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Nucleon-nucleon correlations and the pion-nucleus interaction*

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The scattering of pions from a correlated pair of nucleons is studied in a recently proposed covariant theory of pion-nucleus scattering. In this theory the off-shell effects arising from the correlations can be treated unambiguously. In the covariant theory, "off-shell effects" refers to *both* the usual off-shell effects known from conventional nonrelativistic multiple-scattering theories and to *off-mass-shell* features particular to the relativistic theories. It is shown that these effects, absent in a "fixed-scatterer approximation," require knowledge of the fundamental pion-nucleon scattering amplitudes in regions which are not readily parametrized using on-shell scattering data. In a simplified model we illustrate this point by comparing our dynamical approach to the corresponding fixed-scatterer analysis. We conclude that any reliable estimate of correlation effects in pion-nucleus interaction requires that serious consideration be given to the off-shell aspects of the dynamics.

I. INTRODUCTION

The optical potential for pion-nucleus scattering has been intensively studied in recent years.[1] Having observed that most multiple scattering theories,[2-5] being based completely on nonrelativistic dynamics, are unsuited to treat either the scattering of a relativistic projectile from a nucleus or the scattering of a projectile which can be created or annihilated (as is the case for the pion), we have proposed a covariant approach to the problem in previous works.[6]

It is well known that in the standard theories the calculation of the first-order optical potential requires the knowledge of the single-nucleon density matrix and the calculation of the second-order optical potential requires the knowledge of the nucleon-nucleon correlation function. In this work, we apply our covariant approach to the evaluation of the irreducible kernel corresponding to the scattering of pions from a correlated pair of nucleons. In our approach, the four-momentum at each diagram vertex is a well-defined quantity; our analysis then shows that multiple scattering from correlated pairs leads to expressions containing highly off-shell scattering amplitudes. Some of these off-shell effects in multiple scattering theory have been discussed for nucleon-nucleus scattering in previous publications[7] using potential models and nonrelativistic dynamics. However, the covariant character of our method provides an unambiguous way of using relativistic dynamics to study these off-shell effects and also enables us to observe important new features which are totally ignored in the conventional multiple-scattering theory.

It is important to emphasize that the reference to *off-shell effects* in the covariant theory refers to more general off-shell aspects than those studied in nonrelativistic theories. The relativistic theories contain scattering amplitudes in which the particles are off their mass shells. Therefore these scattering amplitudes depend on a greater number of kinematical variables than the amplitudes which appear in the nonrelativistic theories. Indeed, the nonrelativistic amplitudes may be related to the relativistic amplitudes by restricting the variables which appear in the relativistic amplitudes. (For example, in the relative *four*-momenta, the zeroth component may be taken to be a specific function of the relative three-momentum.[6]) In a future publication we will show how the use of covariant amplitudes for off-mass-shell particles provides an improved treatment of various kinematical features in the case of pion-nucleus scattering.

We present the details of our covariant approach in the following section and discuss its relation to

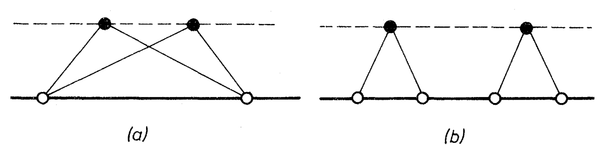

FIG. 1. (a) The double-scattering diagram where the dashed line represents the pion, the light and heavy lines represent respectively the nucleon and the various nuclei. The π-N (off-shell) scattering amplitudes are denoted by the filled circles. The vertex interactions are represented by the open circles. (b) The iteration of the single scattering diagram which must be subtracted from (a) because of the composite nature of the target.

the nonrelativistic scattering theory in Sec. III. In Sec. IV we illustrate the difference between our dynamical approach and the widely employed "fixed scatterer approximation" (FSA) via a model calculation.

In view of the large discrepancies between the results obtained from our dynamical approach and the FSA, we conclude that the off-shell effects are particularly marked in the case of the scattering of pions from correlated pairs. We conclude that the FSA is unreliable for low energy pion-nucleus scattering. Some new aspects of the off-shell effects brought to light by our covariant treatment are summarized in the last section.

II. SCATTERING OF PIONS FROM A CORRELATED PAIR OF NUCLEONS

Since we do not have a satisfactory theory of strong interactions, a discussion of pion-nucleus scattering will necessarily contain various phenomenological elements. In particular, parametrizations of the elementary off-shell pion-nucleon scattering amplitudes are necessary to implement the calculational scheme we propose. Models for these amplitudes are under investigation by several groups.

If we study the propagation of a pion in a nucleus from a space-time point of view, we may think of the pion propagating either forward or backward in time. The elementary pion-nucleon interactions involve either scattering or production (and absorption) amplitudes. If we restrict ourselves to the case in which the intermediate pions move only forward in time, we obtain a development involving only the aforementioned off-shell pion-nucleon scattering amplitudes. Thus it appears desirable, in a phenomenological approach, to clearly separate production processes from scattering processes. Such a separation has the consequence that we do not use a Feynman propagator for the pion between scattering events, but use only that part of the propagator that propagates the pion forward in time.

We now turn to the consideration of that contribution to the pion optical potential arising from the double scattering from a correlated pair of nucleons. The scattering of the pion from a pair of nucleons can be represented by the diagram of Fig. 1(a). Owing to the composite nature of the nucleus, this diagram also contains the iteration of the single scattering term, shown in Fig. 1(b), which in general must be subtracted explicitly. Clearly, the nuclear vertex in Fig. 1(a) contains many-body correlations. However, we will address our attention to an independent-pair approximation, appropriate to the study of the role of pair correlations in pion-nucleus interactions. In this case, we may approximate the complete nuclear vertex by a quasi-three-body model represented by the diagram in Fig. 2, where the upper vertex represents the short-range nucleon-nucleon interaction and the lower vertex describes the interaction between the center of mass of the dinucleon cluster and the residual nucleus (a two-hole state).

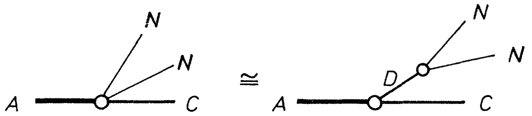

FIG. 2. The quasi-three-body model for the nuclear vertex. The nucleon, dinucleon system, residual nucleus, and the target nucleus are all treated as elementary particles and are denoted respectively by the letters N, D, C, and A.

It is worth noting that owing to the strong correlation contained in the upper vertex function, the momenta of the emerging nucleons are quite large, and therefore the explicit subtraction of Fig. 1(b) need not be made for the process under consideration, given our method of calculation. In other words, for the correlation effects we are studying, the difference of the two diagrams in Fig. 1 is now represented by a single diagram of Fig. 3.

We parametrize the S matrix for pion-nucleus scattering as

$$S = 1 - i2\pi M \tag{2.1}$$

and normalize the free pion state by

$$\langle \vec{k}', \alpha' | \vec{k}, \alpha \rangle = 2\omega_{\vec{k}}\delta(\vec{k}' - \vec{k})\delta_{\alpha'\alpha}, \tag{2.2}$$

where α' (α) are isospin labels. The state of any nucleus is normalized such that

$$\langle \vec{k}', s', n' | \vec{k}, s, n \rangle = \mathfrak{N}(\vec{k})\delta(\vec{k}' - \vec{k})\delta_{s's}\delta_{n'n}, \tag{2.3}$$

where $\mathfrak{N} = (E_{\vec{k}}/M)$ for fermions, and $\mathfrak{N} = 2E_{\vec{k}}$ for bosons. Also, s' (s) and n' (n) denote, respectively, spin labels and internal quantum numbers. Also, we have used the notations $\omega_{\vec{k}} = (\vec{k}^2 + M_\pi{}^2)^{1/2}$, $E_{\vec{k}} = (\vec{k}^2 + M^2)^{1/2}$. The invariant amplitude M corresponding to the second-order irreducible kernel indicated in Fig. 3 then has the following analytic

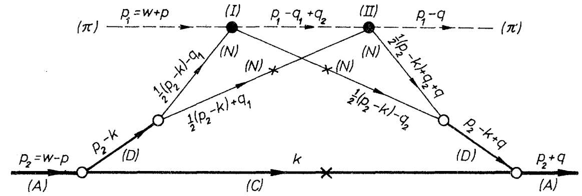

FIG. 3. The irreducible kernel representing the scattering of the pion from a correlated pair of nucleons (See caption of Fig. 1). The lines with a cross represent on-shell particles. The four-momentum W is defined by $W = (W, \vec{0})$.

