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New coupled-reaction-channels formalism for nuclear reactions*

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A set of coupled integral equations (CIE) for multichannel nuclear reactions is considered which appears to have some advantages over the conventional coupled-reaction-channel (CRC) equations when transfer channels must be included. Both the CIE and CRC formalism are based on the assumption that the system has a limited number of cluster type intermediate states available to it. The CIE have a connected kernel while those of the CRC do not. In addition, the troublesome nonorthogonality terms of the CRC formalism have no counterpart in the CIE formalism. The CIE are compared with those of the CRC formalism. The CIE for a simple model for deuteron scattering from a structureless target are presented as an explicit example.

[NUCLEAR REACTIONS Coupled-reaction-channels formalism with no nonorthogonality term and having a connected kernel.]

I. INTRODUCTION

A new method for dealing with the many-body scattering problem has been suggested[1,2] which appears to be a viable alternative to the Faddeev method.[3] This method has been used to derive variational functionals for the many-body transition amplitude.[4]

We will refer to this new method as the "coupled integral equations" (CIE) formalism. The starting point in this method is that suggested by Glöckle and by Takeuchi,[5] the N simultaneous Lippman-Schwinger (LS) equations for the wave function. Here N is the number of partitions of the system into two clusters of particles that can be formed. The specification of the inhomogenieties in all N equations is necessary for a complete characterization of the asymptotic behavior of the wave function. Linear combinations of the N simultaneous LS equations are formed to produce a set of coupled integral equations for the elements of the transition matrix operator. This set of equations can be written as a single matrix equation. It is possible to arrange matters so that the kernel of this matrix integral equation is connected. The fact that the coupled integral equations can be cast in the form of a single matrix equation permits the formal solution to be expressed simply in terms of operator inverses.

In this paper we observe that the use of a truncated channel state expansion to represent the partition Green's function operators that appear in the CIE's leads to a calculational scheme that appears to have the same physical content as the widely used coupled-reaction channels (CRC) formalism. The CIE has the advantage over the CRC in that there are no nonorthogonality terms in the CIE equations and the kernel of the CIE equations is connected. Nonorthogonality terms are present in the CRC equations. They are difficult to evaluate and are generally neglected, although there is evidence[6] that they are not small.

The fact that the CIE formalism has a firm theoretical basis in that it does incorporate the complete set of asymptotic boundary conditions and the kernel of the set of equations is connected means that the conventional methods of constructing successive approximations to the solution of these equations must ultimately converge to the correct result. Thus the CIE appears to be an attractive starting point for theoretical analyses of nuclear reactions.

As an explicit illustration of the kind of expressions that must be evaluated in a CIE analysis, we apply the CIE formalism to a simple model for scattering of a deuteron by a structureless target. The coupled equations for the elastic, stripping, and breakup transition amplitudes are presented. In Sec. II the CRC formalism is presented. The CIE formalism is described in Sec. III. The two methods are compared in Sec. IV. In Sec. V the CIE formalism is applied to a model deuteron scattering problem.

II. COUPLED-REACTION-CHANNEL (CRC) FORMALISM

Let $\phi_a \equiv \phi_{\alpha i}$ be a member of a set of channel states of a many-body system. Each such set is associated with a particular partition (α, β, etc.) of the particles of the system into two groups or

clusters:

$a = \alpha 1,\ \alpha 2,\ \alpha 3,\ \ldots\ ,$

$b = \beta 1,\ \beta 2,\ \beta 3,\ \ldots\ ,$

etc.

The letter a will represent a particular member of the set $\alpha i\ \{i = 1,\ 2,\ \ldots\}$, the letter b will represent a member of the set $\beta j\ \{j = 1,\ 2,\ \ldots\}$, and so on. Each of these sets include a discrete spectrum of two-body channels and a continuum of three- or more-body channels. ϕ_a is a total angular momentum eigenstate that depends on the internal degrees of freedom of the two clusters of partition α and on the direction in space of the relative displacement $\vec{r}_\alpha$ of the centers of mass of the two clusters.

