Microscopic study of the variable-moment-of-inertia model for rare-earth nuclei*

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Based on the microscopic cranking model, the moment-of-inertia parameter J_0 and the force constant C_{VMI} associated with the variable-moment-of-inertia model are calculated microscopically for rare-earth nuclei. Higher-order effects representing quadrupole and hexadecapole centrifugal stretching, proton and neutron Coriolis-antipairing effects, and fourth-order cranking correction are included. The present calculations are able to reproduce the trend and the magnitude of both J_0 and C_{VMI} fairly well with discrepancies ranging from 10 to 40%.

[NUCLEAR STRUCTURE Rare-earth even-even nuclei, calculated moment of inertia and force constant. Variable-moment-of-inertia model, cranking model. Coriolis-antipairing effect, fourth-order cranking, centrifugal stretching.]

I. INTRODUCTION

It is now well established that the quasirotational spectrum plays a central role in the excitations of even-even deformed nuclei.[1,2] The general features of the quasirotational states are as follows: (1) their spins and parities follow the sequence of $0^+, 2^+, 4^+, 6^+, \ldots$, and (2) their energies deviate from the $I(I+1)$ rule as the spins increase. Recently, it was discovered that at very high angular momenta, the rotational energies of some nuclei may exhibit anomalous behavior, the so called back bending.[1-3] We shall not discuss the back-bending phenomenon in this paper, but shall limit our calculations only to those states with moderate high spins. There exist many two-parameter formulas which fit very well the energy levels up to spin $I \sim 12$. Among them we may mention the centrifugal stretching model of Diamond, Stephens, and Swiatecki[4] (which was later extended by Sood[5]), the fourth-order cranking model of Harris,[6] the variable moment-of-inertia model (VMI model) of Mariscotti, Scharff-Goldhaber, and Buck,[7] and the EXP model of Draper.[8] Recently the VMI model has also been extended to high spins by several authors to deal with the back-bending phenomenon.[9,10] Compared to the phenomenological fits, the microscopic calculations of the nuclear rotational energies,[11-17] on the other hand, have only moderate success in reproducing the experimental data. For example, the authors of Refs. 11–15 (Udagawa and Sheline; Chan and Valatin; Bes, Landowne and Mariscotti; Sano and Wakai; Krumlinde) took into consideration the centrifugal stretching and the Coriolis-antipairing effect[18] (CAP effect) and obtained fairly good agreement with the experiment. However, other calculations[16,17] have shown that the fourth-order cranking contribution is as important as the CAP effect and the inclusion of the former makes the theoretical results much worse. Indeed, Marshalek's calculations[16] showed that in general the calculated values of the B coefficient associated with the $I^2(I+1)^2$ correction term in an expansion of the rotational energies is about a factor of 1.5 to 3 too large compared with the experimental data in the rare-earth region. The calculations by Ma and Rasmussen[17] were likewise only able to produce results of the right order of magnitude; however, the quantitative significance of their results is subject to some uncertainty due to the use of a simple basis where the single-particle angular momentum is kept as a good quantum number. More recently, several authors[19-21] have done Hartree-Fock-Bogoliubov variational calculations to study the back-bending phenomenon at high spin states, which, however, will not be discussed here. In summary, the above situations indicate that the microscopic calculation of the rotational energy deserves much further study.

The present calculations are based on the cranking model of Inglis.[22] We follow closely the formulations of Ma and Rasmussen[17] [hereafter referred to as (I)], and make use of the single-particle wave functions of Nilsson *et al.*[23] with the inclusion of both quadrupole (ϵ_2) and hexadecapole (ϵ_4) deformation. Since it has been shown that most of the two-parameter formulas are related to each other,[7,17,24] we shall calculate specifically the parameters associated with the VMI model and the B coefficient

connected with the $I^2(I+1)^2$ term.

The following section will briefly review the formulations developed in (I), the detailed calculations and formulas are given in Sec. III, and the last two sections will give the results and discussions.

II. REVIEW OF MICROSCOPIC THEORY

The VMI model[7] can be expressed as follows:

$$\begin{cases} E_I = \frac{1}{2}C_{\mathrm{VMI}}(J_I - J_0)^2 + \dfrac{I(I+1)}{2J_I} \\ \dfrac{\partial E_I}{\partial J_I} = 0 \end{cases}, \tag{1}$$

where E_I and J_I are, respectively, the energy and moment of inertia of the excited state with spin I. The force constant C_{VMI} and the ground state moment of inertia J_0 are the two parameters which can be determined by a least-squares fit to the experimental energy levels. The VMI model is able to give very good fit for states up to spin $I \sim 12$. Recently Saethre *et al.*[25] have improved the fitting by using a three-parameter and a four-parameter cranking model formulas. The two-parameter VMI model has been shown[7] to be mathematically identical to the Harris fourth-order cranking model. In addition, Klein, Dreizler, and Das[24] have proved that the VMI model and cranking model are equivalent to all orders.

The microscopic derivation of the VMI model has been given in (I) and will be briefly outlined below. One first expresses the total energy of a rotating system as

$$E = \sum_i \tfrac{1}{2}C_i(x - x_i)^2 + \frac{I(I+1)}{2J(x_i)}, \tag{2}$$

where the potential energy is expressed approximately as a sum of harmonic terms, each of which represents contributions from various collective degrees of freedom denoted here by x_i. C_i is the spring constant associated with the ith degree of freedom. The second term is the kinetic energy. The rotational solutions are obtained by minimizing Eq. (2) with respect to various x_i at a given value of spin I. In the present calculation we introduce as collective degrees of freedom the quadrupole and hexadecapole shape deformations ϵ_2 and ϵ_4 involved in the centrifugal stretching effect, the proton and neutron pairing correlation parameters ν_p and ν_n involved in the Coriolis-antipairing effect, and a new collective variable $\eta \equiv \omega^2$ involved in the fourth-order cranking correction where ω is the angular velocity. Thus, we define [see (I) for details]

$$\{x_1, x_2, x_3, x_4, x_5\} \equiv \{\epsilon_2, \epsilon_4, \nu_p, \nu_n, \eta\}, \tag{3a}$$

$$\{C_1, C_2, C_3, C_4, C_5\} \equiv \{C_{22}, C_{44}, C_{\nu p}, C_{\nu n}, C_\eta\}. \tag{3b}$$

We have not included the asymmetric degree of freedom (γ shape vibration), since its contribution is rather insignificant as was shown by the calculations of Marshalek.[16]

In the first-order approximation, Eq. (2) can be reduced to Eq. (1) through a normal coordinate transformation and one obtains[17]

$$C_{\mathrm{VMI}}^{-1} = \sum_i \frac{1}{C_i}\left(\frac{\partial J(x_i)}{\partial x_i}\right)^2_{\{x_{i0}\}}, \tag{4}$$

where x_{i0} is the value of x_i at the ground state; thus $\eta_0 \equiv 0$. The moment of inertia $J(x_i)$ can be expressed as

$$J(x_i) = J_0(x_1, x_2, x_3, x_4) + 2C_\eta(x_1, x_2, x_3, x_4)\eta. \tag{5}$$

The first term is the ground state moment of inertia which can be calculated by the well known cranking formula of Inglis[22] and Belyaev[26]:

$$J_0 = 2\sum_{m_\alpha > 0} \frac{|\langle \alpha' | j_x | \alpha \rangle|^2}{E_{\alpha'} + E_\alpha}(U_{\alpha'}V_\alpha - V_{\alpha'}U_\alpha)^2, \tag{6}$$

where $|\alpha\rangle$ is the deformed single-particle state with α denoting the appropriate quantum numbers, m_α is the magnetic quantum number along the symmetry axis, U_α and V_α are the probability amplitudes in the presence of the pairing interaction, and E_α is the quasiparticle energy.

The Inglis and Belyaev cranking formula (6) is based on the independent quasiparticle approximation. However, a recent calculation by Meyer, Speth, and Vogeler[27] showed that the two correction terms arising from the particle-particle and particle-hole interactions nearly cancel each other. It has also been shown by Rich[28] that correction due to particle-number conservation is also small. Thus it seems that the use of the cranking formula (6) is rather well justified numerically.

The second term of Eq. (5) represents the fourth-order cranking correction which was first studied by Harris[6] and the fourth-order cranking constant C_η can be expressed as[6]

$$C_\eta = 2\sum_{m,n,p}{}' \frac{\langle\psi_0|J_x|\psi_m\rangle\langle\psi_m|J_x|\psi_p\rangle\langle\psi_p|J_x|\psi_n\rangle\langle\psi_n|J_x|\psi_0\rangle}{(\mathcal{E}_m - \mathcal{E}_0)(\mathcal{E}_n - \mathcal{E}_0)(\mathcal{E}_p - \mathcal{E}_0)}$$

$$- 2\sum_{m,n}{}' \frac{\langle\psi_m|J_x|\psi_0\rangle^2\langle\psi_n|J_x|\psi_0\rangle^2}{(\mathcal{E}_m - \mathcal{E}_0)^2(\mathcal{E}_n - \mathcal{E}_0)}, \tag{7}$$

where the ground state ψ_0 is the quasiparticle vacuum state, ψ_m and ψ_n are two-quasiparticle states, and the intermediate state ψ_p can be either two-quasiparticle or four-quasiparticle excitations.