expression[8]:

$$M_{\pi A}^{(a',a)}(p',p\,|\,2W) = -(2\pi i)^{-3}\tfrac{1}{2}A(A-1)\int d^4k\,d^4q_1 d^4q_2\,\bar{\xi}^{(a')}(\vec{p}_2+\vec{q})\langle \underline{A}_{p_2+q}\,|\,V_{CD}\,|\,D_{p_2-k+q};C_k\rangle$$
$$\times G_D(p_2-k+q)\langle D_{p_2-k+q}\,|\,V_{NN}\,|\,N_{\frac{1}{2}p_2-\frac{1}{2}k-q_2};N_{\frac{1}{2}p_2-\frac{1}{2}k+q_2+q}\rangle$$
$$\times G_N(\tfrac{1}{2}p_2-\tfrac{1}{2}k-q_2)G_N(\tfrac{1}{2}p_2-\tfrac{1}{2}k+q_2+q)G_C(k)$$
$$\times\langle \underline{\Pi}^{(\alpha)}_{p_1-q};N_{\frac{1}{2}p_2-\frac{1}{2}k+q_2+q}\,|\,M_{\pi N}(s_{\rm II})\,|\,\Pi^{(\beta)}_{p_1-q_1+q_2};N_{\frac{1}{2}p_2-\frac{1}{2}k+q_1}\rangle$$
$$\times G_\pi(p_1-q_1+q_2)\langle \Pi^{(\beta)}_{p_1-q_1+q_2};N_{\frac{1}{2}p_2-\frac{1}{2}k-q_2}\,|\,M_{\pi N}(s_{\rm I})\,|\,\underline{\Pi}^{(\alpha)}_{p_1};N_{\frac{1}{2}p_2-\frac{1}{2}k-q_1}\rangle$$
$$\times G_N(\tfrac{1}{2}p_2-\tfrac{1}{2}k-q_1)G_N(\tfrac{1}{2}p_2-\tfrac{1}{2}k+q_1)$$
$$\times\langle N_{\frac{1}{2}p_2-\frac{1}{2}k-q_1};N_{\frac{1}{2}p_2-\frac{1}{2}k+q_1}\,|\,V_{NN}\,|\,D_{p_2-k}\rangle G_D(p_2-k)\langle D_{p_2-k};C_k\,|\,V_{CD}\,|\,\underline{A}_{p_2}\rangle\xi^{(a)}(\vec{p}_2)\,. \tag{2.4}$$

In Eq. (2.4), for notational convenience, the on-shell "particles" are underlined. The superscripts are the isospin label of the pions or the spin labels of the nuclear spin wave function ξ. The isospin labels of the nucleons are not explicit but understood. (These will be reintroduced later when they become useful.) The factor $\frac{1}{2}A(A-1)$ is the number of nucleon pairs among A nucleons.

We now proceed to make a reduction of this amplitude. Our procedure will lead to a *covariant* expression which involves only three-dimensional integrations and has the same formal structure as that appearing in the nonrelativistic theory.

We note that for other than very light target nuclei, it is reasonable to neglect the off-shell aspect of the (two-hole state) residual nucleus C, and to write the corresponding propagator as

$$G_C(k)\simeq -2\pi i[\mathfrak{N}_C(\vec{k})]^{-1}\delta(k^0-E_{C,\vec{k}})\Lambda_C^{(+)}(\vec{k})\,, \tag{2.5}$$

with

$$\Lambda_C^{(+)}(\vec{k}) = \sum_c \chi^{(c)}(\vec{k})\bar{\chi}^{(c)}(\vec{k})\,. \tag{2.6}$$

Here, $\Lambda_C^{(+)}(\vec{k})$ is the projection operator for positive energy spinors. The bar over χ represents an appropriate "conjugation" which will provide an invariant normalization of the spinors. For example, in the spin-$\frac{1}{2}$ case, $\bar{\chi}=\chi^\dagger\gamma_0$. Clearly, such on-shell approximations reduce a four-dimensional integration to a three-dimensional one without destroying the invariance of the amplitude.

Taking into account the results of Eqs. (2.5) and (2.6), we now consider the following quantity appearing in Eq. (2.4):

$$Q_1^{(c)(a)} = G_D(p_2-k)\bar{\chi}^{(c)}(\vec{k})\langle D_{p_2-k},\underline{C}_k\,|\,V_{CD}\,|\,\underline{A}_{p_2}\rangle\xi^{(a)}(\vec{p}_2)\,. \tag{2.7}$$

This quantity can be expressed in terms of the variables in the *rest frame* of the initial target nucleus [see Fig. 4(a)] as follows:

$$(Q_1)_\alpha^{(c)(a)} = S_{\alpha\beta}^{(j_D)}(L_{A_1}^{-1})[G_D(W_A+K)]_{\beta\gamma}\bar{\chi}_\eta^{(c)}(W_A-K)$$
$$\times[\langle D_{W_A+K};\underline{C}_{W_A-K}\,|\,V_{CD}\,|\,\underline{A}_{2W_A}\rangle]_{\gamma\eta,\,\delta}\xi_\delta^{(a)}(\vec{0})\,. \tag{2.8}$$

Here $S^{(j)}(L_A)$ represents a nonsingular unitary operator which transforms a spin-j spinor. The four-momentum W_A has only a zeroth component which is defined to be half of the mass M_A of the nucleus A.

We consider correlations only in the S state of relative motion of the nucleons in the di-nucleon D. This system can be either in a singlet or in a triplet spin state. For the triplet state ($j_D=1$), we make the following approximation for $G_D(W_A+K)$:

$$[G_D(W_A+K)]_{\beta\gamma}\simeq[g_D(K\,|\,2W_A)]_{\beta\gamma}$$
$$\equiv\int\frac{d\vec{Q}^2}{-\vec{P}^2-\vec{Q}^2}\delta[(2W'_A-W_A+K)^2-M_D{}^2]$$
$$\times\,\theta(2W'_A-W_A+K^0)\left[-g_{\beta\gamma}+\frac{1}{M_D{}^2}(2W'_A-W_A+K)_\beta(2W'_A-W_A+K)_\gamma\right]$$
$$=\frac{\nu(\vec{K})}{2E_{D,\vec{K}}}\frac{[\Lambda_D(\vec{K})]_{\beta\gamma}}{(-\vec{P}^2-\vec{K}^2)/2m}\,, \tag{2.9}$$

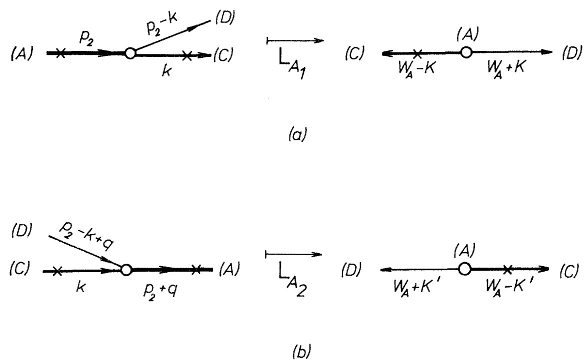

FIG. 4. The relation between the four-momenta in the center-of-mass frame of the pion-nucleus system and in the rest frame of (a) the initial target nucleus and (b) the final target nucleus.

where Λ_D represents the spin-1 projector which we write symbolically as

$$\Lambda_D(\vec{K}) = \sum_d \lambda^{(d)}(\vec{K})\bar{\lambda}^{(d)}(\vec{K}) \tag{2.10}$$

and

$$\nu(\vec{K}) = \frac{E_{C,\vec{K}}E_{D,\vec{K}}}{m(E_{C,\vec{K}}+E_{D,\vec{K}})},$$
$$m = M_C M_D/(M_C+M_D), \tag{2.11}$$

$$2W_A \equiv M_A = (-\vec{P}^2+M_C{}^2)^{1/2}+(-\vec{P}^2+M_D{}^2)^{1/2}, \tag{2.12}$$

and

$$2W'_A = (\vec{Q}^2+M_C{}^2)^{1/2}+(\vec{Q}^2+M_D{}^2)^{1/2}. \tag{2.13}$$

For the case of the singlet state ($j_D=0$) the corresponding expression for $g_D(K\,|\,2W_A)$ is similar to Eq. (2.9), with the Λ_D being replaced by 1. Substituting Eq. (2.9) back into Eq. (2.8), we obtain

$$\begin{aligned}(Q_1)_\alpha^{(c)(a)} &= \sum_d \lambda_\alpha^{(d)}(\vec{p}_2-\vec{k})\frac{\nu(\vec{K})}{2E_{D,\vec{K}}}\left(\frac{-\vec{P}^2-\vec{K}^2}{2m}\right)^{-1}\\ &\quad\times\{\bar{\lambda}_\gamma^{(d)}(\vec{K})\bar{\chi}_\eta^{(c)}(-\vec{K})[\langle D_{W_A+K};\underline{C}_{W_A-K}\,|\,V_{CD}\,|\underline{A}_{2W_A}\rangle]_{\gamma\eta,\,\delta}\,\xi_\delta^{(a)}(\vec{0})\}\\ &= \sum_d \lambda_\alpha^{(d)}(\vec{p}_2-\vec{k})\frac{\nu(\vec{K})}{2E_{D,\vec{K}}}\Psi^{(d)(c);\,(a)}(K),\end{aligned} \tag{2.14}$$

which in matrix notation is

$$Q_1^{(c)(a)} = \sum_d \lambda^{(d)}(\vec{p}_2-\vec{k})[\nu(\vec{K})/2E_{D,\vec{K}}]\Psi^{(d)(c);\,(a)}(K). \tag{2.15}$$

[Note that $K^0=W_A-(\vec{K}^2+M_c{}^2)^{1/2}$.] Similarly, for the following quantity appearing in Eq. (2.4),

$$Q_2^{(a')(c)} = \bar{\xi}^{(a')}(\vec{p}_2+\vec{q})\,\langle\underline{A}_{p_2+q}\,|\,V_{CD}\,|D_{p_2-k+q};\underline{C}_k\rangle\chi^{(c)}(\vec{k})G_D(p_2-k+q), \tag{2.16}$$

we have, after a similar reduction was made in the *rest frame* of the final target nucleus [see Fig. 4(b)],

$$\begin{aligned}Q_2^{(a')(c)} &= \sum_{d'}\frac{\nu(\vec{K}')}{2E_{D,\vec{K}'}}\left(\frac{-\vec{P}^2-\vec{K}'^2}{2m}\right)^{-1}\{\bar{\xi}^{(a')}(\vec{0})\,\langle\underline{A}_{2W_A}\,|\,V_{CD}\,|D_{W_A+K'};\underline{C}_{W_A-K'}\rangle\lambda^{(d')}(\vec{K}')\chi^{(c)}(-\vec{K}')\}\bar{\lambda}^{(d')}(\vec{p}_2-\vec{k}+\vec{q})\\ &= \sum_{d'}[\nu(\vec{K}')/2E_{D,\vec{K}'}]\Psi^{*(a');\,(d')(c)}(K')\bar{\lambda}^{(d')}(\vec{p}_2-\vec{k}+\vec{q}).\end{aligned} \tag{2.17}$$

[Note that $K'^0=W_A-(\vec{K}'^2+M_C{}^2)^{1/2}$.]