Let ψ_a be the scattering state of the system associated with the total energy E and unit incident flux in channel a. Let H be the Hamiltonian operator for the system. Then

$$(E - H)\psi_a = 0\ . \tag{1}$$

The coupled-reaction-channel method is based on consideration of an expansion of the scattering state wave function in terms of the channel states.

$$\psi_a = \sum_b \phi_b F_{ba}(r_\beta)\ . \tag{2}$$

The sum on $b = \beta j$ is understood here to include, in general, various different values of β and of j. If the sum over channels b included only all those channels associated with a particular partition β, including an integral over the continuous spectrum of three- or more-body breakup channels, then Eq. (2) would be an expansion of ψ_a in a complete orthogonal set of channel states. However, this is not feasible in general. In the CRC method the sum on the right of Eq. (2) is taken to include only the most important open channels from all possible partitions. Thus Eq. (2) is to be regarded as an expansion in terms of an incomplete nonorthogonal set of ϕ_b's.

Now we introduce the partition Hamiltonian H_α and the regular channel radial wave function $f_a(r_\alpha)$ normalized to unit incident flux. We write

$$\begin{aligned} 0 &= (E - H_\alpha)\phi_a f_a(r_\alpha) \\ &= \phi_\alpha[E_a - T_\alpha - U_a(r_\alpha)]f_a(r_\alpha)\ , \end{aligned} \tag{3a}$$

$$\begin{aligned} H_\alpha &= T_\alpha + U_\alpha = T_\alpha + \sum_i |\phi_{\alpha i})U_{\alpha i}(\phi_{\alpha i}| \\ &= T_\alpha + \sum_i |\phi_a)U_a(\phi_a|\ , \end{aligned} \tag{3b}$$

where T_α is the kinetic energy operator for the relative motion of the two clusters of partition α and U_a is a potential chosen to provide an approximate representation of the background elastic scattering. Let $V_\alpha = H - H_\alpha$ be the partition α residual interaction. Then

$$\begin{aligned} V_\alpha\psi_b &= (E - H_\alpha)\psi_b \\ &= \phi_a(E_a - T_\alpha - U_a)F_{ab} + (E - H_\alpha)\Lambda_a\psi_b\ , \end{aligned} \tag{4}$$

where

$$\Lambda_a\psi_b = \sum_{c\neq a} \phi_c F_{cb}(r_\gamma)\ . \tag{5}$$

Thus we have the CRC coupled integrodifferential equations

$$\begin{aligned} &(E_a - T_\alpha - U_a)F_{ab} \\ &\qquad = \sum_c\ (\phi_a|\ V_\alpha - (E - H_\alpha)\Lambda_a|\ \phi_c F_{cb}\rangle\ . \end{aligned} \tag{6}$$

The $(E - H_\alpha)\Lambda_a$ term is the nonorthogonality term. It gives a nonvanishing contribution only for channels c which are not associated with partition α. In the case that only channels associated with a single partition are included in the channel sums, the CRC equations reduce to a set of coupled differential equations and no nonorthogonality term appears.

Operating on both sides of Eq. (6) with

$$g_a = (E_\alpha - T_\alpha - U_a)^{-1} \tag{7a}$$

gives

$$\begin{aligned} F_{ab}(r_\alpha) = \delta_{ab} f_b(r_\beta) &+ \sum_c g_a(\phi_a|\ V_\alpha|\ \phi_c F_{cb}(r_\gamma)\rangle \\ &- \sum_{c\neq a} (\phi_a|\ \phi_c F_{cb}(r_\gamma)\rangle\ . \end{aligned} \tag{7b}$$

Multiplication by ϕ_a and summation on a yields, by virtue of Eq. (2),

$$\psi_b = \phi_b f_b + \sum_a \phi_a[g_a(\phi_a|\ V_\alpha|\ \psi_b\rangle - (\phi_a|\ \Lambda_a|\ \psi_b\rangle]\ . \tag{7c}$$

III. COUPLED INTEGRAL EQUATIONS (CIE) FORMALISM

The CIE formalism is based on the Lippman-Schwinger (LS) equations[7] for the scattering wave function. These are a set of N simultaneous integral equations.

$$\psi_a = \phi_a f_a + G_\alpha V_\alpha \psi_a\ , \tag{8a}$$

$$\psi_a = G_\beta V_\beta \psi_a\ , \tag{8b}$$

$$\psi_a = G_\gamma V_\gamma \psi_a\ , \tag{8c}$$

$$\cdots\ ,$$

$$\cdots\ ,$$

$$\psi_a = G_\omega V_\omega \psi_a\ . \tag{8N}$$

The quantity G_β is the partition β Green's function operator defined by

$$G_\beta = (E - H_\beta)^{-1} \ . \tag{9}$$

All but one of the N LS equations are homogeneous. This reflects the asymptotic boundary conditions fulfilled by ψ_a, namely, that ψ_a has incoming flux only in channel a.

