Meson-theoretic potentials and the hypertriton

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A previous result of Gibson and Lehman for the binding energy of ${}^{3}_{\Lambda}$H using a single-term nonlocal separable potential representation of a meson-theoretic ΛN potential is shown to be significantly modified when a two-term separable ΛN potential is used instead.

[NUCLEAR STRUCTURE ${}^{3}_{\Lambda}$H, YN potentials, separable potential three-body calculation, B_Λ.]

I. INTRODUCTION

In a recent article Gibson and Lehman[1] (GL) concluded that the one-boson exchange (OBE) meson-theoretic potential (MTP) of Nagels, Rijken, and DeSwart[2] for the hyperon-nucleon (YN) interaction (called model B) gave a better value for the hypertriton binding energy than did another MTP of these same three authors[2] (called model A), which included an uncorrelated two-pion exchange. Calculations described below show that the GL results which led to this conclusion are a manifestation of the potential forms used to represent the model A and model B interactions, rather than a characteristic of these two meson-theoretic models themselves.

In GL the authors argued that because ${}^{3}_{\Lambda}$H was such a loose structure, the same sort of one-term s-wave nonlocal separable (NLS) potentials which gave reasonably accurate results for the triton could be used in the hypertriton. Further, in GL no allowance was made for any explicit Λ-Σ coupling in the ${}^{3}_{\Lambda}$H calculations, but rather the authors argued that their potentials were "effective" ΛN interactions. Since these effective interactions were matched to the low-energy scattering parameters predicted by MTP's, which in turn were obtained from calculations that included this coupling, they implicitly contained the effects of Λ-Σ coupling. Thus, in GL the ${}^{3}_{\Lambda}$H binding energy was calculated using a different single-channel one-term s-wave Yamaguchi-shape[3] NLS potential for each spin singlet and each spin triplet Λp and Λn potential. For each of these four potentials the potential parameters were fixed by matching the scattering length and effective range it predicted to the corresponding values, in turn, of the model A and model B MTP's. The three-body calculations then yielded a value for B_Λ, the binding energy of the Λ in ${}^{3}_{\Lambda}$H, of 0.70 MeV with model A and 0.28 MeV with model B. The experimental result is $B_\Lambda = 0.15 \pm 0.08$ MeV.

On the basis of previous work it is this author's contention that neglect of explicit Λ-Σ coupling and the use of a one-term NLS potential for each separate ΛN interaction can both lead to significant error in the determination of B_Λ. For example, in 1967 Schick and Hetherington[4] obtained results indicating that B_Λ could be reduced by 50% if a two-term s-wave NLS potential (in which one term represented a long-range attraction and the other a short-range repulsion) were used for each ΛN interaction instead of the use of a single-term potential which yielded the same low-energy ΛN scattering parameters. Further, in 1968 Schick and Toepfer[5] obtained results indicating that B_Λ could be reduced by 50% if a two-channel YN potential that included explicit Λ-Σ coupling were used instead of a one-channel potential which yielded the same low-energy ΛN scattering parameters.

However, the results of Refs. 4 and 5 by themselves are not enough to invalidate the GL conclusion. This is because the ΛN scattering lengths and effective ranges used in these earlier works differ significantly in some particulars from those used in GL. It was decided therefore to recalculate B_Λ using the methods described in Ref. 4 with the ΛN input parameters more closely related to the model A parameters used in GL to determine if values of B_Λ more in line with the GL results for model B may be so obtained. It should be noted that the three-body calculations of Ref. 4 assume a charge-symmetric ΛN potential so that the model A scattering lengths and effective ranges could not be used directly.

What is investigated here, then, is the effect on B_Λ of the short-range (high momentum) part of the ΛN interaction without the use of explicit Λ-Σ coupling. The results obtained indicate that B_Λ is sensitive to the high-momentum part of the ΛN interaction, a region where Λ-Σ coupling effects

TABLE I. Binding energy of the Λ in $^3_\Lambda$H.

ΛN potentials	Core	B_Λ (MeV)
Set 1	None	0.95
	$S=0$	0.63
	$S=0$ and 1	0.58
Set 2	None	0.68
	$S=0$	0.36
	$S=0$ and 1	0.33

are expected to be significant. These results show that the GL conclusion that model B gives a value for B_Λ preferable to that obtainable from model A is unwarranted, but they themselves are not to be taken as yielding the "correct" value for B_Λ predicted by this meson-theoretic model. In the light of the results of Ref. 5, the values for B_Λ obtained here are most likely larger than the correct value.

