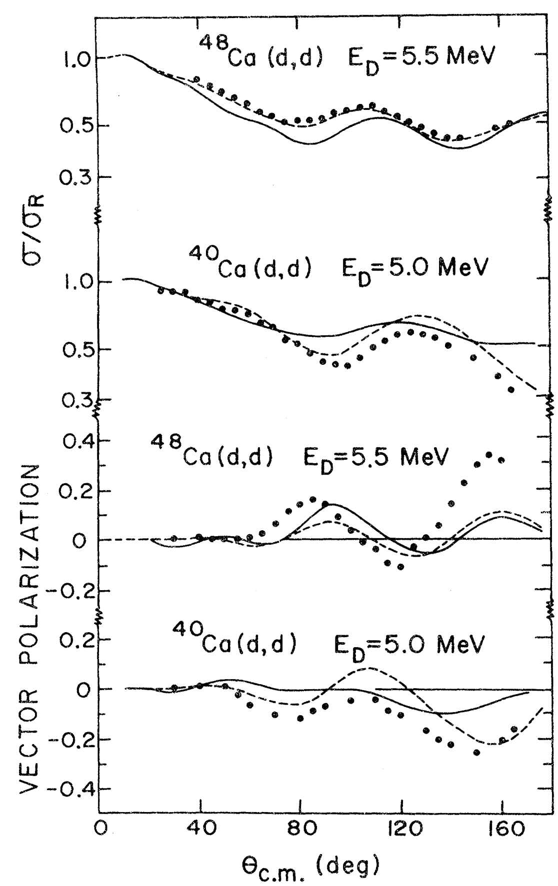

FIG. 2. Deuteron elastic scattering cross sections (as compared to Rutherford) and vector polarization for ^{48}Ca and ^{40}Ca targets at deuteron energies indicated (in MeV). The measurements (solid circles) on ^{40}Ca target are taken from Ref. 11 and Ref. 12 and those on ^{48}Ca from Ref. 8. The solid and dashed curves represent the CC model and the DWBA predictions, respectively.

near the Coulomb region are still scarce. The recent optical model analysis of the polarization data favors the inclusion of a tensor term in the deuteron nucleus interaction.[10] Therefore it is necessary to study the spin-orbit character of the deuteron nucleus interaction caused by the coupling to the stripping channel from the viewpoint of new polarization measurements.

We see from Figs. 1, 2, and 3 that, for incident deuteron energy near the Coulomb barrier, the CC model predictions agree reasonably well with experiments. The agreement on ^{48}Ca is much better than that on ^{40}Ca. One of the reasons for this is that the present CC model is more appropriate for the study ^{48}Ca than ^{40}Ca because of the neglect of the coupling of f stripping. At the lower energy the f stripping, in the reaction ^{40}Ca(d,p)^{41}Ca, becomes comparable to the p stripping, while beyond 10 MeV it is quite negligible. But in the case of ^{48}Ca(d,p)^{49}Ca reaction, f stripping is absent and there are fewer p states. The other reason is that ^{48}Ca core is more featureless than that of ^{40}Ca, from the viewpoint of single transfer reaction. We also note that the experimental cross sections of ^{48}Ca(d, d) and ^{48}Ca(d,p)^{49}Ca reactions at $E_D = 5$ MeV fit well with the CC predictions, when the coupling constant N is set equal to 2 instead of 3, as used in all other cases. Therefore, the 5 MeV results on ^{48}Ca target are shown separately in Fig. 4. It should be stressed that the purpose of the present numerical calculations is to explore how the CC model works near the Coulomb barrier and for different isotopes of calcium rather than to seek a fit to the experimental data.