The corresponding energies are denoted by $\mathcal{E}_0$, $\mathcal{E}_m$, $\mathcal{E}_n$, and $\mathcal{E}_p$. The prime on the summation indicates that the ground state is excluded from the summation.

It is obvious from Eq. (5) that

$$\frac{\partial J}{\partial \eta} = 2C_\eta .$$

Thus, the contribution of the fourth-order cranking in Eq. (4) is simply $4C_\eta$ while the contributions of the other degrees of freedom are given by

$$\frac{1}{C_i}\left(\frac{\partial J_0}{\partial x_i}\right)^2_{\{x_{i0}\}} .$$

The B coefficient associated with the $I^2(I+1)^2$ term in the angular momentum expansion of the rotational energy

$$E_I = \frac{I(I+1)}{2J_0} + BI^2(I+1)^2 + CI^3(I+1)^3 + \cdots \tag{8}$$

can be expressed as

$$B = -\sum_i \frac{1}{8C_i J_0^4}\left(\frac{\partial J}{\partial x_i}\right)^2_{\{x_{i0}\}} . \tag{9}$$

The value of the force constant $C_{\mathrm{VMI}}{}^{-1}$ or the B coefficient indicates the degree to which the spectrum deviates from the $I(I+1)$ rule. Both Eqs. (4) and (9) show that the contributions from various degrees of freedom are all positively added.

A simple relation between C_{VMI} and B can be obtained by combining Eq. (4) with Eq. (9), which yields

$$8BC_{\mathrm{VMI}}J_0^4 = -1 . \tag{10}$$

III. DETAILED CALCULATION AND FORMULAS

A. Single-particle basis and the pairing problem

The deformed single-particle basis used in the present calculations is chosen to be identical to that of Nilsson *et al.*[23] The diagonalization is carried out over the sapce of 11 shells for protons and 12 shells for neutrons. The values of ϵ_2 and ϵ_4 of each nucleus are taken from the work of Nilsson *et al.* and are listed in Table I.

The pairing strength G is chosen to be a smooth function of A as suggested by Nilsson *et al.*

$$GA = g_0 \pm g_1 \frac{N-Z}{A},$$
$$g_0 = 19.2 \text{ MeV}, \tag{11}$$
$$g_1 = 7.4 \text{ MeV},$$

with plus sign for protons and minus sign for neutrons. They also put in a linear surface dependence of G, which may be important for large deformation. The BCS equation is then solved by including $(15Z)^{1/2}$ or $(15N)^{1/2}$ states above and below the proton or neutron Fermi level. The pairing gap parameters Δ_p and Δ_n thus obtained are given in Table I.

The energy of a quasiparticle can be expressed as

$$E_k = (\epsilon_k - \lambda)(U_k^2 - V_k^2) + 2U_k V_k G \sum_{l>0} U_l V_l , \tag{12}$$

where ϵ_k is the single-particle energy and λ is the chemical potential. Following (I) we parametrize the probability amplitudes $\{U_k, V_k\}$ by introducing a pairing correlation parameter ν

$$\left.\begin{matrix} U_k^2 \\ V_k^2 \end{matrix}\right\} = \frac{1}{2}\left\{1 \pm \frac{\epsilon_k - \lambda}{[(\epsilon_k - \lambda)^2 + \nu^2]^{1/2}}\right\} . \tag{13}$$

TABLE I. The quadrupole and hexadecapole deformation parameters ϵ_2 and ϵ_4 are taken from Ref. 23. The energy gap Δ_p and Δ_n are calculated with pairing strength G as given in Eq. (11).

Nucleus	ϵ_2	ϵ_4	Δ_p (MeV)	Δ_n (MeV)
^{152}Sm	0.202	−0.036	1.114	0.975
^{154}Sm	0.227	−0.039	1.024	0.888
^{154}Gd	0.206	−0.029	1.101	1.001
^{156}Gd	0.233	−0.030	1.020	0.935
^{158}Gd	0.245	−0.024	0.980	0.895
^{160}Gd	0.255	−0.015	0.948	0.849
^{160}Dy	0.245	−0.015	0.988	0.934
^{162}Dy	0.256	−0.006	0.945	0.880
^{164}Dy	0.264	0.003	0.910	0.836
^{162}Er	0.242	−0.007	0.989	0.969
^{164}Er	0.254	0.001	0.941	0.906
^{166}Er	0.261	0.010	0.898	0.861
^{168}Er	0.272	0.020	0.847	0.815
^{170}Er	0.273	0.031	0.807	0.786
^{166}Yb	0.246	0.004	1.002	0.926
^{168}Yb	0.255	0.014	0.956	0.883
^{170}Yb	0.265	0.025	0.902	0.835
^{172}Yb	0.270	0.037	0.845	0.799
^{174}Yb	0.266	0.048	0.799	0.739
^{176}Yb	0.258	0.053	0.785	0.661
^{174}Hf	0.258	0.034	0.915	0.822
^{176}Hf	0.256	0.043	0.879	0.734
^{178}Hf	0.250	0.052	0.844	0.672
^{180}Hf	0.243	0.063	0.808	0.561
^{180}W	0.236	0.050	0.870	0.699
^{182}W	0.232	0.060	0.828	0.602
^{184}W	0.216	0.061	0.793	0.735
^{186}W	0.197	0.060	0.777	0.790
^{184}Os	0.213	0.053	0.750	0.690
^{186}Os	0.198	0.055	0.665	0.780
^{188}Os	0.178	0.055	0.592	0.819

If $\nu = \Delta$ (the energy gap Δ is the equilibrium value of ν at ground state), Eq. (12) reduces to the familiar BCS result

$$E_k = [(\epsilon_k - \lambda)^2 + \Delta^2]^{1/2}. \tag{14}$$

In what follows we shall vary ν to calculate the corresponding derivatives of moment of inertia as well as the pairing spring constant for a fixed pairing strength G as given by Eq. (11). For $\nu \neq \Delta$, the BCS gap equation no longer holds and Eq. (14) is not valid. Thus, it is important to use Eq. (12) rather than Eq. (14) as the expression for the quasiparticle energy.

B. Derivatives of the moment of inertia

We shall calculate the derivative of the moment of inertia J_0 with respect to the pairing correlation parameter ν while the average particle number n and the pairing strength G are held fixed. One obtains

$$\left(\frac{\partial J_0}{\partial \nu}\right)_{n,G} = \left(\frac{\partial J_0}{\partial \nu}\right)_{\lambda,G} + \left(\frac{\partial J_0}{\partial \lambda}\right)_{\nu,G}\left(\frac{\partial \lambda}{\partial \nu}\right)_{n,G}, \tag{15}$$

where J_0 is given by Eq. (6) and the average particle number n is given by

$$\sum_{k>0} 2V_k^2 = n. \tag{16}$$

It then follows

$$\left(\frac{\partial J_0}{\partial \nu}\right)_{\lambda,G|\nu=\Delta} = 2\sum_{k,l}\frac{|\langle k|j_x|-l\rangle|^2}{E_k+E_l}(U_kV_l - U_lV_k)\left[(U_l^2 - V_l^2)(U_kU_l + V_kV_l)/E_l - \frac{2U_lV_l(U_kV_l - V_kU_l)}{E_k+E_l}\frac{G}{\Delta}\sum_{m>0}U_mV_m(U_m^2 - V_m^2)^2\right], \tag{17a}$$

$$\left(\frac{\partial J_0}{\partial \lambda}\right)_{\nu,G|\nu=\Delta} = 2\sum_{k,l}\frac{|\langle k|j_x|-l\rangle|^2}{E_k+E_l}(U_kV_l - U_lV_k)\left\{2U_lV_l(U_kU_l + V_kV_l)/E_l + \frac{(U_kV_l - U_lV_k)}{E_k+E_l}\left[(U_l^2 - V_l^2) - 4U_lV_l\frac{G}{\Delta}\sum_{m>0}U_m^2V_m^2(U_m^2 - V_m^2)\right]\right\}, \tag{17b}$$

$$\left(\frac{\partial \lambda}{\partial \nu}\right)_{n,G} = -\sum_{k>0}\tfrac{1}{2}[U_k^2V_k^2(U_k^2 - V_k^2)]\sum_{k>0}U_k^3V_k^3. \tag{17c}$$

Note that in taking the derivative, the quasiparticle energy is given by Eq. (12). After the derivative is taken, its value at $\nu = \Delta$ is then evaluated.

The derivatives of J_0 with respect to the deformation parameters ϵ_2 and ϵ_4 are calculated by finite differences. The mesh point interval is taken to be 0.02 for both ϵ_2 and ϵ_4.

