

Nuclear g factors of the $\frac{21}{2}^-$ and the $\frac{29}{2}^+$ states in ^{211}At †

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The g factors of the $\frac{21}{2}^-$ (1416.3 keV, $T_{1/2}=50$ nsec) and the $\frac{29}{2}^+$ (2641.2 keV, $T_{1/2}=70$ nsec) states in ^{211}At were measured by the time differential spin rotation method. The experimental results corrected for diamagnetic shielding and Knight shift are $g(\frac{21}{2}^-) = +0.917 \pm 0.016$ and $g(\frac{29}{2}^+) = +1.073 \pm 0.031$. The analysis yields an orbital g factor of the proton $g_1 = 1.12 \pm 0.04$. The comparison with g factors of other $(h_{9/2})^n$ proton states in $N=126$ isotones shows a significant deviation from the predicted blocking of core polarization.

[NUCLEAR REACTIONS $^{209}\text{Bi}(\alpha, 2n)$, $E=30, 33$ MeV; measured $\gamma(\theta, H, t)$. ^{211}At] levels deduced g .

I. INTRODUCTION

Nuclear magnetic moments are given in zero order by the single particle (Schmidt) values. The experimental moments, however, deviate more or less from these predictions. The differences can be explained by (i) core polarization contributions, (ii) mesonic exchange effects, and (iii) LS terms, the latter being only a very small correction.

The configuration mixing has been calculated by several authors in first order perturbation theory using a δ -function interaction^{1,2} or more realistic potentials.³⁻⁶ Applying these first order core polarization corrections to the single particle values produces a reasonable agreement between theory and experiment in most cases. For the ^{209}Bi ground state, however, only half of the deviation from the Schmidt moment can be explained by these contributions²⁻⁶; if one takes into account second order terms the agreement with the experiment is even worse.⁷ A convincing improvement is obtained by considering mesonic exchange effects as first demonstrated by Yamazaki *et al.*⁸ for ^{210}Po . Their result—an anomaly of the orbital g factor of the proton $\delta g_1 = +0.1$ —corroborates theoretical predictions by Bloch,⁹ Miyazawa,¹⁰ and de-Shalit.¹¹

In the present experiment we have measured the magnetic moments of the $\frac{21}{2}^-$ and the $\frac{29}{2}^+$ isomeric states in ^{211}At in order to get more experimental evidence of the orbital g factor anomaly δg_1 . A second aim was to investigate the blocking of the core polarization due to $h_{11/2} - h_{9/2}$ proton core excitations. Because of the small predicted effect of a few percent,^{2,12} the Knight shift and diamagnetic corrections (Sec. IV) which are neglected by several authors were estimated in a consistent way.

II. EXPERIMENTAL DETAILS

The magnetic moments of the $\frac{21}{2}^-$ level at 1416.3 keV and of the $\frac{29}{2}^+$ level at 2641.2 keV (see Fig. 1) were determined by the time differential observation of the perturbed angular distribution (TDPAD).¹³ The isomeric states were populated by the nuclear reaction $^{209}\text{Bi}(\alpha, 2n)^{211}\text{At}$ with the 30 MeV α -particle beam provided by the cyclotron of the University of Hamburg. The width of the beam pulses was $\Delta T_0 = 2$ nsec at a repetition time of $T_0 = 93$ nsec. The target was pure metallic Bi (99.999%); it was kept at room temperature during the measurements. No influence of the noncubic lattice structure on the modulation patterns could be observed.

The experimental setup and the electronics were similar to that described in Ref. 15. The γ radiation was detected by two NaI(Tl) scintillators placed asymmetrically to the beam direction at angles θ and $\theta + 90^\circ$ in a plane perpendicular to the magnetic field H .

The γ -ray intensity $I(t)$ detected between the beam pulses is given by (for details see Ref. 13):

$$I(t) = I_0 \exp(-t/\tau) \sum_{0,2,4} A_k P_k[\cos(\omega_L t - \theta)] \quad (1)$$

with the Larmor frequency

$$\omega_L = -(\mu_N/\hbar) gH. \quad (2)$$

The functions P_k are the Legendre polynomials and the coefficients A_k depend on the nuclear alignment and on the multipolarity and spins involved in the γ transition. The g factor of the $\frac{29}{2}^+$ level was

extracted from the intensity ratio

$$R(t) = \frac{I(t, \theta) - I(t, \theta + 90^\circ)}{I(t, \theta) + I(t, \theta + 90^\circ)} = b_2 \cos 2(\omega_L t - \theta) \quad (3)$$

with

$$b_2 = \frac{48A_2 + 20A_4}{64 + 16A_2 + 9A_4}. \quad (4)$$

The small variation of the amplitude b_2 in Eq. (3) with the factor

$$1/[1 + b_4 \cos 4(\omega_L t - \theta)]$$

could be neglected in the present experiment. For the $\frac{21}{2}^-$ state Eq. (3) was modified, taking into account feeding of this state from the isomeric $\frac{29}{2}^+$ level (see Appendix).