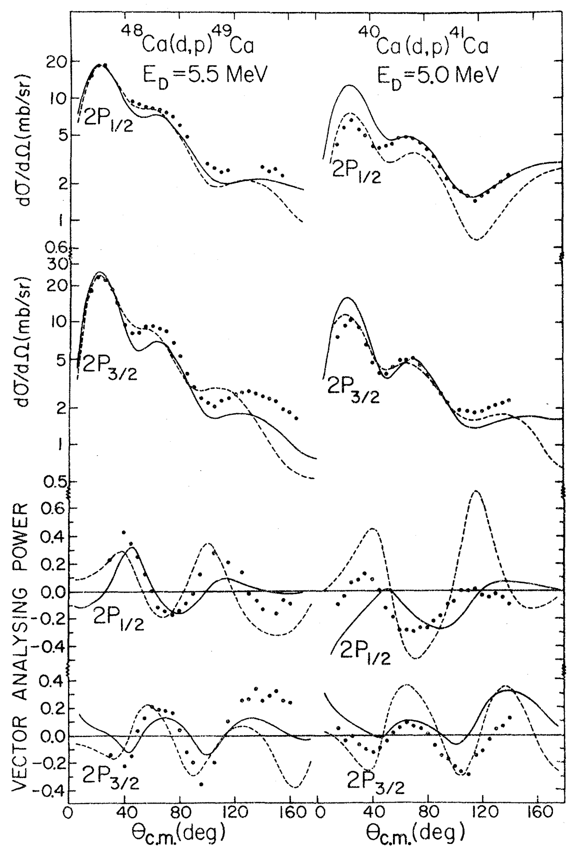

FIG. 3. Stripping differential cross sections and the deuteron vector analyzing power in the reactions ^{48}Ca(d, p)^{49}Ca and ^{40}Ca(d, p)^{41}Ca to $2p_{3/2}$ and $2p_{1/2}$ states of the residual nucleus at deuteron energies indicated (in MeV). The measurements (solid circles) on the ^{40}Ca target are taken from Ref. 11 and Ref. 12 and those on the ^{48}Ca target are taken from Ref. 8. The solid and dashed curves represent the CC model and DWBA predictions, respectively.

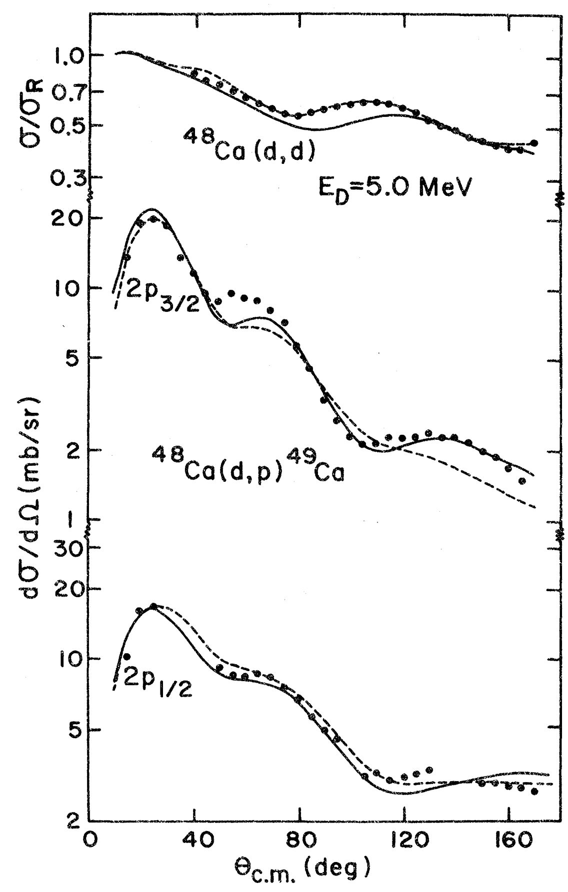

FIG. 4. Deuteron elastic scattering cross sections (as compared to Rutherford) and p-stripping differential cross section for the ^{48}Ca target at 5 MeV deuteron energy. The measurements (solid circles) are taken from Ref. 8. The solid curve represents the CC model predictions for the coupling constant $N = 2$ and the dashed curve indicates the DWBA results.

The spectroscopic factor

$$S=\frac{(d\sigma/d\Omega)_{\text{expt.}}}{(d\sigma/d\Omega)_{\text{theo.}}}$$

has been obtained for each incident deuteron energy. In the sub-Coulomb region one can obtain a more reliable value of the spectroscopic factor due to the low sensitivity of stripping cross sections to the optical potential parameters variations. The spectroscopic factors obtained by the CC method and the DWBA are listed in Table III. The DWBA angular distributions for ^{48}Ca$(d,p)^{49}$Ca reaction are normalized to the main experimental peak, as was done by the previous workers.[8] The values of S in Table III indicate that the final states in ^{49}Ca probably are good single particle states.

TABLE III. Spectroscopic factor.