The various derivatives of the moment of inertia are listed in Table II. The derivatives with respect to pairing are quite stable over the rare-earth region of nuclei. For example, with respect to the neutron pairing, the derivatives fluctuate around $-(36 \pm 10)$ MeV^{-2}, while the derivatives with respect to proton pairing are roughly equal to $-(19 \pm 3)$ MeV^{-2} for nuclei in the region of $A \sim 165$ and $-(11 \pm 2)$ MeV^{-2} for those in the region of $A \sim 187$. The derivatives with respect to the deformation, on the other hand, are quite different as one goes from one nucleus to another. In the case of quadrupole deformation (ϵ_2), the derivatives are largest at the beginning of the rare-earth region and generally decrease towards the end of the region; while in the case of hexadecapole deformation (ϵ_4), the derivatives are strongly negative at the beginning of the rare-earth region and change to positive values near the end. A negative value for the inertia derivative with respect to hexadecapole deformation (ϵ_4) has some interesting consequences. The equilibrium value of ϵ_4 at a given spin I in first-order approximation is given in (I) as

$$\epsilon_4 = \epsilon_{40} + \frac{I(I+1)}{2C_{44}J^2}\frac{\partial J}{\partial \epsilon_4}, \tag{18}$$

where ϵ_{40} is the hexadecapole deformation at ground state. In the beginning of the rare-earth region, the values of both ϵ_{40} and the inertia derivative with respect to ϵ_4 are negative; thus, as the spin goes up, the hexadecapole deformation will increase in magnitude which is just the familiar stretching effect. However, for nuclei in the middle of the rare-earth region, the values of ϵ_{40} become positive while in most cases the inertia derivatives with respect to ϵ_4 still remain negative. Thus, as the spin goes up, the hexadecapole deformation will actually decrease.

Equilibrium values of quadrupole deformation ϵ_2 and of pairing parameters ν_p and ν_n at a given spin I can also be determined by equations similar to

Eq. (18) which then yield the quadrupole stretching and the Coriolis-antipairing effect.

C. Pairing spring constant

The ground state energy can be expressed as

$$\mathcal{E}_0 = \sum_{k>0} \epsilon_k 2V_k^2 - G\left(\sum_{k>0} U_k V_k\right)^2 - G\sum_{k>0} V_k^4 - G\sum_{k\neq l>0} U_k V_k^3 U_l^3 V_l \Big/ \sum_{k>0} U_k^2 V_k^2 , \quad (19)$$

where the first three terms are the BCS ground state energy, and the last term approximately accounts for the fixed-particle-number correction.[29,30] The pairing spring constant C_ν can be obtained by taking the second derivative of $\mathcal{E}_0$ with respect to the pairing correlation parameter ν

$$C_\nu = \left(\frac{\partial^2 \mathcal{E}_0}{\partial \nu^2}\right)_{n,G} = \left(\frac{\partial^2 \mathcal{E}_0}{\partial \nu^2}\right)_\lambda + 2\left(\frac{\partial^2 \mathcal{E}_0}{\partial \lambda \partial \nu}\right)\left(\frac{\partial \lambda}{\partial \nu}\right)_n + \left(\frac{\partial^2 \mathcal{E}_0}{\partial \lambda^2}\right)_\nu \left(\frac{\partial \lambda}{\partial \nu}\right)_n^2 + \left(\frac{\partial \mathcal{E}_0}{\partial \lambda}\right)_\nu \left(\frac{\partial^2 \lambda}{\partial \nu^2}\right)_n . \quad (20)$$

Evaluation of the derivatives of $\mathcal{E}_0$ with respect to λ and ν are straightforward by using Eq. (19). The derivative of λ with respect to ν can also be easily obtained in terms of Eq. (16). The proton and neutron pairing spring constants $C_{\nu p}$ and $C_{\nu n}$ as calculated by Eq. (20) are given in Table III.

It is interesting to note that inclusion of the fixed-particle-number correction in the ground state energy will in general increase the pairing

TABLE II. The moment of inertia and the inertia derivatives, where $D_{x_i} \equiv [(\partial/\partial x_i)2J_0]_{x_{i0}}$.

Nucleus	$D_{\epsilon 2}$ (MeV^{-1})	$D_{\epsilon 4}$ (MeV^{-1})	$D_{\nu p}$ (MeV^{-2})	$D_{\nu n}$ (MeV^{-2})	$2J_p$ (MeV^{-1})	$2J_n$ (MeV^{-1})	$2J_0$ (MeV^{-1})	$2J_{\exp}$ (MeV^{-1})
^{152}Sm	196	−266	−16.68	−32.35	13.62	24.84	40.38	46.8
^{154}Sm	163	−266	−20.17	−46.17	17.16	35.19	54.97	73.0
^{154}Gd	195	−236	−15.51	−29.85	13.26	23.14	38.22	46.6
^{156}Gd	168	−235	−18.49	−41.16	16.33	32.00	50.75	66.6
^{158}Gd	119	−176	−19.88	−37.59	17.45	33.27	53.26	74.8
^{160}Gd	126	−150	−20.98	−40.19	18.20	35.40	56.28	79.4
^{160}Dy	131	−169	−19.48	−35.24	16.55	30.63	49.53	68.6
^{162}Dy	126	−137	−20.55	−38.17	17.60	33.33	53.48	73.8
^{164}Dy	94	−116	−21.25	−43.05	18.33	35.84	56.88	81.2
^{162}Er	148	−147	−16.15	−32.72	15.07	27.99	45.21	58.6
^{164}Er	134	−109	−17.25	−36.35	16.38	31.41	50.17	65.4
^{166}Er	102	−87	−18.22	−41.23	17.41	34.00	53.98	73.8
^{168}Er	85	−35	−19.04	−36.48	18.74	35.11	56.54	75.0
^{170}Er	108	4	−19.93	−40.27	19.59	36.77	59.19	75.6
^{166}Yb	154	−96	−14.94	−35.08	14.16	29.59	45.94	57.8
^{168}Yb	135	−68	−16.52	−39.72	15.54	32.27	50.20	68.4
^{170}Yb	118	−19	−18.14	−36.34	17.20	33.76	53.51	70.8
^{172}Yb	124	7	−19.70	−39.98	18.75	35.96	57.45	75.8
^{174}Yb	89	−4	−21.11	−41.15	19.78	36.83	59.44	78.4
^{176}Yb	49	11	−21.87	−31.59	19.90	35.45	58.12	72.8
^{174}Hf	141	−11	−13.18	−37.80	14.90	34.40	51.77	65.4
^{176}Hf	108	−17	−13.38	−45.53	15.38	38.19	56.25	67.6
^{178}Hf	53	−7	−13.68	−35.14	15.74	36.42	54.77	64.0
^{180}Hf	15	35	−13.88	−33.54	16.18	39.02	57.96	64.2
^{180}W	63	3	−8.69	−36.56	12.34	35.26	49.98	57.6
^{182}W	21	52	−8.94	−35.55	12.90	37.84	53.28	59.6
^{184}W	69	65	−9.91	−28.92	12.88	31.00	46.07	53.6
^{186}W	89	74	−10.87	−22.25	12.55	24.76	39.18	48.6
^{184}Os	49	62	−9.25	−36.92	11.92	34.35	48.58	49.4
^{186}Os	79	68	−10.75	−29.99	12.62	28.53	43.20	43.0
^{188}Os	86	68	−12.97	−22.67	13.29	22.64	37.72	37.4

spring constant by about 20%, hence, reducing the CAP effect. Some of the examples are given in Table IV.

A simple formula for C_ν based on the continuous model is given in (I) which reads

$$C_\nu = \left(2\rho + \frac{2}{\pi\Delta}\right)(1 - \rho G), \tag{21}$$

where ρ is the average nucleon orbital level density. The spring constants given by Eq. (21) [see Table I of (I)] are somewhat larger than those given by the present calculations by about 5 to 15% in the case of protons and 10 to 25% in the case of neutrons. In view of the tremendous numerical work involved in Eq. (20), the simple formula Eq. (21) is indeed a very useful approximation.

D. Shape spring constant

The shape spring constants C_{22} and C_{44} associated with the quadrupole ϵ_2 and hexadecapole ϵ_4 deformation degrees of freedom can be obtained similarly by taking the second derivative of the ground state energy $\mathcal{E}_0$ with respect to ϵ_2 and ϵ_4:

$$C_{22} = \frac{\partial^2 \mathcal{E}_0}{\partial \epsilon_2^{\,2}}, \quad C_{44} = \frac{\partial^2 \mathcal{E}_0}{\partial \epsilon_4^{\,2}}. \tag{22}$$

In applying Eq. (22), the ground state energy $\mathcal{E}_0$ is calculated according to the Strutinsky average prescription as described in Ref. 23 by Nilsson *et al.* The C_{22} and C_{44} are then obtained by finite differences with the mesh point interval taken to be 0.02 both for ϵ_2 and ϵ_4; the results are listed in Table III.

The curvatures C_{22} and C_{44} at the ground state deformation are due to contributions from the liquid drop energy part, the shell correction part, and the pairing energy part, which make up the potential energy surface. The shell correction part gives the largest positive contribution and in fact determines, to the larger extent, the deformation of the ground state nucleus. The pairing effect tends to smooth out the level density and thus acts against the shell effect. It provides a negative contribution to the curvature. The liquid drop energy part in general gives a small positive contribution.