II. PRESENT WORK

The hypertriton binding energy B was determined by the use of a Faddeev type of multiple-scattering analysis for the Λ-d doublet scattering amplitude. Because s-wave NLS potentials were used for each two-body interaction, this analysis yielded a set of coupled one-dimensional integral equations. The $^3_\Lambda$H binding energy B was varied until the Fredholm determinant for this set of integral equations vanished. By definition, then, $B_\Lambda = B-\epsilon$, where $\epsilon = 2.225$ MeV, the deuteron binding energy. For further details of the three-body calculations see Ref. 4.

The average nucleon mass was taken to be 938.9 MeV while a value of 1115.4 MeV was used for the mass of the Λ. The np spin triplet potential was taken to be a single-term Yamaguchi NLS potential, while each of the spin-zero and spin-one ΛN potentials was taken to be either a single-term Yamaguchi potential or a sum of two such potentials. That is, in a relative-momentum-space representation, each two-body potential had the form

$$\langle k' | V | k \rangle = \lambda_1 v_1(k') v_2(k) + \lambda_2 v_2(k') v_2(k) , \qquad (1)$$

where

$$v_j(k) = 1/(\beta_j^2 + k^2), \quad j = 1, \ 2 , \qquad (2)$$

with $\lambda_2 = 0$ for the np potential and $\lambda_2 \geq 0$ for the ΛN potentials. The np parameters λ_1 and β_1 were fixed by matching the deuteron binding energy (2.225 MeV) and the triplet np scattering length (5.3858 fm). When a "no-core" (i.e., $\lambda_2 = 0$) model was used for a particular ΛN interaction, the two parameters λ_1 and β_1 were determined by matching given values of the scattering length and effective range. When a "with core" (i.e., $\lambda_2 > 0$) model was used, in addition to fitting a given scattering length and effective range, the core range parameter $1/\beta_2$ was chosen arbitrarily and the requirement that the phase shift vanish at some given ΛN center-of-mass energy E_0 was also imposed.

The model A low-energy ΛN scattering parameters are[2]:

$$\begin{aligned} a^s_{p\Lambda} &= -2.16 \pm 0.26 \text{ fm}, & r^s_{p\Lambda} &= 2.03 \pm 0.10 \text{ fm} , \\ a^t_{p\Lambda} &= -1.36 \pm 0.07 \text{ fm}, & r^t_{p\Lambda} &= 2.31 \pm 0.08 \text{ fm} , \\ a^s_{n\Lambda} &= -2.67 \pm 0.35 \text{ fm}, & r^s_{n\Lambda} &= 2.04 \pm 0.10 \text{ fm} , \\ a^t_{n\Lambda} &= -1.02 \pm 0.05 \text{ fm}, & r^t_{n\Lambda} &= 2.55 \pm 0.10 \text{ fm} . \end{aligned} \qquad (3)$$

In the calculations reported on here, two different sets of ΛN scattering parameters were used to represent these model A values. For Set 1 the ΛN singlet scattering length and effective range, and triplet scattering length and effective range, were taken to be given by, respectively,

$$\begin{aligned} a^s &= -2.415 \text{ fm}, & r^s &= 2.035 \text{ fm} , \\ a^t &= -1.19 \text{ fm}, & r^t &= 2.43 \text{ fm} , \end{aligned} \qquad (4)$$

while for Set 2

$$a^s = -2.11 \text{ fm}, \quad r^s = 2.035 \text{ fm} , \qquad (5)$$

and the triplet parameters were the same as those in Set 1.

The Set 1 parameters were obtained merely by averaging the model A values of the corresponding Λp and Λn parameters. When used to determine "no-core" $S=0$ and $S=1$ ΛN NLS potentials which were then used in the $^3_\Lambda$H calculation, this set yielded $B_\Lambda = 0.95$ MeV, as is shown in Table I. If the above averaging had given exactly the charge symmetric part of the model A parameters, the GL model A result $B_\Lambda = 0.70$ MeV would have been obtained.

Because the Set 1 parameters, when used with a no-core form of ΛN potential, yielded a rather large value of B_Λ, which result might be thought to prejudice any conclusions drawn, the magnitude of a^s was reduced so that the calculation of B_Λ yielded a value closer to the GL model A result. A lucky guess for a^s gave the Set 2 input parameters, which when used in the $^3_\Lambda$H calculation yielded $B_\Lambda = 0.68$ MeV, as shown in line 4 of Table I. Note that from Eq. (3) the Set 2 value of a^s is just the average of the largest (i.e., least negative) values of $a^s_{p\Lambda}$ and $a^s_{n\Lambda}$ allowed by the quoted error range.

A repulsive core was introduced into each of

TABLE II. ΛN NLS potential parameters and phase shifts.

