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Rule for the spins of the lowest state with $T = |T_Z|$ of doubly odd nuclei having j^n configuration

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Two empirical rules given earlier have been modified to one rule for the spins of the lowest state with $T = |T_Z|$ for the odd-odd nuclei where the valence nucleons are filling the same subshell. The recent measurement on ${}^{22}_{9}F$ follows the new rule. Predictions for the unknown spins have been made. Some remarks have been made about the odd-even nuclei.

[NUCLEAR STRUCTURE Rule for spins of doubly odd nuclei with $A = 2-92$, compared with existing data, predicted spins for unknown cases.]

Recently the β decay of ^{22}F has been measured by Davids *et al.*[1] They have assigned $J^\pi = 4^+$ to the ground state of ^{22}F. This assignment is in contradiction to the empirical rules given by the author in an earlier paper.[2] These rules were modified by the author and replaced by one rule. The known value of the given spin of ^{22}F, which was at that time 3^+, did not agree with prediction of the new rule. However, the measurement of Davids *et al.* is in agreement. This modification has not been published so far and in view of this agreement is being described in this note.

In the earlier paper[1] the author gave two empirical rules on the ground state spins of doubly odd nuclei having the last neutron and proton filling the same subshell. The existence of two rules was quite unsatisfactory because all the nuclei were put in two groups rather arbitrarily. It was not possible to unify the two rules because five odd-odd nuclei with $T_Z = 0$ (^{34}Cl, ^{42}Sc, ^{46}V, ^{50}Mn, and ^{54}Co) have their ground state as characterized by $T = 1$ and $J = 0$. With the recent data[3,4] available on ^{42}Sc, ^{46}V, ^{50}Mn, and ^{54}Co, it has been possible to unify the two empirical rules into one taking into account the lowest $T_Z = 0$ state in these nuclei. The rule is: The spin J of the lowest state with the isobaric spin $T = |T_Z|$ in a doubly odd nucleus with n valence nucleons filling the same (l, j) subshell is given by

$$J = \tfrac{1}{2}n + T - 1 + \delta(0, T) + \tfrac{1}{2}\delta(0, T)(2j - \tfrac{1}{2}n)[1 + (-)^{l+j+1/2}], \quad (1)$$

where $\delta(0, T)$ is the Kronecker δ symbol.

For $T = 0$ and $l + j + \frac{1}{2}$ odd we get

$$J = \tfrac{1}{2}n . \quad (1a)$$

Table I contains all the data given by expression (1a). For $T = 0$ and $l + j + \frac{1}{2}$ even we get

$$J = 2j . \quad (1b)$$

Presently ${}^{14}_{7}N$, ${}^{34}_{17}Cl$, and ${}^{38}_{19}K$ are the only nuclei covered under this rule. Their measured spins[7] for the lowest $T = 0$ state are 1^+, 3^+, and 3^+ which are in agreement with the expression (1b). The spins of ^{14}N and ^{38}K are predicted properly by the expression (1a) also. Therefore, ^{34}Cl is the only case covered by the expression (1b) alone. The nuclei in the $1f_{5/2}$ subshell will provide a crucial test for the validity of expression (1b). The nuclei are ${}^{66}_{33}As$, ${}^{70}_{35}Br$, and ${}^{74}_{37}Rb$. The predicted spins for all these will be 5^+ for the lowest $T = 0$ state. All the nuclei mentioned above are highly deficient in neutrons and will be accessible through heavy ion reactions. For $T \neq 0$ we get

$$J = \tfrac{1}{2}n + T - 1 . \quad (1c)$$

The expression (1c) explains all the available data on the ground state (g.s.) spins of odd-odd nuclei when the last neutron and proton are occupying the same subshell. The present rule predicts the

TABLE I. Spins of the lowest $T = |T_Z|$ state in odd-odd nuclei when $T = 0$ and $l + j + \frac{1}{2}$ is odd. Expression (1a) $J = \frac{1}{2}n$. Data on ^{42}Sc are from Ref. 3. Data on ^{46}V, ^{50}Mn, ^{54}Co and ^{58}Cu are from Ref. 4. The rest of the data are from Ref. 7.

$J = \frac{1}{2}n$	Subshells				
$\pi = +$	$1s_{1/2}$	$1p_{3/2}$	$1d_{5/2}$	$1f_{7/2}$	$2p_{3/2}$
1^+	${}^{2}_{1}H$ 1^+	${}^{6}_{3}Li$ 1^+	${}^{18}_{9}F$ 1^+	${}^{42}_{21}Sc$ 1^+	${}^{58}_{29}Cu$ 1^+
3^+		${}^{10}_{3}Li$ 1^+	${}^{22}_{11}Na$ 3^+	${}^{46}_{23}V$ 3^+	...
5^+			${}^{26}_{13}Al$ 5^+	${}^{50}_{25}Mn$ (5^+)	
7^+				${}^{54}_{27}Co$ 7^+	

TABLE II. Ground state spins of odd-odd nuclei when $T \neq 0$ expression (1c) $J=\frac{1}{2}n-T+1$. Data on ^{70}As and ^{90}Nb have been taken from Refs. 5 and 6. The rest of the data have been taken from Ref. 7.