Equations (2.15) and (2.17) define, respectively, the relative wave functions Ψ and Ψ^* for the pair C-D in the rest frame of the target nucleus. They satisfy a covariant equation of motion which was constructed to be of the Schrödinger form. This wave function is a generalization of the standard nonrelativistic wave function. Since two of the three "particles" involved at each vertex [A-(CD)] are on shell, it follows that each of these wave functions depends only on one invariant. We choose it to be the relative four-momentum of the pair (CD), K and K' respectively, as in Eqs. (2.15) and (2.17). (Note that the values of K^0 and K'^0 are fixed by having the nucleus C on shell.)

Further reduction of the four-dimensional integrations in Eq. (2.4) can be achieved by putting two of the four nucleons (marked by crosses in Fig. 3) on the mass shell. To do this, we first consider the following quantity appearing in Eq. (2.4):

$$\begin{aligned}Q_3^{(d)} &= G_N(\tfrac{1}{2}p_2-\tfrac{1}{2}k+q_1)G_N(\tfrac{1}{2}p_2-\tfrac{1}{2}k-q_1)\\ &\quad\times\langle N_{\frac{1}{2}p_2-\frac{1}{2}k+q_1};N_{\frac{1}{2}p_2-\frac{1}{2}k-q_1}\,|\,V_{NN}\,|D_{p_2-k}\rangle\lambda^{(d)}(\vec{p}_2-\vec{k}),\end{aligned} \tag{2.18}$$

where the spin function $\lambda^{(d)}$ results from using Eq. (2.15). In terms of the variables defined in the *rest frame* of the initial D system, this quantity can be expressed as follows:

$$(Q_3)^{(d)}_{\alpha\lambda}=S^{(1/2)}_{\alpha\beta}(L_{D_1}{}^{-1})[G_N(W_D+\kappa)]_{\beta\gamma}S^{(1/2)}_{\lambda\mu}(L_{D_1}{}^{-1})[G_N(W_D-\kappa)]_{\mu\nu}[\langle N_{W_D+\kappa};N_{W_D-\kappa}|V_{NN}|D_{2W_D}\rangle]_{\gamma\nu,\epsilon}\lambda^{(d)}_\epsilon(\vec{0}). \tag{2.19}$$

Our reduction scheme then consists in using the following Green's function[6] to approximate the two nucleon propagators in Eq. (2.19):

$$\begin{aligned}G_N(W_D+\kappa)G_N(W_D-\kappa)&\simeq g(\kappa|2W_D)\\&=(-2\pi i)\delta[(W_D-\kappa)^2-M_N{}^2]\theta(W_D-\kappa^0)2M_N\Lambda^{(+)}_N(-\vec{k})\\&\quad\times\int\frac{d\vec{q}\,'^2}{-\vec{p}^2-\vec{q}\,'^2}\delta[(2W'_D-W_D+\kappa)^2-M_N{}^2]\theta(2W'_D-W_D+\kappa^0)[\gamma\cdot(2W'_D-W_D+\kappa)+M_N]\\&=(-2\pi i)\frac{M_N}{E_{N,\vec{k}}}\frac{\delta[\kappa^0-\Delta(\vec{k})]}{(-\vec{p}^2-\vec{k}^2)/M_N}\sum_r u^{(r)}(\vec{k})\bar{u}^{(r)}(\vec{k})\sum_s u^{(s)}(-\vec{k})\bar{u}^{(s)}(-\vec{k}),\end{aligned} \tag{2.20}$$

with

$$W_D=\tfrac{1}{2}M_D=(-\vec{p}^2+M_N{}^2)^{1/2}, \tag{2.21}$$

$$W'_D=(\vec{q}\,'^2+M_N{}^2)^{1/2}, \tag{2.22}$$

$$\Delta(\vec{k})=W_D-E_{N,\vec{k}}=\tfrac{1}{2}M_D-(\vec{k}^2+M_N{}^2)^{1/2}. \tag{2.23}$$

It is clear that by this reduction procedure the nucleon having the momentum $W_D-\kappa$ is now on the mass shell and all the spinors in Eq. (2.18) have positive energy. In addition, the presence of a new δ function in Eq. (2.20) will reduce one four-dimensional integration in Eq. (2.4) to a three-dimensional one. Using Eqs. (2.19) and (2.20), we obtain

$$\begin{aligned}(Q_3)^{(d)}_{\alpha\lambda}&=(-2\pi i)(M_N/E_{N,\vec{k}})\delta(\kappa^0-\Delta(\vec{k}))\sum_{r,s}S^{(1/2)}_{\alpha\beta}(L_{D_1}{}^{-1})u^{(r)}_\beta(\vec{k})u^{(s)}_\lambda(\tfrac{1}{2}\vec{p}_2-\tfrac{1}{2}\vec{k}+\vec{q}_1)\bar{u}^{(r)}_\gamma(\vec{k})\bar{u}^{(s)}_\nu(-\vec{k})\\&\quad\times[\langle N_{W_D+\kappa};\underline{N}_{W_D-\kappa}|V_{NN}|D_{2W_D}\rangle]_{\gamma\nu,\epsilon}\lambda^{(d)}_\epsilon(\vec{0})\left(\frac{M_N}{-\vec{p}^2-\vec{k}^2}\right)\\&=(-2\pi i)(M_N/E_{N,\vec{k}})\delta(\kappa^0-\Delta(\vec{k}))\sum_{r,s}S^{(1/2)}_{\alpha\beta}(L_{D_1}{}^{-1})u^{(r)}_\beta(\vec{k})u^{(s)}_\lambda(\tfrac{1}{2}\vec{p}_2-\tfrac{1}{2}\vec{k}+\vec{q}_1)\psi^{(r)(s);(d)}(2W_D,\kappa),\end{aligned} \tag{2.24}$$

or in a more compact notation

$$Q^{(d)}_3=(-2\pi i)(M_N/E_{N,\vec{k}})\delta(\kappa^0-\Delta(\vec{k}))\sum_{r,s}S^{(1/2)}_{\alpha\beta}(L_{D_1}{}^{-1})u^{(r)}(\vec{k})u^{(s)}(\tfrac{1}{2}\vec{p}_2-\tfrac{1}{2}\vec{k}+\vec{q}_1)\psi^{(r)(s);(d)}(2W_D,\kappa). \tag{2.25}$$

By applying the same reduction procedure in the rest frame of the final D system to the quantity related to the other $[(NN)\text{-}D]$ vertex function, we obtain

$$\begin{aligned}Q^{(d')}_4&=\bar{\lambda}^{(d')}(\vec{p}_2-\vec{k}+\vec{q})\langle D_{p_2-k+q}|V_{NN}|N_{\frac{1}{2}p_2-\frac{1}{2}k-q_2};N_{\frac{1}{2}p_2-\frac{1}{2}k+q_2+q}\rangle G_N(\tfrac{1}{2}p_2-\tfrac{1}{2}k-q_2)G_N(\tfrac{1}{2}p_2-\tfrac{1}{2}k+q_2+q)\\&=(-2\pi i)(M_N/E_{N,\vec{k}'})\delta(\kappa'^0-\Delta(\vec{k}'))\sum_{r',s'}\psi^{*(d');(r')(s')}(2W_D,\kappa')\,\bar{u}^{(r')}(\tfrac{1}{2}\vec{p}_2-\tfrac{1}{2}\vec{k}-\vec{q}_2)\bar{u}^{(s')}(\vec{k}')S^{(1/2)}(L_{D_2}{}^{-1}).\end{aligned} \tag{2.26}$$

Here the variable κ' is the relative momentum of the nucleon pair, as illustrated in Fig. 5(b), and $\Delta(\vec{k}')=W_D-E_{N,\vec{k}'}$. Equations (2.24) and (2.26) define the two-nucleon "defect functions," ψ and ψ^*. Since only one of the three "particles" in the correlation vertex is on the mass shell, it follows that the wave function so defined must depend on two invariants. We choose one of them to be the relative four-momentum of the pair and the other to be the square of the total four-momentum of the pair. The dependence on the second invariant is usually less significant than on the first invariant. Again, we emphasize that these wave functions satisfy a covariant equation of motion of Schrödinger form.

Combining all the formulas, we obtain from Eq.