The Strutinsky normalization replaces the smooth part of the eigenenergy summation by the liquid drop energy. Due to the inadequacy of volume normalization of the single-particle potential well, the former has a much stronger curvature than that of the liquid drop part, as is obvious from the fact that its value would be infinite at $\epsilon_2 = 1.5$ (which is of course quite far away from the ground state deformation of $\epsilon_2 \sim 0.25$), whereas the liquid drop energy would be finite. Hence, one would expect the value of curvature calculated in a scheme with the employment of Strutinsky normalization to be smaller than the value calculated without it. This is indeed borne out by our de-

TABLE III. The spring constants associated with various degrees of freedom.

Nucleus	C_{22} (MeV)	C_{44} (MeV)	$C_{\nu p}$ (MeV^{-1})	$C_{\nu n}$ (MeV^{-1})	C_η (MeV^{-3})
^{152}Sm	760	1205	3.67	3.88	27.32
^{154}Sm	966	1519	3.74	3.27	37.68
^{154}Gd	725	1222	3.72	3.93	23.61
^{156}Gd	899	1453	3.75	3.50	31.48
^{158}Gd	1019	1556	3.78	4.18	24.81
^{160}Gd	1092	1620	3.81	4.13	28.44
^{160}Dy	989	1450	3.36	4.18	23.79
^{162}Dy	1108	1592	3.36	4.19	27.36
^{164}Dy	1215	1711	3.40	3.50	33.25
^{162}Er	926	1318	3.73	4.22	19.27
^{164}Er	1043	1503	3.66	4.24	22.38
^{166}Er	1175	1663	3.62	3.59	27.44
^{168}Er	1205	1780	3.58	4.40	17.75
^{170}Er	1181	2005	3.57	4.18	23.34
^{166}Yb	956	1390	3.93	4.27	21.21
^{168}Yb	1064	1517	3.83	3.69	26.56
^{170}Yb	1097	1645	3.66	4.40	19.49
^{172}Yb	1113	1714	3.46	4.25	25.66
^{174}Yb	1220	1710	3.34	4.03	30.56
^{176}Yb	1275	1709	3.39	5.20	19.89
^{174}Hf	1032	1637	4.10	4.56	20.90
^{176}Hf	1178	1643	4.20	3.71	31.15
^{178}Hf	1243	1603	4.29	4.92	21.16
^{180}Hf	1300	1600	4.39	2.19	14.27
^{180}W	1238	1600	4.15	4.76	18.34
^{182}W	1280	1595	4.18	2.49	12.16
^{184}W	1255	1450	4.37	5.25	15.29
^{186}W	1175	1225	4.60	5.37	12.46
^{184}Os	1280	1663	2.45	3.06	13.53
^{186}Os	1225	1488	2.06	5.14	16.28
^{188}Os	1188	1325	1.77	5.36	13.12

TABLE IV. The first four columns list pairing spring constants with and without fixed-particle-number correction (PBCS and BCS). The fifth and sixth columns give separately the two- and four-quasiparticle contributions to the fourth-order cranking constant.

Nucleus	$C_{\nu p}$ (MeV^{-1}) BCS	PBCS	$C_{\nu n}$ (MeV^{-1}) BCS	PBCS	C_η (MeV^{-3}) 2QP	4QP
^{154}Sm	3.17	3.74	3.07	3.27	49.8	−12.2
^{158}Gd	3.21	3.78	3.43	4.18	35.2	−10.4
^{162}Dy	3.07	3.36	3.50	4.19	37.7	−10.4
^{166}Er	3.09	3.62	3.36	3.59	37.7	−10.3
^{170}Yb	3.18	3.66	3.60	4.40	28.8	−9.35
^{174}Hf	3.44	4.10	3.70	4.56	30.0	−9.06
^{180}W	3.32	4.15	3.40	4.76	27.3	−8.97
^{184}Os	2.45	2.45	2.89	3.06	22.3	−8.76

tailed calculation which shows that the Strutinsky normalization generally reduces the values by about 20%.

On comparison with Table I of (I), our present results for C_{22} are about 40 to 100% larger. The first reason is that we are currently using a finer set of grids, $\Delta\epsilon_2 = 0.02$, as compared with $\Delta\epsilon_2 = 0.05$ used in the older calculation. Thus, the new calculations are less likely to suffer from the problem of anharmonic effects in the potential energy surface which, in the present case, will tend to reduce the effective value of the curvature. The second and probably the main reason behind the discrepancy is that the older calculation used a surface-independent pairing force, whereas the new calculation has a pairing force dependent on the surface area. For most properties of the nucleus near the ground state, this difference does not present significant discrepancy. But for such higher-order effect as the curvature, we find it does make a difference. When we calculated the pairing energy contribution, we found that the new calculation with a surface-dependent pairing force gives a smaller negative value than the old calculation, and the change is sufficient to account for the discrepancy between the two results. A detailed discussion was made by other authors[23,31] on the choice of these two versions of the pairing force. We have not pursued the question regarding which is the more appropriate form of pairing force to be used. However, as will be seen later, the contribution of centrifugal stretching effect to the VMI force constant is very small when compared with the contributions from Coriolis-antipairing and fourth-order cranking effects, so that either choice of the pairing force will have little effect on our final results and conclusions.

E. Fourth-order cranking constant C_η

The evaluation of the fourth-order cranking constant C_η given by Eq. (7) is rather tedious, since now one has to calculate the contributions from the four-quasiparticle intermediate states as well as those from the two-quasiparticle states. Fortunately, we are able to reduce Eq. (7) to a sum of quadratic terms; as a result, the numerical work is considerably simplified. We shall quote the final result below while the derivation will be given in the Appendix.

$$
\begin{aligned}
C_\eta = -4 \sum_{\substack{m_p>0 \\ m_q = m_p - 1 \\ m_{q'} = m_p + 1}} \sum_{t_q,\, t_{q'}} (E_q + E_{q'}) \left(\sum_{t_p} \langle p|j_x|q\rangle \langle p|j_x|q'\rangle \frac{(U_p V_q - V_p U_q)(U_p V_{q'} - U_{q'} V_p)}{(E_p + E_q)(E_p + E_{q'})} \right)^2 \\
- 2 \sum_{m_p = m_{p'} > 0} \sum_{t_p,\, t_{p'}} (E_p + E_{p'}) \left(\sum_{\substack{t_q \\ m_q = m_p \pm 1}} \frac{\langle p|j_x|q\rangle \langle p'|j_x|q\rangle (U_p V_q - V_p U_q)(U_{p'} V_q - V_{p'} U_q)}{(E_p + E_q)(E_{p'} + E_q)} \right)^2 \\
+ 4 \sum_{\substack{m_p>0 \\ m_q = m_p - 1 \\ m_{q'} = m_p + 1}} \sum_{t_q,\, t_{q'}} \frac{1}{E_q + E_{q'}} \left[\sum_{t_p} \langle p|j_x|q\rangle \langle p|j_x|q'\rangle \left(\frac{(U_p V_q - V_p U_q)(U_p U_{q'} + V_p V_{q'})}{E_p + E_q} \right.\right. \\
\left.\left. + \frac{(U_p V_{q'} - V_p U_{q'})(U_p U_q + V_p V_q)}{E_p + E_{q'}} \right) \right]^2 \\
+ 2 \sum_{m_p = m_{p'} > 0} \sum_{t_p,\, t_{p'}} \frac{1}{E_p + E_{p'}} \left[\sum_{\substack{t_q \\ m_q = m_p \pm 1}} \langle p|j_x|q\rangle \langle p'|j_x|q\rangle \left(\frac{(U_p V_q - V_p U_q)(U_{p'} U_q + V_{p'} V_q)}{E_p + E_q} \right.\right. \\
\left.\left. + \frac{(U_{p'} V_q - V_{p'} U_q)(U_p U_q + V_p V_q)}{E_{p'} + E_q} \right) \right]^2 , \qquad (23)
\end{aligned}
$$

where m_p is the magnetic quantum number of the particle in state $|p\rangle$, t_p denotes the quantum numbers other than m_p, and E_p is the corresponding quasiparticle energy. The first two terms represent the contributions from four-quasiparticle intermediate states while the last two terms represent those from two-quasiparticle intermediate states, as can readily be seen from the form of the products of the U, V coefficients. The fourth-order cranking constants C_η calculated in terms of Eq. (23) are listed in Table III. Our calculations show that the two-quasiparticle contribution is always positive while the four-quasiparticle contribution is always negative. Furthermore, the former is generally about three to four times larger than the latter in magnitude. Some of the ex-

amples are given in Table IV.