III. RESULTS AND ANALYSIS

The delayed γ -ray spectrum following the reaction $^{209}\text{Bi}(\alpha, 2n)^{211}\text{At}$ at an α energy of 33 MeV is shown in Fig. 2. The 688.9 keV and the 713.3 keV γ transitions could not be resolved by the NaI(Tl) detectors. Bergström *et al.*¹⁴ report an intensity ratio of these transitions $I(688.9)/I(713.3) = 2.6$. Combining this ratio with the theoretical anisotropies¹⁶ $b_2(688.9) = -0.22$ and $b_2(713.3) = +0.45$, one expects a nearly isotropic angular distribution of the sum peak. The magnetic moment of the $\frac{29}{2}^+$ state was therefore extracted from the intensity

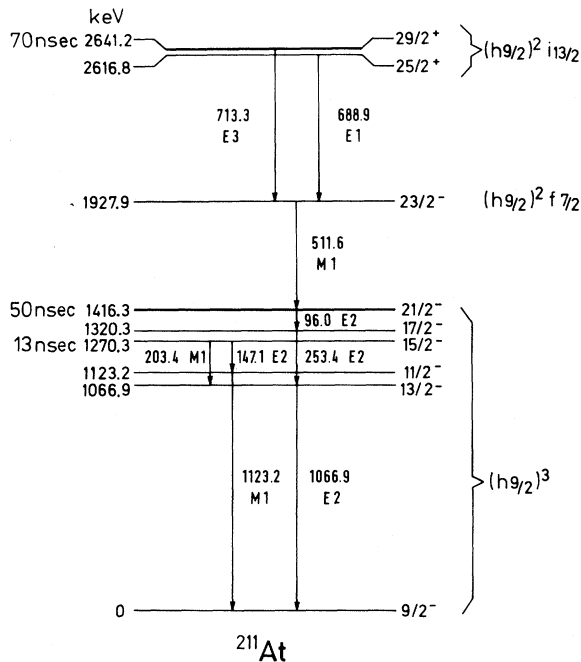


FIG. 1. Low energy part of the level scheme of ^{211}At (from Ref. 14). Only the delayed γ transitions which are of interest in the present experiment are shown.

TABLE I. Results of the TDPAD experiment on ^{211}At .

Level	E	E_α	T_0	H_{ext}	Observed γ transition	g_{uncorr}
I^π	(keV)	(MeV)	(nsec)	(kOe)	(keV)	
$\frac{29}{2}^+$	2641.2	33	89.06	27.9	511.6	1.059 ± 0.031
$\frac{21}{2}^-$	1416.3	30	93.45	21.0	1066.9	0.900 ± 0.018
		33	89.06	27.9		0.910 ± 0.016

modulation of the 511.6 keV transition, favored by the fact that almost no annihilation radiation was produced by the nuclear reaction.

The modulation of the intensity ratio $R(t)$ of the 511.6 keV γ radiation at an external magnetic field of 27.9 kOe is shown in Fig. 3. The experimental data were fitted by Eq. (3). The result is

$$g_{\text{uncorr}}(\frac{29}{2}) = +1.059 \pm 0.031.$$

The 511.6 keV γ peak contains a substantial admixture of the Compton background of the 1066.9 keV radiation, which has a different modulation frequency due to the different g factor of the $\frac{21}{2}^-$ state. The influence on the spin rotation pattern and on the g factor of the $\frac{29}{2}^+$ level is studied in detail in Ref. 17. Because of the large uncertainties of the experimental parameters entering into this estimation the correction concerning the Compton contribution was not applied.

The isomeric $\frac{21}{2}^-$ state decays via a stretched $E2$ cascade, thus leaving the initial alignment produced by the nuclear reaction nearly undisturbed.¹⁶ The measurement of the magnetic moment of the $\frac{21}{2}^-$ level could therefore be carried out by detecting

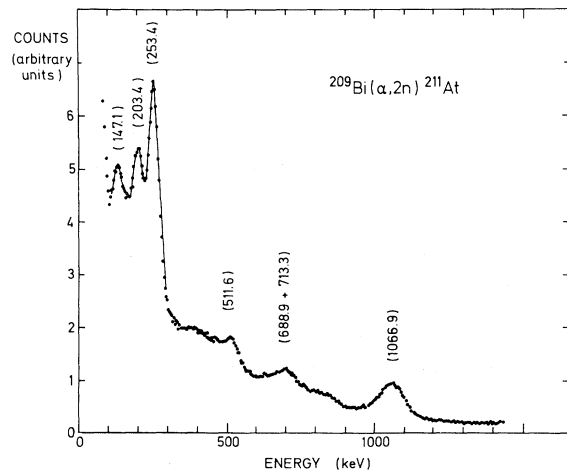


FIG. 2. Delayed γ -ray spectrum following the nuclear reaction $^{209}\text{Bi}(\alpha, 2n)^{211}\text{At}$ at an α energy of 33 MeV with in a time window from 10 to 80 nsec after the beam pulse. The γ energies given in brackets are taken from Ref. 14.

the 1066.9 keV γ transition, taking advantage of the much lower background of this line compared to the 253.4 keV γ peak.

Figure 4 shows the intensity modulation of the 1066.9 keV γ radiation at two magnetic fields of 21.0 and 27.9 kOe. The data were fitted by Eq. (A5), where the feeding from the isomeric $\frac{29}{2}^+$ level is included. According to this feeding the measured modulation pattern is no longer determined by the Larmor frequency of the $\frac{21}{2}^-$ state alone, but is also influenced significantly by the Larmor precession of the $\frac{29}{2}^+$ level. The details are given in the Appendix. The result for the g factor is

$$g_{\text{uncorr}}(\frac{21}{2}^-) = +0.905 \pm 0.016.$$

The result of Baba *et al.*¹⁸ for the g factor of the $\frac{21}{2}^-$ state, $g = 0.920 \pm 0.025$, agrees with our present experiment.