(2.4) the following reduced invariant amplitude:

$$\hat{M}_{\pi A}^{(a',a)}(p',p\,|\,2W)=\tfrac{1}{2}A(A-1)\int d^3\vec{\mathrm{K}}\,d^3\vec{\kappa}\,d^3\vec{\kappa}'\sum_{r's'rs}\sum_{d'dc\beta}\left\{\Psi^{*(a');\,(d')(c)}(K')\left[\frac{\nu(\vec{\mathrm{K}}')}{\mathfrak{N}_C(\vec{\mathrm{K}}')2E_{D,\vec{\mathrm{K}}'}}\right]^{1/2}\right\}$$
$$\times\left\{\left(\frac{\nu(\vec{\mathrm{K}}')}{2E_{D,\vec{\mathrm{K}}'}}\right)^{1/2}\psi^{*(d');\,(r')(s')}(2W'_D,\kappa')\left(\frac{M_N}{E_{N,\vec{\kappa}'}}\right)\right\}\bar{u}^{(s')}(\vec{\kappa}')S^{(1/2)}(L_{D_2}{}^{-1})$$
$$\times\langle\underline{\Pi}^{(\alpha')}_{p_1-q};N_{\frac{1}{2}p_2-\frac{1}{2}k+q_2+q}\,|M_{\pi N}(s_{\mathrm{II}})|\,\Pi^{(\beta)}_{p_1-q_1+q_2};\underline{N}_{\frac{1}{2}p_2-\frac{1}{2}k+q_1}\rangle\,u^{(s)}(\tfrac{1}{2}\vec{\mathrm{p}}_2-\tfrac{1}{2}\vec{\mathrm{k}}+\vec{\mathrm{q}}_1)$$
$$\times[2\omega_{\vec{\mathrm{p}}_1-\vec{\mathrm{q}}_1+\vec{\mathrm{q}}_2}(p_1^0-q_1^0+q_2^0-\omega_{\vec{\mathrm{p}}_1-\vec{\mathrm{q}}_1+\vec{\mathrm{q}}_2}+i\epsilon)]^{-1}\,\bar{u}^{(r)}(\tfrac{1}{2}\vec{\mathrm{p}}_2-\tfrac{1}{2}\vec{\mathrm{k}}-\vec{\mathrm{q}}_2)$$
$$\times\langle\Pi^{(\beta)}_{p_1-q_1+q_2};\underline{N}_{\frac{1}{2}p_2-\frac{1}{2}k-q_2}\,|M_{\pi N}(s_{\mathrm{I}})|\,\underline{\Pi}^{(\alpha)}_{p_1};N_{\frac{1}{2}p_2-\frac{1}{2}k-q_1}\rangle\,S^{(1/2)}(L_{D_1}{}^{-1})u^{(r)}(\vec{\kappa})$$
$$\times\left\{\left(\frac{M_N}{E_{N,\vec{\kappa}}}\right)\psi^{(r)(s);\,(d)}(2W_D,\kappa)\left(\frac{\nu(\vec{\mathrm{K}})}{2E_{D,\vec{\mathrm{K}}}}\right)^{1/2}\right\}\left\{\left[\frac{\nu(\vec{\mathrm{K}})}{\mathfrak{N}_C(\vec{\mathrm{K}})2E_{D,\vec{\mathrm{K}}}}\right]^{1/2}\Psi^{(d)(c);\,(a)}(K)\right\}. \quad (2.27)$$

In keeping with our discussion at the beginning of this section, we use only the retarded part of the pion propagator in Eq. (2.27). We have also used the relation $d^4q_1d^4q_2=d^4\kappa\,d^4\kappa'$ and the fact that the values of the zeroth components k^0, κ^0, and κ'^0 were determined previously by the reductions. The four-momenta of the pions and the nucleons in Eq. (2.27) are therefore related to the integration variables $\vec{\mathrm{K}}$, $\vec{\kappa}$, and $\vec{\kappa}'$ by appropriate Lorentz transformations (see Figs. 4 and 5). Owing to the large masses of the nuclei A, C, and D, these Lorentz transformations are very close to the corresponding Galilean transformations. In other words, the vectors $\vec{\mathrm{K}}$, $\vec{\mathrm{K}}'$, $\vec{\kappa}$, and $\vec{\kappa}'$ are very close to those calculated with nonrelativistic kinematics. [$\vec{\kappa}\simeq\vec{\mathrm{q}}_1$, $\vec{\kappa}'\simeq\vec{\mathrm{q}}_2+\frac{1}{2}\vec{\mathrm{q}}$, $\vec{\mathrm{K}}\simeq(A-2)\vec{\mathrm{p}}_2/A-\vec{\mathrm{k}}$, and $\vec{\mathrm{K}}'\simeq(A-2)\times(\vec{\mathrm{p}}_2+\vec{\mathrm{q}})/A-\vec{\mathrm{k}}$.]

In Eq. (2.27), the pion-nucleon scattering amplitudes as well as the wave functions all involve off-shell quantities. In our covariant approach, the *invariant energy* associated with each process is unambiguously defined. For example, we have $s_{\mathrm{I}}=(p_1+\frac{1}{2}p_2-\frac{1}{2}k-q_1)^2$ and $s_{\mathrm{II}}=(p_1+\frac{1}{2}p_2-\frac{1}{2}k+q_2)^2$. In this respect, the notation $M_{\pi N}(s_{\mathrm{I}})$ and $M_{\pi N}(s_{\mathrm{II}})$ is redundant. However, we adopt this notation so as to facilitate the comparison of our analysis with that based on the FSA.

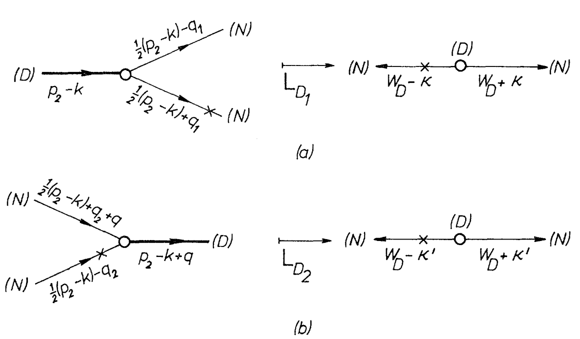

FIG. 5. The relation between the four-momenta in the center-of-mass frame of the pion-nucleus system and in the rest frame of (a) the initial dinucleon system and (b) the final dinucleon system.

III. RELATION TO NONRELATIVISTIC SCATTERING THEORY

For simplicity, we discuss forward scattering, $q=0$, and consider the case where the nuclei A, C, and D are all of spin zero. Therefore, the factors $[\mathfrak{N}_A(\vec{\mathrm{p}})]^{-1/2}$ and $[\mathfrak{N}_C(\vec{\mathrm{k}})]^{-1/2}$, etc., are equal to $(2E_{A,\vec{\mathrm{p}}})^{-1/2}$ and $(2E_{C,\vec{\mathrm{k}}})^{-1/2}$, etc., respectively. For the forward scattering, a simple parametrization of the invariant amplitude is obtained in the laboratory frame, which now coincides with the rest frames of both the initial and final nuclei A. (See Fig. 6.)

The magnitude of the momentum $\vec{\mathrm{K}}$ is small compared with that of the relative momentum $\vec{\kappa}$ of the correlated pair. Consequently, we neglect the dependence of the πN amplitude on $\vec{\mathrm{K}}$ and separate the $d^3\vec{\mathrm{K}}$ integration from the rest of the expression. With the following normalization of the wave function Ψ

$$\int d^3\vec{\mathrm{K}}\nu(\vec{\mathrm{K}})(2E_{D,\vec{\mathrm{K}}}2E_{C,\vec{\mathrm{K}}}2M_A)^{-1}\Psi^*(\vec{\mathrm{K}})\Psi(\vec{\mathrm{K}})=1\,, \quad (3.1)$$

we can rewrite the forward scattering amplitude

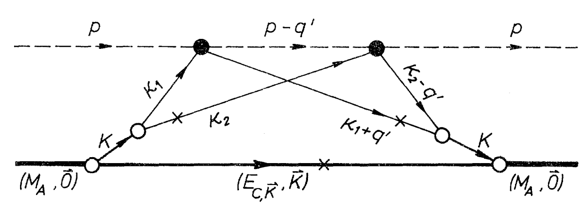

FIG. 6. Forward scattering of pions by a correlated pair in a finite nuclear system (see captions for Figs. 1 to 3). Here $q'=(q'^0,\vec{\mathrm{q}}')$, $p=(\omega_{\vec{\mathrm{p}}},\vec{\mathrm{p}})$, $K=(M_A-E_{C,\vec{\mathrm{K}}},-\vec{\mathrm{K}})$, $\kappa_1=(K^0-E_{N,\vec{\kappa}},\vec{\kappa})$, and $\kappa_2=(E_{N,\vec{\kappa}},-\vec{\kappa})$.

as follows:

$$
\begin{aligned}
\hat{M}_{\pi A}(p,p\,|\,2W) = {} & \tfrac{1}{2}A(A-1)\int d^3\vec{q}\,'d^3\vec{\kappa}\sum_{\beta i j i' j'}\sum_{r s r' s'}(2M_A)g_{(i',j')}\tilde{\psi}^{*(r',i')(s',j')}(\vec{\kappa}+\vec{q}\,')(M_N/E_{N,\vec{\kappa}+\vec{q}\,'})^{1/2}\bar{u}^{(s')}(-\vec{\kappa}-\vec{q}\,') \\
& \times\langle \underline{\Pi}_p^{(\alpha)};N_{\kappa_2-q'}^{(j')}\,|M_{\pi N}(s_{\rm II})\,|\,\Pi_{p-q'}^{(\beta)};\underline{N}_{\kappa_2}^{(j)}\rangle\, u^{(s)}(-\vec{\kappa})(M_N/E_{N,\vec{\kappa}})^{1/2}(2\omega_{\vec{p}-\vec{q}\,'})^{-1} \\
& \times(\omega_{\vec{p}}-\Delta_M-E_{N,\vec{\kappa}}-E_{N,\vec{\kappa}+\vec{q}\,'}+2M_N-\omega_{\vec{p}-\vec{q}\,'}+i\epsilon)^{-1}(M_N/E_{N,\vec{\kappa}+\vec{q}\,'})^{1/2}\bar{u}^{(r')}(\vec{\kappa}+\vec{q}\,') \\
& \times\langle \Pi_{p-q'}^{(\beta)};\underline{N}_{\kappa_1+q'}^{(i')}\,|M_{\pi N}(s_{\rm I})\,|\,\underline{\Pi}_p^{(\alpha)};N_{\kappa_1}^{(i)}\rangle\, u^{(r)}(\vec{\kappa})(M_N/E_{N,\vec{\kappa}})^{1/2}g_{(i,j)}\tilde{\psi}^{(r,i)(s,j)}(\vec{\kappa})\,,
\end{aligned}
\tag{3.2}
$$

with

$$
\begin{aligned}
s_{\rm I} &= (p+\kappa_1)^2\,, \\
s_{\rm II} &= (p-q'+\kappa_2)^2\,,
\end{aligned}
\tag{3.3}
$$

and

$$\kappa_1 = (M_A - M_C - E_{N,\vec{\kappa}}, \vec{\kappa})\,,$$

$$\kappa_2 = (E_{N,\vec{\kappa}}, -\vec{\kappa})\,.$$

In Eq. (3.2) the nucleon isospin labels are explicitly indicated, as are the isospin coupling coefficients $g_{(i',j')}$ and $g_{(i,j)}$ describing the coupling of the nucleon pair in the dinucleon state. In addition, we used the following relation in writing the denominator of the pion propagator:

$$q'^0 = -M_A + M_C + E_{N,\vec{\kappa}} + E_{N,\vec{\kappa}+\vec{q}\,'}\,, \tag{3.4}$$

$$q'^0 \equiv \Delta_M - 2M_N + E_{N,\vec{\kappa}} + E_{N,\vec{\kappa}+\vec{q}\,'}\,. \tag{3.5}$$

