

Neutron resonance spectroscopy:  $^{177}\text{Hf}^\dagger$ H. I. Liou,\* J. Rainwater, G. Hacken, and U. N. Singh  
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(Received 4 March 1975)

High resolution neutron time of flight spectroscopy measurements were made for a sample enriched in  $^{177}\text{Hf}$  using the Columbia University Nevis synchrocyclotron. The measurements used the 202.05 m flight path for transmission measurements and the 39.37 m flight path for capture measurements. We obtained  $(E_0, g\Gamma_n^0)$  resonance parameters for 176 levels in  $^{177}\text{Hf}$  to 700 eV and 12 levels in  $^{178}\text{Hf}$  to 720 eV. We also obtained  $\Gamma_\gamma$  values for 25 levels in  $^{177}\text{Hf}$ , giving  $\langle\Gamma_\gamma\rangle=65$  meV. The  $s$  wave strength function for  $^{177}\text{Hf}$  is found to be  $(2.70\pm 0.25)$ . Most  $s$  levels were detected to 100 eV, but many were missed above 100 eV. An analysis favors  $\langle D_0\rangle=(2.22\pm 0.13)$  eV for the "true"  $s$  level spacing in  $^{177}\text{Hf}$ . The results to 100 eV are in reasonable agreement with predictions for statistical orthogonal population ensembles.

NUCLEAR REACTIONS  $^{177}\text{Hf}(n, n)$ ,  $(n, \gamma)$ ,  $E=3-700$  eV; measured  $\sigma_i(E)$ ; deduced  $E_0, g\Gamma_n^0, \Gamma_\gamma, S_0, \langle D_0\rangle, S_0(J=4)/S_0(J=3)$ ; orthogonal ensemble tests.

## I. INTRODUCTION

This is one of a series<sup>1-18</sup> of papers reporting the results of high resolution neutron time of flight spectroscopy measurements using the Columbia University Nevis synchrocyclotron. We report results using 202.05 m path transmission measurements and 39.57 m path capture measurements on a sample of  $\text{HfO}_2$  enriched in  $^{177}\text{Hf}$  to 74.35% abundance. By using previously established information<sup>19</sup> on the resonance level assignments for the Hf isotopes, we were able to assign the resonance parameters  $(E_0, g\Gamma_n^0)$  for 176 levels in  $^{177}\text{Hf}$  from 3 to 700 eV. The transmission sample had  $(1/n)=940$  b/atom for  $^{177}\text{Hf}$ , which was much too thin to provide information on the energy dependence of the between level  $\sigma$  vs  $E$ . We also obtained  $\Gamma_\gamma$  values for 25 levels in  $^{177}\text{Hf}$ . Our measured  $s$  wave strength function  $S_0$  is considerably larger than we obtained for  $^{175}\text{Lu}$ <sup>17</sup> or other nuclei having atomic weights near to 177. All the observed levels are  $l=0$ , since a Bayes's theorem analysis of our data indicates that the probability is  $<10^{-3}$  for  $10^4 S_1 \leq 6$  that any  $p$  levels will have been detected. The binding energy for an extra neutron in  $^{177}\text{Hf}$  is 7.265 MeV.

The  $s$  levels in  $^{177}\text{Hf}$  ( $I^\pi = \frac{7}{2}^-$ ) are expected to form two populations having  $J=3$  and 4 of nearly equal abundances. We have not attempted to determine favored  $J$  values since the two possible spin weight factors,  $g = \frac{7}{16}$  or  $\frac{9}{16}$ , are nearly equal. We obtain  $g\Gamma_n^0$  for the "strength" of each level, and  $\Gamma_\gamma$  values in 25 favorable cases. Natural Hf has  $>5\%$  abundance for  $A=176, 177, 178, 179$ , and 180. The data for our enriched  $^{177}\text{Hf}$  sample revealed 5, 11, 26, and 4 previously known levels

due to  $^{176}\text{Hf}$ ,  $^{178}\text{Hf}$ ,  $^{179}\text{Hf}$ , and  $^{180}\text{Hf}$ , respectively, to 700 eV. Levels are listed for each of the stable Hf isotopes in the latest BNL-325,<sup>19</sup> with 7 to 17 each for the even isotopes ( $A=174, 176, 178, 180$ ), 132 levels for  $^{177}\text{Hf}$ , and 106 levels for  $^{179}\text{Hf}$ .

