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Neutron resonance spectroscopy: The separated isotopes of Dy†

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Results are given for the neutron resonance parameters of the Dy isotopes (160 to 164) based on 202.05 m transmission and 39.536 m capture self-indication flight path time of flight measurements using the Columbia-Nevis synchrocyclotron. The $A = 161$ to 164 samples were ~ 92 to 98% enriched. The ^{160}Dy sample was 69.5% enriched. We give E_0 and $(g)\Gamma_n^0$ values for: ^{160}Dy—64 levels to 2 keV; ^{161}Dy—251 levels to 1 keV; ^{162}Dy—142 levels to 16 keV; ^{163}Dy—114 levels to 1 keV; ^{164}Dy—116 levels to 21 keV. The $\langle\Gamma_\gamma\rangle$ values and (numbers) of Γ_γ values were 108 meV (9), 112 meV (26), 112 meV (16), 113 meV (39), and 114 meV (5), respectively. A shape fit to the strong asymmetric level at 147 eV in ^{164}Dy gave $R' = 7.5$ fm. The $l = 0$ strength functions, $10^4 S_0$, are (2.00 ± 0.36), (1.73 ± 0.17), (1.88 ± 0.25), (2.02 ± 0.30), and (1.70 ± 0.25), respectively. Using our threshold sensitivities for level detection vs E, and various statistical tests, indicates the following numbers of missed s levels to the E_{max} where most s levels were seen as: ^{160}Dy—2 to 900 eV; ^{161}Dy—5 to 140 eV; ^{162}Dy—2 to 3 keV and 6 to 4.5 keV; ^{163}Dy—2 to 220 eV; and ^{164}Dy—2 to 5 keV. No p levels were included for 160, 161, 163, but a few were present for 162 and 164, giving $10^4 S_1 = (1.1 \pm 0.4)$ (^{162}Dy) and (1.3 ± 0.3) (^{164}Dy). Various statistical tests suggest where the missed s levels should be located to give good agreement with the tests, giving $\langle D_0\rangle = (27.3 \pm 1.7)$, (2.67 ± 0.13), (64.6 ± 1.9), (6.85 ± 0.54), and (147 ± 9) eV, respectively.

[NUCLEAR REACTIONS ^{160,161,162,163,164}Dy (n,n), (n,γ), $E = 1$ eV–few keV; measured $\sigma_t(E)$; deduced E_0, $g\Gamma_n^0$, Γ_γ, S_0, $\langle D_0\rangle$, S_1; various statistical tests.]

I. INTRODUCTION

This is one of a series[1–14] of papers reporting the results of high resolution neutron time of flight spectroscopy measurements using a 202.05 m transmission flight path and a capture γ ray detector at 39.536 m with the Columbia University Nevis synchrocyclotron (S.C.). We present the results of measurements using Dy_2O_3 samples ~92% to 98% enriched in ^{161}Dy, ^{162}Dy, ^{163}Dy, and ^{164}Dy and a 69.5% enriched ^{160}Dy sample. We obtained resonance parameters for 64 levels in ^{160}Dy to 2 keV, 251 levels in ^{161}Dy to 1 keV, 142 levels in ^{162}Dy to 16 keV, 114 levels in ^{163}Dy to 1 keV, and 116 levels in ^{164}Dy to 21 keV. Our separated isotope samples were too thin to give results for the between level cross section behavior.

The latest (1973) edition of BNL-325[15] includes only our preliminary results for ^{163}Dy. It otherwise reflects the present status of knowledge of the resonance parameters for the Dy isotopes, excluding the results given in this paper. Parameters are given there for 6 levels in ^{156}Dy (0.06%) to 30 eV, 3 levels in ^{158}Dy (0.10%) to 86 eV, 4 levels in ^{160}Dy (2.3%) to 85 eV, 44 levels in ^{161}Dy (18.9%) to 138 eV, 14 levels in ^{162}Dy (25.5%) to 861 eV, and 8 levels in ^{164}Dy (28.2%) to 1316 eV. For ^{163}Dy (24.9%), 23 levels had previously been reported to 262 eV. The isotopic abundances in natural Dy are in parentheses. References to previous measurements are given on pp. 66-10 of Ref. 15. The other main results for the Dy isotopes 161, 162, 163, and 164 are those of Mughabghab and Chrien.[16] While our data extended to much higher energies than our highest reported energy levels for each isotope, the data do not favor reliable analysis above the energies of the last reported resonances. The samples all had <0.1% of ^{156}Dy or ^{158}Dy, so none of their levels were seen in our data. The $A = 161$ to 164 Dy samples were sufficiently enriched that there was no difficulty in subtracting levels due to "impurity" isotopes. The ^{160}Dy sample had 17.83, 6.45, 3.55, and 2.67%, respectively, of ^{161,162,163,164}Dy. Since essentially all of the numerous ^{161}Dy and ^{162}Dy levels were seen in the ^{160}Dy data, this reduced our ability to include weak ^{160}Dy levels. Other impurities in the samples were negligible.

Measurements on the Dy isotopes are important since Dy lies in the region of the split 4s maximum of the $l = 0$ strength function where the p strength functions are expected to be smaller, so the observed level populations are nearly complete s populations, with little or no p level contamination. This is important for tests[1–5] of theories of the level spacing systematics for single populations such as the Dyson-Mehta (DM)

Δ test etc. These results for the Dy isotopes help to support our previous results for ^{166}Er [1] and other even-even nuclei[2,4,5] having $150 < A < 190$, supporting the validity of the DM theory for a statistical orthogonal ensemble (OE) of levels (single population).

For each reported resonance, we obtain the level energy E_0, the reduced neutron width Γ_n^0 (even A) or $g\Gamma_n^0$ (odd A), and in favorable cases, the radiation widths Γ_γ. We thus obtain the $l = 0$ strength functions S_0 for each isotope based on statistically large level samples. A few of the weak levels seen in ^{162}Dy and ^{164}Dy were probably $l = 1$ levels, permitting us to give estimates of the p strength functions S_1 for these two isotopes. The binding energies for an extra neutron to ^{160,161,162,163,164}Dy are 6.451, 8.195, 6.272, 7.655, and 5.715 MeV, respectively (the approximate excitation of compound nuclei for these resonances). The even A isotopes have $I = 0$. ^{161}Dy and ^{163}Dy both have $I = \frac{5}{2}$, with (+) and (−) parities, respectively.

II. EXPERIMENTAL DETAILS

These measurements were made during the same cyclotron "run" as for ^{232}Th and ^{238}U [3] and for the Cd isotopes.[8] Details of the operation of the S.C., the 202.05 m (transmission) and 39.536 m (capture) flight paths and detector stations, the time of flight (tof) analyzers, etc., are given in previous papers.[1,3] Cyclotron operation was at a 70 Hz cycle rate using a 16 000 channel time of flight analyzer. Channel widths of 40 ns were used above ~1200 eV with progressively increased widths at lower energies to ~15 eV for the 202.05 m measurements. Channel widths of 40 ns were used above ~300 eV, with progressively increased widths to ~0.9 eV for the 39.536 m measurements.

The samples were all prepared in a rectangular 32×127 mm format. The sample thicknesses used were as follows: One sample of ^{160}Dy, $(1/n) = 1126$ b/atom of ^{160}Dy; three samples of ^{161}Dy, $(1/n) = 134.2$, 200.6, and 608 b/atom of ^{161}Dy; four samples of ^{162}Dy, $(1/n) = 144.2$, 215.7, 652.3, and 1305 b/atom of ^{162}Dy; one sample of ^{163}Dy, $(1/n) = 712.5$ b/atom of ^{163}Dy; four samples of ^{164}Dy, $(1/n) = 94.9$, 142.3, 427.8, and 855.7 b/atom of ^{164}Dy. For the 39.536 m path measurements, we had 10^6 cyclotron bursts (~4 h) counting periods for the thickest sample of each isotope at the detector in addition to counting using the thinner samples, and the 202.05 m transmission measurements. The 39.536 m detector was for capture γ rays from samples positioned at the center of the detector. These measurements are the most sensitive for detecting weak (mainly capture) levels.

III. RESULTS FOR THE RESONANCE PARAMETERS

The methods of analysis for level parameters are the same as for the Er, Yb, W, etc. isotopes. Details are given in earlier papers.[1,4,5]

Our main results are given in Tables I–V where the level energies and the reduced neutron level widths, $\Gamma_n^0 = \Gamma_n (1\ \text{eV}/E_0)^{1/2}$ or $(g\Gamma_n^0)$, are given for $A = 160$ to 164 (Dy). We also (not tabulated) obtained Γ_γ values for many levels as follows: (a) 9 levels in ^{160}Dy to 730 eV with values from (96 ± 20) to (120 ± 20) meV and $\langle \Gamma_\gamma \rangle = 108$ meV; (b) 26 levels in ^{161}Dy to 276 eV with values from (96 ± 20) to (120 ± 20) meV and $\langle \Gamma_\gamma \rangle = 112$ meV; (c) 16 levels in ^{162}Dy to 1835 eV with values from

TABLE I. Resonance energies and $\Gamma_n^0 = \Gamma_n\ (1\ \text{eV}/E)^{1/2}$ values for the levels in ^{160}Dy.