IV. RESULTS AND DISCUSSIONS

A. Moment of inertia

The ground state moment of inertia J_0 is calculated according to Eq. (6). In addition, we have followed Nilsson and Prior[32] and increased the calculated values by 5% which represents approximately the effects of the coupling between the shells N and $N+2$, due to the j_x operator. This is because the Nilsson wave functions of Ref. 23 are expressed in the stretched coordinates and the j_x operator in these stretched coordinates will give rise to a term which will couple the shells N and $N+2$. The results are listed in Table II and plotted in Fig. 1. From Fig. 1, it is seen that the trends of the experimental moment of inertia are well reproduced by the calculations. The calculated magnitudes, however, are generally too small by 10 to 40%, the average discrepancy being 25%. This disagreement in magnitude seems to be somewhat too large compared to a similar calculation by Nilsson and Prior[32] where the calculated J_0 are generally 10 to 30% too small, the average discrepancy being 20%. But it should be pointed out that in the present calculation, the single-particle states and the parameters G, ϵ_2, and ϵ_4 are all taken directly from the works of Nilsson *et al.*[23] without any readjustment. One may, for example, obtain very good agreement with the experimental data by choosing $g_0 = 18.0$ MeV instead of 19.2 MeV in Eq. (11). We shall return to this question at the end of this section.

B. Force constant C_{VMI}

The force constant C_{VMI}^{-1} associated with the VMI model of Eq. (1) is calculated using Eq. (4) and the results are listed in Table V and plotted

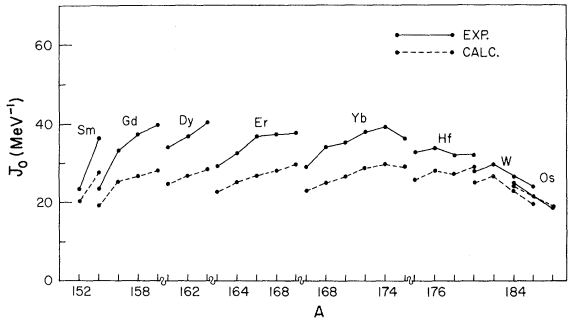

FIG. 1. The moment of inertia J_0. The theoretical values are calculated with the single-particle states and pairing strength [Eq. (11)] as given by Nilsson *et al.* (Ref. 23). The experimental values are taken from Mariscotti *et al.* (Ref. 7).

TABLE V. The force constant C_{VMI}^{-1} and the separate contributions from various higher-order effects. The experimental values of C_{VMI}^{-1} are taken from Ref. 7. We define $K_{x\,i} \equiv (\partial/\partial x_i\, J_0)^2/C_i$. All units are in MeV^{-3}.

Nucleus	K_{22}	K_{44}	K_{vp}	K_{vn}	$4C_\eta$	C_{VMI}^{-1} (Calc.)	C_{VMI}^{-1} (exp.)
^{152}Sm	13	15	18.98	67.45	109.53	224.0	595
^{154}Sm	7	12	27.20	163.11	150.73	360.0	229
^{154}Gd	13	11	16.16	56.68	94.46	191.3	544
^{156}Gd	8	10	22.78	121.15	125.94	287.9	338
^{158}Gd	3	5	26.13	84.55	99.24	217.9	245
^{160}Gd	4	4	28.85	97.89	113.76	248.5	215
^{160}Dy	4	5	28.25	74.28	95.15	206.7	219
^{162}Dy	4	3	31.43	86.97	109.44	234.8	195
^{164}Dy	2	2	33.23	132.55	132.99	302.8	238
^{162}Er	6	4	17.48	63.43	77.07	168.0	255
^{164}Er	4	2	20.29	77.95	89.50	193.7	197
^{166}Er	2	1	22.96	118.26	109.76	254.0	240
^{168}Er	2	0	25.35	75.57	70.98	173.9	110
^{170}Er	2	0	27.80	96.92	93.36	220.1	132
^{166}Yb	6	2	14.19	71.98	84.83	179.0	255
^{168}Yb	4	1	17.79	106.91	106.23	235.9	258
^{170}Yb	3	0	22.48	75.03	77.96	178.5	160
^{172}Yb	3	0	28.05	93.90	102.64	227.6	213
^{174}Yb	2	0	33.33	105.01	122.26	262.6	108
^{176}Yb	0	0	35.23	48.01	79.57	162.8	128
^{174}Hf	5	0	10.60	78.40	83.61	177.6	215
^{176}Hf	2	0	10.66	139.57	124.61	276.8	170
^{178}Hf	1	0	10.90	62.80	84.66	159.4	135
^{180}Hf	0	0	10.97	128.56	57.06	196.6	73
^{180}W	1	0	4.55	70.13	73.35	149.1	188
^{182}W	0	0	4.79	126.76	48.65	180.2	98
^{184}W	1	1	5.61	39.81	61.16	108.6	102
^{186}W	2	1	6.42	23.04	49.84	82.3	93
^{184}Os	1	1	8.74	111.23	54.14	176.1	180
^{186}Os	1	1	14.04	43.71	65.11	124.9	162
^{188}Os	2	1	23.82	23.96	52.49	103.3	196

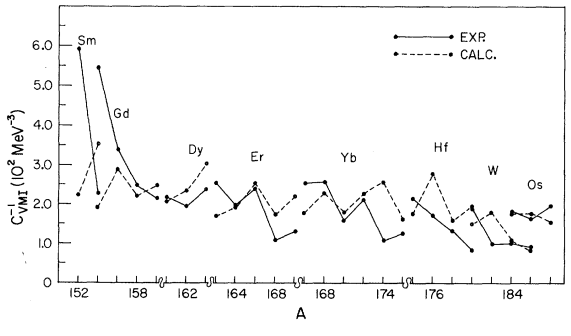

FIG. 2. The force constant C_{VMI}^{-1}. The theoretical values are calculated with the single-particle states and pairing strength [Eq. (11)] as given in Nilsson *et al.* (Ref. 23). The experimental values are taken from Mariscotti *et al.* (Ref. 7). Note the large discrepancies at neutron number $N = 90$, 104 and 108.

in Fig. 2. The contributions to C_{VMI}^{-1} from various sources are also given separately in Table V. One notices first that, except for nuclei with neutron number $N=90$ and 92, both quadrupole and hexadecapole centrifugal stretching contribute very little to the energy deviation from the $I(I+1)$ rule. Typically they amount to only a few percent of the total contribution and hence, in most cases can be entirely neglected. This result is consistent with other microscopic calculations and also with experimental observations.[33] It is important to note that the contributions of hexadecapole stretching are comparable with those of quadrupole stretching. Hence, they should both be taken into account in other relevant analyses, such as the study[16] of change of nuclear mean-square radius $\Delta\langle r^2\rangle$ or the study of the deviation of the transition probability from the rigid rotor formula.

It is shown in Table V that the neutron Coriolis-antipairing and the fourth-order cranking corrections are the two largest contributions and are comparable with each other. The proton Coriolis-antipairing term is relatively smaller and amounts to about 10 to 20% of the total contribution. In general, the present results are very different quantitatively from those of (I). However, many qualitative discussions of (I) are still valid.

We observe in Fig. 2 that, except for nuclei with neutron number $N=90$, 104, and 108, both the experimental trend and magnitude of the force constant C_{VMI}^{-1} are fairly well reproduced in general by the present calculation. In most cases the discrepancy ranges from 10 to 40%; the average discrepancy for all nuclei (excluding those with $N=90$) is about 34%.

The large discrepancies of the calculated force

TABLE VI. The results of calculation B where neutron levels have been shifted according to Eq. (24).

Nucleus	Δ_n (MeV)	$\frac{\partial}{\partial\nu_n}2J_0$ (MeV^{-2})	$C_{\nu n}$ (MeV^{-1})	C_η (MeV^{-3})	$2J_n$ (MeV^{-1})	$2J_0$ (MeV^{-1})	C_{VMI}^{-1} (MeV^{-3})	$-B_{\mathrm{theo}}$ (eV)	$-B_{\mathrm{exp}}$ (eV)
^{152}Sm	0.975	−32.35	3.88	27.32	24.84	40.38	224.0	169	195
^{154}Sm	0.888	−46.17	3.27	37.68	35.19	54.97	353.0	77.3	14.9
^{154}Gd	1.001	−29.85	3.93	23.61	23.14	38.22	191.3	179	180
^{156}Gd	0.935	−41.16	3.50	31.48	32.00	50.75	287.9	86.8	33.8
^{158}Gd	0.895	−37.59	4.18	24.81	33.27	53.26	217.9	54.2	17.5
^{160}Gd	0.849	−40.19	4.13	28.44	35.40	56.28	248.5	49.5	11.8
^{160}Dy	0.934	−35.24	4.18	23.79	30.63	49.53	206.7	68.7	20.7
^{162}Dy	0.880	−38.17	4.19	27.36	33.33	53.48	234.8	57.4	12.0
^{164}Dy	0.836	−43.05	3.50	33.25	35.84	56.88	302.8	57.9	12.0
^{162}Er	0.969	−32.72	4.22	19.27	27.99	45.21	168.0	80.4	40.0
^{164}Er	0.906	−36.35	4.24	22.38	31.41	50.17	193.7	61.1	19.9
^{166}Er	0.861	−41.23	3.59	27.44	34.00	53.98	254.0	59.8	17.4
^{168}Er	0.815	−36.48	4.40	17.75	35.11	56.54	173.9	34.0	6.74
^{170}Er	0.771	−44.06	3.48	25.57	38.84	61.35	271.7	38.4	10.1
^{166}Yb	0.926	−35.08	4.27	21.21	29.59	45.94	179.0	80.4	49.5
^{168}Yb	0.883	−39.72	3.69	26.56	32.27	50.20	235.9	74.3	22.9
^{170}Yb	0.813	−37.41	4.38	19.58	35.34	55.17	183.8	39.7	12.7
^{172}Yb	0.786	−43.59	3.61	27.80	37.86	59.44	273.9	43.9	10.4
^{174}Yb	0.789	−36.41	4.70	26.46	34.99	57.52	211.6	38.7	8.31
^{176}Yb	0.746	−29.64	4.95	17.55	33.14	55.70	149.8	31.1	14.6
^{174}Hf	0.825	−41.11	3.98	22.85	35.61	53.04	213.2	53.9	21.0
^{176}Hf	0.803	−39.93	4.42	26.53	35.30	53.21	208.9	52.1	14.7
^{178}Hf	0.770	−32.51	4.62	18.10	33.19	51.38	141.5	40.6	15.1
^{180}Hf	0.721	−30.89	3.68	13.82	33.48	52.13	131.0	35.5	6.36
^{180}W	0.795	−33.53	4.55	15.19	31.88	46.44	128.1	55.1	39.9
^{182}W	0.747	−31.94	3.84	11.13	32.48	47.65	115.7	44.9	16.8
^{184}W	0.749	−31.75	4.75	14.65	31.09	46.18	119.2	52.4	24.4
^{186}W	0.759	−27.90	4.78	14.91	26.98	41.52	109.8	73.9	34.2
^{184}Os	0.807	−32.37	4.08	11.75	29.71	43.71	121.9	66.8	59.2
^{186}Os	0.809	−31.47	4.76	15.16	27.93	42.58	128.7	78.3	88.2
^{188}Os	0.814	−26.63	5.09	14.92	23.84	38.99	121.3	105	174