IV. DIAMAGNETIC AND KNIGHT SHIFT CORRECTIONS

The magnetic field at the position of the nucleus differs from the applied external field. In our case there are essential two contributions: the diamagnetic shielding σ and the Knight shift K . Thus the effective magnetic field is given by

$$H_{\text{eff}} = H_{\text{ext}}(1 + \sigma)(1 + K). \quad (5)$$

The diamagnetic shielding factors for Rn^0 and Ra^{2+} are taken from relativistic calculations by Feiock and Johnson²⁴:

$$\sigma(\text{Rn}^0) = -0.0191 \quad \text{and} \quad \sigma(\text{Ra}^{2+}) = -0.0204.$$

The corresponding values for Po^{4+} and At^{5+} are obtained by extrapolating the sequence $\sigma(\text{Tl}^{1+})$,

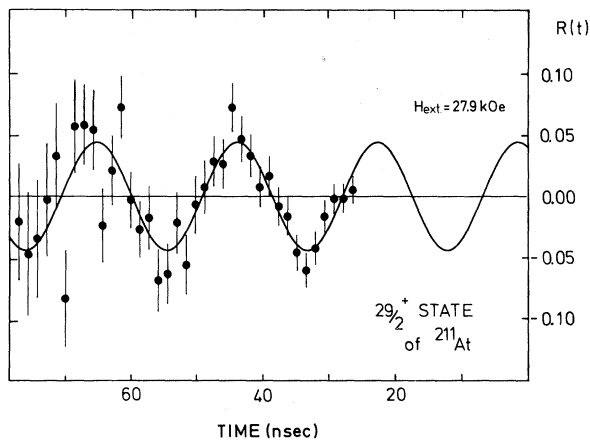


FIG. 3. Modulation of the intensity ratio of the 511.6 keV γ radiation. The solid line is a least-squares fit of Eq. (3) to the experimental data.

$\sigma(\text{Pb}^{2+})$, and $\sigma(\text{Bi}^{3+})$ given in Ref. 24:

$$\sigma(\text{Po}^{4+}) = -0.0179 \quad \text{and} \quad \sigma(\text{At}^{5+}) = -0.0185.$$

The values of $\sigma(\text{Ra}^{2+})$, $\sigma(\text{Th}^{4+})$, and $\sigma(\text{U}^{6+})$ yield

$$\sigma(\text{Fr}^{1+}) = -0.0197.$$

There exist no experimental data on the Knight shift of Po, At, Fr, and Ra. Therefore one has to estimate these Knight shifts by extrapolating known data. Since the magnetic moments are measured on impurity atoms in these cases a further problem arises from the fact that the Knight shifts K are changed by a factor $(1 + \Delta K/K)$ (Ref. 25).

The extrapolated Knight shifts for Po, At, Fr, and Ra summarized in Table II are based on data given by Drain,²⁶ Warren²⁷ and the U. S. Alloy Data Center²⁸ for Cu, Ga, As; Cd, In, Sn, Sb, Te; Tl, Pb, Bi and for the alkali metals. The values in Table II include the Knight shift changes $\Delta K/K$ for the systems $\text{Rn}(\underline{\text{Hg}})$, $\text{Fr}(\underline{\text{Hg}})$, $\text{Po}(\underline{\text{Pb}})$, $\text{Ra}(\underline{\text{Pb}})$ and $\text{At}(\underline{\text{Bi}})$. For details see Ref. 17

V. DISCUSSION

The shell model additivity relation for g factors holds rather well for all high-spin many-particle

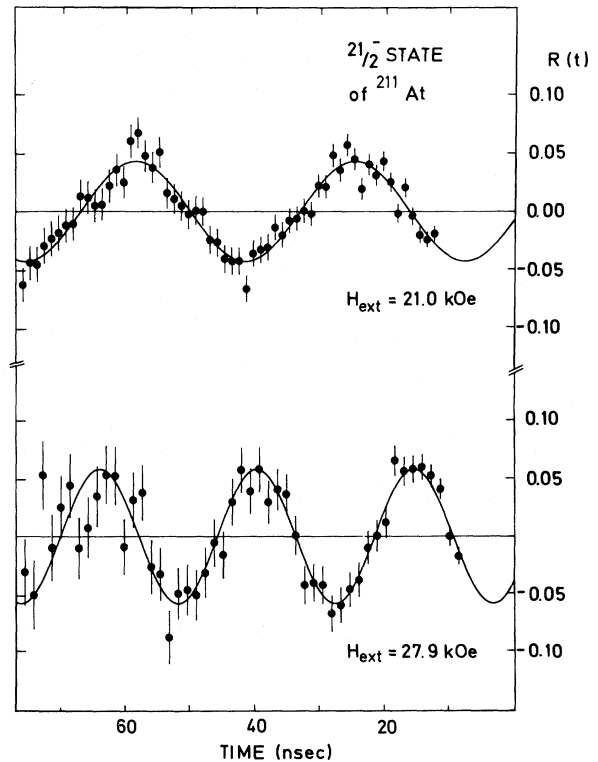


FIG. 4. Modulation of the intensity ratio of the 1066.9 keV γ radiation at magnetic fields of 21.0 and 27.9 kOe. The solid lines are least-squares fits of Eq. (A5) (see Appendix) to the data.

TABLE II. Experimental g factors of $(h_{9/2})^n$ proton states in $N=126$ isotones.

Nucleus	Level I^π	Reaction	Target	Solute Knight shift		Diamagnetic shielding	g_{uncorr}	Ref.	g_{corr}
				$K_0 = K \left(1 + \frac{\Delta K}{K}\right)$					
^{209}Bi	$\frac{9}{2}^-$	(g.s. NMR)		a		-0.0174	0.8976	19	0.9132
^{210}Po	8^+	$^{208}\text{Pb}(\alpha, 2n)$	Solid Pb	+0.010 (2)		-0.0179	0.910 (6)	b	0.917 (7)
	11^-						1.107 (16)	8	1.116 (17)
^{211}At	$\frac{21}{2}^-$	$^{209}\text{Bi}(\alpha, 2n)$	Solid Bi	+0.006 (2)		-0.0185	0.905 (16)	Present work 18	0.917 (16)
	$\frac{29}{2}^+$						1.059 (31)		Present work
^{212}Rn	8^+	$^{204}\text{Hg}(^{12}\text{C}, 4n)$	Liquid Hg	0.000 (5)		-0.0191	0.894 (10)	35	0.911 (12)
^{213}Fr	$\frac{29}{2}^+$	$^{204}\text{Hg}(^{14}\text{N}, 5n)$	Liquid Hg	0.017 (10)		-0.0197	1.04 (1)	23	1.043 (14)
^{214}Ra	8^+	$^{206}\text{Pb}(^{12}\text{C}, 4n)$	Solid Pb	0.024 (13)		-0.0204	0.882 (4)	34	0.878 (12)

^a g_{uncorr} was measured in an ionic environment.