To make this set of simultaneous integral equations more tractable we convert them into a matrix equation for the vector $(V_\alpha\psi_a,\ V_\beta\psi_a, \ldots, V_\alpha\psi_a)$. Let $\{W_{\alpha\beta}\}$ be the elements of an N by N matrix such that

$$\sum_{\beta=1}^{N} W_{\alpha\beta} = 1 \ . \tag{10}$$

Then we can form the following N linear combinations of the N LS equations:

$$V_\gamma\psi_a = V_\gamma W_{\gamma\alpha}\phi_a f_a + \sum_\beta V_\gamma W_{\gamma\beta} G_\beta V_\beta \psi_a \qquad (\gamma = 1, 2, \ldots, N). \tag{11}$$

Equation (11) then can be given the following matrix form

$$\zeta = VW\eta + VWG\zeta \ , \tag{12a}$$

where the vectors ζ and η have elements

$$\zeta_\gamma = V_\gamma\psi_a \ , \tag{12b}$$

$$\eta_\gamma = \delta_{\gamma\alpha}\phi_a f_a \tag{12c}$$

and the matrices V and G have elements

$$V_{\alpha\gamma} = \delta_{\alpha\gamma} V_\gamma \ , \tag{12d}$$

$$G_{\alpha\gamma} = \delta_{\alpha\gamma} G_\gamma \ . \tag{12e}$$

Equation (11) or, equivalently, Eqs. (12) are the coupled integral equations of the CIE formalism. The formal solution of these equations is then

$$\zeta = (1 - VWG)^{-1} VW\eta \ . \tag{13}$$

It has been pointed out[2] that if W is chosen such that

$$\begin{aligned} &W_{\beta,\beta+1} = 1 \quad \beta = 1, 2, \ldots, N-1 \ , \\ &W_{N,1} = 1, \\ &W_{\alpha,\beta} = 0 \quad \text{for all other } (\alpha, \beta) \ , \end{aligned} \tag{14}$$

then the kernel VWG is connected, so henceforth this choice will be assumed. One consequence of this choice is that $(VWG)^N$ is a diagonal (in partition space) matrix. Thus, if we iterate Eq. (12a) $N-1$ times we find

$$\begin{aligned} \zeta &= \sum_{n=0}^{N-1} (VWG)^n VW\eta + (VWG)^N \zeta \\ &= [1 - (VWG)^N]^{-1} \sum_{n=0}^{N-1} (VWG)^n VW\eta \ , \end{aligned} \tag{15}$$

where now the kernel is diagonal in partition space so that the equations for the various components of ζ are uncoupled.

A set of coupled integrodifferential equations based on the CIE formalism integral equation can be formed using the procedure suggested by Hahn, Kouri, and Levin.[8] One merely makes the substitution

$$\psi_a = \sum_\beta W_{\gamma\beta}\chi_{\beta a} \tag{16}$$

in Eq. (11) with the result

$$\chi_{\beta a} = \delta_{\beta\alpha}\phi_a f_a + \sum_\delta G_\beta V_\beta W_{\gamma\delta}\chi_{\delta a} \ . \tag{17}$$

Then operation by the inverse of G_β gives a set of coupled differential equations for the wave function components χ:

$$(E - H_\beta)\chi_{\beta a} = V_\beta \sum_\delta W_{\gamma\delta}\chi_{\delta a} \ . \tag{18}$$

To convert these multidimensional differential equations into six dimensional integrodifferential equations, we make a channel state expansion of each wave function component χ:

$$\chi_{\beta a} = \sum_i \phi_{\beta i} J_{\beta i,a}(r_\beta) \ . \tag{19}$$

Substitution of this expansion into Eq. (18) gives

$$(E_b - T_\beta - U_b) J_{ba} = \sum_\delta \sum_j W_{\gamma\delta}(\phi_b | V_\beta | \phi_{\delta j} J_{\delta j,a}\rangle \ . \tag{20}$$