Equation (3.4) is given by the conservation of the zeroth component of the four-momentum, while Eq. (3.5) defines the mass defect Δ_M.

The result of the FSA can now be obtained from the more general expression, Eq. (3.2). Apart from the different multiplicative factors arising from the use of noninvariant normalization for states, in the FSA the momenta of the struck nucleous are set equal to zero; also, the binding energy of the nucleons is ignored. Further, *each* of the intermediate nucleons appearing in the diagram is placed on its mass shell; it is then easy to see that energy conservation at each internal process is no longer possible if we allow momentum transfer to the struck nucleons. Because of the limitations imposed by the FSA, the total energy associated with each πN scattering processes can no longer be determined by first principles and in fact these are arbitrarily chosen to have the values corresponding to the incoming pion impinging on a motionless free nucleon. We shell denote the energies so determined by $\bar{s}$. Consequently, the forward scattering amplitude in FSA is given by

$$
\begin{aligned}
\hat{M}_{\pi A}^{\rm FSA}(p,p\,|\,2W) = {} & \tfrac{1}{2}A(A-1)\int d^3\vec{q}\,'\sum_{\beta i j i' j'}\sum_{r s r' s'}g_{(i',j')}g_{(i,j)} \\
& \times(2M_A)(M_N/E_{N,\vec{q}\,'})\langle \underline{\Pi}_{\vec{p}}^{(\alpha)};\underline{N}_{-\vec{q}\,'}^{(s',j')}\,|M_{\pi N}(\bar{s})\,|\,\Pi_{\vec{p}-\vec{q}\,'}^{(\beta)};\underline{N}_{\vec{0}}^{(s,j)}\rangle \\
& \times\frac{(\omega_{\vec{p}}-q'^0+\omega_{\vec{p}-\vec{q}\,'})/2\omega_{\vec{p}-\vec{q}\,'}}{(\omega_{\vec{p}}-q'^0)^2-\omega_{\vec{p}-\vec{q}\,'}^2+i\epsilon}\langle \Pi_{\vec{p}-\vec{q}\,'}^{(\beta)};\underline{N}_{\vec{q}\,'}^{(r',i')}\,|M_{\pi N}(\bar{s})\,|\,\underline{\Pi}_{\vec{p}}^{(\alpha)};\underline{N}_{\vec{0}}^{(r,i)}\rangle\, C_{\rm corr}^{(r,i;s,j)(r',i';s',j')}(\vec{q}\,')\,,
\end{aligned}
\tag{3.6}
$$

where $C_{\rm corr}(\vec{q}\,')$ is a pair correlation function and

$$\bar{s} = (\omega_{\vec{p}} + M_N)^2 - \vec{p}^{\,2}\,. \tag{3.7}$$

The quantity q'^0, which is inherited from our covariant approach and represents the energy part of the relativistic four-momentum transfer to the pion, loses its meaning in the potential scattering theory within the framework of which the FSA is originally formulated. Henceforth, we set it to be zero. Indeed, with $q'^0 = 0$, Eq. (3.6) becomes identical, in the nonrelativistic limit for the kinematic factors, with the FSA amplitude used in the literature.

After this discussion of the approximations leading to the FSA expression for the πA amplitude, we return to a more general discussion of the result of our dynamical analysis and compare that result to other schemes. In the nonrelativistic limit, we use in Eq. (3.2), $(M_N/E_{N,\vec{\kappa}}) \simeq (M_N/E_{N,\vec{\kappa}+\vec{q}\,'}) \simeq 1$ and $(2M_N - E_{N,\vec{\kappa}} - E_{N,\vec{\kappa}+\vec{q}\,'}) \simeq -\vec{\kappa}^2/2M_N - (\vec{\kappa}+\vec{q}\,')^2/2M_N$. We see that there is now a correspondence between Eq. (3.2) and the Goldstone diagram shown in Fig. 7(a). Also, if we apply our covariant analysis to the Feynman diagram obtained from the diagram in Fig. 3 by crossing the external pion lines and again keep only the retarded part of the pion propagator, we then, in the nonrelativistic limit for the kinematics, obtain a result corre-

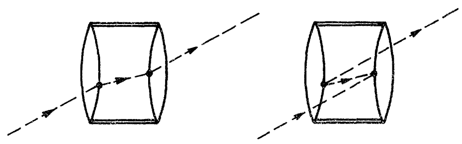

FIG. 7. Goldstone diagrams representing the scattering of pions from a correlated nucleon pair in nuclear matter.

sponding to that given by the other "Goldstone" diagram shown in Fig. 7(b). It is possibly worth remarking on how the Goldstone diagrams of Fig. 7 could be generated. One procedure would involve the construction of a pseudointeraction which, when calculated in lowest order, would generate the pion-nucleon scattering and production amplitudes. In Fig. 7 the heavy black dots would then represent these amplitudes while the horizontal double lines would be nucleon-nucleon reaction matrices calculated in the nuclear medium. We may criticize this procedure on several grounds, probably the most serious criticism being the general ambiguity as to the specification of the off-shell aspects of the phenomenological amplitudes when they are inserted in diagrams such as those of Fig. 7. However, when these diagrams are generated via the procedure we have described, the off-shell aspects of the pion-nucleon invariant amplitudes are clearly defined. It is possible to show that the diagram of Fig. 7(a) can be given a precise meaning in a nonrelativistic theory of pion-nucleus interactions. Indeed, it will correspond to the influence of correlations in the target on the second term of the multiple-scattering series for the optical potential. The process shown in Fig. 7(b) does not appear as a proper Goldstone diagram unless we introduce production amplitudes in the Hamiltonian and, as noted above, that procedure is ambiguous.

In the nonrelativistic theory of nucleon-nucleus scattering we usually work with potential models of the interaction. Scattering amplitudes may then be introduced through the summation of ladders of potential interactions between the particles. In this approach the energy variable appearing in the scattering amplitudes is obtained unambiguously as in the energy denominator arising from a diagram such as that of Fig. 7(a). Similar potential models of pion-nucleon scattering are presently under intensive investigation and these nonrelativistic models have been used in the calculation of pion-nucleus scattering. To the extent that these nonrelativistic potential models are reasonable, we may discuss the correspondence of our relativistic theory and the nonrelativistic analysis. It is the correspondence with the nonrelativistic theory which provides some justification of the procedure we have used to evaluate the Feynmann diagrams of Figs. 3 or 6. Indeed, referring to these figures, we see that we have chosen to put some of the particles internal to the diagram on their mass shells in a particular manner. Other choices for the reductions of the vertex functions of the correlated pair are possible. For example, in Fig. 3 (or Fig. 6) the particle entering the first filled circle and that leaving the last filled circle could have been placed on its mass shell. Still another reduction, such as that used by Blankenbecler and Sugar, would have *neither* of the particles of the correlated pair on their mass shells. Each of these possible reduction schemes leads to quite different values of q'^0 and the invariant energies s. Correspondingly, the energy denominator of the pion propagator and the dynamic off-shell aspects of the πN scattering process will differ depending upon the reduction procedure used. The values for these quantities obtained using the method of this paper are given in Eqs. (3.3) and (3.4) for the forward scattering case. It can be shown that the first of the above-mentioned two other possible reduction schemes will produce s values of πN scattering processes completely irrelevant to the binding of nucleon, while the second scheme yields a pion propagator which has no correspondence to the Goldstone diagram of nonrelativistic many-body theory.

As remarked previously, our procedure for placing particles on their mass shell in the evaluation of the Feynman diagrams reproduces energy denominators, Eq. (3.4), and scattering amplitudes having the same off-shell structure as those obtained from the nonrelativistic theory. This correspondence between the diagrams of the relativistic theory is limited to only a few diagrams, since the nonrelativistic potential models do not admit production processes such as those required for the evaluation of Fig. 7(b). However, the correspondence we have exhibited gives us confidence that the reduction scheme we have adopted is sensible. In the next section we introduce an elementary model for some pion-nucleon off-shell amplitudes in order that we may compare the dynamical approach with the FSA analysis.