^b Weighted average of results from Refs. 8, 20, 21, and 22.

states of isomeric nuclei in the lead region in spite of the large deviations of the magnetic moments from their single particle values.¹² This fact permits to renormalize nearly all corrections to the g factor of a single particle state into an effective one-particle magnetic moment operator with approximately state-independent effective parameters. The expectation value μ of the most general magnetic moment operator $\hat{\mu}$ for a single particle state with $j = l \pm \frac{1}{2}$ is given by²⁹

$$\mu = j \left(g_l + \delta g_l \pm \frac{(g_s + \delta g_s) - (g_l + \delta g_l)}{2l+1} \right) + g_p \langle i^2 (sY^2)^1 \rangle_z. \quad (6)$$

The parameters δg_l and δg_s denote the deviations

from the respective g factors g_l and g_s of the free nucleon.

The quantities δg_l , δg_s , and g_p have their origins in first order (fo) and higher order (ho) $M1$ core polarization, in the mesonic exchange currents, and in the two-body spin-orbit interaction.³⁰ They may be expressed by

$$\begin{aligned} \delta g_l &= \delta g_l^{\text{mes}} + \delta g_l^{\text{cp}}(\text{ho}), \\ \delta g_s &= \delta g_s^{\text{mes}} + \delta g_s^{\text{cp}}(\text{fo}) + \delta g_s^{\text{cp}}(\text{ho}) + \delta g_s^{\text{LS}}, \\ g_p &= \delta g_p^{\text{mes}} + \delta g_p^{\text{cp}}(\text{fo}) + \delta g_p^{\text{cp}}(\text{ho}) + \delta g_p^{\text{LS}}. \end{aligned} \quad (7)$$

The contributions from core polarization δg^{cp} and from the LS force δg^{LS} are highly state dependent.³¹ Therefore they cannot be renormalized

TABLE III. Deduced g factors of $h_{9/2}$ and $i_{13/2}$ proton states in $N=126$ isotones.

Nucleus	I^π	g_{corr}^a	Main configuration	Deduced			
				single particle state (j, l)	$g_{\text{exp}}(j, l)$	$g_{\text{sp}} - g(j, l)^b$	$\delta g^{\text{cp}}(\text{fo})^c$
^{209}Bi	$\frac{9}{2}^+$	+0.9132	$h_{9/2}$	$h_{9/2}$	+0.9132	-0.330	+0.203
^{210}Po	8^+	+0.917 (7)	$[(h_{9/2})^2]_{8^+}$	$h_{9/2}$	+0.917 (7)	-0.334	+0.182
	11^-	+1.116 (17)	$[h_{9/2} \otimes i_{13/2}]_{11^-}$	$i_{13/2}$	+1.299 (33)	+0.054	-0.169
^{211}At	$\frac{21}{2}^-$	+0.917 (16)	$[(h_{9/2})^3]_{(21/2)^-}$	$h_{9/2}$	+0.917 (16)	-0.334	+0.162
	$\frac{29}{2}^+$	+1.073 (31)	$[(h_{9/2})^2]_{8^+} \otimes i_{13/2}]_{(29/2)^+}$	$i_{13/2}$	+1.307 (79)	+0.046	-0.154
^{212}Rn	8^+	+0.911 (12)	$[(h_{9/2})^2]_{8^+}$	$h_{9/2}$	+0.911 (12)	-0.328	+0.140
^{213}Fr	$\frac{29}{2}^+$	+1.043 (14)	$[(h_{9/2})^2]_{8^+} \otimes i_{13/2}]_{(29/2)^+}$	$i_{13/2}$	+1.214 (36)	+0.139	-0.124
^{214}Ra	8^+	+0.878 (12)	$[(h_{9/2})^2]_{8^+}$	$h_{9/2}$	+0.878 (12)	-0.295	+0.097

^a See Table II.

^b $g_{\text{sp}}(h_{9/2}) = +0.583$; $g_{\text{sp}}(i_{13/2}) = +1.353$.

^c Calculated according to Noya *et al.* (Ref. 2) with a harmonic oscillator potential and an interaction strength parameter $C=40$ MeV.

into Eq. (6). In order to get rid of this state dependency the quantities δg^{cp} and δg^{LS} are subtracted from the experimental single particle values $g_{\text{exp}}(j, l)$, i.e.,

$$\Delta g(j, l) = g_{\text{exp}}(j, l) - g_{\text{sp}} - \delta g^{\text{cp}}(\text{fo}) - \delta g^{\text{LS}}, \quad (8)$$

where the Schmidt value g_{sp} is also subtracted for convenience. The contributions δg^{cp} and δg^{LS} are not accessible to experimental determination. They can be obtained by calculation only: δg^{cp} is usually calculated by configuration mixing theories (e.g. Refs. 2-5), and δg^{LS} by a formula given by Chemtob.³² Using expression (8), Eq. (6) can be written as

$$\Delta g(j, l) = \delta g_l \pm \frac{\delta \tilde{g}_s - \delta g_l}{2l+1} + \delta g_p \frac{1}{j} \langle i^2 (sY^2)^l \rangle_s \quad (9)$$

with $\delta \tilde{g}_s = \delta g_s^{\text{mes}} + \delta g_s^{\text{cp}}(\text{ho})$ and $\delta g_p = \delta g_p^{\text{mes}} + \delta g_p^{\text{cp}}(\text{ho})$. The state dependency of the quantities δg_l , $\delta \tilde{g}_s$, and δg_p is small,³³ and will be neglected here.

