Is there evidence of three-body forces from violation of the Koltun energy sum rule?

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The suggested failure of the model independent energy sum rule of Koltun in a recent $(e, e'p)$ experiment is studied in light of current Hartree-Fock theory. It is shown that the use of (i) density dependent forces or (ii) partial occupation probabilities compensates for the restricted momentum and energy range of the experimental proton spectral function, and can explain the sum rule violation.

[NUCLEAR STRUCTURE Correction to Koltun's energy sum rule calculated for ^{12}C, ^{28}Si, ^{40}Ca, and ^{58}Ni using effective density dependent or three-body forces.]

I. INTRODUCTION

The Hartree-Fock approximation equates the total energy of the nucleus to half the sum of the single particle and kinetic energies of all nucleons. This relation was tested by Köhler[1] and Elton[2] and found to be in gross disagreement with empirical observations. This was attributed to the fact that Koopmans's[3] theorem which equates the single particle energies (ϵ_i) and the separation energies ($-\epsilon_i = S_i > 0$) is not valid in nuclear physics.[4] The difference $R_i = -\epsilon_i - S_i$ was termed the rearrangement energy.

Two years ago Koltun[5] derived a model independent sum rule valid for the most general two-body forces. Appropriately corrected[6] for recoil energies this sum rule states that

$$\Delta \equiv \frac{1}{Z}E_Z - \tfrac{1}{2}\left[-\langle S\rangle_\pi + \langle T\rangle_\pi - \frac{1}{A-1}\langle T\rangle_\pi\right] = 0 \ . \qquad (1)$$

Here E_Z is the total energy of the protons,[5] and S the average proton separation energy. The $\langle T\rangle_\pi$ represent the average kinetic energy of the protons in the nucleus. The quantities are related to the spectral function

$$P_\pi(k, S) = \sum_f |\langle A-1, f|a_\pi(k)|A\rangle|^2 \delta(S - E_f) \qquad (2)$$

of the protons as follows

$$n = \int P_\pi(k, S) dS d^3k \ ,$$

$$\langle S\rangle = n^{-1} \int P_\pi(k, S) S \, dS d^3k \ , \qquad (3)$$

$$\langle T\rangle = n^{-1} \int P_\pi(k, S) \frac{k^2}{2m} dS d^3k \ .$$

Here E_f is the energy of the final state relative to the target.

Recently Bernheim *et al.* using $(e, e'p)$ reactions measured the spectral function for the range $0 < S < 80$ MeV and $\hbar k < 300$ MeV/c in ^{12}C,[6] ^{28}Si, ^{40}Ca, and ^{58}Ni.[7] They tested the energy sum rule for the protons and found typical violations $\Delta = -3$ MeV (see Table I) instead of zero. Bernheim *et al.* gave two possible reasons for this violation. (i) Three-body forces contribute to the total binding energy and the energy sum rule has to be generalized. (ii) The proton spectral function has some strength at separation energies S higher than 80 MeV which is the upper limit of the experimental measurement.

The purpose of this note is to discuss to what extent one can derive from such a limited measurement the existence of a bare of effective three-body force between nucleons.

The range of the momenta $\hbar k$ exhausted by the measurement was restricted to be less than 300 MeV/c. This range covers roughly the momenta contained in occupied single particle shell model wave functions (see for example Fig. 2 of Ref. 6). Short range correlations (SRC) are expected to induce the major changes at relative momenta of

TABLE I. Test of the energy dependent sum rule of Koltun (Ref. 5) and its analyses in Hartree-Fock theory. The second column displays the average binding energy per proton (Ref. 5). Columns three and four list the average kinetic $\langle T\rangle_\pi$ and separation energy $\langle S\rangle_\pi$ per proton calculated from the experimental (Refs. 6 and 7) spectral function $P_\pi(k, S)$ according to Eq. (3). Δ indicates the failure of the model independent sum rule as defined in Eq. (1). Only the statistical errors are given. $\langle H_3\rangle$ is the contribution of the three-body term of the Skyrme 2 force (Ref. 11). According to the sum rule including three-body forces the term $-\frac{1}{2}\langle H_3\rangle_\pi$ given in column 6 should be equal to Δ. Column 7 displays minus half of the average proton rearrangement energy $-\frac{1}{4}\langle \partial G/\partial\rho\rangle_\pi$ calculated with the MDI 4 force (Ref. 16). Again this should be equal to Δ. Column 8 lists the average occupation probability $\bar{P}_l$ Eq. (6) needed to fulfill sum rule Eq. (9) using $\langle T\rangle_\pi$ and $\langle S\rangle_\pi$ from Refs. 6 and 7. One recognizes from columns 6 and 7 that the generalized sum rules Eqs. (4) and (7) are qualitatively fulfilled, because Δ is roughly equal to $-\frac{1}{2}\langle H_3\rangle_\pi$ and $-\frac{1}{4}\langle \partial G/\partial\rho\rangle_\pi$. In ^{12}C, ^{28}Si, and ^{58}Ni we have approximated the deformed HF solution by a spherical one. One expects that this still yields the right order of magnitude for the three-body contribution or the density dependent part of the force. Furthermore the experimental $\bar{P}_l$ listed agrees approximately with values derived from theoretical consideration (Refs. 14 and 17). The discrepancies for ^{40}Ca are not understood. They may, however, be connected with the experimental data for the spectral function (Ref. 7). The last column lists the value n/Z [see Eq. (3)] obtained from experiment (Refs. 6 and 7).