ΛN potentials	$\lambda_1/(20\pi)^3$ (MeV^2)	$\lambda_2/(20\pi)^3$ (MeV^2)	β_1^{-1} (fm)	β_2^{-1} (fm)	Phase shifts (deg) at c.m. energies 40 MeV	80 MeV	160 MeV
Set 1	−3.660 91	0.0	0.525 734	⋯	32.7	25.7	17.5
$S=0$	−866.771	1903.77	0.266 243	0.2225	29.5	17.8	0.0
Set 2	−3.820 00	0.0	0.512 414	⋯	31.8	25.4	17.5
$S=0$	−2912.53	10 554.8	0.239 904	0.2000	28.8	17.6	0.0
Sets 1 and 2	−3.017 01	0.0	0.513 992	⋯	24.4	20.2	14.1
$S=1$	−299.316	697.814	0.280 352	0.2225	22.1	13.9	0.0

the Set 1 ΛN potentials by requiring that for each spin state the core range parameter be $\beta_2^{-1}=0.2225$ fm and that the ΛN phase shift vanish at a c.m. energy $E_0=160$ MeV. From the work of Kashef and Schick[6] in which a sum of NLS Yamaguchi potentials was used to represent a single-channel local ΛN potential with a hard core, this range parameter is of a reasonable size, while the phase shift condition should yield a potential which, if anything, is not repulsive enough at small distances.[7] The Set 2 repulsive cores were defined in the same way, except in the $S=0$ state $\beta_2^{-1}=0.2$ fm was used, as no fit to the other parameters could be obtained with the larger value of this parameter. The potential parameters for each of the ΛN potentials used are given in Table II. Also given in this table are the ΛN phase shifts at c.m. energies of 40, 80, and 160 MeV for each of these potentials.

The results obtained here for B_Λ are shown in the rightmost column of Table I. Lines 1, 2, and 3 of this table show the effect on B_Λ of using Set 1 ΛN scattering parameters with, respectively, no-cores, a singlet core only, and both a singlet and a triplet core. Lines 4, 5, and 6, respectively show the same results for the Set 2 ΛN scattering parameters. As is well known, the ΛN singlet interaction provides most of B_Λ so that the relative insensitivity of B_Λ to a core in the ΛN triplet interaction is not surprising.

As may be seen from Table I, for the Set 1 parameters the use of a core in both spin states reduces B_Λ by about 40%, while for the Set 2 parameters a 50% reduction in B_Λ is achieved, the absolute reduction being about the same (≈0.36 MeV) in each case. A similar reduction in the GL model A result would yield $B_\Lambda \approx 0.35$ MeV, which is well within 0.1 MeV of the GL model B result. Now it is no doubt true that the use of ΛN potentials with cores would also reduce the model B value of B_Λ, probably to around 0.15 MeV. However, the use of explicit Λ-Σ coupling is (from Ref. 5) expected to reduce both the model A and model B values of B_Λ until both would be in good agreement with the experimental value.

In any case, the results of Table I show that a change in the high-momentum part of the ΛN potentials can lead to significant changes in B_Λ. Thus the neglect of both repulsive cores and explicit Λ-Σ coupling in the GL model of the ΛN interaction makes unwarranted the GL conclusions as to which meson-theoretic model is better.

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[1]B. F. Gibson and D. R. Lehman, Phys. Rev. C 10, 888 (1974), hereafter referred to as GL.
[2]M. M. Nagels, T. A. Rijken, and J. J. DeSwart, in *Few Particle Problems in the Nuclear Interaction*, edited by I. Šlaus, S. A. Moszkowski, R. P. Haddock, and W. R. H. van Oers (North-Holland, Amsterdam, 1972), pp. 42–45.
[3]Y. Yamaguchi, Phys. Rev. 95, 1628 (1954).
[4]L. H. Schick and J. H. Hetherington, Phys. Rev. 156, 1602 (1967).
[5]L. H. Schick and A. J. Toepfer, Phys. Rev. 170, 946 (1968).
[6]Bayesteh Ghaffary Kashef and L. H. Schick, Phys. Rev. D 3, 2661 (1971).
[7]In Ref. 6 the $S=0$ ΛN phase shift calculated from the local hard core potential vanished at about 90 MeV. With the values of a^s, r^s, and β_2 used here it was not possible to make the ΛN phase shifts calculated from a sum of Yamaguchi potentials vanish at an energy much below 160 MeV. No doubt the use of a core potential which came in more strongly at high momentum relative to its low-momentum behavior, such as a shape $k^2/(\beta_2^2+k^2)^2$, would cure this difficulty. In any case, the cores used here are certainly not too strong.