E_D		^{40}Ca$(d,p)^{41}$Ca $2p_{1/2}$	$2p_{3/2}$	E_D		^{48}Ca$(d,p)^{49}$Ca $2p_{1/2}$	$2p_{3/2}$
2.0	CC	0.87	1.3	2.5	CC	1.34	1.26
	DWBA	$\cdots$	1.04		DWBA	1.00	1.00
5.0	CC	0.50	0.63	5.0	CC	1.06	0.92
	DWBA	0.87	0.89		DWBA	1.00	1.00
				5.5	CC	1.00	0.93
					DWBA	1.00	1.00

V. j DEPENDENCE

Measurements of the differential cross sections and the vector analyzing powers in (d,p) reaction for spin zero target have shown a strong dependence of the angular distributions upon the total angular momentum j of the captured neutron. This j dependence of the $l=1$ angular distribution in the ^{40}Ca$(d,p)^{41}$Ca reaction, for incident deuteron energy above the Coulomb barrier, was earlier observed by Lee *et al.*[5] and later by Haeberli and his group at Wisconsin,[13] and is particularly shown by a deep minimum for $p_{1/2}$ capture around 100°. For incident deuteron energy near the Coulomb barrier, (d,p) angular distributions for ^{40}Ca and ^{48}Ca targets also show a $p_{3/2}$-$p_{1/2}$ j dependence. The most striking feature of the experimental (d,p) angular distribution in this energy region is that the $p_{3/2}$ cross section also has a deeper minimum around 50° both for ^{40}Ca and ^{48}Ca targets. This may be recognized as forward angle j dependence. However, at backward angles the minimum near 100° is deeper for the $p_{3/2}$ transition than that for the $p_{1/2}$ one, for the ^{48}Ca$(d,p)^{49}$Ca reaction. But for the ^{40}Ca$(d,p)^{41}$Ca reaction at 5 MeV deuteron energy, the deep minimum for $p_{1/2}$ transition around 100°, though less pronounced, is still observed (Fig. 3). These new measurements make possible severe tests of stripping theories. At the same time they provide a simple means of determining the spin of the final state of the residual nucleus.

The inadequacies of the DWBA theory to explain the j dependence have led to different approaches. On the one hand, there have been efforts to improve the present form of the DWBA theory such as inclusion of the deuteron D wave in the calculation by Johnson and Santos[14]; on the other hand, entirely new stripping theories have been proposed such as the weakly bound projectile model,[15] the Butler, Hewitt, McKellar, and May (BHMM) model,[16] and the models of Johnson and Soper[17] and Rawitscher,[18] which consider the deuteron breakup effects.

It is likely that the j dependence in (d,p) reactions is partly due to a large coupling which exists between the deuteron channel and the stripping and the breakup channels. The j dependence produced by the breakup effects has not been investigated so far. However, the possibility of obtaining j dependence as a result of coupling between the incident deuteron channel and the stripping channel is explored here. To see how j dependence comes about in the framework of the present CC model, it is desirable to examine the expression for the stripping cross section.

The stripping amplitude in the CC model has the form

$$S^{\mu_0}(jJ_cM_c)=(4\pi)^{1/2}\kappa_D^{-1}\sum_{LI\mathfrak{L}''}\langle I\mu_0\,|\,LJ_cMM_c\rangle\langle I\mu_0\,|\,\mathfrak{L}''10\mu_0\rangle[J_c][\mathfrak{L}''][j][1]\,i^{(\mathfrak{L}''-L)}\exp(i\sigma_{\mathfrak{L}''}+i\sigma_L)Q^{\mathfrak{L}''}{}_{jJ_cLI}Y_L^M(\theta,\varphi)\,, \tag{9}$$

with

$$Q^{\mathfrak{L}''}{}_{jJ_cLI}=\sum_{J\mathfrak{L}}(2J+1)[L][l][\mathfrak{L}](-)^{\mathfrak{L}}W(L\tfrac{1}{2}Ij;JJ_c)\begin{pmatrix}L & l & \mathfrak{L}\\ 0 & 0 & 0\end{pmatrix}\begin{Bmatrix}J & j & I\\ L & l & \mathfrak{L}\\ \frac{1}{2} & \frac{1}{2} & 1\end{Bmatrix}R^{\mathfrak{L}''}_{LJj,\mathfrak{L}I}\,, \tag{10}$$

where R's are the radial overlap integrals

$$R^{\mathcal{L}''}_{LJj;\mathcal{L}I} = -\frac{2m_p}{\hbar^2}\frac{1}{\kappa_p}\frac{D_0}{\sqrt{4\pi}}\int_0^\infty F^{(u)}_{LJj}\, u_j F^{\mathcal{L}''}_{\mathcal{L},I}\, dr\,, \tag{11}$$