E_0 (eV)	Γ_n^0 (meV)	E_0 (eV)	Γ_n^0 (meV)	E_0 (eV)	Γ_n^0 (meV)	E_0 (eV)	Γ_n^0 (meV)
10.45±0.03	5.1 ±0.6	398.79±0.35	10.5 ±1.5	989.3±0.4	2.8 ±0.6	1541.7±0.4	3.3 ±0.8
20.47±0.05	6.9 ±0.7	430.10±0.39	5.5 ±0.7	1053.5±1.0	0.40±0.18	1592.1±0.4	9.5 ±1.8
34.90±0.02	0.20±0.02	522.17±0.26	5.8 ±0.9	1085.9±0.4	21 ±3	1625.3±1.8	0.32±0.32
73.14±0.07	0.76±0.07	577.44±0.31	1.5 ±0.2	1103.0±1.1	0.75±0.21	1648.3±0.7	7.6 ±1.5
85.56±0.13	9.8 ±1.0	589.97±0.40	0.35±0.09	1133.6±1.1	4.9 ±1.0	1710.3±0.4	7.3 ±1.7
115.62±0.11	0.19±0.03	625.56±0.44	0.32±0.08	1139.2±1.1	0.24±0.15	1721.0±0.4	6.3 ±1.4
136.35±0.14	2.2 ±0.3	653.60±0.47	0.22±0.06	1161.5±1.1	0.65±0.18	1746.2±2.1	1.0 ±0.3
155.68±0.17	6.8 ±0.8	679.52±0.39	28 ±4	1201.3±1.2	0.89±0.20	1763.6±2.1	0.62±0.24
178.27±0.21	2.3 ±0.3	703.62±0.41	33 ±5	1244.3±1.2	1.28±0.23	1785.6±2.1	1.3 ±0.5
202.20±0.26	2.2 ±0.3	729.55±0.43	4.2 ±0.6	1279.4±1.3	0.48±0.25	1829.7±2.2	3.5 ±1.2
246.32±0.22	0.56±0.10	736.41±0.44	46 ±7	1313.1±0.5	7.7 ±1.4	1861.6±0.5	30 ±7
271.94±0.25	0.26±0.04	817.20±0.51	1.12±0.21	1367.4±1.4	0.49±0.24	1874.9±0.5	3.7 ±1.4
320.66±0.16	0.73±0.17	845.21±0.54	3.8 ±1.0	1401.4±1.5	3.6 ±0.8	1899.6±0.5	6.4 ±2.1
341.15±0.28	21 ±2	868.60±0.72	5.3 ±1.0	1411.7±1.5	2.9 ±0.8	1937.8±0.5	14 ±4
379.29±0.33	15.0 ±1.5	889.14±0.74	0.60±0.17	1437.2±0.3	4.7 ±1.1	1944.3±0.5	5.4 ±2.0
394.29±0.34	3.0 ±0.5	971.99±0.85	0.22±0.13	1495.3±0.3	5.2 ±1.3	1994.3±2.5	5.6 ±2.2

TABLE II. Resonance parameters for ^{161}Dy.

E_0 (eV)	$g\Gamma_n^0$ (meV)	E_0 (eV)	$g\Gamma_n^0$ (meV)	E_0 (eV)	$g\Gamma_n^0$ (meV)	E_0 (eV)	$g\Gamma_n^0$ (meV)
2.71±0.02	0.21 ±0.02	191.00±0.29	0.21 ±0.02	450.20±0.42	0.75 ±0.19	720.90±0.43	1.1 ±0.3
3.68±0.02	0.47 ±0.05	192.94±0.30	0.048±0.024	454.85±0.43	0.36 ±0.09	724.30±0.43	0.41±0.11
4.33±0.02	0.28 ±0.03	194.44±0.30	0.72 ±0.07	458.91±0.43	1.03 ±0.14	729.67±0.43	0.81±0.22
7.74±0.03	0.10 ±0.01	197.28±0.31	0.73 ±0.07	462.37±0.44	1.7 ±0.3	736.19±0.44	0.88±0.22
10.26±0.03	0.034±0.006	202.81±0.27	0.77 ±0.14	472.33±0.45	0.87 ±0.18	738.16±0.44	1.4 ±0.3
10.85±0.03	0.073±0.006	206.01±0.27	1.25 ±0.14	475.72±0.46	1.5 ±0.3	740.36±0.44	0.51±0.18
12.65±0.04	0.008±0.002	208.50±0.27	0.37 ±0.07	477.35±0.46	2.2 ±0.5	743.91±0.45	0.55±0.18
14.31±0.05	0.82 ±0.05	210.48±0.27	0.90 ±0.28	483.08±0.60	0.18 ±0.05	746.58±0.45	0.73±0.22
16.67±0.06	0.86 ±0.05	211.97±0.27	0.49 ±0.21	485.31±0.60	0.064±0.064	749.95±0.45	0.62±0.18
18.48±0.06	0.91 ±0.07	214.15±0.27	1.37 ±0.14	490.59±0.60	0.095±0.072	755.50±0.58	0.15±0.07
20.24±0.04	0.023±0.003	224.43±0.37	0.15 ±0.05	497.20±0.49	0.76 ±0.13	759.58±0.59	0.21±0.11
25.22±0.04	0.12 ±0.01	227.78±0.15	0.86 ±0.13	500.64±0.25	0.17 ±0.06	763.69±0.59	0.24±0.14
29.04±0.04	0.23 ±0.02	235.46±0.16	0.98 ±0.13	503.60±0.25	0.35 ±0.11	766.64±0.47	0.23±0.07
29.92±0.04	0.084±0.007	238.98±0.16	0.14 ±0.02	504.97±0.25	0.32 ±0.11	770.84±0.47	0.52±0.18
35.74±0.04	0.20 ±0.02	240.77±0.16	0.14 ±0.02	506.83±0.25	0.58 ±0.18	773.66±0.47	0.76±0.22
37.71±0.05	0.73 ±0.07	242.42±0.17	0.49 ±0.07	514.04±0.26	0.88 ±0.18	776.26±0.48	1.4 ±0.4
38.51±0.05	1.18 ±0.13	245.25±0.17	0.83 ±0.13	517.24±0.26	0.84 ±0.22	780.06±0.48	2.8 ±0.6
43.27±0.06	0.99 ±0.08	251.77±0.17	0.10 ±0.01	519.57±0.26	0.70 ±0.18	784.14±0.48	0.82±0.21
45.14±0.06	0.92 ±0.07	256.81±0.23	0.053±0.019	526.91±0.27	0.52 ±0.13	786.55±0.49	0.27±0.14
50.86±0.06	0.34 ±0.03	258.74±0.18	0.34 ±0.06	529.83±0.27	0.23 ±0.09	790.97±0.62	0.1 ±0.1
51.72±0.06	1.39 ±0.14	261.13±0.18	0.61 ±0.10	535.08±0.27	0.82 ±0.17	797.54±0.63	0.17±0.10
55.19±0.06	0.54 ±0.05	263.73±0.18	2.1 ±0.04	539.58±0.28	1.03 ±0.26	803.87±0.64	0.07±0.07
59.57±0.06	0.34 ±0.04	265.62±0.19	0.44 ±0.09	543.45±0.28	0.11 ±0.04	807.74±0.51	0.33±0.11
61.41±0.06	0.47 ±0.05	267.81±0.19	0.25 ±0.09	545.40±0.28	0.77 ±0.17	812.80±0.51	0.98±0.25
63.64±0.06	0.29 ±0.04	275.66±0.20	3.0 ±0.4	553.01±0.29	0.47 ±0.13	818.08±0.66	0.21±0.07
67.55±0.07	0.006±0.006	283.55±0.21	0.19 ±0.02	555.01±0.29	0.15 ±0.06	822.02±0.66	0.17±0.09
73.17±0.08	0.21 ±0.02	287.64±0.27	0.12 ±0.02	559.54±0.37	0.085±0.085	831.38±0.53	0.94±0.24
77.07±0.08	0.27 ±0.03	291.89±0.22	0.12 ±0.03	562.09±0.29	1.05 ±0.21	834.02±0.53	0.29±0.14
78.09±0.09	0.033±0.005	293.64±0.22	0.34 ±0.06	566.06±0.30	0.67 ±0.13	836.13±0.53	1.6 ±0.5
82.27±0.09	0.14 ±0.02	294.97±0.22	0.070±0.023	574.65±0.39	0.071±0.071	845.58±0.69	0.27±0.09
85.07±0.09	0.30 ±0.03	299.92±0.23	1.1 ±0.2	581.56±0.31	1.5 ±0.3	850.62±0.55	0.72±0.17
88.77±0.10	0.72 ±0.08	302.37±0.29	0.13 ±0.04	584.49±0.31	0.21 ±0.08	856.08±0.55	0.41±0.14
91.12±0.10	0.050±0.006	305.45±0.24	2.7 ±0.5	587.17±0.40	0.062±0.062	860.48±0.55	1.5 ±0.4
93.29±0.11	1.1 ±0.1	311.75±0.24	0.79 ±0.11	591.17±0.41	0.066±0.066	864.09±0.56	0.44±0.21
95.23±0.11	0.16 ±0.02	314.78±0.25	0.90 ±0.17	594.40±0.41	0.28 ±0.12	867.16±0.56	0.41±0.17
101.35±0.11	0.72 ±0.09	315.76±0.25	0.44 ±0.11	595.62±0.41	0.25 ±0.12	879.02±0.57	0.15±0.07
102.42±0.11	0.029±0.009	319.61±0.25	1.03 ±0.14	599.64±0.33	3.3 ±0.7	881.60±0.57	2.3 ±0.5
104.12±0.11	0.027±0.009	328.68±0.26	0.077±0.011	602.18±0.42	0.16 ±0.10	885.34±0.58	0.84±0.27
104.98±0.11	0.31 ±0.04	331.31±0.27	0.17 ±0.02	607.59±0.42	0.26 ±0.13	892.86±0.75	0.31±0.12
110.50±0.12	0.47 ±0.06	337.60±0.27	5.7 ±0.8	614.38±0.34	1.33 ±0.24	897.00±0.60	0.12±0.06
112.43±0.13	0.48 ±0.06	343.53±0.28	0.70 ±0.11	620.26±0.34	1.6 ±0.3	900.00±0.76	0.24±0.11
113.44±0.14	0.081±0.019	349.53±0.38	0.17 ±0.02	624.85±0.34	0.20 ±0.06	903.20±0.30	1.1 ±0.3
118.42±0.14	0.34 ±0.04	361.95±0.30	3.4 ±0.4	630.02±0.35	0.52 ±0.20	910.23±0.30	0.73±0.17
120.51±0.14	0.34 ±0.04	378.74±0.40	0.10 ±0.02	632.80±0.35	0.17 ±0.17	914.31±0.31	3.1 ±0.8
124.65±0.15	2.7 ±0.3	381.24±0.33	0.21 ±0.07	635.30±0.45	0.23 ±0.12	918.10±0.31	1.5 ±0.4
127.54±0.15	0.13 ±0.02	383.05±0.33	1.9 ±0.4	641.78±0.35	1.8 ±0.6	922.85±0.31	0.30±0.12
131.16±0.16	0.35 ±0.04	388.94±0.43	0.13 ±0.02	645.18±0.36	1.4 ±0.3	925.46±0.31	1.2 ±0.4
138.38±0.17	0.82 ±0.09	392.41±0.34	1.6 ±0.2	648.60±0.36	0.47 ±0.16	933.22±0.31	0.39±0.13
142.80±0.18	0.39 ±0.06	396.35±0.35	0.87 ±0.10	651.97±0.47	0.15 ±0.05	941.07±0.32	0.2 ±0.1
144.83±0.20	0.16 ±0.02	400.11±0.45	0.042±0.025	657.11±0.47	0.098±0.098	942.97±0.32	0.85±0.26
149.42±0.20	0.15 ±0.02	402.93±0.45	0.095±0.045	664.96±0.48	0.07 ±0.07	948.07±0.32	1.2 ±0.3
153.80±0.21	0.35 ±0.04	404.07±0.36	0.075±0.040	670.49±0.38	2.5 ±0.6	955.30±0.65	0.39±0.10
156.83±0.22	0.25 ±0.03	415.33±0.37	0.88 ±0.15	672.21±0.38	1.08 ±0.23	958.22±0.33	1.1 ±0.3
162.59±0.23	2.0 ±0.2	420.01±0.38	0.54 ±0.10	674.90±0.49	0.18 ±0.09	963.15±0.84	0.20±0.09
166.60±0.24	1.47 ±0.15	424.76±0.39	1.8 ±0.3	677.60±0.49	0.15 ±0.15	965.24±0.33	0.15±0.08
168.61±0.24	2.5 ±0.2	428.23±0.39	1.2 ±0.2	680.68±0.39	1.4 ±0.4	972.18±0.33	1.2 ±0.3
170.09±0.25	0.29 ±0.05	432.54±0.40	0.28 ±0.09	691.90±0.40	0.68 ±0.15	975.68±0.34	0.61±0.22
172.78±0.25	0.65 ±0.08	435.11±0.40	0.38 ±0.10	697.32±0.41	0.80 ±0.23	978.52±0.34	1.9 ±0.6
175.29±0.26	0.53 ±0.15	438.51±0.40	0.12 ±0.07	698.94±0.41	0.38 ±0.15	980.87±0.34	0.23±0.11
176.53±0.26	0.56 ±0.17	440.12±0.41	0.12 ±0.07	704.52±0.52	0.11 ±0.11	989.34±0.34	2.1 ±0.6
179.05±0.27	0.90 ±0.07	442.56±0.41	0.27 ±0.10	709.25±0.53	0.068±0.068	992.75±0.34	0.21±0.10
183.69±0.28	1.5 ±0.2	445.22±0.41	0.85 ±0.19	712.91±0.42	0.45 ±0.15	996.16±0.88	0.09±0.09
189.31±0.29	1.02 ±0.07	447.70±0.42	0.66 ±0.14	714.79±0.42	1.3 ±0.4		