	E_Z/Z (MeV)	$\langle T\rangle_\pi$ (MeV)	$\langle S\rangle_\pi$ (MeV)	Δ (MeV)	$-\frac{1}{2}\langle H_3\rangle_\pi$ (MeV)	$-\frac{1}{4}\left\langle\frac{\partial G}{\partial\rho}\right\rangle_\pi$ (MeV)	$\bar{P}_l$	n/Z
^{12}C	−6.93	16.9	23.4	−2.9 ± 0.5	−2.93	−2.15	0.85	0.60
^{28}Si	−6.84	17.0	24.5	−2.8 ± 0.6	−3.61	−2.65	0.86	0.81
^{40}Ca	−6.51	16.6	27.8	−0.7 ± 0.5	−3.81	−2.81	0.97	0.80
^{58}Ni	−6.95	18.7	24.8	−3.8 ± 0.7	−4.06	−2.97	0.83	0.93

500 MeV/c or higher. As far as the limitation of the separation energies to $S < 80$ MeV is concerned, both the total occupation numbers and the shell model analysis of the spectral function[6,7] show that the normally occupied shell model orbits have been essentially covered by the experiment.

Now Hartree-Fock (HF) theory with its limitation to single particle Slater determinants basically is also confined to a description of the "shell model region." In theory, however, one knows how to renormalize the nucleon-nucleon interaction to account for this restriction. Three possibilities shall be discussed: (i) The effective force becomes density dependent due to the short range correlations.[8-12] (ii) This density dependence can also be considered as an effective three-body force.[11] (An example of such an effective three-body force is the "Skyrme 2" force[11].) (iii) Partial occupation probabilities [13-15,17,18] are considered in the renormalized Brueckner-Hartree-Fock theory.

II. REARRANGEMENT ENERGY

One way of defining an effective force in an independent single particle space containing only momenta below ≃300 MeV/c and energies below ≈80 MeV is to renormalize the short range correlations (SRC) into a density dependence. Such density dependent forces[8] take into account 2p2h intermediate states as described by the Bethe-Goldstone equation. Due to the Pauli operator and the starting energy dependence the reaction matrix (which is equivalent to the effective force) gets density dependent. With such forces the energy sum rule evaluated in the Hartree-Fock model is of the following form:

$$E_Z/Z = \tfrac{1}{2}\left[-\langle S\rangle_\pi + \langle T\rangle_\pi\left(1-\frac{1}{A-1}\right)\right] - \tfrac{1}{4}\left\langle\frac{\partial G}{\partial p}\right\rangle_\pi . \quad (4)$$

The last term is minus half the average proton rearrangement energy. $G(1, 2, \rho)$ is an effective density dependent force, like the Skyrme[11] or the MDI[10] forces. The symbol $\rho[\frac{1}{2}(\vec{r}_1+\vec{r}_2)]$ denotes the density distribution of the nucleus. In the self-consistent single particle representation $|i\rangle$, $|k\rangle$, $|l\rangle$ ··· the density is parametrized by the diagonal density matrix elements $\rho_{ik} = \langle i|\rho|k\rangle = \delta_{i,k}\rho_i$ with values 1 or 0. In deriving Eq. (4) we used a modification of the HF theory (see e.g., Refs. 13 and 14) which includes the rearrangement energy in the definition of the single particle energy, whence $-\epsilon_i = S_i$.

$$\left\langle\frac{\partial G}{\partial\rho}\right\rangle_\pi = \frac{Z}{A}\sum_{lki}\left\langle lk\left|\frac{\partial G}{\partial\rho_i}\right|lk\right\rangle\rho_l\rho_k\rho_i . \quad (5)$$

Expression (5) has been used to calculate the rearrangement correction. But for later use in Sec. IV we want to show that this expression can be approximately modified:

$$\sum_{lk}\left\langle lk\left|\frac{\partial G}{\partial\rho_i}\right|lk\right\rangle\rho_l\rho_k \simeq 2\sum_{lk}\left\langle lk\left|\frac{\partial G}{\partial W_{lk}}\right|lk\right\rangle\rho_l\rho_k \times\langle li|G(W)|li\rangle \quad (6)$$

$$\simeq 2\sum_l\langle il|G(W)|il\rangle\rho_l(P_l-1)$$

with

$$P_l \simeq 1 + \sum_k \left\langle lk \left| \frac{\partial G}{\partial W_{lk}} \right| lk \right\rangle \rho_k \ .$$