IV. COMPARISON OF THE FSA AND THE DYNAMICAL APPROACH VIA A MODEL CALCULATION

For the sake of illustration it is useful to choose some model for the correlation function and for the off-shell pion-nucleon amplitudes. Given such a model we can compare the FSA with our dynamical analysis. One example which may be treated relatively simply is the case in which the pion-

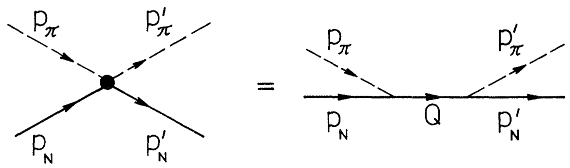

FIG. 8. Elementary model for the pion-nucleon scattering amplitude based on the nucleon pole diagram.

nucleon scattering is in the channel which may contain a single nucleon. In this case we may attempt to approximate the full amplitude by the nucleon pole term. (See Fig. 8; for simplicity we neglect the corresponding crossed diagrams.)

Limiting ourselves to this simple model, we can argue that the correlations introduce large values for κ and therefore the invariant energy in the π-N system, s_{I} of Eq. (3.3), has values that can be in the vicinity of the pole at $s_{\mathrm{I}} = M_N^2$. Thus we expect large differences between the FSA [cf. Eq. (3.6)] and the dynamical approach. Similar large differences would be expected in other π-N channels and could be elucidated by making models of the off-shell amplitudes in those channels.

In our simple model, we write the pion-*nucleon* scattering amplitudes of Eq. (3.2) using four-momenta defined by $\kappa_1 = (M_A - M_C - E_{N,\vec{\kappa}}, \vec{\kappa})$ and $\kappa_2 = (E_{N,\vec{\kappa}}, -\vec{\kappa})$, as follows:

$$\bar{u}^{(r')}(\vec{\kappa}+\vec{q}\,')\langle \Pi^{(\beta)}_{p-q'}; \underline{N}^{(i')}_{\kappa_1+q'} | M_{\pi N}(s_I) | \underline{\Pi}^{(\alpha)}_{p}; N^{(i)}_{\kappa_1}\rangle u^{(r)}(\vec{\kappa})$$

$$= \bar{u}^{(r')}(\vec{\kappa}+\vec{q}\,') v[(p-\kappa_1-2q')^2; (p-q')^2, M_N^2]\, G\, \frac{\gamma_5[\gamma\cdot(p+\kappa_1)+M_N]\gamma_5}{(p+\kappa_1)^2 - M_N^2 + i\epsilon}\, G v[(p-\kappa_1)^2; M_\pi^2, \kappa_1^2]$$

$$\times\{\chi^{\dagger(i')}(\hat{\phi}^{*(\beta)}\cdot\vec{\tau}_{\mathrm{I}})(\vec{\tau}_{\mathrm{I}}\cdot\hat{\phi}^{(\alpha)})\chi^{(i)}\}u^{(r)}(\vec{\kappa}) \qquad (4.1)$$

and

$$\bar{u}^{(s')}(-\vec{\kappa}-\vec{q}\,')\langle \underline{\Pi}^{(\alpha)}_{p}; N^{(j')}_{\kappa_2-q'} | M_{\pi N}(s_{\mathrm{II}}) | \Pi^{(\beta)}_{p-q'}; \underline{N}^{(j)}_{\kappa_2}\rangle u^{(s)}(-\vec{\kappa})$$

$$= \bar{u}^{(s')}(-\vec{\kappa}-\vec{q}\,') v[(p-\kappa_2+q')^2; M_\pi^2, (\kappa_2-q')^2]\, G\, \frac{\gamma_5[\gamma\cdot(p-q'+\kappa_2)]\gamma_5}{(p-q'+\kappa_2)^2 - M_N^2 + i\epsilon}\, G v[(p-q'-\kappa_2)^2; (p-q')^2, M_N^2]$$

$$\times\{\chi^{\dagger(j')}(\hat{\phi}^{*(\alpha)}\cdot\vec{\tau}_{\mathrm{II}})(\vec{\tau}_{\mathrm{II}}\cdot\hat{\phi}^{(\beta)})\chi^{(j)}\}, \qquad (4.2)$$

where $(\hat{\phi}^*\cdot\vec{\tau})$ and $(\vec{\tau}\cdot\hat{\phi})$ are the isotopic factors. In Eqs. (4.1) and (4.2), G^2 is related to the renormalized pion-nucleon coupling constant.[9] The v's are the phenomenological form factors. Using p_π and p_N to denote respectively the four-momentum of the pion and the nucleon, we can construct a Yamaguchi-type form factor in the following covariant form:

$$v[(p_\pi - p_N)^2; p_\pi^2, p_N^2] = \frac{\lambda^2 + t_0'}{\lambda^2 + (st')/M_N^2}\,. \qquad (4.3)$$

Here $s = (p_\pi + p_N)^2$, t' is a new invariant defined by

$$t'[(p_\pi - p_N)^2; p_\pi^2, p_N^2] = \frac{(p_\pi\cdot p_N)^2 - p_\pi^2 p_N^2}{(p_\pi + p_N)^2} \qquad (4.4)$$

and $t_0' = M_\pi^2(M_\pi^2/4M_N^2 - 1)$ is the value of t' corresponding to the specification $(p_\pi + p_N)^2 = p_N^2 = M_N^2$ and $p_\pi^2 = M_\pi^2$. From Eq. (4.4) we see that the invariant t' reduces to the square of the pion-nucleon relative three-momentum in the c.m. frame of the pion-nucleon system. The form factor v in Eq. (4.3) depends on three Lorentz invariants, say $(p_\pi\cdot p_N)$, p_π^2, and p_N^2, in the case when all three particles are off the mass shell.

Again for the sake of illustration, we may assume that the correlated pair (or the dinucleon D) is in a spin singlet ($S=0$) and isospin triplet ($T=1$) state. The wave function in Eq. (3.2) can then be expressed as:

$$\tilde{\psi}^{*(r',i')(s',j')}(\vec{\kappa}+\vec{q}\,')\tilde{\psi}^{(r,i)(s,j)}(\vec{\kappa})$$

$$= \sum_{M_T} C^{\frac{1}{2}\,\frac{1}{2}\,1}_{i'\,j'\,M_T}\, C^{\frac{1}{2}\,\frac{1}{2}\,1}_{i\,j\,M_T}\, C^{\frac{1}{2}\,\frac{1}{2}\,0}_{r'\,s'\,0}\, C^{\frac{1}{2}\,\frac{1}{2}\,0}_{r\,s\,0}$$

$$\times\tilde{\psi}^{*S=0}(\vec{\kappa}+\vec{q}\,')\tilde{\psi}^{S=0}(\vec{\kappa})\,. \qquad (4.5)$$

Accordingly, the isospin coupling factors in Eq. (3.2) are given by $g_{(i,j)} = g_{(i',j')} = C^{1\;1\;0}_{M_T\,-M_T\,0}$ for an isospin-zero target nucleus A. By introducing Eqs. (4.1)–(4.5) into Eq. (3.2), we finally obtain the following forward scattering amplitude:

$$\hat{M}_{\pi A}(p,p|2W)=\tfrac{1}{2}A(A-1)\int d^3\vec{q}\,'d^3\vec{\kappa}(2M_A)\left\{\sum_{M_T}[C^{1\ \ 1\ 0}_{M_T-M_T\ 0}]^2\langle 1,M_T|(\hat{\phi}^{*(\alpha)}\cdot\vec{\tau}_{\rm II})(\vec{\tau}_{\rm II}\cdot\vec{\tau}_{\rm I})(\vec{\tau}_{\rm I}\cdot\hat{\phi}^{(\alpha)})|1,M_T\rangle\right\}$$
$$\times\left(\frac{E_{N,\vec{\kappa}+\vec{q}\,'}+M_N}{2E_{N,\vec{\kappa}+\vec{q}\,'}}\right)\langle(\tfrac{1}{2},\tfrac{1}{2})00|[A_{\rm II}(p,\kappa_2,q')+i\vec{\sigma}_{\rm II}\cdot\vec{\rm B}_{\rm II}(p,\kappa_2,q')]$$
$$\times[A_{\rm I}(p,\kappa_1,q')+i\vec{\sigma}_{\rm I}\cdot\vec{\rm B}_{\rm I}(p,\kappa_1,q')]|(\tfrac{1}{2},\tfrac{1}{2})00\rangle$$
$$\times\left(\frac{E_{N,\vec{\kappa}}+M_N}{2E_{N,\vec{\kappa}}}\right)\tilde{\psi}^{*S=0}(\vec{\kappa}+\vec{q}\,')\tilde{\psi}^{S=0}(\vec{\kappa})G^4[s_{\rm I}-M_N^2+i\epsilon]^{-1}[s_{\rm II}-M_N^2+i\epsilon]^{-1}$$
$$\times v[(p-\kappa_2+q')^2;M_\pi^2,(\kappa_2-q')^2]\,v[(p-q'-\kappa_2)^2;(p-q')^2,M_N^2]$$
$$\times v[(p-\kappa_1-2q')^2;(p-q')^2,M_N^2]\,v[(p-\kappa_1)^2;M_\pi^2,\kappa_1^2]$$
$$\times(2\omega_{\vec{p}-\vec{q}\,'})^{-1}[\omega_{\vec{p}}+2M_N-\Delta_M-E_{N,\vec{\kappa}}-E_{N,\vec{\kappa}+\vec{q}\,'}-\omega_{\vec{p}-\vec{q}\,'}+i\epsilon]^{-1},\qquad(4.6)$$