TABLE III. Resonance parameters for ^{162}Dy. An "a" before the energy indicates that the level is likely to be p level by a Bayes's theorem analysis. For p levels a p reduced neutron width $g\Gamma_n^1 \approx \Gamma_n^0$ (357 keV/E) should be used.

E_0 (eV)	Γ_n^0 (meV)	E_0 (eV)	Γ_n^0 (meV)	E_0 (eV)	Γ_n^0 (meV)	E_0 (eV)	Γ_n^0 (meV)
5.45±0.02	9.4 ±1.3	2240.9±0.6	20 ±3	4657.0±8.9	1.4± 0.9	8729.9± 4.5	16± 4
71.10±0.20	49 ±2	2275.6±0.6	17 ±2	4785.6±1.8	6.9± 1.2	8946.1± 4.7	42± 7
117.22±0.11	0.94±0.06	2341.5±0.6	16 ±2	4844.6±9.5	1.9± 1.1	9084.3± 4.8	13± 3
207.97±0.26	1.8 ±0.1	2363.4±0.7	8.6 ±1.2	5032.2±2.0	6.1± 1.0	9296.6± 4.9	57±10
223.29±0.15	2.5 ±0.2	2492.6±3.5	0.84±0.30	5084.4±2.0	17 ± 3	9488.5± 5.1	50± 9
269.36±0.20	40 ±3	2526.8±0.7	4.4 ±0.6	5159.7±2.1	64 ±10	9657.8± 5.2	26± 5
357.02±0.30	1.4 ±0.1	2576.2±0.7	3.3 ±0.5	5234.6±2.1	9.0± 1.7	9720.5± 5.3	11± 3
412.75±0.37	7.4 ±0.7	2673.4±0.8	14 ±2	5375.3±2.2	3.4± 0.7	9834.3± 5.4	13± 4
470.32±0.45	0.37±0.05	2796.8±0.8	51 ±8	5502.7±2.3	16 ± 3	9923.0± 5.4	10± 3
529.83±0.27	12.2 ±1.3	2847.6±0.9	15 ±2	5518.4±2.3	13 ± 3	10276 ± 6	58±12
632.80±0.70	62 ±8	2885.9±0.9	20 ±3	5616.2±2.3	87 ±12	10359 ± 6	18± 8
685.96±0.40	17 ±1	2944.2±0.9	2.8 ±0.6	5679.0±2.4	7.6± 2.3	10546 ± 6	9± 6
716.47±0.42	22 ±2	2957.2±0.9	2.8 ±0.6	5733.3±2.4	26 ± 4	10642 ± 6	23±10
766.42±0.47	31 ±3	a3028.9±4.7	0.33±0.18	5776.3±2.4	11 ± 3	10772 ± 6	11± 6
866.60±0.56	85 ±7	3080.8±4.8	0.76±0.36	6057.9±2.6	31 ± 6	10956 ± 7	30± 9
952.08±0.33	6.3 ±0.6	3183.4±1.0	2.1 ±0.5	6076.0±2.6	7.7± 2.3	11258 ± 7	16± 5
1005.2 ±0.4	1.6 ±0.2	3243.8±1.0	13 ±2	6099.4±2.6	6.4± 2.0	11739 ± 7	28± 8
1066.5 ±0.4	0.95±0.18	a3270.9±5.2	0.31±0.21	6217.2±2.7	16 ± 3	11813 ± 7	17± 6
1110.6 ±0.4	7.8 ±0.9	3327.3±1.1	4.5 ±0.7	6353.5±2.8	8.8± 2.5	12012 ± 7	25± 7
1261.4 ±0.5	24 ±3	3363.9±1.1	4.0 ±0.7	6545.0±2.9	15 ± 4	12404 ± 8	50± 9
1360.2 ±0.3	3.5 ±0.5	a3513.8±5.8	0.44±0.25	6591.7±3.0	8.6± 2.5	12564 ± 8	74±16
a1387.5 ±1.5	0.13±0.07	3554.4±1.2	40 ±5	6704.5±3.0	8.9± 3.1	12626 ± 8	28± 7
1431.6 ±0.3	40 ±5	3624.0±6.1	0.58±0.42	6896.4±3.2	4.8± 1.8	12918 ± 8	33± 9
a1483.3 ±1.6	0.19±0.10	3666.0±1.2	8.4 ±1.2	6964.3±3.2	10 ± 4	13163 ± 8	25± 9
1526.2 ±0.3	5.6 ±0.8	3798.9±1.3	47 ±6	7060.8±3.3	39 ± 6	13196 ± 8	35±13
1567.3 ±0.4	16 ±2	3894.6±6.8	0.62±0.40	7142.7±3.3	10 ± 4	13372 ± 9	84±17
1680.7 ±2.0	0.32±0.10	a3966.9±7.0	0.41±0.29	7460.4±3.6	4.7± 2.3	13726 ± 9	32± 8
1724.5 ±0.4	2.6 ±0.4	4012.7±1.4	22 ±3	7590.9±3.7	64 ±11	14053 ± 9	25± 7
1794.9 ±0.4	0.42±0.14	4081.8±1.5	22 ±5	7643.7±3.7	35 ± 7	14219 ± 9	39±11
1835.3 ±0.5	5.1 ±0.7	4096.1±1.5	6.1 ±1.6	7745.4±3.8	24 ± 5	14671 ±10	54±17
1935.9 ±0.5	21 ±2	4114.9±1.5	41 ±6	7925.8±3.9	7.0± 3.4	14848 ±10	57±16
1950.2 ±2.4	1.6 ±0.4	4239.0±1.5	13 ±2	8130.6±4.0	58 ±10	15068 ±10	68±21
2003.2 ±0.5	6.0 ±0.9	4273.3±1.6	8.0 ±1.2	8236.0±4.1	8.5± 3.9	15298 ±11	39±13
a2047.6 ±2.6	0.2 ±0.1	4393.5±1.6	3.2 ±0.9	8333.0±4.2	11 ± 4	15534 ±11	30±10
2081.5 ±0.5	3.9 ±0.7	4452.3±1.7	31 ±5	8450.5±4.3	44 ± 9	15814 ±11	43±14
2181.4 ±0.6	1.6 ±0.3	4554.9±3.4	1.0 ±0.7				