In the following sections we discuss two topics of main interest: (i) the blocking of core polarization in the $h_{9/2}$ proton shell, which was analyzed previously by several authors^{6,12,34-36} on other $(h_{9/2})^n$ proton states in the $N=126$ isotones, and (ii) mesonic and higher order core polarization effects, which cause an anomaly in the orbital g factor of the proton. For this purpose, it is necessary to deduce from the $\frac{21}{2}^-$ and $\frac{29}{2}^+$ many-particle states in ^{211}At by use of the additivity relation the experimental single particle g factors $g_{\text{exp}}(j, l)$ for protons in the $h_{9/2}$ and $i_{13/2}$ states.

A. $1h_{9/2}$ proton g factor

The $\frac{21}{2}^-$ state belongs to the lowest seniority $\nu=3$ states in ^{211}At and is formed by three $1h_{9/2}$ protons coupling to $[(h_{9/2})^3]_{21/2-}$. A number of experimental and theoretical investigations^{14,37,38} proved that the admixtures of states belonging to other configurations and to collective excitations are surprisingly small. Thus, the additivity relation $g(h_{9/2}^1) = g(h_{9/2}^3)$ yields for the experimental single particle g factor of the $1h_{9/2}$ proton in ^{211}At $g_{\text{exp}}(h_{9/2}) = +0.917(16)$. For comparison the g factors of $h_{9/2}$ protons deduced from other $[(h_{9/2})^n]_J$ states in the $N=126$ isotone region are given in Table III.

B. $1i_{13/2}$ proton g factor

The $\frac{29}{2}^+$ state in ^{211}At has been studied extensively by Bergström *et al.*¹⁴. From reduced transition probabilities they obtained the following configuration of this state:

$$|\frac{29}{2}^+\rangle = 1.00 |[(h_{9/2})^2]_{3^+} \otimes |\frac{13}{2}^+\rangle$$

with

$$|\frac{13}{2}^+\rangle = 0.94 |i_{13/2}\rangle + 0.32 |f_{7/2} \otimes 3^-\rangle + 0.05 |[i_{13/2} \otimes 3]_{7/2} \otimes 3^-\rangle.$$

Using this wave function together with the additivity rule, the decoupling procedure yields for the g factor of the $1i_{13/2}$ proton $g_{\text{exp}}(i_{13/2}) = +1.307(79)$. The calculation of this value was carried out using the g factor of the $[\pi(h_{9/2})^2]_{3^+}$ state in ^{210}Po (see Table III), the g factor $g(f_{7/2}) = 1.47$ for the $f_{7/2}$ state, which is simply the $f_{7/2}$ Schmidt value corrected for core polarization, and the magnetic moment $\mu(3^-) = 1.65\mu_N$ for the octupole vibration.³⁹ The g factor of the small $|[i_{13/2} \otimes 3]_{7/2} \otimes 3^-\rangle$ admixture in the wave function was computed by use of the magnetic moment of the $\pi i_{13/2}$ state deduced from the 11^- state in ^{210}Po .

C. Configuration mixing and blocking of $M1$ core polarization

The $M1$ core polarization contributions δg^{cp} were calculated up to first order by the configuration mixing theory of Noya, Arima, and Horie,² neglecting the fact that the states in question are seniority $\nu=3$ states. The calculations were carried out with a δ function like residual interaction, using an oscillator potential and an interaction strength parameter $C=40$ MeV. The values $\delta g^{\text{cp}}(\text{fo})$ are given in Table III. For the $1h_{9/2}$ states $\delta g^{\text{cp}}(\text{fo})$ accounts for approximately half the difference between the values $g_{\text{exp}}(j, l)$ and the Schmidt value, whereas this difference is overestimated for the $1i_{13/2}$ states in ^{210}Po and ^{211}At , and reproduced for the g factor of the $1i_{13/2}$ state in ^{213}Fr .

The configuration mixing theory predicts a slight increase (enhancement) or decrease (blocking) of core polarization, depending on the number of nucleons in the unfilled states. These effects have been observed nicely in the $1f_{5/2}$ and $1f_{7/2}$ proton and neutron shells.^{30,40} For $1h_{9/2}$ protons, deduced from n -particle states $[1\pi(h_{9/2})^n]_J$ in the $N=126$ isotones, the blocking of core polarization is given by

$$\delta g^{\text{cp}}(h_{9/2}^n) = \frac{9-n}{8} \delta g_{\pi}^{\text{cp}}(^{209}\text{Bi}) + \delta g_{\nu}^{\text{cp}}(^{209}\text{Bi}). \quad (10)$$

The quantities $\delta g_{\pi}^{\text{cp}}(^{209}\text{Bi})$ and $g_{\nu}^{\text{cp}}(^{209}\text{Bi})$ are the contributions due to the proton particle-hole excitation $1\pi h_{11/2} \rightarrow 1\pi h_{9/2}$, and for the neutron particle-hole excitation $1\nu i_{13/2} \rightarrow 1\nu i_{11/2}$, respectively.