This result is seen to be identical with CRC result shown in Eq. (6) except for the presence of the elements of W and the fact that there is no nonorthogonality term. Recall that this results from using

$$\begin{aligned} \psi_a &= \sum_\beta \sum_j W_{\gamma\beta}\phi_{\beta j} J_{\beta j,a}(r_\beta) \\ &= \sum_j \phi_{\gamma+1,j} J_{\gamma+1\,j,a}(r_\beta) \end{aligned} \tag{21}$$

in place of Eq. (2) and substituting this into the CIE formalism coupled integral equations instead of the Schrödinger equation.

IV. COMPARISONS OF THE CRC AND CIE FORMALISMS

Let us introduce the channel state expansion for the partition Green's function operators

$$\begin{aligned} G_\alpha &= \sum_{i=1}^{\infty} |\phi_{\alpha i})g_{\alpha i}(r_\alpha, r'_\alpha)(\phi_{\alpha i}| \\ &= \sum_{i=1}^{\infty} |\phi_a)g_a(r_\alpha, r'_\alpha)(\phi_a| \ , \end{aligned} \tag{22}$$

where the sum is understood to include an integral over the continuum of three- or more-body breakup channels. By using the subscript αi and summing over the index i we mean to indicate that the sum includes only those channels associated with partition α. This is in contrast to our use of a (or b or c) as a subscript and the summation over that index to indicate a summation over channels from all partitions.

Substituting the channel state expansion into Eq. (11) leads to the following form of the CIE:

$$\mathcal{F}_{ca}(r) = \mathfrak{f}_{ca}(r) + \sum_b \int dr' \mathcal{K}_{cb}(r, r') \mathcal{F}_{ba}(r') \ , \tag{23a}$$

$$\mathcal{F}_{ca}(r) = \langle \phi_c \delta(r_\gamma - r) | \ V_\gamma \ | \psi_a \rangle \ , \tag{23b}$$

$$\mathfrak{f}_{ca}(r) = \langle \phi_c \delta(r_\gamma - r) | \ V_\gamma \ | \phi_a f_a \rangle W_{\gamma\alpha} \ , \tag{23c}$$

$$\mathcal{K}_{cb}(r, r') = \int dr'' \langle \phi_c \delta(r_\gamma - r) | \ V_\gamma \ | \phi_b \delta(r_\beta - r'') \rangle \times W_{\gamma\beta} g_b(r'', r) \ . \tag{23d}$$

The solution of this set of coupled integral equations would then be used to evaluate the elements of the transition matrix

$$T_{ab} = \langle \phi_a f_a | \ V_\alpha \ | \psi_b \rangle = \int dr \, r^2 f_a^*(r) \mathcal{F}_{ab}(r) \ . \tag{24}$$

In this form the CIE method may be readily compared with the CRC result of Eq. (7) which can be written in the following form:

$$F_{ca}(r) = \delta_{ca} f_a(r) + \sum_b \int dr' K_{cb}(r, r') F_{ba}(r') \ , \tag{25a}$$

where

$$K_{cb}(r, r') = \int dr'' g_c(r, r'') \times \langle \delta(r_\gamma - r'') \phi_c | \ V_\gamma \ | \delta(r_\beta - r') \phi_b \rangle + (1 - \delta_{cb}) \langle \delta(r_\gamma - r) \phi_c | \ \delta(r_\beta - r') \phi_b \rangle \ . \tag{25b}$$

The solution of the CRC equations would be used in the following expression to evaluate the elements of the transition matrix.

$$T_{ab} = \sum_c \langle \phi_a f_a | \ V_\alpha \ | \phi_c F_{ca} \rangle = \sum_c \int dr'_\alpha \int dr'_\gamma f_a(r'_\alpha)^* \times \langle \delta(r_\alpha - r'_\alpha) \phi_a | \ V_\alpha \ | \delta(r_\gamma - r'_\gamma) \phi_c \rangle F_{ca}(r'_\gamma) \ . \tag{26}$$

We see that the equations have a fairly similar form. The CIE equations are for the transition functions (= the residual interactions times the wave function) while the CRC equations are for the components of the wave function. There is no counterpart of the nonorthogonality term in the kernel of the CIE equations. This must be regarded as an important advantage. The other important advantage that the CIE formalism has over the CRC is that the kernel $\mathcal{K}$ of the CIE formalism is connected[8] while the kernel K of the CRC formalism is not.