(100 ± 20) to (130 ± 25) meV and $\langle \Gamma_\gamma \rangle = 112$ meV; (d) 39 levels in ^{163}Dy with values from (70 ± 20) to (160 ± 35) meV and $\langle \Gamma_\gamma \rangle = 113$ meV; and (e) 5 levels in ^{164}Dy with values from (105 ± 25) to (120 ± 20) meV and $\langle \Gamma_\gamma \rangle = 114$ meV. The variations in the Γ_γ values may be mainly due to experimental uncertainties in the evaluations. The $\langle \Gamma_\gamma \rangle$ values are all consistent with a common value in the range 108 to 114 meV.

Figure 1 shows a fit of transmission T vs E for the exceptionally strong level at 147 eV in ^{164}Dy using a single Breit-Wigner formula which includes the interference of σ_{pot} and σ_{res} and includes Doppler broadening and the energy dependence of Γ_n for the level. The experimental points are many channel averages. The choices $R' = 7.5$ fm ($\sigma_{\text{pot}} = 4\pi R'^2$) and other parameters, as indicated, give a good fit to the (interference) asymmetric level shape from ~130 to 190 eV. There are no other strong ^{164}Dy levels nearby.

IV. SYSTEMATICS OF THE RESULTS

The plots of the number of observed levels N vs energy E for each isotope are given in Figs. 2(a) to 2(e). In each case, it is seen that there is a reduced slope at higher energies than for the lowest part. For ^{162}Dy, the slope decreases monotonically above ~4.5 keV. For the other isotopes, the upper slopes remain nearly constant, but less than that at lower energies (as shown). The upper energies at which the 39.536 m (capture) data were used were 1 keV for ^{161}Dy and ^{163}Dy, 2 keV for ^{160}Dy, and 5 keV for ^{162}Dy and ^{164}Dy. These are also the upper energy limits for all analysis except for ^{162}Dy and ^{164}Dy for which a much larger fraction of weak s levels were missed at higher energies.

The plots of $\sum \Gamma_n^0$ or $\sum (g\Gamma_n^0)$ are given in Figs. 3(a) to 3(e). These plots are insensitive to missed weak levels. The slopes give the values of the s

TABLE IV. Resonance parameters for ^{163}Dy.

E_0 (eV)	$g\Gamma_n^0$ (meV)	E_0 (eV)	$g\Gamma_n^0$ (meV)	E_0 (eV)	$g\Gamma_n^0$ (meV)	E_0 (eV)	$g\Gamma_n^0$ (meV)
1.72±0.01	0.76 ±0.08	213.74±0.27	0.23 ±0.03	459.21±0.43	0.70±0.14	736.14±0.56	0.36±0.09
5.81±0.02	0.011±0.002	224.15±0.30	6.2 ±0.8	465.32±0.44	0.51±0.09	741.69±0.44	0.40±0.11
16.23±0.03	2.7 ±0.2	233.54±0.32	0.24 ±0.03	479.10±0.46	1.6 ±0.2	747.15±0.57	0.12±0.05
19.65±0.03	0.099±0.009	250.55±0.18	0.57 ±0.07	483.55±0.47	1.8 ±0.2	756.05±0.46	0.95±0.22
35.79±0.04	0.74 ±0.05	261.13±0.19	3.3 ±0.4	504.59±0.25	2.6 ±0.3	764.30±0.60	0.32±0.14
50.27±0.06	0.25 ±0.03	268.01±0.19	4.2 ±0.5	516.34±0.26	0.66±0.13	769.93±0.60	0.12±0.06
55.85±0.08	1.9 ±0.1	274.17±0.20	0.92 ±0.11	519.83±0.26	0.48±0.09	776.55±0.61	0.50±0.11
58.97±0.08	5.5 ±0.4	281.06±0.21	0.92 ±0.12	533.18±0.27	4.3 ±0.7	794.40±0.63	0.96±0.21
66.11±0.10	0.54 ±0.05	288.85±0.22	3.7 ±0.5	542.22±0.28	5.2 ±0.9	809.51±0.51	4.6 ±1.1
72.00±0.11	0.19 ±0.05	296.00±0.29	0.17 ±0.04	564.29±0.29	1.3 ±0.2	812.29±0.51	0.88±0.28
75.48±0.10	0.14 ±0.01	297.77±0.23	2.5 ±0.4	569.02±0.30	2.6 ±0.3	823.04±0.52	0.84±0.28
78.99±0.13	0.88 ±0.11	307.10±0.46	0.046±0.029	580.79±0.31	1.83±0.25	830.86±0.53	1.5 ±0.3
86.30±0.14	0.070±0.016	323.08±0.25	7.8 ±1.1	594.52±0.32	2.0 ±0.3	846.62±0.69	0.22±0.12
94.08±0.08	1.1 ±0.1	324.55±0.26	7.5 ±1.1	601.25±0.32	3.0 ±0.4	851.16±0.54	2.7 ±0.6
105.88±0.10	3.1 ±0.5	326.93±0.26	0.97 ±0.19	615.71±0.34	1.4 ±0.2	857.75±0.70	1.0 ±0.3
107.18±0.10	1.4 ±0.3	329.73±0.26	0.47 ±0.10	620.77±0.34	2.0 ±0.3	864.92±0.56	0.88±0.27
120.33±0.12	0.45 ±0.05	342.86±0.28	0.62 ±0.11	632.10±0.35	4.2 ±0.7	873.91±0.57	2.0 ±0.5
126.58±0.13	0.79 ±0.13	348.32±0.29	6.4 ±1.0	637.17±0.35	3.1 ±0.6	899.65±0.30	3.0 ±0.7
127.46±0.13	0.58 ±0.09	368.64±0.31	4.4 ±0.5	641.96±0.36	3.2 ±0.6	918.71±0.31	1.5 ±0.3
135.31±0.14	0.24 ±0.03	374.96±0.32	0.098±0.036	646.26±0.36	2.4 ±0.4	929.79±0.31	3.4 ±0.8
143.38±0.15	0.64 ±0.08	382.16±0.33	0.13 ±0.04	652.20±0.47	0.11±0.05	935.75±0.80	0.62±0.33
144.97±0.15	1.91 ±0.25	387.01±0.33	0.62 ±0.09	660.40±0.48	0.37±0.10	939.49±0.32	4.2 ±1.3
155.02±0.17	3.5 ±0.3	390.44±0.34	1.4 ±0.2	686.95±0.39	0.46±0.11	949.67±0.32	2.2 ±0.6
163.81±0.19	0.48 ±0.05	400.34±0.35	0.70 ±0.15	695.71±0.40	3.0 ±0.5	957.75±0.83	0.58±0.19
177.18±0.21	0.15 ±0.03	403.15±0.35	0.085±0.040	709.78±0.53	0.15±0.08	967.35±0.84	0.77±0.26
185.09±0.22	0.19 ±0.04	411.08±0.37	13.3 ±1.5	712.49±0.42	1.4 ±0.2	972.84±0.34	3.7 ±1.0
188.95±0.23	0.12 ±0.02	420.56±0.38	0.068±0.024	721.32±0.43	1.6 ±0.2	980.52±0.86	0.21±0.13
202.90±0.25	0.62 ±0.11	429.38±0.39	1.11 ±0.15	732.05±0.44	0.35±0.09	996.60±0.88	0.86±0.29
205.26±0.26	1.9 ±0.2	454.84±0.43	2.2 ±0.3				

TABLE V. Resonance parameters for ^{164}Dy. The significance of an "a" before the resonance energy is the same as in Table III.