For better comparison the g factors $g_{\text{exp}}(h_{9/2})$, deduced from $[1\pi(h_{9/2})^n]_J$ states in ^{209}Bi , ^{210}Po , ^{211}At , ^{212}Rn , and ^{214}Ra are depicted in Fig. 5. Within the experimental errors, there is no difference between the $1h_{9/2}$ g factors from ^{209}Bi to

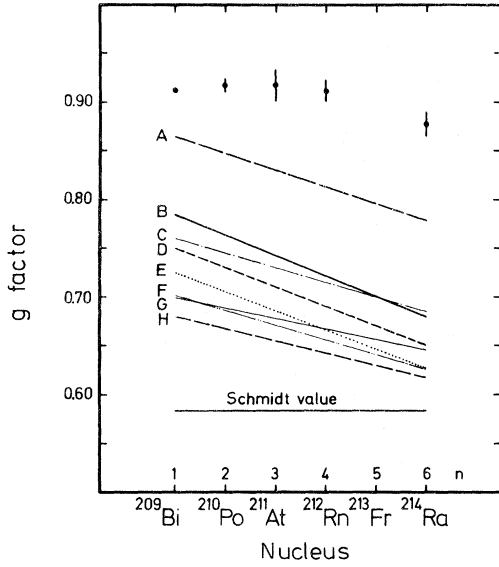


FIG. 5. Experimental g factors of $(h_{9/2})^n$ proton states in $N=126$ isotones compared to theoretical predictions obtained from configuration mixing theories using different residual interactions: A, Arita (Ref. 6); B, calculated according to Noya *et al.* (Ref. 2); C, G, Mavromatis and Zamick (Ref. 4); D, E, F, H, Blomqvist *et al.* (Ref. 5).

^{212}Rn , i.e., these nuclei do not show any blocking effect. Only the g factor of the $h_{9/2}$ proton deduced from the 8^+ state in ^{214}Ra is smaller by about 4%. This, however, may be attributed to a higher amount of admixtures to the wave function of this particular 8^+ state compared with the 8^+ states in ^{210}Po and ^{212}Rn . Such an assumption is also corroborated by the life time of the 8^+ state in ^{214}Ra , which is appreciably longer than the lifetimes of the 8^+ states in ^{210}Po and ^{212}Rn .

The lines in Fig. 5 show the blocking predictions

according to Eq. (10) for several different values of $\delta g_{\pi}^{\text{cp}}(^{209}\text{Bi})$ and $\delta g_{\nu}^{\text{cp}}(^{209}\text{Bi})$, which were calculated (see Table IV) by Blomqvist, Freed, and Zetterström⁵ with a Gillet, a Kim-Rasmussen, and a Brückner potential, by Mavromatis and Zamick⁴ with a Kallio-Kolltveit interaction and a Hamada-Johnston potential, by Arita⁶ with a Rosenfeld interaction, and finally according to Noya *et al.*² by a δ -function residual interaction. Depending on the potentials used in the calculations for $\delta g^{\text{cp}}(^{209}\text{Bi})$, the blocking effect demands a linear decrease of the $1h_{9/2}$ g factors from ^{209}Bi to ^{214}Ra between 7% and 14%. This decrease of the $h_{9/2}$ g factors is not observed. However, there should be no doubt about the existence of the blocking effect also in the lead region, since it is a consequence of the exclusion principle. Therefore, one is compelled to assume other mechanisms, which are at least partially compensating the blocking effect. Arita⁶ discussed two-particle excitations of $(h_{9/2})^n$ protons to the nearby $f_{7/2}$ and $i_{13/2}$ orbits. In a preliminary estimate he shows that the g factors are increased by these second order effects. Blomqvist *et al.*⁵ suggest the presence of a collective oscillation other than 1^+ , which is coupled strongly to the spin of the odd proton. One also has to keep in mind that the blocking of core polarization expressed by Eq. (10) is valid for seniority $\nu=1$ states only. The seniority quantum numbers of the $(h_{9/2})^n$ states considered here, however, range from $\nu=1$ to $\nu=3$.

D. Contributions from mesonic exchange currents and higher order core polarization

In order to determine from the g factors measured in ^{211}At the quantities δg_l , $\delta \tilde{g}_s$, and δg_p , which were defined in Eq. (9), it is necessary to calculate the quantities $\Delta g(h_{9/2}, ^{211}\text{At})$ and

TABLE IV. Summary of first order core polarization contributions to the g factor of ^{209}Bi (ground state) calculated with different residual interactions. Only those $\delta g_g^{\text{cp}}(^{209}\text{Bi})$ values are given here, where a separation into proton part $\delta g_{\pi}^{\text{cp}}$ and neutron part $\delta g_{\nu}^{\text{cp}}$ was carried out.

$g_{\text{sp}} - g_{\text{exp}}(h_{9/2})$	π	ν	δg_g^{cp} (First order)		Ref.
			Sum	Potential	
-0.330	+0.082	+0.033	+0.115	Kallio-Kolltveit	4
	+0.122	+0.056	+0.178	Hamada-Johnston	
	+0.102	-0.004	+0.098	Gillet	5
	+0.118	0.00	+0.118	Kim-Rasmussen I	
	+0.156	-0.013	+0.142	Kim-Rasmussen II	
	+0.158	+0.009	+0.167	Brückner	
	+0.135	+0.146	+0.281	Rosenfeld	6
	+0.169	+0.034	+0.203	δ function	Present work

$\Delta g(i_{13/2}, {}^{211}\text{At})$ [see Eq. (8)]. As shown in the preceding section, the parameters δg^{cp} are subject to large theoretical uncertainties (cf. column 4 of Table IV). The values $g_{\text{exp}}(h_{9/2})$ do not show any blocking of core polarization in this mass region. Therefore, disregarding this effect, the present evaluation procedure is carried out with δg^{cp} values calculated for the $h_{9/2}$ and $i_{13/2}$ states in ${}^{209}\text{Bi}$. Four different sets of δg^{cp} values for the $h_{9/2}$ and $i_{13/2}$ states in ${}^{209}\text{Bi}$ are available and are used for the computation of $\Delta g(j, l)$: three sets are calculated up to first order with the configuration mixing theory using different potentials,^{2, 41, 42} and one is obtained by the theory of finite Fermi systems.⁴³ Insertion of $\Delta g(h_{9/2}, {}^{211}\text{At})$ and $\Delta g(i_{13/2}, {}^{211}\text{At})$ into Eq. (9) yields two conditional equations for the determination of the three parameters δg_l , $\delta \tilde{g}_s$, and δg_p . Since the main interest lies in the determination of the anomaly δg_l of the proton orbital g factor here, the assumption $\delta g_p \equiv 0$ will be made. This is justified by the following reasons: (i) the magnitude of δg_p is expected to be small as has been shown from experimental data in the rubidium region,³⁰ and (ii) the influence of the tensor term $\delta g_p(1/j)\langle i^2(sY^2)^1 \rangle_z$ on δg_l is very small, since it is of the same order of magnitude but of opposite sign for $h_{9/2}$ and $i_{13/2}$ states. A variation of δg_p in the range $-2 \leq \delta g_p \leq +2$ affects δg_l only by ± 0.01 .