In both cases the basic approximation that must be made in practical applications is the truncation of the channel sums. Thus, except for the fact that the nonorthogonality term will be neglected in a CRC calculation, CIE calculations and CRC calculations rely on the same physical model. This model is a kind of a cluster model that permits the system only cluster type intermediate states.

The CRC integral equation displayed in Eq. (25) would be completely equivalent to the LS equation shown in Eq. (8a) if the channel subscripts c and b were to refer only to partition α channels and all such channels were included in the channel sums. In truncating the partition α channel sums, the CRC formalism adds contributions from other partitions. This compensates in some degree for the truncation. Thus in the CRC, where the partition Green's function operator truncation is accompanied by the insertion of additional terms and the neglect of nonorthogonality terms, it is not clear that the consequences of the truncation will be similar to those of the simple truncation used in the CIE.

The CRC formalism has been invoked as the theoretical basis for the perturbation expansions used in the analysis of multistep transfer reactions. Let us compare CRC expressions for multistep transition amplitudes with those predicted by the LS equations. Toward this end we rewrite Eq. (7) to read

$$\psi_b = \phi_b f_b + \sum_\alpha G'_\alpha V_\alpha \psi_b + \sum_\alpha N'_\alpha \psi_b \ , \tag{27a}$$

$$G'_\alpha = \sum_i{}' | \ \phi_{ai} \rangle (E_{ai} - T_\alpha - U_{ai})^{-1} (\phi_{ai} | \ , \tag{27b}$$

$$N'_\alpha = \sum_i{}' | \ \phi_{ai} \rangle (\phi_{ai} | \ \Lambda_{ai} \ , \tag{27c}$$

where the prime serves to indicate that the channel sum has been truncated. Neglecting the nonorthogonality term and repeatedly iterating Eq.

(21a) gives the perturbation expansion

$$\Psi_b \cong \left[1+\sum_\alpha G'_\alpha V_\alpha + \left(\sum_\alpha G'_\alpha V_\alpha\right)^2 + \cdots\right]\phi_b f_b \tag{28}$$

leading to the transition amplitude sum

$$T_{ab} \cong \langle \phi_a f_a | V_\alpha + V_\alpha \sum_\gamma G'_\gamma V_\gamma + V_\alpha \left(\sum_\gamma G'_\gamma V_\gamma\right)^2 + \cdots | \phi_b f_b \rangle \ . \tag{29}$$

This then is the CRC multistep expansion. The transition amplitude is represented as a sum of one-step, two-step, three-step, etc. terms involving all possible combinations of intermediate states.

Now let us turn to the LS equations shown in Eq. (8) and see what sort of perturbation expansion they produce. The CRC integral equation, Eq. (27), is after all merely an approximate representation of the LS equations. If we combine the LS equations to form the CIE and iterate the CIE as shown in Eqs. (12) or (15) using the W matrix given by Eq. (14), then the result is equivalent to successively iterating the N LS equations in cyclic order. The result we get depends on the order we assign to the various partitions. For example, if we start with Eq. (8a) and successively iterate using Eqs. (8b), (8c), ... (8N), (8a), (8b), ... in that order we will find

$$\psi_a = \{1 + Q_\alpha + {Q_\alpha}^2 + \cdots\}\phi_a f_a \ , \tag{30a}$$

$$Q_\alpha = G_\alpha V_\alpha G_\beta V_\beta G_\gamma V_\gamma \ldots G_\omega V_\omega \ . \tag{30b}$$

On the other hand if we had started the same procedure with Eq. (8c) instead of Eq. (8a), we would have found

$$\psi_a = G_\gamma V_\gamma G_\delta V_\delta \cdots G_\omega V_\omega \times \{1 + Q_\alpha + Q_\alpha^2 + \cdots\}\phi_a f_a \ . \tag{31}$$