E_0 (eV)	Γ_n^0 (meV)	E_0 (eV)	Γ_n^0 (meV)	E_0 (eV)	Γ_n^0 (meV)	E_0 (eV)	Γ_n^0 (meV)
146.97±0.32	68 ±4	2395.8±0.7	8.6 ± 1.2	5211.9±2.1	180 ±30	13280± 9	34± 7
a 227.57±0.38	0.024±0.012	2536.6±0.7	13 ± 2	5592.1±2.3	21 ± 4	13466± 9	23± 4
450.41±0.42	12 ±1	a2723.2±4.0	0.52± 0.17	5702.5±2.4	3.2± 0.9	13762± 9	22± 4
a 479.39±0.59	0.073±0.023	a2804.4±4.2	0.30± 0.11	5923.3±5.0	136 ±26	14076± 9	14± 4
536.30±0.28	5.0 ±0.4	2871.9±0.9	47 ± 8	6096.8±2.6	23 ± 5	14141± 9	66±13
548.76±0.28	0.29 ±0.04	2968.7±0.9	59 ± 7	6109.8±2.6	27 ± 5	14298± 9	22± 5
a 740.91±0.56	0.11 ±0.06	a3048.2±0.9	0.31± 0.14	6280.7±2.7	87 ±13	14690±10	69±13
804.24±0.50	0.31 ±0.04	a3100.0±4.8	0.34± 0.14	6444.7±2.9	29 ± 4	14882±10	34± 7
853.89±0.55	19 ±2	3209.0±5.1	0.94± 0.25	6621.1±3.0	47 ± 7	15335±11	35± 7
a 925.90±0.80	0.099±0.033	3281.6±1.0	3.8 ± 0.5	6854.2±3.1	14 ± 2	15455±11	51±10
a 940.98±0.81	0.16 ±0.07	a3416.8±5.6	0.80± 0.21	6996.3±3.2	12 ± 2	15550±11	34± 7
983.06±0.34	3.8 ±0.5	a3460.5±1.1	0.95± 0.20	7432.2±3.5	45 ± 7	15846±11	17± 5
1052.0 ±0.38	7.1 ±0.9	3480.6±1.1	32 ± 5	7695.2±3.7	87 ±14	16179±11	45± 9
1207.7 ±0.46	20 ±2	3520.3±1.2	1.2 ± 0.2	7958.7±3.9	10 ± 2	17194±13	28± 7
a1286.5 ±1.3	0.12 ±0.07	a3621.0±6.1	0.28± 0.17	8090.6±4.0	34 ± 6	17744±13	14± 5
1323.6 ±0.5	25 ±3	3734.4±1.3	12 ± 2	8252.4±4.1	48 ± 8	17900±13	170±30
a1405.8 ±1.5	0.17 ±0.04	a3911.6±6.9	0.54± 0.32	8368.6±4.2	4.0± 1.6	18212±14	41± 8
a1567.4 ±1.8	0.25 ±0.06	a4095.8±7.3	0.94± 0.63	8763.5±4.5	8.1± 1.9	18307±14	33±10
1588.1 ±0.4	5.5 ±0.8	4135.2±1.5	7.6 ± 1.2	9358.3±5.0	11 ± 3	18368±14	96±22
1644.7 ±1.9	0.57 ±0.10	4231.5±1.5	34 ± 5	9400.5±5.0	15 ± 3	18637±14	45±13
a1709.0 ±0.4	0.24 ±0.06	a4348.8±8.0	0.77± 0.53	9702.1±5.2	24 ± 4	18685±14	80±22
1896.2 ±0.9	46 ±7	a4389.2±8.1	0.44± 0.44	10974 ±6	66 ±10	19127±15	46± 9
1960.2 ±0.5	28 ±3	4445.8±1.6	75 ±10	11530 ±7	63 ± 9	19295±15	48±10
a2038.7 ±0.5	0.64 ±0.13	4757.6±3.6	2.0 ± 0.9	11753 ±7	30 ± 5	19728±15	30± 9
2065.9 ±2.6	1.01 ±0.22	4780.1±1.8	22 ± 4	11890 ±7	19 ± 4	19804±15	67±20
a2250.7 ±3.0	0.17 ±0.08	4810.1±1.9	156 ±23	12411 ±8	84 ±14	20317±16	46±13
a2285.1 ±0.6	0.65 ±0.13	a4916.0±1.9	1.2 ± 0.4	12894 ±8	17 ± 4	20573±16	44±13
2319.3 ±0.6	39 ±6	4954.9±1.9	2.4 ± 0.7	12942 ±8	53 ±11	20917±17	63±21
a2352.6 ±3.2	0.16 ±0.12	5169.9±2.0	29 ± 6	13060 ±8	260 ±40	21151±17	65±21

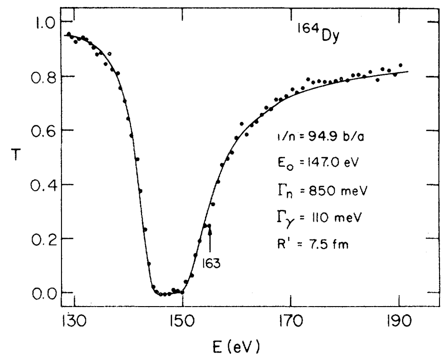

FIG. 1. Result of a shape fit to the transmission data of the ^{164}Dy separated isotope ($1/n = 94.9$ b/atom) for the strong level at 147.0 eV using a Breit-Wigner single level formula with the indicated parameters. The Doppler effect and the $(E)^{1/2}$ variation of Γ_n with E are included. The choice $R' = 7.5$ fm gives a good fit to the interference asymmetry. The arrow shows where a strong resonance at 155.02 eV in the low abundance ^{163}Dy impurity in the ^{164}Dy sample produces a small level shape distortion in the measured curve.

strength functions and permit an evaluation of the sensitivity of the S_0 values to the upper energy limit (of importance when comparing results by different experimenters). It also permits an examination for possible intermediate structure effects. Figure 3(b) for ^{161}Dy shows the most nearly constant slope, with smaller fluctuations even than the $(2/n)^{1/2}$ fractional value expected theoretically for a single channel Porter-Thomas (PT) distribution. Figure 3(c) for ^{162}Dy, shows a smooth increase in $\sum \Gamma_n^0$ after a short but much steeper initial rise. The subsequent smaller slope corresponds to $10^4 S_0 = 1.48$, vs (1.88 ± 0.25), our final choice for the full region which gives a little less weight to the highest energy region where the Γ_n values are less certain. Figures 3(d) and 3(e) have roughly constant slopes for $\sum g\Gamma_n^0$. A minimum for S_0 for ^{163}Dy would result for $E_{max} = 250$ eV and a maximum for $E_{max} = 410$ eV. Similarly, Fig. 3(e) shows that $E_{max} = 4500$ eV gives a low S_0 and 6600 gives a high value, reflecting the disproportionate contribution of levels between 4500 and 6600 eV for ^{164}Dy. The coarsest $\sum \Gamma_n^0$ plot is for ^{160}Dy [Fig. 3(a)] where there are large increases near 350 and 700 eV. It has the shape nearest to what one expects when intermediate structure effects are present in S_0 vs E. However, its N and Z are not near closed shell values where intermediate structure effects are most likely.

Figures 4(a) to 4(e) show histograms of the observed distributions of $(\Gamma_n^0)^{1/2}$ or $(g\Gamma_n^0)^{1/2}$, and the comparisons with the best fit single channel Porter-Thomas distributions. In each case, the results for two choices of upper energy limits are shown, since more of the weak levels are missed at higher energies. The choices of N for the curves are normalized to what we believe to be the correct number of s levels for the given

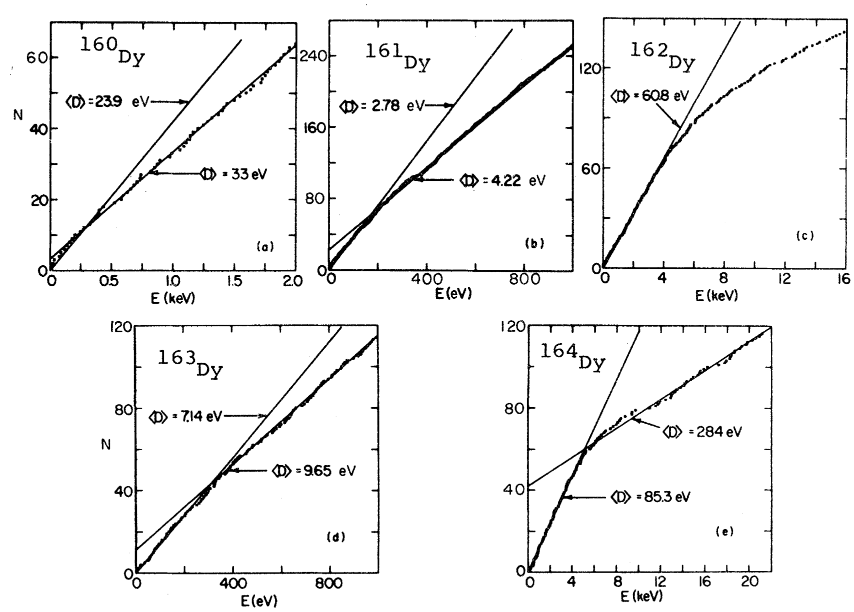

FIG. 2. Plots of the cumulative counts of observed levels vs energy for (a) ^{160}Dy, (b) ^{161}Dy, (c) ^{162}Dy, (d) ^{163}Dy, (e) ^{164}Dy. Each indicated $\langle D\rangle$ value in a plot represents only the slope of a visually fitted straight line for a chosen energy interval. Our final choices of $\langle D\rangle$ values for $l = 0$ are given in Table VII.