The results of δg_l and $\delta \tilde{g}_s$ for ${}^{211}\text{At}$ are summarized in Table V together with the values δg_l and

$\delta \tilde{g}_s$ obtained from corresponding data of ${}^{210}\text{Po}$. In both cases the quantities $\delta \tilde{g}_s$ are subject to large errors. They are rather small with a tendency to negative values. This is compatible with results obtained in the rubidium region.³⁰ Because of the high angular moments involved ($l=5$ and $l=6$) and the factor $1/(2l+1)$ in Eq. (9), the values δg_l are rather insensitive in respect to different δg^{cp} values as long as the values of $\delta g^{\text{cp}}(h_{9/2})$ and $\delta g^{\text{cp}}(i_{13/2})$ are shifted by the same amount. Variation of only one of these quantities results in a rather large alteration of δg_l . The excellent agreement between $\delta g_l({}^{210}\text{Po})$ and $\delta g_l({}^{211}\text{At})$ is only mirroring the good agreement between the experimental single particle g factors in ${}^{210}\text{Po}$ and ${}^{211}\text{At}$. A similar evaluation carried out by Nagamiya and Yamazaki⁴⁴ for ${}^{210}\text{Po}$ yields $\delta g_l = 0.11(2)$.

The contributions $\delta g^{\text{cp}}(\text{ho})$ from higher order core polarization to the orbital g factor have been estimated by Blomqvist⁵ and by Mavromatis and Zamick⁴ for ${}^{209}\text{Bi}$ to be about $\delta g^{\text{cp}}(\text{ho}) \approx -0.05$, and by Bertsch⁷ to be about $\delta g^{\text{cp}}(\text{second order}) \approx -0.13$. Assuming the same value for ${}^{210}\text{Po}$ and ${}^{211}\text{At}$, the mesonic contribution δg_l^{mes} to the orbital g factor of a proton in the $N=126$ isotones is $\delta g_l^{\text{mes}} \approx +0.17$ to $+0.25$. This is in good agreement with the value $\delta g_l^{\text{mes}} \approx +0.15$ obtained from g factors in the rubidium region, and compares well with one pion exchange potential (OPEP) calculations, which yield $\delta g_l^{\text{OPEP}} \approx +0.1$,^{10, 32, 45} and $\delta g_l^{\text{OPEP}} \approx +0.25$.⁴⁶

TABLE V. Results for δg_l and $\delta \tilde{g}_s$ from the semiempirical analysis of g factors in ${}^{210}\text{Po}$ and ${}^{211}\text{At}$.

Nucleus	State (j, l)	$g_{\text{exp}}(j, l)^a$	δg^{cp}	Ref.	$\delta g^{LS} b$	$\Delta g(j, l)^c$	$\delta g_p \equiv 0^c$ δg_l	$\delta \tilde{g}_s$
${}^{210}\text{Po}$	$h_{9/2}$	+0.917 (7)	+0.203	2	+0.010	0.121 (22)	0.13 (2)	0.2 (3)
			+0.190	41		0.134 (20)		
			+0.176	42		0.148 (19)		
			+0.150	43		0.174 (17)		
	$i_{13/2}$	+1.299 (33)	-0.185	2	-0.007	0.138 (38)	0.13 (2)	0.1 (3)
			-0.175	41		0.128 (37)		
			-0.092	42		0.045 (34)		
			-0.125	43		0.078 (35)		
${}^{211}\text{At}$	$h_{9/2}$	+0.917 (16)	+0.203	2	+0.010	0.121 (26)	0.13 (5)	0.3 (5)
			+0.190	41		0.134 (25)		
			+0.176	42		0.148 (24)		
			+0.150	43		0.174 (22)		
	$i_{13/2}$	+1.307 (79)	-0.185	2	-0.007	0.146 (81)	0.14 (5)	0.1 (5)
			-0.175	41		0.136 (81)		
			-0.092	42		0.053 (80)		
			-0.125	43		0.086 (80)		

^a See Table III.

^b Calculated for protons by the formula (Ref. 32) $\delta g^{LS} = \pm 0.05(2j+1)/(2j+2)$ with $j=l \pm \frac{1}{2}$.

^c See Eqs. (8) and (9); the errors include the experimental errors and an estimated error of 10% for δg^{cp} and δg^{LS} .

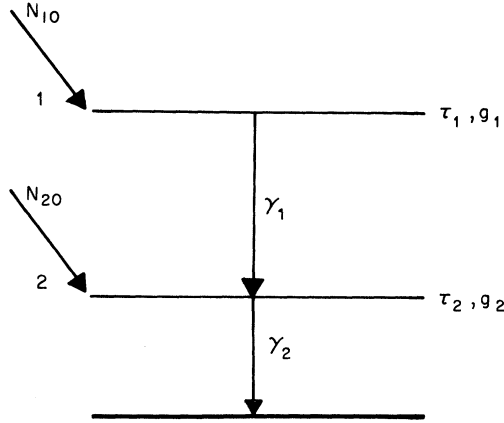


FIG. 6. Definition of the symbols used in the feeding calculation.