In Eq. (6) we used the fact that the total rearrangement energy is composed of the starting energy rearrangement, Pauli rearrangement, and orbital rearrangement.[13,14] The first two contributions are often called Brueckner rearrangement, because Brueckner[4] studied this effect first. Müther, Faessler, and Goeke[13] showed that in ^{16}O the starting (or occupation) rearrangement always yields more than 74% of the total rearrangement energy for a single particle state (see Table 2 of Ref. 13). Equation (6) includes the starting energy rearrangement only. In addition we introduced the partial occupation probability[15] P_l.

The rearrangement correction $-\frac{1}{4}\langle \partial G/\partial \rho \rangle_\pi$ calculated with the force[16] MDI 4 is given in Table I. With the exception of ^{40}Ca the agreement is satisfactory. (The discrepancy in ^{40}Ca is hard to understand, if it is not an experimental effect.) The error given in Table I for the violation Δ of the two-body sum rule does not include theoretical uncertainties e.g., coming from the corrections for the distortion of the outgoing proton.

III. EFFECTIVE THREE-BODY FORCE

Restricting the range of the momenta to less than 300 MeV/c can also be partially compensated for by the introduction of an effective three-body force. Vautherin and Brink[11] showed that a linear density dependence is equivalent to a three-body δ force in the Hartree-Fock approach. For such a case the energy sum rule has to be generalized to

$$\frac{1}{Z} E_Z = \tfrac{1}{2}\left[-\langle S \rangle_\pi + \langle T \rangle_\pi \left(1 - \frac{1}{A-1} \right) - \langle H_3 \rangle_\pi \right] . \qquad (7)$$

Here

$$\langle H_3 \rangle_\pi = \tfrac{1}{2}\langle H_3 \rangle = \frac{1}{12} \sum_{ikl} \langle ikl | H_3 | ikl - ilk - kil + lik + kli - lki \rangle \rho_i \rho_k \rho_l$$

is the average contribution of the three-body force per proton. The sum rule is therefore fulfilled if Δ of Eq. (1) equals $-\frac{1}{2}\langle H_3 \rangle_\pi$. Table I shows this quantity calculated from the three-body part H_3 of Skyrme 2 with the corresponding Hartree-Fock wave function. Again the agreement with the experimental values Δ is quite satisfactory.

IV. PARTIAL OCCUPATION PROBABILITIES

A third way of taking into account only a finite momentum space lies in the introduction of partial occupation probabilities P_l of Eq. (6). This is in first order equivalent with taking into account starting energy rearrangement as shown in Eq. (6). The total energy and the "single particle" energies are given in this approach by[17,18]:

$$\langle H \rangle = \sum_l \tilde{t}_l \rho_l + \tfrac{1}{2} \sum_{lk} \langle lk | \tilde{G}(W) | lk \rangle \rho_l (2 - P_l) \rho_k \ ,$$

$$\epsilon_i = \tilde{t}_i + \sum_k \langle ik | \tilde{G}(W) | ik \rangle \rho_k P_k \ . \qquad (8)$$

If one introduces $d_l \equiv 1 - P_l$ the energy sum rule in the HF approximation[19] has the form:

$$\Delta (= -\tfrac{1}{2}\bar{\epsilon}_{\text{rear}}) = \frac{E_Z}{Z(1+\bar{d})} - \frac{1}{2Z} \sum_{i(=\pi)<F} \frac{1-d_i}{1+\bar{d}} \tilde{t}_i - S_i \ . \qquad (9)$$

The tilde in Eqs. (8) and (9) indicates that the correction for the center of mass motion has been included [$\tilde{t}_i \equiv t_i(1 - 1/A)$].

We determined the average occupation probability $P_l = 1 - \bar{d}$ in Table I by equating the right hand side of Eq. (9) to zero. This yields an average experimental occupation probability in agreement with theoretical values[13-15,17,18] (again apart of ^{40}Ca).

V. CONCLUSION

We therefore conclude that measurements[6] of the spectral function $P(k, S)$ in the restricted range of the momentum k and the missing energy ($\hbar k \leq 300$ MeV; $S \leq 80$ MeV) can give no information about the bare three-body force. However, it is possible to extract from them the strength of an effective three-body force, the strength of the density dependence of an effective interaction, or the average partial occupation probability $\bar{P}_l$. As we now see all three descriptions are approximately equivalent as explanations of the "failure" of the model independent energy sum rule[5] applied to the measurements of Refs. 6 and 7.

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