where

$$A_{\rm I}(p,\kappa_1,q')\equiv(M_N-Q^0_{\rm I})+\frac{(\vec{\kappa}+\vec{q}\,')\cdot(\vec{p}+\vec{\kappa})}{E_{N,\vec{\kappa}+\vec{q}\,'}+M_N}+\frac{(\vec{p}+\vec{\kappa})\cdot\vec{\kappa}}{E_{N,\vec{\kappa}}+M_N}-\frac{(M_N+Q^0_{\rm I})[(\vec{\kappa}+\vec{q}\,')\cdot\vec{\kappa}]}{(E_{N,\vec{\kappa}+\vec{q}\,'}+M_N)(E_{N,\vec{\kappa}}+M_N)},\qquad(4.7)$$

$$A_{\rm II}(p,\kappa_2,q')\equiv(M_N-Q^0_{\rm II})-\frac{(\vec{\kappa}+\vec{q}\,')\cdot(\vec{p}-\vec{q}\,'-\vec{\kappa})}{E_{N,\vec{\kappa}+\vec{q}\,'}+M_N}-\frac{(\vec{p}-\vec{q}\,'-\vec{\kappa})\cdot\vec{\kappa}}{E_{N,\vec{\kappa}}+M_N}-\frac{(M_N+Q^0_{\rm II})[(\vec{\kappa}+\vec{q}\,')\cdot\vec{\kappa}]}{(E_{N,\vec{\kappa}+\vec{q}\,'}+M_N)(E_{N,\vec{\kappa}}+M_N)},\qquad(4.8)$$

$$\vec{\rm B}_{\rm I}(p,\kappa_1,q')\equiv\frac{(\vec{\kappa}+\vec{q}\,')\times(\vec{p}+\vec{\kappa})}{E_{N,\vec{\kappa}+\vec{q}\,'}+M_N}+\frac{(\vec{p}+\vec{\kappa})\times\vec{\kappa}}{E_{N,\vec{\kappa}}+M_N}-\frac{(M_N+Q^0_{\rm I})[(\vec{\kappa}+\vec{q}\,')\times\vec{\kappa}]}{(E_{N,\vec{\kappa}+\vec{q}\,'}+M_N)(E_{N,\vec{\kappa}}+M_N)},\qquad(4.9)$$

$$\vec{\rm B}_{\rm II}(p,\kappa_2,q')\equiv-\frac{(\vec{\kappa}+\vec{q}\,')\times(\vec{p}-\vec{q}\,'-\vec{\kappa})}{E_{N,\vec{\kappa}+\vec{q}\,'}+M_N}-\frac{(\vec{p}-\vec{q}\,'-\vec{\kappa})\times\vec{\kappa}}{E_{N,\vec{\kappa}}+M_N}-\frac{(M_N+Q^0_{\rm II})[(\vec{\kappa}+\vec{q}\,')\times\vec{\kappa}]}{(E_{N,\vec{\kappa}+\vec{q}\,'}+M_N)(E_{N,\vec{\kappa}}+M_N)},\qquad(4.10)$$

and

$$s_{\rm I}=(p+\kappa_1)^2,\quad s_{\rm II}=(p-q'+\kappa_2)^2,\quad Q^0_{\rm I}\equiv\omega_{\vec{p}}+2M_N-\Delta_M-E_{N,\vec{\kappa}},\quad Q^0_{\rm II}\equiv\omega_{\vec{p}}+2M_N-\Delta_M-E_{N,\vec{\kappa}+\vec{q}\,'}.\qquad(4.11)$$

In Eq. (4.6), the quantity inside the curly bracket represents the isospin average and is equal to $\frac{1}{3}$. The spin matrix element is simply $(A_{\rm II}A_{\rm I}+\vec{\rm B}_{\rm II}\cdot\vec{\rm B}_{\rm I})$. We note further that in the pionic-atom limit, $\vec{p}\simeq 0$ and $\omega_{\vec{p}}\simeq M_\pi$, Eqs. (4.7)–(4.10), and the form factors v in Eq. (4.6) all have very simple expressions.

Since the FSA essentially reduces the dynamics to a potential scattering problem, there are therefore various ambiguities in passing from our dynamical result to the FSA result. Adopting the scheme outlined in Sec. III we obtain (after the isospin average) the invariant FSA amplitude

$$\hat{M}^{\rm FSA}_{\pi A}(p,p|2W)=\tfrac{1}{6}A(A-1)\int d^3\vec{q}\,'G^4(\bar{s}-M_N^2+i\epsilon)^{-2}(2M_A)$$
$$\times[\tilde{A}_{\rm II}(p,\bar{\kappa}_2,\bar{q}')\tilde{A}_{\rm I}(p,\bar{\kappa}_1,\bar{q}')+\vec{\tilde{\rm B}}_{\rm II}(p,\bar{\kappa}_2,\bar{q}')\cdot\vec{\tilde{\rm B}}_{\rm I}(p,\bar{\kappa}_1,\bar{q}')]$$
$$\times\frac{(\omega_{\vec{p}}+\omega_{\vec{p}-\vec{q}\,'})/2\omega_{\vec{p}-\vec{q}\,'}}{\omega_{\vec{p}}^2-(\vec{p}-\vec{q}\,')^2-M_\pi^2+i\epsilon}\,v[(p-\bar{\eta}_2)^2;M_\pi^2,M_N^2]\,v[(p-\bar{q}'-\bar{\kappa}_2)^2;(p-\bar{q}')^2,M_N^2]$$
$$\times v[(p-\bar{q}'-\bar{\eta}_1)^2;(p-\bar{q}')^2,M_N^2]\,v[(p-\bar{\kappa}_1)^2;M_\pi^2,M_N^2]\,C^{S=0}_{\rm corr}(\vec{q}\,'),\qquad(4.12)$$

where we have introduced the new notation $\bar{\kappa}_1=\bar{\kappa}_2=(M_N,\vec{0})$, $\bar{\eta}_1=(E_{N,\vec{q}\,'},\vec{q}\,')$, $\bar{\eta}_2=(E_{N,\vec{q}\,'},-\vec{q}\,')$, $\bar{q}'=(0,\vec{q}\,')$, and $\bar{s}=(p+\bar{\kappa}_1)^2$. These quantities are, respectively, the counterpart of κ_1, κ_2, κ_1+q', κ_2-q', q', and $s_{\rm I}$ (and $s_{\rm II}$) of the dynamical approach. The functions $\tilde{A}$ and $\tilde{B}$ are defined as follows:

$$\tilde{A}_{\rm I}(p,\bar{\kappa},\bar{q}')\equiv-p^0+(\vec{p}\cdot\vec{q}\,')/(E_{N,\vec{q}\,'}+M_N),\qquad(4.13)$$

$$\tilde{A}_{\rm II}(p,\bar{\kappa},\bar{q}')\equiv-p^0-(\vec{p}-\vec{q}\,')\cdot\vec{q}\,'/(E_{N,\vec{q}\,'}+M_N),\qquad(4.14)$$

$$\vec{\tilde{\rm B}}_{\rm I}(p,\bar{\kappa},\bar{q}')\equiv\vec{q}\,'\times\vec{p}/(E_{N,\vec{q}\,'}+M_N),\qquad(4.15)$$

$$\vec{\tilde{\rm B}}_{\rm II}(p,\bar{\kappa},\bar{q}')\equiv-\vec{q}\,'\times\vec{p}/(E_{N,\vec{q}\,'}+M_N).\qquad(4.16)$$

The form factors of Eq. (4.12) are still defined by Eq. (4.3). In particular, in the pionic-atom limit, where the magnitude of the pion momentum $|\vec{p}|$ can be treated as small compared to the masses of the particles involved, we have $\bar{s} \cong (M_\pi + M_N)^2$, and the pion propagator is approximately equal to $1/(\vec{q}'^2)$. Further, the quantity $C_{\rm corr}^{S=0}(\vec{q}')$ is the usual correlation function appearing in the fixed scatter approximation, and may be expressed in terms of the "defect functions" to a good approximation by

$$C_{\rm corr}^{S=0}(\vec{q}') \simeq \int d^3\vec{\kappa}\, \tilde{\psi}^{*S=0}(\vec{\kappa}+\vec{q}')\tilde{\psi}^{S=0}(\vec{\kappa}). \qquad (4.17)$$

We note that Eq. (4.12), the FSA, still contains certain off-shell features with respect to the pion-nucleon scattering amplitudes. This is due to the fact that the pion propagates off its mass shell between successive scatterings. (For overlapping scattering centers these "off-shell" effects influence the result.) We emphasize, however, that most of the off-shell aspects of our *dynamical* calculation are lost in the FSA. For example, in the pionic atom limit we have $\bar{s} \simeq (M_\pi + M_N)^2 > M_N^2$, while in Eq. (3.2)

$$\begin{aligned} s_{\rm I} &= (M_\pi + M_A - M_C - E_{N,\vec{\kappa}})^2 - \vec{\kappa}^2 \\ &= (M_\pi - \Delta_M + 2M_N)(M_\pi - \Delta_M + 2M_N - 2E_{N,\vec{\kappa}}) + M_N^2 . \end{aligned} \qquad (4.18)$$

Then taking $\Delta_M = 40$ MeV, we have $s_{\rm I} \lesssim M_N^2$ for $\kappa \gtrsim 1.5\ \text{fm}^{-1}$. From various nuclear matter calculations[10] one finds that values of $\kappa \simeq 3\ \text{fm}^{-1}$ are important and therefore in that case $s_{\rm I} \ll M_N^2$. By comparing Eqs. (4.13) to (4.16) with Eqs. (4.8) to (4.11), we also note that the absence of nucleon motion in the FSA changes the interaction strength considerably from the dynamical method.