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APPENDIX

In the present experiment two isomeric levels with mean lives τ_1 , and τ_2 of the same order of magnitude are simultaneously populated by the nuclear reaction. Since the state 1 (see Fig. 6) decays into state 2 the intensity modulation of the radiation γ_2 cannot be described any more by Eq. (1) of Sec. II. In this case it is rather composed of two contributions:

$$I(t) = I^{\text{dir}}(t) + I^{\text{feed}}(t), \quad (\text{A1})$$

where $I^{\text{dir}}(t)$ denotes that part of the γ_2 intensity coming from the direct population of level 2, and $I^{\text{feed}}(t)$ the contribution due to feeding from level 1.

The intensity $I^{\text{feed}}(t)$ is given by

$$\begin{aligned} I^{\text{feed}}(t) &= \frac{N_{10}}{\tau_1 \tau_2} \int_0^t \exp\left(-\frac{t_f}{\tau_1}\right) \exp\left[-\frac{(t-t_f)}{\tau_2}\right] \sum_{k \text{ even}} b_k \cos k[\omega_1 t_f - \theta + \omega_2(t-t_f)] dt_f \\ &= \frac{N_{10}}{\tau_2 - \tau_1} \left[\exp\left(-\frac{t}{\tau_2}\right) \sum_{k \text{ even}} b_k \cos k \phi_k \cos k(\omega_2 t - \theta + \phi_k) - \exp\left(-\frac{t}{\tau_1}\right) \sum_{k \text{ even}} b_k \cos k \phi_k \cos k(\omega_1 t - \theta + \phi_k) \right] \end{aligned} \quad (\text{A2})$$

with

$$\phi_k = \frac{1}{k} \arctan \left[\frac{k(\omega_1 - \omega_2)\tau_1 \tau_2}{\tau_2 - \tau_1} \right],$$

where N_{10} denotes the number of nuclei excited at $t=0$; they experience the interaction frequency ω_1 ; the fraction $N_1(t) = N_{10} \exp(-t/\tau_1)$ decays at $t=t_f$ into the level 2.

The pulse repetition time T_0 of the ion beam and the mean lives τ_i were of the same order in the present experiment. Therefore the summation over the intensities produced by different beam pulses had to be carried out; the procedure was exactly the same as that used in the stroboscopic method (SOPAD).⁴⁷ The intensity (A2) thus becomes

$$\begin{aligned} I^{\text{feed}}(t) &= \frac{N_{10}}{\tau_2 - \tau_1} \sum_{i=1,2} (-1)^i \exp\left(-\frac{t}{\tau_i}\right) \\ &\quad \times \sum_{k \text{ even}} W_k^i b_k \cos k \phi_k \cos k(\omega_i t - \theta + \phi_k + \Omega_k^i) \end{aligned} \quad (\text{A3})$$

with

$$W_k^i = -\frac{1}{k} \arctan \left[\frac{\exp(-T_0/\tau_i) \sin k \omega_i T_0}{1 - \exp(-T_0/\tau_i) \cos k \omega_i T_0} \right]$$

and

$$\Omega_k^i = [1 - 2 \exp(T_0/\tau_i) \cos k \omega_i T_0 + \exp(-2T_0/\tau_i)]^{-1/2}.$$

The contribution $I^{\text{dir}}(t)$ already given in Eq. (1) of Sec. II is also corrected for the stroboscopic conditions of the experiment:

$$I^{\text{dir}}(t) = \frac{N_{20}}{\tau_2} \exp\left(-\frac{t}{\tau_2}\right) \sum_{k \text{ even}} W_k^2 b_k \cos k(\omega_2 t - \theta + \Omega_k^2). \quad (\text{A4})$$

The only approximation made in Eqs. (A3) and (A4) is that the same coefficients b_k were used in both expressions, which is justified by the stretched γ cascade between both levels in the present case. The corresponding values of b_k for other cases can be taken from the literature.¹⁶

By inserting Eqs. (A3) and (A4) into Eq. (A1), one can form the ratio $R(t)$ corresponding to Eq. (3) of Sec. II and obtain ($k_{\max} = 2$ is assumed)

$$R(t) = b_2 \left\{ \frac{\exp(-t/\tau_2)}{\tau_2 [1 - \exp(-T_0/\tau_2)]} + \frac{B}{\tau_2 - \tau_1} \left(\frac{\exp(-t/\tau_2)}{1 - \exp(-T_0/\tau_2)} - \frac{\exp(-t/\tau_1)}{1 - \exp(-T_0/\tau_1)} \right) \right\}^{-1} \\ \times \left\{ (W_2^2/\tau_2) \exp(-t/\tau_2) \cos 2(\omega_2 t - \theta + \Omega_k^2) + \frac{B}{\tau_2 - \tau_1} \cos 2\phi_2 \left[W_2^2 \exp(-t/\tau_2) \cos 2(\omega_2 t - \theta + \phi_2 + \Omega_2^2) \right. \right. \\ \left. \left. - W_1^2 \exp(-t/\tau_1) \cos 2(\omega_1 t - \theta + \phi_2 + \Omega_1^2) \right] \right\}, \quad (\text{A5})$$

where $B = N_{10}/N_{20}$ denotes the ratio of the population probabilities. For the special case of vanishing feed-in of level 2, that is $B \equiv 0$, Eq. (A5) reduces to the simple expression Eq. (3) of Sec. II.

[†]Preliminary results of this experiment were previously reported by H. Ingwersen *et al.* at the International Conference on Nuclear Moments and Nuclear Structure, Osaka, Japan, 1972 [J. Phys. Soc. Jpn. Suppl. **34**, 288, 289 (1973)].

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