$J=\frac{1}{2}n-T+1$	Subshells						
$\pi=+$	$1p_{3/2}$	$1d_{5/2}$	$1d_{3/2}$	$1f_{7/2}$	$2p_{3/2}$	$1f_{5/2}$	$1g_{9/2}$
2^+	$^{8}_{3}Li$ 2^+	$^{20}_{9}F$ 2^+	^{36}Cl 2^+	^{44}Sc 2^+	$^{60}_{29}Cu$ 2^+	...	...
4^+		$^{24}_{11}Na$ 4^+	$^{46}_{21}Sc$ 4^+			$^{70}_{33}As$ 4	...
		$^{24}_{13}Al$ (4^+)	$^{48}_{23}V$ 4^+				
		$^{22}_{9}F$ 4^+					
6^+			$^{48}_{21}Sc$ 6^+				
			$^{50}_{23}V$ 6^+				
			$^{52}_{25}Mn$ 6^+				...
8^+							$^{90}_{41}Nb$ 8^+
							$^{92}_{43}Tc$ $(8, 9^+)$

spin of ^{22}F to be $J^{\pi}=4^+$ in agreement with the measurement of Davids *et al.* Table II lists all the available data.

A theoretical proof of the expression (1) is highly desirable but it is difficult to conceive it within the framework of the $j-j$ coupling shell model alone. The reason is that we expect similar spectra for the particle-particle and hole-hole nuclei under the $j-j$ coupling model while the expression (1) does not always give the same spin for the particle-particle and hole-hole nuclei. The L-S coupling shell model without any spin-orbit coupling predicts the ground state spins of odd-odd nuclei to be 1^+. This is experimentally so only for the j^2 configuration. The proof of the expression may therefore come from an intermediate coupling model.

The rule (1) can be used to predict spins of many unknown nuclei with $(2p_{3/2})^n$, $(1f_{5/2})^n$, and $(1g_{9/2})^n$ configurations. $^{68}_{33}As$ and $^{88}_{41}Nb$ are the nuclei which have been produced but their ground state spins have not been measured. [A recent work[8] assigns to the g.s. spin of ^{68}As, values of $(1,2,3)^{\pm}$ which is a very wide choice.] The predictions for these nuclei will be 2^+ and 6^+. After the $1g_{9/2}$ subshell the order of neutron and proton subshells is not the same; therefore, nuclei with the last neutron and proton in the same subshell will not be occurring.

It will be very important to see if there is any correlation between the ground state spins of odd-even nuclei and the odd-odd nuclei. We have found that all the odd nuclei with a j^n configuration have their spins equal to j, the only exceptions being the nuclei with $j \geq \frac{5}{2}$, $4j-1-4T \geq n \geq 4j-7$ and $j-2 \geq T \geq \frac{1}{2}$. For these nuclei the ground state spin is given by

$$J=\tfrac{1}{4}n-\tfrac{1}{2}(-1)^{1/2n+T}T. \qquad (2)$$

The only data available to confirm expression (2) are up to the $1f_{7/2}$ subshell. These data are listed

TABLE III. Ground state spins of odd-even nuclei when $j \geq \frac{5}{2}$, $4j-1-4T \geq n \geq 4j-7$ and $j-2 \geq T \geq \frac{1}{2}$, $J=\frac{1}{2}n-\frac{1}{2}(-)^{n+T}T$. All the data are from Ref. 7.

$J=\frac{1}{4}n-\frac{1}{2}(-)^{1/2n+T}T$		Subshells	
T	$J_{\rm cal}$	$1d_{5/2}$	$1f_{7/2}$
$\frac{1}{2}$	$\frac{1}{2}$	$^{19}_{9}F$ $\frac{1}{2}^+$, $^{19}_{9}Ne$ $\frac{1}{2}^+$	
$\frac{1}{2}$	$\frac{3}{2}$	$^{21}_{10}Ne$ $\frac{3}{2}^+$, $^{21}_{11}Na$ $\frac{3}{2}^+$	$^{47}_{23}V$ $(\frac{3}{2})^-$
		$^{23}_{11}Na$ $\frac{3}{2}^+$, $^{23}_{12}Mg$ $\frac{3}{2}^+$	
$\frac{1}{2}$	$\frac{5}{2}$		^{49}Cr $(\frac{5}{2})^-$, ^{51}Mn $(\frac{5}{2},\frac{7}{2})^-$
$\frac{3}{2}$	$\frac{5}{2}$		$^{47}_{22}Ti$ $\frac{5}{2}^-$

in Table III. If the spins given by expression (1) and (2) are labeled as J_{o-o} and J_{o-e} we can write

$$J_{o-o}=2J_{o-e}-1+\delta(0,T)+\tfrac{1}{2}\delta(0,T)(2j-\tfrac{1}{2}n)[1+(-)^{l+j+1/2}]. \qquad (3)$$

Thus there seems to be some relationship between the expressions (1) and (2) but it is restricted only to a small fraction of odd-even nuclei.

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