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Symmetry coefficient and the isospin-spin dependence of the single-particle potential

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The symmetry coefficient a_τ for nuclear matter with a given neutron excess has been calculated within the framework of the independent particle model using two effective interactions determined in an earlier paper. The interactions referred to as Set 1 and Set 2 have been found to give $a_\tau = 44.89$ MeV and $a_\tau = 43.16$ MeV, respectively. The isospin-spin dependent part of the single-particle potential U_1 has also been estimated using an explicit expression due to Brueckner and Dabrowski. The corresponding values of $U_1(k_F)$ are 158.48 and 102.64 MeV for the interactions of Set 1 and Set 2, respectively. These values of a_τ and $U_1(k_F)$ compare reasonably well with those obtained by others.

We have calculated the symmetry coefficient a_τ using the two effective interactions.[1] The rearrangement contribution $\Delta\epsilon_R$ to a_τ has been calculated using the approximations of Brueckner and Dabrowski[2] (BD). Since the isospin-spin dependence of the single-particle potential is closely related to the symmetry coefficient a_τ, we have also calculated the isospin dependent part of the single-particle potential U_1 using an explicit expression due to BD.

For the neutron excess parameter $\alpha = (N-Z)/A$, the symmetry coefficient a_τ is the coefficient of α^2 and, including the isospin flip term[3] S, is given by

$$a_\tau = \frac{1}{6}\frac{\hbar^2 k_F^{\,2}}{m} + \frac{1}{6}k_F\left(\frac{\partial U}{\partial k_m}\right)_{k_m=k_F} + S\,. \tag{1}$$

For the interactions[1] of set 1 and set 2, S can be calculated. The value of $U(k_m)$ for the interaction of set 1 is

$$U(k_m) = [-83.32 + 36.91(k_m/k_F)^2]\,\text{MeV}\,, \tag{2}$$

and for the interaction of set 2 is

$$U(k_m) = [-101.38 + 52.68(k_m/k_F)^2]\,\text{MeV}\,. \tag{3}$$

Equations (2) and (3) and the values of S when substituted in Eq. (1) will give an explicit expression for a_τ.

Since our interactions have intrinsic dependence on k_F, we must also add to Eq. (1) the rearrangement part of the symmetry coefficient $\Delta\epsilon_R$. Following BD, the rearrangement part of the symmetry coefficient is

$$\Delta\epsilon_R = \tfrac{1}{2}(\Delta_0\epsilon_R + \Delta_1\epsilon_R)\,, \tag{4}$$

where

$$U_R(k_F) = 2Ck_F^{\,5}/9\pi^2\,, \tag{5}$$

$$\Delta_0\epsilon_R = -\tfrac{2}{3}U_R(k_F)\,, \tag{6}$$

$$\Delta_1\epsilon_R = 14Ck_F^{\,5}/27\pi^2\,. \tag{7}$$

The above expressions for both the sets of interactions remain the same except for C, which is different.

For $k_F = 1.35\ \text{fm}^{-1}$ which corresponds to the saturation density for the interactions,[1] the kinetic energy part of a_τ is 12.77 MeV, $U(k_m) = 12.30$ MeV, $S = 9.51$ MeV, $\Delta_0\epsilon_R = -8.24$ MeV, $U_R(k_F) = 12.37$ MeV, $\Delta_1\epsilon_R = 28.84$ MeV, $\Delta\epsilon_R = 10.30$ MeV, and consequently taking into consideration the effect of $\Delta\epsilon_R$, $a_\tau = 44.89$ MeV for the interactions of set 1. For the interactions of set 2, $U(k_m) = 17.56$ MeV, $S = 6.62$ MeV, $U_R(k_F) = 7.44$ MeV, $\Delta_0\epsilon_R = -5.96$ MeV, $\Delta_1\epsilon_R = 17.36$ MeV, $\Delta\epsilon_R = 6.2$ MeV and thus $a_\tau = 43.16$ MeV.

Following[2] BD, the single-particle potential U of a nucleon with momentum k_m to a linear approximation in the neutron excess parameter α is given by

$$U(k_m^{\pm}) = U_0(k_m) \pm \tfrac{1}{4}\alpha U_1(k_m)\,. \tag{8}$$

The relation between U_1 and a_τ at the Fermi surface is[2]

$$a_\tau = \frac{1}{6}\frac{\hbar^2 k_F^{\,2}}{m} + \frac{1}{6}k_F\left[\frac{\partial U_0(k_m)}{\partial k_m}\right]_{k_m=k_F} + \frac{1}{2}\left[\tfrac{1}{4}U_1(k_F)\right]. \tag{9}$$

In Eq. (9), $U_0(k_m)$ usually includes the rearrangement part U_R [see BD, Eq. (56)]. However, in our case, the inclusion of U_R in Eq. (9) will not make any difference as U_R given by Eq. (5) is independent of k_m. Thus it permits us to use $U(k_m)$ given by Eq. (2) and Eq. (3) for $U_0(k_m)$ in Eq. (9). Of course, it does not apply to Eq. (8), where U_R has to be included to get the correct neutron (proton) single-particle potential. The isospin part of the single-particle potential $U_1(k_F)$ for the interaction of set 1 and with rearrangement effects is 158.48 MeV. The values of a_τ and $U_1(k_F)$ without rearrangement effects are $a_{\tau_{\text{nr}}} = 34.58$ MeV and $U_1(k_F)_{\text{nr}} = 76.08$ MeV. Similarly, for the interaction of set 2 and with rearrangement effects, $U_1(k_F)$

$=102.64$ MeV. The corresponding values without rearrangement effects are $a_{\tau_{nr}}=36.96$ MeV and $U_1(k_F)_{nr}=52.96$ MeV.

By considering a pure volume symmetry coefficient Green[4] finds $a_\tau=23.52$ MeV, whereas considering also a surface part of the symmetry coefficient, Green finds $a_\tau=30.34$ MeV and Cameron[5] finds $a_\tau=31.45$ MeV. Dabrowski[6] refers to the experimental value of $a_\tau=28-32$ MeV. Our values of a_τ are somewhat higher than those referred to above.

The value of the isospin dependent part of the single-particle potential U_1 is in close agreement with the upper limit of the known values of $U_1(k_F)$. An estimate of the difference between the neutron and proton single-particle potentials due to symmetry effect for a large nucleus like ^{208}Pb can be made using the value $U_1(k_F)$. For ^{208}Pb $\alpha=0.21$ and $\frac{1}{4}U_1(k_F)\ \alpha=8$ MeV, and $\frac{1}{4}U_1(k_F)_{nr}\ \alpha=4$ MeV for the interaction of set 1. These values show that the 1s single particle energies for neutrons and protons in ^{208}Pb should differ due to symmetry effect by about 16 MeV with rearrangement and 8 MeV with no rearrangement. This latter value is comparable to the 10 MeV value as calculated by Moszkowski.[7] For the interaction of set 2, we get $\frac{1}{4}U_1(k_F)=5$ MeV, which is in excellent agreement with that calculated by Moszkowski.[7]

At the end we must mention that the agreements attained for a_τ and $U_1(k_F)$ are very impressive in view of the apparent sensitivity of these quantities on the nature of the interactions used. The results may improve further if we suitably modify our interactions.

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