J_c is the channel spin defined by $\vec{J}_c = \vec{S} + \vec{j}$. Other symbols have the usual meaning and are defined in Ref. 3. The differential cross section is given by

$$\left(\frac{d\sigma}{d\Omega}\right)^{\mu_0}_{(d,p)} = \frac{1}{3}\frac{\beta_j}{\beta_D}\sum_{J_c M_c} |S^{\mu_0}(jM_cJ_c)|^2\,, \tag{12}$$

where

$$\beta_j = \frac{\kappa_p^j}{m_p} \text{ and } \beta_D = \frac{\kappa_D}{m_D}\,.$$

The formal difference between the DWBA and the CC expressions for the stripping amplitude can be seen easily from the above expression. The coefficient of Y_L^M in Eq. (9) is a sum over $\mathcal{L}''$ and I. More values of I contribute to this sum of $j=\frac{3}{2}$ in

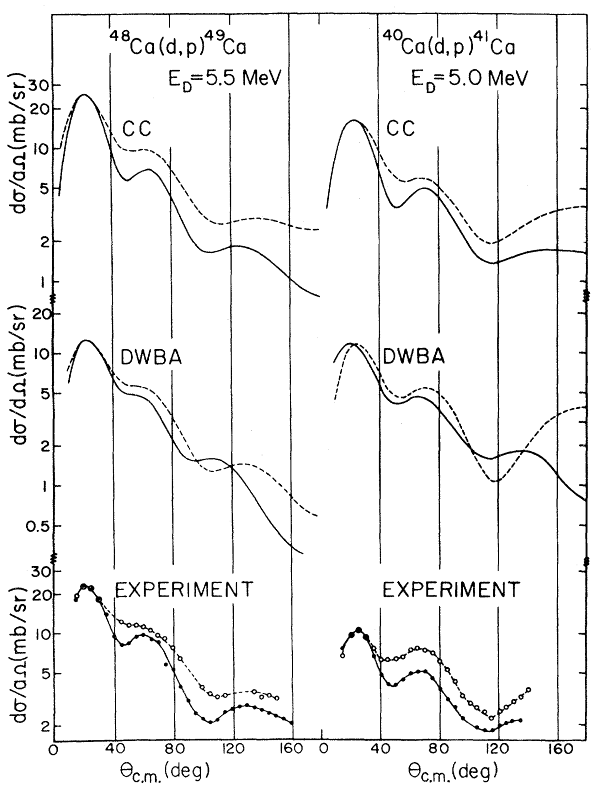

FIG. 5. j dependence of the stripping differential cross section. The $p_{1/2}$ stripping angular distributions (dashed curve), are normalized to the $p_{3/2}$ main peak (solid curve). The open and the closed circles represent $p_{1/2}$ and $p_{3/2}$ measured angular distributions, respectively.

the CC case than in the DWBA case because of the coupling rules. Further, the stripping amplitudes S^{μ_0} are very numerous, since the channel spin J_c takes on two values (0 and 1) for $j=\frac{1}{2}$ and two more values (1 and 2) for $j=\frac{3}{2}$. Of course, another difference between the CC and the DWBA calculation, which is not dependent on the spin considerations and which is probably the major one, stems from the fact that the radial overlap integral R defined in Eq. (11) has a numerically different value in the two types of calculations, since the value of the deuteron radial functions $F_{\mathcal{L},I}$ are different at small distances even though they are very similar asymtotically, in order to give rise to the same elastic cross section. Summarily, we may say that the CC calculation may show some j dependence in the stripping differential cross section and the vector analyzing power.

In order to see this, we have plotted the $p_{3/2}$ and the $p_{1/2}$ stripping differential cross sections in such a manner that the $p_{1/2}$ distributions are normalized to the $p_{3/2}$ main peak. Figures 5 and 6 show a comparison of the j dependence respectively in the stripping differential cross section and the vector analyzing power for ^{40}Ca and ^{48}Ca targets for incident deuteron energy around 5 MeV. The dashed curves represent $p_{1/2}$ angular distributions and the solid curve the $p_{3/2}$ one. We see that the forward angle j dependence obtained from the CC method is more close to experiment than than obtained from the DWBA. Similarly, for the ^{48}Ca$(d,p)^{49}$Ca reaction the CC results seem to have a backward angle j dependence which is closer to the experimental one than the DWBA. At this low energy, however, the possibility that the obtained j dependence is simply the Q-value effect may not be excluded. One should do at least a calculation in which some hypothetical Q value is taken so that the final $p_{1/2}$ and $p_{3/2}$ states have the same excitation energy.