constants which occurred at neutron number $N=90$, 104, and 108 deserve more careful study. Perhaps the 90-neutron nuclei are so close to being shape unstable that the present model of stable deformation may be somewhat questionable. This argument, however, cannot be applied to nuclei with $N=104$ and 108, all of which appear to be good rotors. We then compare the calculated Nilsson single-neutron levels around $N=104$ and 108 with those deduced semiempirically by Ogle *et al.*[34] and are not surprised to find some discrepancies between them. Consequently, we make the following preliminary neutron level shifts:

$$\text{Calculation B:} \begin{cases} [512, \tfrac{5}{2}^-]_n + 0.05\hbar\omega \\ [510, \tfrac{1}{2}^-]_n - 0.05\hbar\omega \quad \text{for } A \geq 170. \\ [512, \tfrac{3}{2}^-]_n - 0.05\hbar\omega \end{cases} \tag{24}$$

With the above neutron level shifts and assuming further that the wave function and the quadrupole and hexadecapole stretching are the same, we repeat the calculation on the moment of inertia and the force constant which will be called calculation B while the previous calculation without level shifts will be called calculation A. The results of calculation B are listed in Table VI. In general, the results of calculation B are similar to those of calculation A except for nuclei around neutron number $N=104$ and 108. Note in Table VI that the moments of inertia from calculation B change only slightly over the results of calculation A. On the other hand, the force constants of calculation B are considerably improved over the results of calculation A around $N=104$ and 108, as can be seen in Fig. 3. The serious discrepancies of calculation A which occurred at $N=104$ and 108 are now much reduced; in most cases, both the trend and magnitude of the experimental force constants are now fairly well reproduced. However, in addition to the 90-neutron nuclei there still remain three nuclei ($^{170}_{102}$Er, $^{174}_{104}$Yb, and $^{180}_{108}$Hf) whose calculated force constants are a factor of 2 too large compared to the experiment. The average discrepancy for all nuclei (excluding $N=90$) is now 29%. Thus the agreement between the theoretical and experimental values of the force constant is comparable to that of the moment-of-inertia calculations.

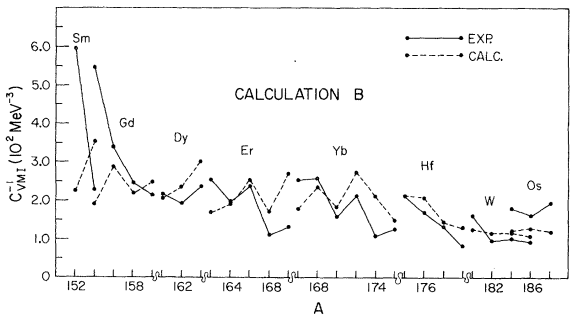

FIG. 3. Same as Fig. 2, except in these calculations the neutron levels have been shifted according to Eq. (24).

The above results indicate that the force constant C_{VMI} which represents the higher-order effects in the rotational energy calculation is much more sensitive to the single-particle levels than the moment of inertia. We have no intention here to do a detailed searching for better single-neutron levels. Instead, the emphasis is to indicate that by removing the discrepancy of the Nilsson neutron levels around $N=104$ and 108 (although only in a preliminary way), the calculated results of the force constant C_{VMI}^{-1} can be considerably improved.

C. *B* coefficients

The B coefficient associated with the $I^2(I+1)^2$ term in an expansion of the rotational energy can be evaluated either by Eq. (9) or in a more straightforward way, since we already know J_0 and C_{VMI}^{-1}, by Eq. (10). The results of calculation B are given in Table VI and plotted in Fig. 4. The experimental B coefficients are obtained by a least-squares fit to the first three excited states by using the first three terms in Eq. (8) with J_0 taken from Ref. 7. It is seen in Fig. 4 that the trend of the B coefficient is fairly well reproduced; the calculated magnitudes, however, are generally too large by a factor of 2 to 5. Thus, the agreements are much worse than those of the force constant C_{VMI}^{-1} although both of them represent the higher-order effects. The reason is easy to understand, because the B coefficient depends on the

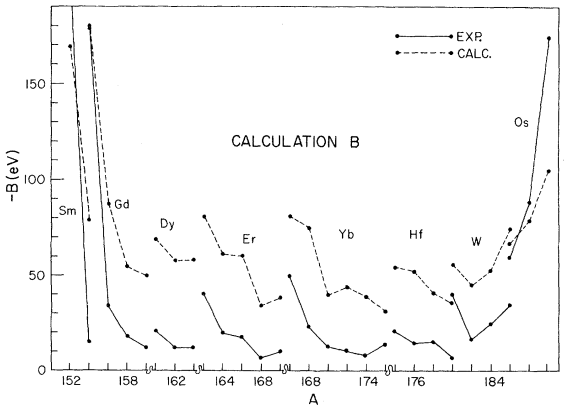

FIG. 4. The B coefficient. The theoretical values are calculated with the pairing strength as given in Eq. (11) and with the neutron levels shifted according to Eq. (24).

inverse fourth power of the moment of inertia J_0 according to Eq. (10). Our calculated J_0 are roughly 10 to 40% smaller while our calculated C_{VMI}^{-1} are roughly 10 to 40% larger; so combined they yield the B values by a factor 2 to 5 too large.

Because of this J_0^{-4} dependence, it seems that in order to get reasonable agreement for B coefficient one probably should first fit the moment of inertia as accurately as possible. We have mentioned in the beginning of this section that very good agreements of the moment of inertia could be achieved provided one uses a reduced pairing strength $g_0 = 18.0$ MeV instead of 19.2 MeV in Eq. (11). The values of J_0, B, and C_{VMI}^{-1} calculated with $g_0 = 18.0$ MeV and without neutron levels shifts are listed in Table VII. In addition, the second set of results of J_0 and B of Marshalek[16] which are obtained by adjusting the pairing strength so as to exactly reproduce the experimental moment of inertia are also included for comparison. Our results are roughly similar to those of Marshalek at the middle of the rare-earth region, though discrepancies occur at both ends of this region. Note also in Table VII that our B values are now improved over those obtained previously with $g_0 = 19.2$ MeV, although they are still too large by a factor of 1.5 to 3 in general. On the other hand, however, the force constants C_{VMI}^{-1} in Table VII are much worse than those obtained previously with $g_0 = 19.2$ MeV.

TABLE VII. The moment of inertia, B coefficient, and the force constant calculated with $g_0 = 18.0$ MeV in Eq. (11), the results of Marshalek (Ref. 16) are also listed for comparison.