To illustrate the differences between the two approaches we have discussed here, we use a simple model for the correlation functions. Following Grange and Preston[10] we note that it is reasonable to write, with $\kappa = |\vec{\kappa}|$,

$$\tilde{\psi}^{S=0}(\vec{\kappa}) \propto \frac{(2\pi)^{1/2}}{\kappa} \frac{c}{\Gamma^2 + (\kappa - \bar{\kappa})^2}, \qquad (4.19)$$

where $\bar{\kappa} = 3.7\ \text{fm}^{-1}$, $c = 0.42$, and $\Gamma = 1.25\ (\text{fm})^{-1}$. Now in the evaluation of Eq. (4.6) or Eq. (4.12) we encounter the product $\tilde{\psi}^{S=0}(\vec{\kappa}) \times \tilde{\psi}^{*S=0}(\vec{\kappa}+\vec{q}')$. Since Eq. (4.19) implies that the defect function is represented by a function that is peaked in momentum space, we do not introduce a large error by making the approximation

$$\begin{aligned} &\tilde{\psi}^{S=0}(\vec{\kappa})\tilde{\psi}^{*S=0}(\vec{\kappa}+\vec{q}') \\ &\simeq \frac{(2\pi)^{1/2}}{\bar{\kappa}} \frac{c}{\Gamma} \delta(\kappa - \bar{\kappa}) \frac{(2\pi)^{1/2}}{|\vec{\kappa}+\vec{q}'|} \frac{c}{\Gamma^2 + (|\vec{\kappa}+\vec{q}'| - \bar{\kappa})^2}. \end{aligned} \qquad (4.20)$$

We note that introducing the δ function for the variable $|\vec{\kappa}+\vec{q}'|$ rather than κ will yield the same result as the approximation defined in Eq. (4.20).

Since the pionic atom limit ($|\vec{p}| = 0$) has been explicitly used in the framework of FSA to derive the so-called Lorentz-Lorenz effect in pion-nucleus scattering,[11] we believe that a comparison between the dynamical approach and the FSA in that limit deserves our attention. In the $\vec{p} = 0$ limit the FSA amplitude becomes a real quantity, while the scattering amplitude due to the dynamical approach remains complex, although its imaginary part is negligibly small with respect to its real part. To show the detailed structure of these two amplitudes, we compare in Figs. 9 to 11 the integrand in Eq. (4.12) and the real part of the integrand in Eq. (4.6) as functions of the intermediate pion momentum transfer $|\vec{q}'|$. We see that both integrands acquire larger magnitudes when the damping parameter λ in the pion-nuclearn form factor increases. In fact, the larger the value of the parameter λ the less important is the cutoff effect of the form factor. Our calculations show also that the integrand due to FSA (dashed curves) is negative at low $|\vec{q}'|$ and becomes positive for very large values of $|\vec{q}'|$. The change of the sign is associated with the increasing importance of the $\vec{q}'$ term in Eq. (4.14) as $\vec{q}'$ increases. On the con-

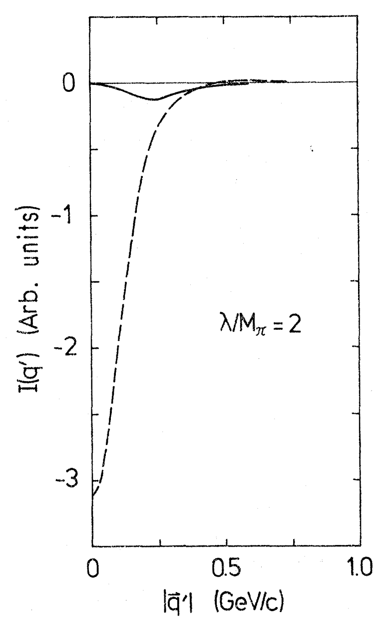

FIG. 9. The integrands of the forward scattering amplitude as function of intermediate pion momentum transfer $|\vec{q}'|$, in the limit $\vec{p} \simeq 0$. The dashed curves are FSA results and the solid curves are the real parts of the dynamical approach. Calculations are done with $\Delta_M = 40$ MeV and $\lambda/M_\pi = 2$.

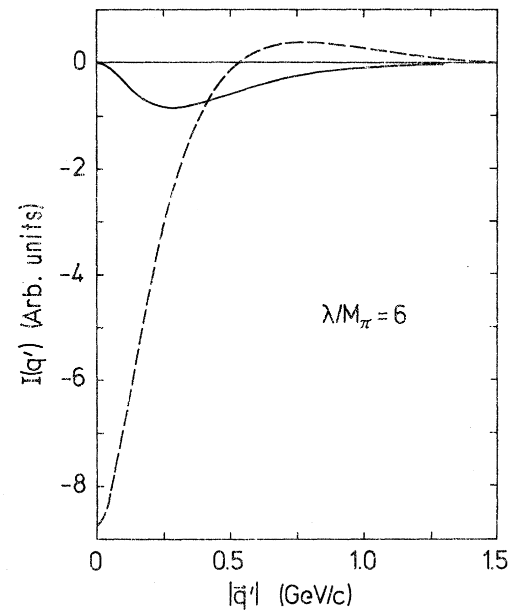

FIG. 10. Same as Fig. 9 with $\lambda/M_\pi = 6$.

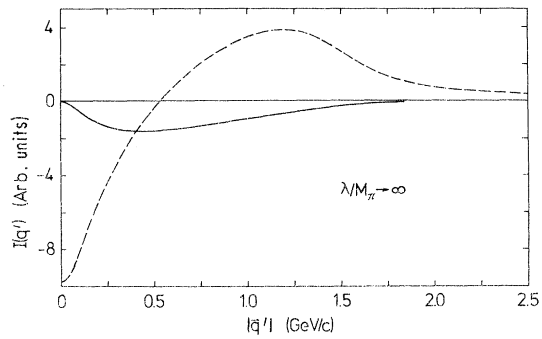

FIG. 11. Same as Fig. 9 with $\lambda/M_\pi \to \infty$.

trary, the real part of the integrand of the dynamical approach (solid curves) is negative at all $|\vec{q}\,'|$ values. This difference in the asymptotic behaviors will persist in the cases with not too large $|\vec{p}|$. On the other hand, the discrepancy between the behavior at $|\vec{q}\,'| = 0$ of these two approaches changes drastically when the $\vec{p} = 0$ limit is removed. In fact, for $\vec{p} \neq 0$ the FSA pion propagator is no longer equal to $-(\vec{q}\,')^{-2}$ and does not cancel the $(\vec{q}\,')^2$ coming from the volume element, $d^3\vec{q}\,'$; the FSA integrand will then also start from zero at $|\vec{q}\,'| = 0$ as does the intrgrand of dynamical approach. Finally, we note that for very high-energy incoming pions such that $|\vec{p}| >> |\vec{\kappa}|$, we may expect a similarity between these two different approaches.

V. CONCLUSION

We have considered the contribution to the pion-optical potential due to the pion scattering from a correlated nucleon pair. In our formalism we see that the strong nucleon-nucleon correlation provides the struck nucleon with a large momentum and also takes it very far off its mass shell. Consequently, the s values for the pion-nucleon scattering amplitudes appearing in the analysis of this part of the pion-nucleus interaction are mostly in a region very different from the one corresponding to on-shell scattering data. These features of the problem are usually ignored in any current analysis which uses nonrelativistic scattering theory where, as we have emphasized earlier, the s values are ambiguous.

In a phenomenological analysis of pion-nucleus scattering using a nonrelativistic separable potential model for the πN interaction, the energy denominator and form factors are usually parametrized in a manner appropriate to particles on their mass shells; for example, the FSA uses the parameter $\bar{s}$ rather than s. In this respect, our covariant approach provides a more satisfactory scheme in the sense that the extension to off-mass-shell particles can be made unambiguously in the framework of a phenomenological covariant model. However, we have seen that owing to the off-shell nature of the particles involved, the form factors for πN scattering depend on more than one relativistic invariant. [See Eq. (4.4).] In other words, the use of *covariant* form factors in a phenomenological analysis requires more detailed information for the off-shell dynamics of πN scattering processes. (Although unlimited choices of specific analytic structures for form factors exist, the number of independent Lorentz invariants on which these form factors depend is determined by the number of off-shell particles, as dictated by the general principles of relativistic kinematics.) The various choices of form factors are equivalent to using various models of off-shell dynamics in a phenomenological analysis within a chosen covariant reduction scheme.

As mentioned previously, we have used the nonrelativistic theory to provide some guidance as to the nature of the best reduction scheme. It is now clear that in the nonrelativistic limit of kinematics, our covariant pion-nucleus scattering amplitudes has the same formal structure as the one generated from the corresponding Goldstone diagram. The covariant amplitude still contains ingredients such as the covariant off-shell form factors, which cannot be supplied *a priori* by a nonrelativistic many-body scattering theory, not to mention a potential scattering theory.

We have also pointed out that in order to compare our approach to widely used nonrelativistic approaches, we are obliged to leave out the part of our covariant amplitude involving elementary pion production or absorption amplitudes. These amplitudes are related to pion-nucleon scattering amplitudes by "crossing." This aspect may be important in pion-nucleus scattering because of the small mass of the pion; some consideration is being given to this question.

In view of the large differences between our dynamical approach and the FSA, shown in the results of our model calculation, we believe that many conclusions, for example the derivation of Lorentz-Lorenz effect in pion-nucleus scattering based upon nonrelativistic potential scattering theory, should be seriously reexamined.

One may expect that the discrepancies between these two approaches will diminish for very high energy pions. However, at these ultra relativistic energies the use of a potential scattering model, such as the FSA, becomes dubious. For example, amplitudes involving production and annihilation of pions not contained in the potential scattering model may be important. Clearly, only in a covariant scattering formalism such as the one we advocate here can these relativistic aspects of the dynamics be taken into account. We recognize, however, that many of the amplitudes required in a fully covariant approach are only poorly known and more work is needed in this respect.

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