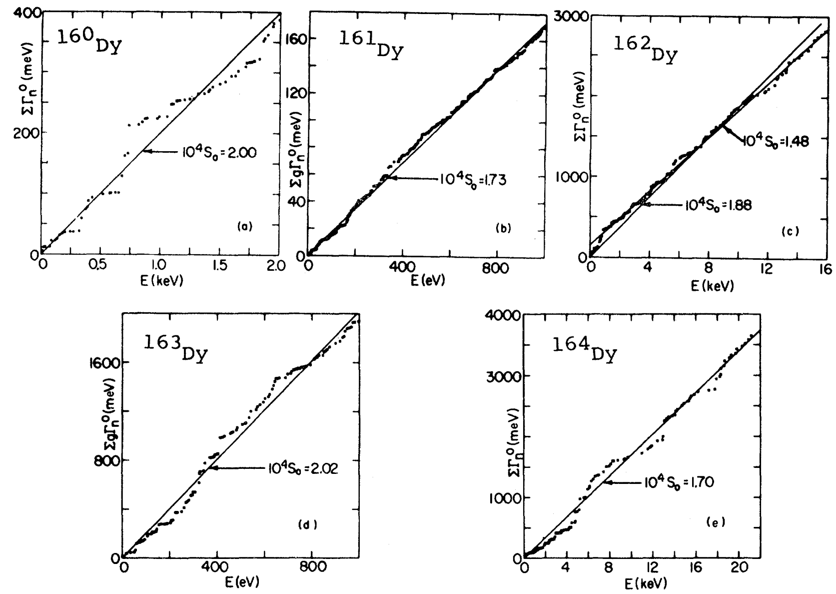

FIG. 3. Plots of $\Sigma\Gamma_n^0$ or $\Sigma g\Gamma_n^0$ vs energy for (a) ^{160}Dy, (b) ^{161}Dy, (c) ^{162}Dy, (d) ^{163}Dy, (e) ^{164}Dy. The slopes of the fitted straight lines give the s strength functions.

E_{max} values by fits to the upper parts of their corresponding $(g\Gamma_n^0)^{1/2}$ histograms. They are also normalized to our final choices for the s strength functions S_0. The fits are generally satisfactory for the region beyond the first or second histogram intervals where one notes that a few weak levels have been missed, except for ^{164}Dy, where an excess of weak levels indicates the presence of some p levels in the sample.

Figures 5(a) to 5(e) show the comparison of the histograms of observed nearest neighbor level spacings with the Wigner formula. The energy intervals used are the smaller intervals used in Fig. 4, above which a significantly larger fraction

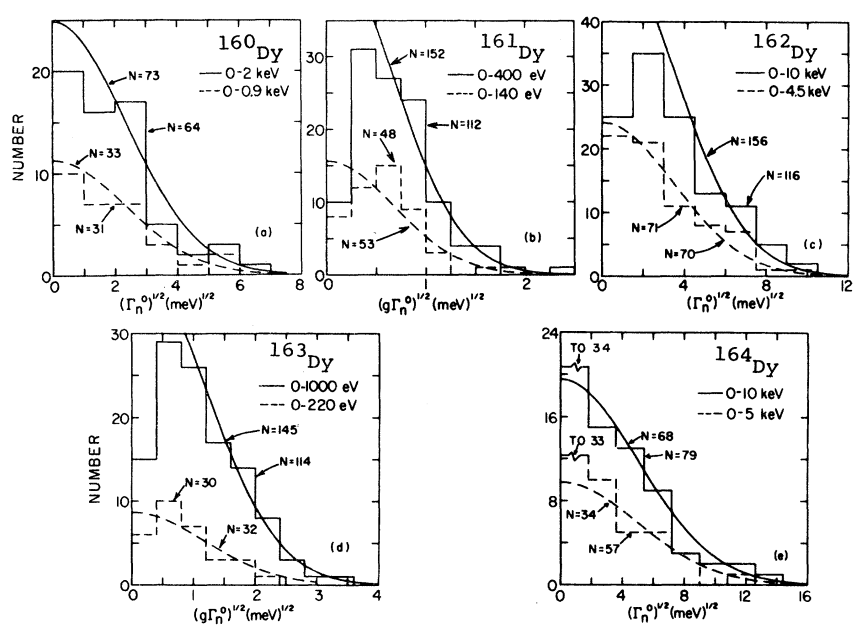

FIG. 4. Histograms of $(\Gamma_n^0)^{1/2}$ or $(g\Gamma_n^0)^{1/2}$ values for (a) ^{160}Dy, (b) ^{161}Dy, (c) ^{162}Dy, (d) ^{163}Dy, (e) ^{164}Dy. In each isotope case, the results for two choices of upper energy limits are shown. The upper part of each histogram is fitted with a Porter-Thomas single channel curve which is normalized to our final choice for the s strength function S_0.

of weak s levels are missed. The solid histograms are for all observed levels, while the dashed histograms are for our corrected s population choices after deleting p levels and attempting to correct for missed weak s levels.

The analysis to identify probable p levels and the number of weak s levels missed was made as follows. Using the final S_0 and an approximate mean s level spacing $\langle D\rangle$, we calculate (from our threshold sensitivity for detecting levels) the expected number of missed weak s levels vs energy for a single channel PT distribution. For ^{160}Dy this also includes the effect of "impurity" levels (^{161}Dy mainly) preventing the detection of weak ^{160}Dy levels. The implied number of missed s levels and upper energy in each case were: (a) two in ^{160}Dy to 900 eV; (b) five in ^{161}Dy to 140 eV; (c) two in ^{162}Dy to 3 keV and six to 4.5 keV; (d) two in ^{163}Dy to 220 eV; and (e) two in ^{164}Dy to 5 keV. Fits to the upper parts of the $(g\Gamma_n^0)^{1/2}$ distributions [Figs. 4(a) to 4(e)] give an estimate of the total true number of s levels for the intervals, and thus the $\langle D\rangle$ values for s levels. After correcting the histograms for the implied numbers of missed weak s levels, the first histogram boxes were studied to obtain estimates of the resulting excesses of weak levels (which are probably p levels). They suggest that some p levels are present foı ^{162}Dy and ^{164}Dy, 7 p levels in ^{162}Dy to 4.5 keV and 25 p levels in ^{164}Dy to 5 keV. This reflects the trend also found for other element even isotopes in this mass region, to see more p levels as A increases. This is partly due to an increase in $\langle D\rangle$ which increases $\langle\Gamma_n^1\rangle$ for p levels even for fixed S_1, and is partly due to a tendency for S_1 to increase with A for fixed Z in this mass region.

The next step is to use the experimental threshold sensitivities vs E for observing weak levels to calculate the expected number of p levels which we should see, vs energy, for various choices of 10^4S_1. For ^{160}Dy, ^{161}Dy, and ^{163}Dy choices of 10^4S_1 <3.0 suggest that no p levels should have been seen. Here we assume mean level spacings $\langle D\rangle_p = 0.5\ \langle D\rangle_s$ for odd A ($I=\frac{5}{2}$) and $\langle D\rangle_p = 0.33\ \langle D\rangle_s$ for even A, and assume the same $\langle g\Gamma_n^1\rangle$ value for each possible J choice for a given A nucleus. A Bayes's theorem test[17] was also applied to each isotope. We sum the probabilities that our observed weak levels are $l=1$ to obtain an independent expectation value for the number of p levels observed vs assumed S_1. It gives the same conclusion for ^{160}Dy, ^{161}Dy, and ^{163}Dy that probably no p levels were included.

For $10^4S_1 = 0.9$, 1.1, and 1.3, the Bayes test predicts mean numbers of ^{162}Dy p levels included to 4.5 keV, as 5.2, 6.6, and 7.7, respectively, vs 7 from the preceding reasoning. The threshold test for the same set of S_1 values yields 3.9, 7.0,

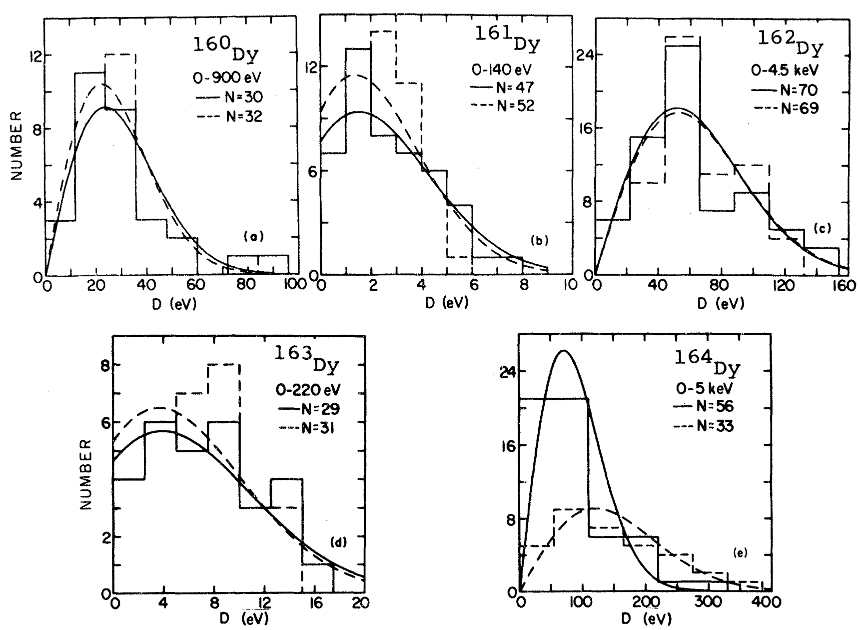

FIG. 5. Plots of the nearest neighbor level spacing histograms and the comparison Wigner distributions (normalizing to the same number of spacings as the histogram) for (a) ^{160}Dy, (b) ^{161}Dy, (c) ^{162}Dy, (d) ^{163}Dy, (e) ^{164}Dy. In each plot, the solid histogram corresponds to the observed spacing distribution, while the dashed histogram is for our final s population choice. For ^{161}Dy and ^{163}Dy, the theoretical curves are for a merged two-population Wigner distribution with the relative densities in the ratio of $(2J+1)$ values.