	This calculation			Marshalek	
Nucleus	$2J_0$ (MeV^{-1})	$-B$ (eV)	C_{VMI}^{-1} (MeV^{-3})	$2J_0$ (MeV^{-1})	$-B$ (eV)
^{152}Sm	55.57	99.8	476	46.77	221
^{154}Sm	78.02	50.4	934	72.31	37.5
^{154}Gd	52.17	109	403	46.30	160
^{156}Gd	70.69	56.9	711	66.53	43.0
^{158}Gd	69.66	32.4	382	74.96	26.7
^{160}Gd	73.42	29.4	427	79.05	25.4
^{160}Dy	66.06	42.7	407	68.45	37.4
^{162}Dy	70.74	34.6	433	74.02	33.4
^{164}Dy	78.00	39.6	733	81.37	26.0
^{164}Er	65.54	36.9	340	65.19	43.7
^{166}Er	73.17	43.5	623	73.96	33.7
^{168}Er	71.40	19.3	251	74.96	25.0
^{170}Er	75.56	21.2	346	75.13	30.0
^{170}Yb	68.33	25.5	278	70.82	31.8
^{172}Yb	73.90	25.0	373	75.93	30.0
^{174}Yb	76.61	28.1	485	77.82	27.8
^{176}Yb	71.03	20.8	265	72.46	27.7
^{176}Hf	73.30	39.5	570	67.43	42.6
^{178}Hf	66.44	26.1	254	64.10	44.0
^{184}W	54.67	29.8	133	53.48	68.5
^{186}W	46.41	50.4	117	48.40	99.1

We seem to be in a very interesting situation. On the one hand, our calculation with pairing strength $g_0 = 19.2$ MeV is able to reproduce fairly well J_0 and C_{VMI}; however, it yields very poor B coefficients. On the other hand, very good J_0 and improved values of B (but still too large by a factor of 1.5 to 3) could be obtained from calculation with the reduced pairing strength $g_0 = 18.0$ MeV, but now the C_{VMI} becomes very poor. We feel that an accurate microscopic theory should be able to reproduce both force constant and the B coefficient correctly. For calculations involving as many approximations as these, however, we suggest that the force constant C_{VMI} is a more meaningful quantity to be compared with. The reasons are as follow: (1) Because of the J_0^{-4} dependence, the large discrepancy of the calculated B coefficient may be misleading since it may essentially be a result of small to moderate deviation in J_0. It is also probably misleading for one to obtain better B coefficient by adjusting the pairing strength alone in order to reduce the error arising from the J_0^{-4} dependence, because in doing so, the force constant C_{VMI} will become unduly worse. (2) It is well known that the expansion of the rotational energy in terms of the angular velocity ω^2 is much better than the expansion in terms of the angular momentum $I(I+1)$. Thus, the force constant C_{VMI} also appears to be a more physically significant parameter than the B coefficient.

V. SUMMARY

Based on the microscopic cranking model, the present calculations are able to reproduce fairly well the moment of inertia J_0 and the force constant C_{VMI} associated with the VMI model with discrepancy ranging from 10 to 40% in general. On the other hand, the calculated B coefficients are quantitatively poor, which resemble the calculations of Marshalek.[16] However, as we have mentioned, one must use care in interpreting the large discrepancy of the B coefficient because of its J_0^{-4} dependence.

We have taken into account the fixed-particle-number correction for the potential energy; obviously it will also affect the moment of inertia and the fourth-order cranking calculations. According to the calculation of Rich,[28] the fixed-particle-number correction will reduce the inertia derivative with respect to pairing by 10 to 20%. Since the force constant C_{VMI}^{-1} is proportional to

the square of the inertia derivative, this will cause 20 to 50% reduction of the Coriolis-antipairing effect, which is in the right direction of improvement. We feel that the present approach is not accurate enough to study nuclear rotation at very high spin; to do that, the perturbation treatment on ωJ_x probably will have to be avoided altogether.

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APPENDIX

We first express the J_x operator in the quasiparticle representation as

$$J_x = J_{11} + J_{20} \tag{A1}$$

with

$$\langle p|J_{11}|q\rangle = \langle p|j_x|q\rangle(U_pU_q + V_pV_q)\,, \tag{A2a}$$

$$\langle 0|J_{20}|pq\rangle = (-1)^{m_q+1/2}\langle p|j_x|-q\rangle(U_pV_q - V_pU_q)\,, \tag{A2b}$$

where $|0\rangle$ is the quasiparticle vacuum state. Note that J_x only operates between states with J_z components differed by ± 1.

Consider now the contribution of the four-quasiparticle excitations to the first term in Eq. (7)

$$C'_\eta(4\mathrm{QP}) = \frac{1}{8}\,2 \sum_{\substack{(pq)\pm1\\(p'q')\pm1\\(p_1q_1)\pm1\\p\neq q\neq p'\neq q'}} \frac{\langle 0|J_{20}|pq\rangle}{(E_p+E_q)} \frac{\langle pq|J_{20}|pq,p'q'\rangle\langle pq,p'q'|J_{20}|p_1q_1\rangle\langle p_1q_1|J_{20}|0\rangle}{(E_p+E_q+E_{p'}+E_{q'})(E_{p_1}+E_{q_1})}\,,$$

where $(pq)\pm1$ denotes a complete set of two-quasiparticle states $|pq\rangle$ with J_z component $m_p+m_q=\pm1$. Since for each $|pq\rangle$ there is a corresponding state $|qp\rangle$, the factor $\frac{1}{8}$ thus accounts for the double counting of pq, $p'q'$, and p_1q_1. Writing the nonzero terms of the above expression explicitly one obtains

$$C'_\eta(4\mathrm{QP}) = \frac{1}{2} \sum_{\substack{(pq)\pm1\\(p'q')\pm1\\p\neq q\neq p'\neq q'}} \left(\frac{\langle 0|J_{20}|pq\rangle^2\langle 0|J_{20}|p'q'\rangle}{(E_p+E_q)^2(E_{p'}+E_{q'})} - \frac{\langle 0|J_{20}|pq\rangle\langle 0|J_{20}|p'q'\rangle\langle 0|J_{20}|pq'\rangle\langle 0|J_{20}|p'q\rangle}{(E_p+E_q)(E_p+E_{q'})(E_{p'}+E_q)} \right.$$

$$\left. - \frac{\langle 0|J_{20}|pq\rangle\langle 0|J_{20}|p'q'\rangle\langle 0|J_{20}|pp'\rangle\langle 0|J_{20}|qq'\rangle}{(E_p+E_q)(E_p+E_{p'})(E_q+E_{q'})} \right).$$

We notice first that the constraint $p\neq q\neq p'\neq q'$ can be dropped because the additional terms thus created will cancel each other. Secondly, the third term may be made equal to the second term by exchanging p' with q'. It then follows

$$C'_\eta(4\mathrm{QP}) = \frac{1}{2} \sum_{\substack{(pq)\pm1\\(p'q')\pm1}} \frac{\langle 0|J_{20}|pq\rangle^2\langle 0|J_{20}|p'q'\rangle^2}{(E_p+E_q)^2(E_{p'}+E_{q'})} - \frac{1}{2} \sum_{\substack{(pq)\pm1\\(p'q')\pm1}} \frac{\langle 0|J_{20}|pq\rangle\langle 0|J_{20}|p'q'\rangle\langle 0|J_{20}|pq'\rangle\langle 0|J_{20}|p'q\rangle}{(E_p+E_q)(E_{p'}+E_{q'})(E_p+E_{q'})(E_{p'}+E_q)}$$

$$\times(E_p+E_{p'}+E_q+E_{q'})\,. \tag{A3}$$

The first term in this equation obviously cancels the second term in Eq. (7); we thus defined the full contributions of the four-quasiparticle excitations as

$$C_\eta(4\mathrm{QP}) = C'_\eta(4\mathrm{QP}) + \text{second term of Eq. (7)}$$

$$= \text{second term of Eq. (A3)}.$$

The summation of the second term in Eq. (A3) can be separated into four terms

$$\sum_{\substack{(pq)\pm1\\(p'q')\pm1}} = \sum_{\substack{(pq)_1\\(p'q')_1}} + \sum_{\substack{(pq)_1\\(p'q')_{-1}}} + \sum_{\substack{(pq)_{-1}\\(p'q')_1}} + \sum_{\substack{(pq)_{-1}\\(p'q')_{-1}}}\,.$$

The first term yields

$$\sum_{\substack{(pq)_1\\(p'q')_1}} = -\frac{1}{2} \sum_{\substack{m_p=\pm 1/2,\pm 3/2,\dots\\ m_{p'}=m_p\\ m_q=m_{q'}=-m_p+1}} \sum_{\substack{t_p,t_{p'}\\ t_q,t_{q'}}} \frac{\langle 0|J_{20}|pq\rangle\langle 0|J_{20}|p'q'\rangle\langle 0|J_{20}|pq'\rangle\langle 0|J_{20}|p'q\rangle}{(E_p+E_q)(E_{p'}+E_{q'})(E_p+E_{q'})(E_{p'}+E_q)}(E_p+E_{p'}+E_q+E_{q'})$$

$$= -\sum_{\substack{m_p=\pm 1/2,\pm 3/2,\dots\\ m_{p'}=m_p\\ m_q=-m_p+1}} \sum_{t_p,t_{p'}} (E_p+E_{p'})\left(\sum_{t_q} \frac{\langle 0|J_{20}|pq\rangle\langle 0|J_{20}|p'q\rangle}{(E_p+E_q)(E_{p'}+E_q)}\right)^2,$$

where we have applied the conditions

$$m_p+m_{q'}=\pm 1\,, \quad m_{p'}+m_q=\pm 1\,.$$

The other three terms can be evaluated in a similar way, and we obtain finally

$$C_\eta(4\mathrm{QP}) = -4 \sum_{\substack{m_p>0\\ m_q=-m_p+1\\ m_{q'}=-m_p-1}} \sum_{t_q,t_{q'}} (E_q+E_{q'})\left(\sum_{t_p} \frac{\langle 0|J_{20}|pq\rangle\langle 0|J_{20}|pq'\rangle}{(E_p+E_q)(E_p+E_{q'})}\right)^2$$

$$-2 \sum_{m_{p'}=m_p>0} \sum_{t_p,t_{p'}} (E_p+E_{p'})\left(\sum_{\substack{t_q\\ m_q=-m_p\pm 1}} \frac{\langle 0|J_{20}|pq\rangle\langle 0|J_{20}|p'q\rangle}{(E_p+E_q)(E_{p'}+E_q)}\right)^2. \tag{A4}$$

Substitution of Eq. (A2) into Eq. (A4) then yields the first two terms in Eq. (23).