The associated transition amplitude is

$$T_{ab} = \langle \phi_a f_a | V_\alpha G_\gamma V_\gamma \cdots G_\omega V_\omega \{1 + Q_\beta + Q_\beta^2 + \cdots\} | \phi_b f_b \rangle \ . \tag{32}$$

There are infinitely many other expansions of the transition amplitude that can be constructed by iterating Eqs. (8) in various sequences. For example, one might have

$$T_{ab} = \langle \phi_a f_a | V_\alpha G_\gamma V_\gamma \cdots G_\omega V_\omega \{1 + G_\beta V_\beta + G_\beta V_\beta G_\delta V_\delta + G_\beta V_\beta G_\delta V_\delta G_\beta V_\beta + G_\beta V_\beta G_\delta V_\delta G_\beta V_\beta G_\lambda V_\lambda G_\mu V_\mu G_\nu V_\nu + G_\beta V_\beta G_\delta V_\delta G_\beta V_\beta G_\lambda V_\lambda G_\mu V_\nu G_\nu V_\nu G_\beta V_\beta + \cdots\} | \phi_b f_b \rangle \ . \tag{33}$$

The pattern that any such expansion must conform to is (a) the product outside the bracket can have any number of factors of GV except that $G_\beta V_\beta$ must not be included, β being the partition to which the incident channel b belongs; (b) each new term of the series must include the previous term as an initial factor; (c) every time the factor $G_\beta V_\beta$ appears it must appear by itself; and (d) two subsequent terms both lacking a final factor of $G_\beta V_\beta$ may not appear. Ultimately, one term in the series must include all N different possible factors of $G\ V$ in order that the series not contain an infinite set of unlinked diagrams.

The exact perturbation expansion is thus seen to differ from the approximate CRC expansion in that (a) there is no nonorthogonality term to neglect, (b) the partition Green's function operators G_α, G_β, etc. appear instead of the truncated partition Green's function operators G'_α, G'_β, etc., and (c) rather than being a sum over all possible diagrams as in the CRC expansion, the LS equations expansion is a sum of a particular, though fairly arbitrary, sequence of diagrams. It is especially noteworthy that the first term of the LS equations expansion can be of arbitrarily high order. Thus the transition amplitude for a particular multistep process can be the lowest order term of a particular perturbation expansion. In that case, however, the amplitude for other multistep processes of the same order or lower order may not appear in the expansion at all.

Although the CRC expansion is approximate in nature and is not convergent because of the unlimited number of unlinked diagrams it contains, it nevertheless has the advantage that a simple physical interpretation of each term is possible. The LS equations expansion, on the other hand, is formally exact and will converge for sufficiently small residual interactions V_α, V_β, etc. Physical interpretation of individual terms of the LS equations expansion, however, is problematic in view of the arbitrariness of its structure.

V. SCATTERING OF A MODEL DEUTERON BY A STRUCTURELESS TARGET

Consider a system consisting of three spinless, structureless particles: a neutron n, a proton p, and a target t. We suppose these particles to interact with each other via two-body local potentials V_{pn}, V_{pt}, V_{nt}, capable of supporting a single s-wave bound state each. The two-body subsystems will be identified by capital letters: $p+n=D$, $p+t=P$, $n+t=N$. The Hamiltonian of the system is then

$$H = T + V_{pn} + V_{pt} + V_{nt} \ , \tag{34a}$$

$$T = T_{np} + T_{Dt} = T_{nt} + T_{pN} = T_{pt} + T_{nP} \ . \tag{34b}$$

The partition Hamiltonians are defined to be

$$H_D = T + V_{pn} + U_{Dt} \ , \tag{35a}$$

$$H_p = T + V_{nt} + U_{pN} \ , \tag{35b}$$

$$H_n = T + V_{pt} + U_{nP} \tag{35c}$$

so that the residual interactions are

$$V_D = V_{pt} + V_{nt} - U_{Dt} \ , \tag{36a}$$

$$V_p = V_{pn} + V_{pt} - U_{pN} \ , \tag{36b}$$

$$V_n = V_{pn} + V_{nt} - U_{nP} \ . \tag{36c}$$

The partition D channel states are solutions of

$$(\epsilon_{Dk} - T_{pn} - V_{pn})\phi_{Dk}(\vec{r}_{np}, \hat{r}_{Dt}) = 0 \ , \tag{37a}$$