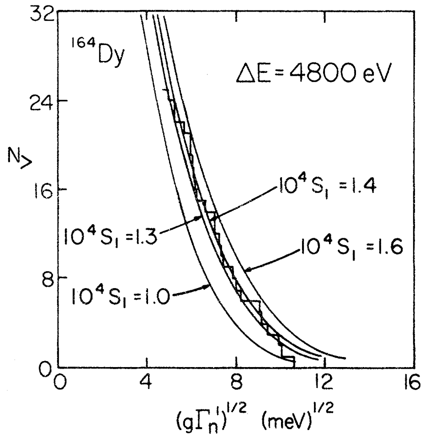

FIG. 6. Cumulative distribution of $(g\Gamma_n^1)^{1/2}$ values for the 25 levels of ^{164}Dy most apt to be $l=1$ from the Bayes theorem test. Four integral Porter-Thomas curves are plotted for comparison, which are normalized to $10^4 S_1 = 1.0$, 1.3, 1.4, and 1.6, respectively. The mean level spacing $\langle D\rangle_p$ is assumed equal to $\frac{1}{3}$ of $\langle D\rangle_s$. The effective energy interval ΔE is the observed interval (5 keV) minus the portions (~4%) blocked by the strong s levels.

and 10.6 expected p levels included. We choose $10^4 S_1 = (1.1 \pm 0.4)$ for our final result for ^{162}Dy. For the limit values, $10^4 S_1 = 0.7$ and 1.5, the above tests predict 3.4 and 8.5 p levels included (Bayes) and 1.6 and 14.4 p levels included (threshold test). For $10^4 S_1 = 1.1$, seven observed levels are chosen as p levels by the Bayes test, all of those having probability >0.6 for $l=1$.

For ^{164}Dy, the choices $10^4 S_1 = 0.9$, 1.3, and 1.7 yield 22.4, 26.3, and 28.2 expected p levels to 5 keV (Bayes test), and 17.5, 26.1, and 32.7 expected p levels (threshold test). About 4% of the p levels in ^{164}Dy will have their detection blocked by s levels, so the effective energy range for the threshold test is 4.8 keV. We choose $10^4 S_1 = (1.3 \pm 0.3)$ for ^{164}Dy. Figure 6 shows comparisons with the cumulative count of levels having $(g\Gamma_n^1)^{1/2}$ values larger than a lower limit vs the lower limit for the 25 levels most apt to be $l=1$ from the Bayes theorem test. The value 25 cannot be very much in error. The relative probability that a given level be p vs s is insensitive to the choice of S_1. For the choice $10^4 S_1 = 1.3$, the cut is at $p_p \geq 0.71$ for being $l=1$. If we delete the four levels in Fig. 6 having $0.79 \geq p_p \geq 0.71$ this removes the upper four levels in Fig. 6 having $(g\Gamma_n^1)^{1/2} > 9.4$ meV$^{1/2}$ without changing things below that. There are four levels having $0.71 \geq p_p \geq 0.39$ for being $l=1$. If these are included in Fig. 6, the added levels have $(g\Gamma_n^1)^{1/2} = 10.2$, 10.9, 11.0, and 11.8 meV$^{1/2}$. The best fit remains for $10^4 S_1$ between 1.0 and 1.6. The levels in ^{162}Dy and ^{164}Dy which are treated as p levels have the symbol "a" before the energies in Tables III and V.

The above discussion indicates that we believe that a few very weak s levels were missed for each isotope. We now consider the logic by which we judge where these missed s levels were most likely to actually be situated. For identification purposes, we call the s populations with these added, the "corrected" s populations. In this process, we use the statistical tests appropriate to a statistical orthogonal ensemble,[1] which we believe to give a correct description for a single complete s population. Tests supporting the theory are weakened in significance to the extent that such "corrections" are required. We are here mainly using the theory to test the data rather than vice versa. There is a certain arbitrariness in where we deduce that s levels were probably missed, but we try to do it as "reasonably" as possible. Table VI lists the results of tests for the Dyson-Mehta (DM) Δ statistic and the theoretical value Δ_{DM} and for the correlation coefficient ρ, for adjacent nearest neighbor spacings, and ρ_{OE} expected for the OE theory.[1] The first group in Table VI shows the results for all observed levels to the indicated upper energies. The second group shows the results after p levels are deleted as discussed above. The last group shows the results after some "missed s levels" are added to the second sets as indicated below. Here Δ is the mean square deviation of the staircase plot of N vs E from a best fit straight line,[18] with $\Delta_{DM} = (1/\pi^2)(\ln n - 0.0687)$ for n consecutive

TABLE VI. Results of OE statistical tests.

Isotope	E_{max} (eV)	n	Δ	Δ_{DM}	ρ	ρ_{OE}
			All observed levels			
^{161}Dy	140	48	0.76	0.63 ± 0.22	−0.22	−0.25 ± 0.12
^{163}Dy	220	30	0.50	0.53 ± 0.22	−0.21	−0.25 ± 0.15
^{160}Dy	900	31	0.66	0.34 ± 0.11	0.01	−0.27 ± 0.17
^{162}Dy	3000	49	0.45	0.39 ± 0.11	−0.16	−0.27 ± 0.13
^{162}Dy	4500	71	0.52	0.42 ± 0.11	−0.21	−0.27 ± 0.11
^{164}Dy	5000	57	1.08	0.40 ± 0.11	−0.13	−0.27 ± 0.12
			After subtracting p levels			
^{162}Dy	3000	46	0.46	0.38 ± 0.11	−0.17	−0.27 ± 0.14
^{162}Dy	4500	64	0.81	0.41 ± 0.11	−0.19	−0.27 ± 0.12
^{164}Dy	5000	32	0.73	0.34 ± 0.11	−0.31	−0.27 ± 0.17
			Corrected s level sets			
^{161}Dy	140	53	0.62	0.65 ± 0.22	−0.17	−0.25 ± 0.11
^{163}Dy	220	32	0.34	0.55 ± 0.22	−0.21	−0.25 ± 0.15
^{160}Dy	900	33	0.39	0.35 ± 0.11	−0.25	−0.27 ± 0.16
^{162}Dy	3000	48	0.34	0.39 ± 0.11	−0.29	−0.27 ± 0.13
^{162}Dy	4500	70	0.40	0.42 ± 0.11	−0.28	−0.27 ± 0.11
^{164}Dy	5000	34	0.41	0.35 ± 0.11	−0.33	−0.27 ± 0.16

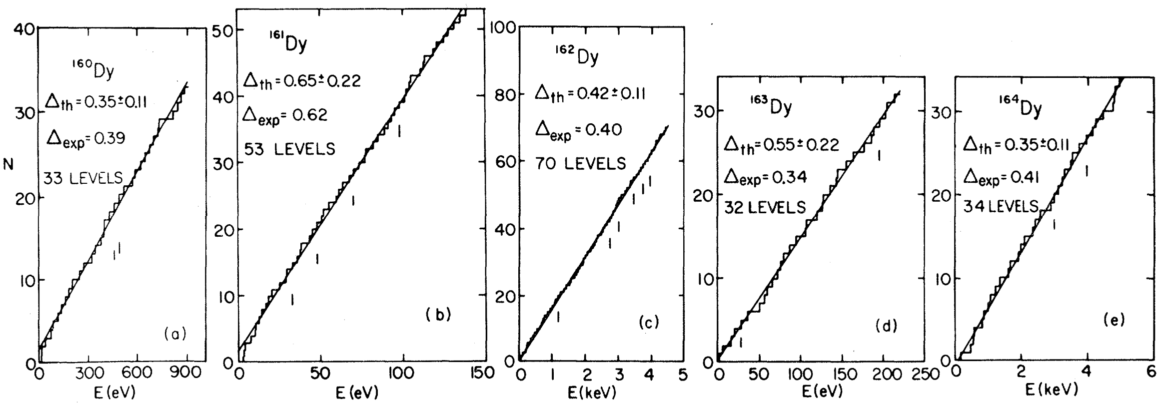

FIG. 7. Plots of the Dyson-Mehta Δ test applied to our final choice s level sets for (a) ^{160}Dy to 900 eV, (b) ^{161}Dy to 140 eV, (c) ^{162}Dy to 4500 eV, (d) ^{163}Dy to 220 eV, (e) ^{164}Dy to 5000 eV. The vertical lines below the DM best fit straight lines indicate the position at which "extra missed s levels" were added.

levels of an orthogonal ensemble. The last group of comparisons is to be regarded as tests of the plausibility of our estimates of the numbers and approximate positions of missed s levels.