It is observed that at higher energies the j dependence changes character (i.e., the $p_{1/2}$ minima becomes deeper). However, this is not reproduced by the CC calculation. For example, for 11 MeV deuterons the CC calculation does not show any j dependence at large angle as discussed previously.[3] The lack of the large angle j dependence of the above mentioned result will be due to the opaqueness of the nucleus in the deuteron channel, caused by the large imaginary part of the deuteron potential employed in the above calculation. It seems that some other mechanism sets in at higher energies. Breakup could, indeed, be a likely candidate, because breakup increases with higher energies and the d-state breakup component increases in importance relative to the s-state breakup at higher energies.

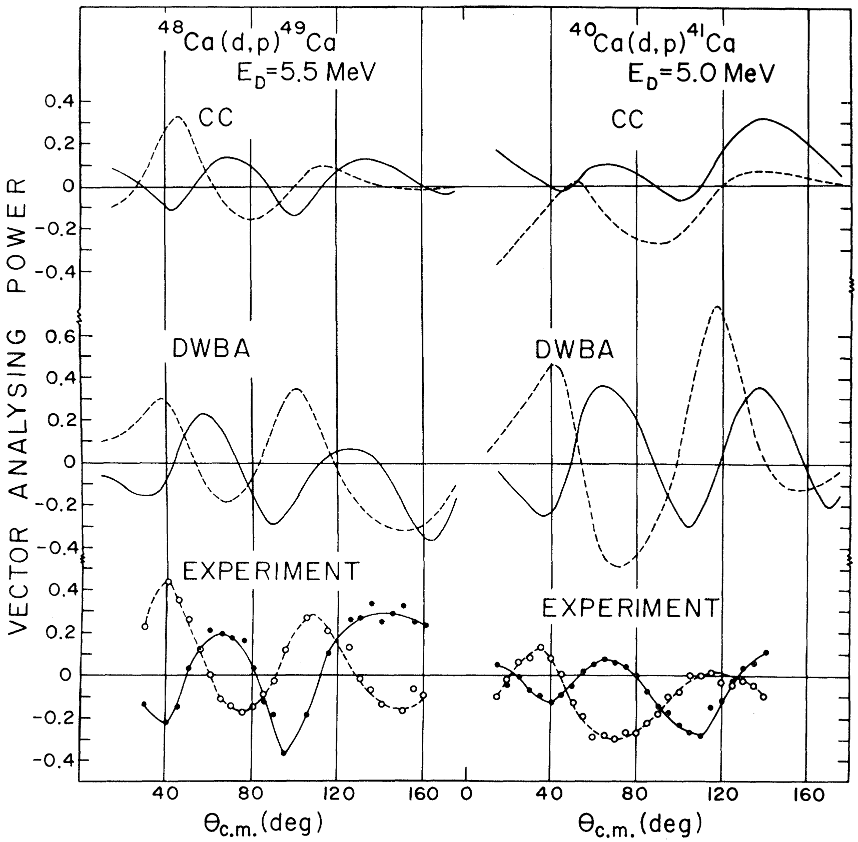

FIG. 6. j dependence of the vector analyzing power. The $p_{1/2}$ and $p_{3/2}$ distributions are shown by dashed and solid curves, respectively. The open and closed circles represent $p_{1/2}$ and $p_{3/2}$ measurements of the vector analyzing power.

VI. SUMMARY AND CONCLUSIONS

In summary, our conclusions are as follows. The CC model tested here reproduces much of the systematic behavior of the measurements for the two $l=1$ transitions studied in the reactions ^{48}Ca-$(d,p)^{49}$Ca and ^{40}Ca$(d,p)^{41}$Ca for incident deuteron energy near the Coulomb barrier. The shapes of the angular distribution and the vector analyzing power are well reproduced. In the DWBA analysis, it is necessary to have some energy dependence of the optical potential parameters to fit the data. On the other hand, in the CC model the potential parameters are energy independent. This behavior reflects the energy-dependent feedback of the stripping channel to the deuteron channel, and lends credibility to the CC method of calculation. In the CC formulation the same set of parameters are used both for ^{48}Ca and ^{40}Ca targets. However, the agreement of the theory with the experiment for the ^{48}Ca target is somewhat better than that for ^{40}Ca. One of the reasons for this may be that the stripping to the $f_{7/2}$ state ignored in the calculation does not occur in ^{48}Ca, whereas f stripping is comparable to p stripping in ^{40}Ca. The value of the spectroscopic factor in the Coulomb region indicates that the final states of ^{49}Ca are good single particle states. The CC calculation shows j dependence which is closer to the experimental than the DWBA. It is unfortunate that the present form of the CC model does not include breakup effects and the deuteron D waves. Further inclusion of these effects appears to be necessary.

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