where $k=0$ is the bound state and the rest of the spectrum is continuous. $\hat{r}_{Dt}$ is the unit vector in the direction of $\vec{r}_{Dt}$. The corresponding equations for the other channels are

$$(\epsilon_{pk} - T_{nt} - V_{nt})\phi_{pk}(\vec{r}_{nt}, \hat{r}_{pN}) = 0 \ , \tag{37b}$$

$$(\epsilon_{nk} - T_{pt} - V_{pt})\phi_{nk}(\vec{r}_{pt}, \hat{r}_{nP}) = 0 \ . \tag{37c}$$

Again $k=0$ is the bound state and the rest of the spectrum is continuous. In practical calculations one will generally approximate the continuous spectrum by a discrete one.

Our objective here is to present explicit expressions for the components of the CIE integral equation, Eq. (23), for the deuteron scattering model just described. We suppose that the incident channel is $D0$, ground state deuterons incident on the target. The source function $f_{c,D0}$ defined by Eq. (23c) then is calculated from the unit incident current regular optical model wave function which is a solution of

$$(E_{D0} - T_{Dt}^{(l_0)} - U_{Dt}) f_{D0}(r_{Dt}) = 0 \ , \tag{38a}$$

$$E_{D0} = E - \epsilon_{D0} \ , \tag{38b}$$

$$T_{Dt}^{(l_0)} = -\frac{\hbar^2}{2m_{Dt}} \left\{ \frac{1}{r_{Dt}} \frac{d^2}{dr_{Dt}^2} r_{Dt} - \frac{l_0(l_0+1)}{r_{Dt}^2} \right\} \ , \tag{38c}$$

where $l_0\hbar$ is the orbital angular momentum of the incident channel and m_{Dt} is the reduced mass of the $D+t$ system.

$$f_{pk,D0}(r) = \langle \phi_{pk}\delta(r_{pN} - r) | V_{pn} + V_{pt} - U_{pN} | \phi_{D0} f_{D0} \rangle, \tag{39a}$$

$$f_{nk,D0}(r) = 0 \ , \tag{39b}$$

$$f_{D0,D0}(r) = 0 \ , \tag{39c}$$

where we have chosen the channel sequence to be $D \to p \to n \to D$. The nonvanishing kernels are

$$K_{pk,Dl}(r,r') = \int dr'' \langle \phi_{pk}\delta(r_{kN} - r) | V_{pn} + V_{pt} - U_{pN} | \phi_{Dl}\delta(r_{Dt} - r'') \rangle g_{Dl}(r'', r') \tag{40}$$

and $K_{Dl,nm}$ and $K_{nm,pk}$. The partition radial Green's function for channel Dl is

$$g_{Dl}(r'', r') = -x_{Dl}(r_<) y_{Dl}(r_>) \ , \tag{41}$$

where x_{Dl} and y_{Dl} are solutions of

$$(E_D - T_{Dt}^{(l_l)} - U_{Dt}) \frac{x(r_{Dt})}{r_{Dt}} = 0 \tag{42}$$

such that x_{Dl} is regular at the origin and has the asymptotic behavior

$$x_{Dl}(r) \left(\frac{2m_{Dt}}{\hbar^2 k_D} \right)^{1/2} \sin(k_D r - l_l \tfrac{1}{2}\pi + \delta_{Dl}) \ , \tag{43a}$$

$$\frac{\hbar^2 k_{Dl}^2}{2m_{Dt}} = E_{Dl} \ , \tag{43b}$$

and y_{Dl} has the asymptotic behavior

$$y_{Dl}(r) \to \left(\frac{2m_{Dt}}{\hbar^2 k_{Dl}} \right)^{1/2} \exp i\left(k_{Dl} r - \frac{l_l \pi}{2} + \delta_{Dl} \right) \ . \tag{43c}$$

The integrations required for the evaluation of the kernels and the source functions are six dimensional ones of the sort encountered in finite range distorted-wave Born approximation calculations. These calculations are difficult but not beyond reach.

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