The contribution of the two-quasiparticle excitations to the first term in Eq. (7) is

$$C_\eta(2\mathrm{QP}) = \frac{1}{8}\,2 \sum_{\substack{(pq)\pm 1\\ (p'q')0,\pm 2\\ (p_1q_1)\pm 1}} \frac{\langle 0|J_{20}|pq\rangle\langle pq|J_{11}|p'q'\rangle\langle p'q'|J_{11}|p_1q_1\rangle}{(E_p+E_q)(E_{p'}+E_{q'})} \frac{\langle p_1q_1|J_{20}|0\rangle}{(E_{p_1}+E_{q_1})}\,.$$

Since

$$\langle pq|J_{11}|p'q'\rangle = \delta_{pp'}\langle q|J_{11}|q'\rangle + \delta_{qq'}\langle p|J_{11}|p'\rangle - \delta_{pq'}\langle q|J_{11}|p'\rangle - \delta_{qp'}\langle p|J_{11}|q'\rangle$$

one obtains

$$C_\eta(2\mathrm{QP}) = 2 \sum_{\substack{(pq)\pm 1\\ (pq')0,\pm 2\\ (pq_1)\pm 1}} \frac{\langle 0|J_{20}|pq\rangle\langle q|J_{11}|q'\rangle\langle q'|J_{11}|q_1\rangle\langle pq_1|J_{20}|0\rangle}{(E_p+E_q)(E_p+E_{q'})(E_p+E_{q_1})}$$

$$+2 \sum_{\substack{(pq)\pm 1\\ (pq')0,\pm 2\\ (p'q')\pm 1}} \frac{\langle 0|J_{20}|pq\rangle\langle q|J_{11}|q'\rangle\langle p|J_{11}|p'\rangle\langle p'q'|J_{20}|0\rangle}{(E_p+E_q)(E_p+E_{q'})(E_{p'}+E_{q'})}\,.$$

Recalling that J_x only operates between states with J_z components differed by ± 1, we get

$$C_\eta(2\mathrm{QP}) = \left\{4 \sum_{\substack{(pq)_1\\ (pq')_0\\ (pq_1)_1}} + 4 \sum_{\substack{(pq)_1\\ (pq')_0\\ (pq_1)_{-1}}} + 4 \sum_{\substack{(pq)_1\\ (pq')_2\\ (pq_1)_1}}\right\} \left(\frac{\langle 0|J_{20}|pq\rangle\langle q|J_{11}|q'\rangle\langle q'|J_{11}|q_1\rangle\langle pq_1|J_{20}|0\rangle}{(E_p+E_q)(E_p+E_{q'})(E_p+E_{q_1})}\right)$$

$$+\left\{4 \sum_{\substack{(pq)_1\\ (pq')_0\\ (p'q')_1}} + 4 \sum_{\substack{(pq)_1\\ (pq')_0\\ (p'q')_{-1}}} + \sum_{\substack{(pq)_1\\ (pq')_2\\ (p'q')_1}}\right\} \left(\frac{\langle 0|J_{20}|pq\rangle\langle q|J_{11}|q'\rangle\langle p|J_{11}|p'\rangle\langle p'q'|J_{20}|0\rangle}{(E_p+E_q)(E_q+E_{q'})(E_{p'}+E_{q'})}\right), \tag{A5}$$

where we have used the concise notation

$$\Big\{\sum_1+\sum_2+\sum_3\Big\}\ (A)=\sum_1 A+\sum_2 A+\sum_3 A\,.$$

We regroup the third and the sixth term in Eq. (A5) where the J_z components of the two-quasiparticle intermediate states are $m_p+m_{q'}=2$, and obtain

$$C'_\eta(2\mathrm{QP})=4\sum_{\substack{m_p=\pm1/2,\pm3/2,\dots\\ m_{q'}=-m_p+2}}\sum_{t_p,t_{q'}}\frac{1}{E_p+E_{q'}}\Big(\sum_{\substack{t_q\\ m_q=-m_p+1}}\frac{\langle 0|J_{20}|pq\rangle\langle q|J_{11}|q'\rangle}{E_p+E_q}\Big)^2$$

$$+4\sum_{\substack{m_p=\pm1/2,\pm3/2,\dots\\ m_{q'}=-m_p+2}}\sum_{t_p,t_{q'}}\frac{1}{E_p+E_{q'}}\Big(\sum_{\substack{t_q\\ m_q=-m_p+1}}\frac{\langle 0|J_{20}|pq\rangle\langle q|J_{11}|q'\rangle}{E_p+E_q}\Big)\Big(\sum_{\substack{t_q\\ m_q=m_p-1}}\frac{\langle 0|J_{20}|qq'\rangle\langle p|J_{11}|q\rangle}{E_{q'}+E_q}\Big)\,.$$

The above expression can be rewritten finally as

$$C'_\eta(2\mathrm{QP})=4\sum_{\substack{m_p\geq3/2\\ m_{q'}=-m_p+2}}\sum_{t_p,t_{q'}}\frac{1}{E_p+E_{q'}}\Big(\sum_{\substack{t_q\\ m_q=-m_p+1}}\frac{\langle 0|J_{20}|pq\rangle\langle q|J_{11}|q'\rangle}{E_p+E_q}+\sum_{\substack{t_q\\ m_q=m_p-1}}\frac{\langle 0|J_{20}|qq'\rangle\langle p|J_{11}|q\rangle}{E_{q'}+E_q}\Big)^2\,. \tag{A6}$$

We then regroup the remaining four terms in Eq. (A5) where the J_z components of the two-quasiparticle intermediate states are $m_p+m_{q'}=0$, and obtain

$$C''_\eta(2\mathrm{QP})=4\sum_{\substack{m_p=\pm1/2,\pm3/2,\dots\\ m_{q'}=-m_p}}\sum_{t_p,t_{q'}}\frac{1}{E_p+E_{q'}}\Big[\Big(\sum_{\substack{t_q\\ m_q=-m_p+1}}\frac{\langle 0|J_{20}|pq\rangle\langle q|J_{11}|q'\rangle}{E_p+E_q}\Big)^2$$

$$+\Big(\sum_{\substack{t_q\\ m_q=-m_p+1}}\frac{\langle 0|J_{20}|pq\rangle\langle q|J_{11}|q'\rangle}{E_p+E_q}\Big)\Big(\sum_{\substack{t_q\\ m_q=-m_p-1}}\frac{\langle 0|J_{20}|pq\rangle\langle q|J_{11}|q'\rangle}{E_p+E_q}\Big)$$

$$+\Big(\sum_{\substack{t_q\\ m_q=-m_p+1}}\frac{\langle 0|J_{20}|pq\rangle\langle q|J_{11}|q'\rangle}{E_p+E_q}\Big)\Big(\sum_{\substack{t_q\\ m_q=m_p+1}}\frac{\langle 0|J_{20}|qq'\rangle\langle p|J_{11}|q\rangle}{E_q+E_{q'}}\Big)$$

$$+\Big(\sum_{\substack{t_q\\ m_q=-m_p+1}}\frac{\langle 0|J_{20}|pq\rangle\langle q|J_{11}|q'\rangle}{E_p+E_q}\Big)\Big(\sum_{\substack{t_q\\ m_q=m_p-1}}\frac{\langle 0|J_{20}|qq'\rangle\langle p|J_{11}|q\rangle}{E_q+E_{q'}}\Big)\Big]$$

which can be rewritten finally as

$$C''_\eta(2\mathrm{QP})=2\sum_{\substack{m_p>0\\ m_{q'}=-m_p}}\sum_{t_p,t_{q'}}\frac{1}{E_p+E_{q'}}\Big(\sum_{\substack{t_q\\ m_q=-m_p\pm1}}\frac{\langle 0|J_{20}|pq\rangle\langle q|J_{11}|q'\rangle}{E_p+E_q}+\sum_{\substack{t_q\\ m_q=m_p\pm1}}\frac{\langle 0|J_{20}|qq'\rangle\langle q|J_{11}|p\rangle}{E_q+E_{q'}}\Big)^2\,. \tag{A7}$$

Substitution of Eq. (A2) into Eqs. (A6) and (A7) then yields the last two terms in Eq. (23).

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