For ^{161}Dy to 140 eV, ^{163}Dy to 220 eV, and ^{162}Dy to 4.5 keV, we see that the Δ and ρ values for the unmodified experimental level sets agree with the theoretical values to within the statistical uncertainty of the theoretical values. For ^{160}Dy and ^{164}Dy, the agreement is poor. Figure 5(a) for ^{160}Dy to 900 eV suggests that the two largest observed nearest neighbor spacings are unlikely. Adding an s level to the center of the largest spacing (at 476 eV) decreases Δ from 0.66 to 0.47. Adding a second at the center of the next largest spacing (at 777 eV) gives $\Delta = 0.40$ which is within the limits $\Delta_{DM} = (0.35 \pm 0.11)$, but ρ becomes +0.014. Our final choice puts the two extra levels symmetrically in the largest spacing to give $\Delta = 0.39$ and $\rho = -0.25$, in good agreement with theory, thus a plausible guess. We note that the value of Δ is not significantly changed, while ρ can be more strongly changed, if the "added s levels" are placed asymmetrically in the larger spacing intervals.

Figure 5(b) for ^{161}Dy shows no unexpectedly large spacings before extra s levels are added. If extra s levels are added, one at the center of each of the five largest spacing intervals [Fig. 5(b)], a shortage of larger spacings results, but the resulting Δ and ρ values are in good agreement with OE theory. This suggests that these are suitable regions for five missed s levels.

Removing p levels from the ^{162}Dy set maintains a good Δ fit to 3 keV, but not to 4.5 keV. Figure 5(c) suggests that a few, but not six of the largest spacings should be broken up. The centers of the nine largest spacing intervals, in decreasing size, are at 3459, 1186, 3732, 2428, 4177, 3019, 2735, 4333, and 3954 eV. If six missed s levels are inserted at the centers of the 1st, 2nd, 3rd, 6th, 7th, and 9th largest intervals as underlined, good fits to the theory are obtained for Δ and ρ.

For ^{163}Dy the fit was already good, and there was no excess of large spacings to begin with, as shown in Fig. 5(d). Two missed s levels are to be added. The centers of the three largest spacings were at 28, 43, and 196 eV. Adding at 28 and 196 eV gave good values for Δ and ρ.

For ^{164}Dy the main correction was the deletion of 25 p levels which reduced Δ from 1.08 to 0.73 (to 5 keV). The "corrected set" in Table VI placed the two extra levels at the centers of the largest

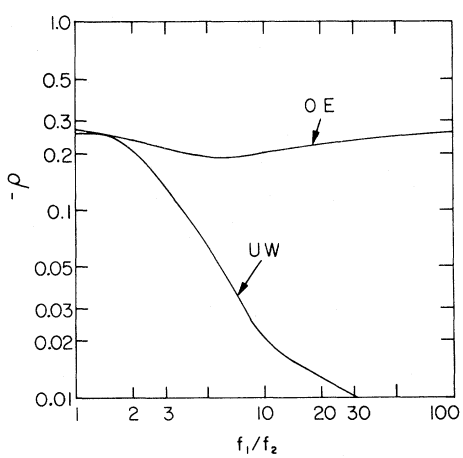

FIG. 8. Monte Carlo calculations of ρ vs f_1/f_2 for a level population having two independent level sequences randomly merged, where ρ is the correlation coefficient of the nearest neighbor level spacings, and f_1/f_2 is the ratio of the two level densities. Shown are the results for both the OE case and the UW case.

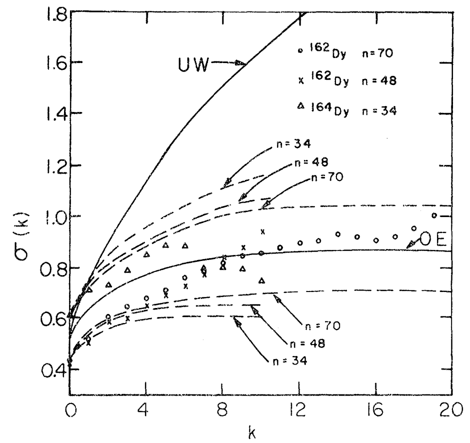

FIG. 9. Comparison of $\sigma(k)$ values for our "corrected" s level sets for ^{162}Dy to 4500 eV ($n=70$) and to 3000 eV ($n=48$) as well as for ^{164}Dy to 5000 eV ($n=34$) with Monte Carlo results for OE and UW theories. $\sigma(k)$ is the standard deviation from the mean for the level spacings having k levels between (in units of $\langle D_0\rangle$). The dashed curves give the 10 and 90% confidence limits for the OE case.

TABLE VII. Dy isotope average resonance parameters.

Isotope	$\langle\Gamma_\gamma\rangle$ (meV)	$\langle D_0\rangle$ (eV)	$10^4 S_0$	$10^4 S_1$
160	108	27.3 ± 1.7	2.00 ± 0.36	
161	112	2.67 ± 0.13	1.73 ± 0.17	
162	112	64.6 ± 1.9	1.88 ± 0.25	1.1 ± 0.4
163	113	6.85 ± 0.54	2.02 ± 0.30	
164	114	147 ± 9	1.70 ± 0.25	1.3 ± 0.3

interval (3935 eV) and the 9th largest (3089 eV), giving good values for Δ and ρ. If they are placed at the centers of the two largest intervals (3935 and 2704 eV), we still obtain a good value for Δ (=0.40) which suggests that these are suitable regions to add the extra levels [see Fig. 5(e)]. The value $\rho=-0.39$ is a little large negative, but the ρ test is too sensitive to where the levels are placed in the interval to be as meaningful.

Figures 7(a) to 7(e) show the staircase plots of N vs E and the best DM straight line fits for our "corrected" s level sets of the Dy isotopes. The vertical lines in each case indicate the positions at which missed s levels were added to obtain the fits.

The magnitudes and uncertainties of the ρ values in Table VI are based on Monte Carlo studies using the results of Dyson Brownian motion model OE calculations.[1] For a single OE population, Mehta found[19] that this should be $\rho=-0.271$, in agreement with our Monte Carlo result which also yields the expected standard deviation for ρ. The odd A case involves two merged s populations for the two J values. In order to obtain predictions for this situation, we extended our Monte Carlo calculations to include two merged s populations, each obeying OE theory, with various possible values f_1/f_2 for the two level densities. Calculations were made for 25 different f_1/f_2 ratios, in each case using 450 merged sets of 90 levels (total) for the OE case. We also treated sets where adjacent level spacings followed a Wigner distribution, with no long or short range correlations between the spacings. This we call the uncorrelated Wigner (UW) distribution. For the UW distribution, out Monte Carlo calculations used 1000 sets of 100 merged levels for each choice of f_1/f_2. These new theoretical results for the predicted ρ vs f_1/f_2 for the OE and UW cases are shown in Fig. 8. The standard deviations for the OE case are approximately fitted by $0.82/(n-1)^{1/2}$, for n levels and for $1 \leq f_1/f_2 \leq 3$. The UW case is included as a most plausible alternative to the full OE theory (as previously discussed[1] in our Er paper). The standard deviation of ρ is approximately $0.89/(n-1)^{1/2}$ for the UW case ($1 \leq f_1/f_2 \leq 3$).

Figure 9 shows our usual test of $\sigma(k)$ vs k for the corrected s level sets for ^{162}Dy and ^{164}Dy. $\sigma(k)$ is the standard deviation from the mean for the spacings of levels having k levels in between, in units of $\langle D\rangle$ for nearest neighbor spacings. In the figure, we show the predicted average values of $\sigma(k)$ for the OE and the UW cases, as well as the 10% and 90% confidence limits for the OE case for $n=34$, 48, and 70. As previously discussed, this mainly confirms the consistency of our choices (for missed s levels) with the OE theory rather than providing tests of the OE theory.

The final choices for $10^4 S_0$, $10^4 S_1$, $\langle D_0\rangle$, and $\langle\Gamma_\gamma\rangle$ for each isotope are listed in Table VII. This is a region of A near the minimum of the split $4s$ peak in the s strength function (see p. xxi of Ref. 15) as are the Gd isotopes for which we also have results.[10] The $\langle D_0\rangle$ values were essentially obtained from the Dyson-Mehta Δ statistic fit for the corrected s level sets as shown in Figs. 7(a)–7(e). Detailed discussions about the quoted uncertainties of $\langle D_0\rangle$ and $10^4 S_0$ are given in Refs. 1 and 4. Mughabghab and Chrien[16] obtain $10^4 S_0=(1.85\pm0.15)$, (2.0 ± 0.5), and (1.7 ± 0.2), from σ_{av} measurements over the energy intervals, 5 to 15 keV, for ^{161}Dy, ^{162}Dy, and ^{163}Dy. These agree with our values to within the combined uncertainties.

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