The main previous results for  $^{177}\text{Hf}$  (and  $^{179}\text{Hf}$ ) are due to Fuketa and Harvey<sup>20</sup> using the ORNL fast chopper, and Fuketa, Russell, and Hockenbury<sup>21</sup> using the RPI electron linac. In addition, Coceva, Corvi, Giacobbe, and Stefanon<sup>22</sup> (Geel), from analysis for the systematics of the resonance capture  $\gamma$  spectra for levels in  $^{177}\text{Hf}$  and  $^{179}\text{Hf}$ , have assigned resonance  $J$  values to 99 levels in  $^{177}\text{Hf}$  and 85 levels in  $^{179}\text{Hf}$ . Rohr and Weigmann<sup>23</sup> reported  $\sigma_\gamma$  measurements, using the Geel electron linac. They determined  $\Gamma_n$  for 89 levels, and  $\Gamma_\gamma$  for 16 levels in  $^{177}\text{Hf}$  to 300 eV where the spin assignments were known. Below 300 eV we observed all but one of the  $^{177}\text{Hf}$  levels of Ref. 19, and we include three  $^{177}\text{Hf}$  levels not in Ref. 19. We assign all observed resonances to  $^{177}\text{Hf}$  when not assigned to known levels in other Hf isotopes of  $A=176, 178, 179$ , or 180 in Ref. 19. Between 300 and 700 eV we have 76 levels due to  $^{177}\text{Hf}$  vs 30 levels listed in Ref. 19. In some cases above 300 eV, it is difficult to correlate the energies of the previously reported  $^{177}\text{Hf}$  resonances from poorer resolution measurements with specific ones of our levels. However, our  $g\Gamma_n^0$  values for levels below 300 eV agree well with those of Ref. 19 except in a few cases. We also obtained energies and  $\Gamma_n^0$  values for all of the 12 levels in  $^{178}\text{Hf}$  to 720 eV which are listed in Ref. 19. (Results from Refs. 20 and 21.) Our sample had 17.17% abundance of  $^{178}\text{Hf}$ . Our  $\Gamma_n^0$  values for  $^{178}\text{Hf}$  generally agree with those of Ref. 19 to within quoted uncertainties.

TABLE I. Resonance parameters of  $^{177}\text{Hf}$ . An asterisk (\*) indicates level parameters quoted from BNL-325 (Ref. 19).

$E_0$ (eV)	$g\Gamma_n^0$ (meV)	$E_0$ (eV)	$g\Gamma_n^0$ (meV)	$E_0$ (eV)	$g\Gamma_n^0$ (meV)	$E_0$ (eV)	$g\Gamma_n^0$ (meV)
*1.098±0.02	0.92 ±0.02	115.03 ±0.03	0.24 ±0.03	238.64±0.26	1.4 ±0.3	446.57±0.52	1.6 ±0.4
*2.380±0.004	2.9 ±0.1	121.17 ±0.09	0.21 ±0.03	240.71±0.26	0.84±0.13	449.43±0.53	0.66±0.14
5.894±0.016	0.87 ±0.12	122.72 ±0.10	0.070±0.018	248.74±0.27	0.72±0.10	453.81±0.68	0.14±0.08
6.572±0.017	1.7 ±0.2	123.68 ±0.10	0.52 ±0.05	259.65±0.29	0.06±0.02	457.36±0.54	4.0 ±0.6
8.868±0.019	1.1 ±0.1	126.17 ±0.10	0.054±0.013	264.48±0.30	2.2 ±0.3	466.58±0.71	0.19±0.09
10.95 ±0.02	0.073±0.009	131.59 ±0.11	2.3 ±0.2	267.57±0.31	1.3 ±0.2	470.77±0.56	0.88±0.28
13.67 ±0.02	0.092±0.014	134.00 ±0.11	0.20 ±0.03	272.22±0.31	1.8 ±0.4	472.17±0.56	1.5 ±0.4
13.94 ±0.02	0.37 ±0.05	136.15 ±0.12	0.032±0.010	273.16±0.32	0.45±0.15	474.99±0.57	2.3 ±0.5
21.95 ±0.03	0.23 ±0.04	137.42 ±0.12	0.80 ±0.09	284.69±0.34	5.3 ±0.8	478.70±0.58	1.5 ±0.3
22.23 ±0.02	0.085±0.015	141.12 ±0.12	1.1 ±0.1	287.85±0.34	0.31±0.06	481.58±0.74	0.33±0.11
23.41 ±0.02	0.18 ±0.02	143.15 ±0.19	0.18 ±0.04	294.19±0.35	0.12±0.03	488.60±0.60	4.8 ±0.7
25.62 ±0.03	0.041±0.006	143.71 ±0.19	0.71 ±0.13	298.47±0.36	2.1 ±0.3	498.51±0.61	1.3 ±0.4
26.99 ±0.03	0.26 ±0.03	145.53 ±0.13	0.30 ±0.03	302.37±0.37	0.09±0.03	507.15±0.63	1.6 ±0.4
31.61 ±0.03	0.027±0.005	148.52 ±0.13	0.75 ±0.07	306.96±0.30	3.3 ±0.5	512.19±0.64	1.5 ±0.4
32.77 ±0.04	0.14 ±0.01	151.15 ±0.13	0.023±0.009	310.92±0.38	0.24±0.06	520.71±0.65	0.61±0.18
36.08 ±0.04	0.25 ±0.03	152.86 ±0.20	0.071±0.018	313.60±0.39	0.45±0.11	523.02±0.66	1.4 ±0.3
36.94 ±0.04	0.89 ±0.08	155.94 ±0.14	0.10 ±0.03	319.85±0.40	0.84±0.11	525.46±0.66	3.1 ±0.7
43.03 ±0.04	0.43 ±0.03	159.97 ±0.15	0.17 ±0.03	323.60±0.41	1.2 ±0.2	533.20±0.86	0.27±0.09
45.09 ±0.05	0.29 ±0.02	162.98 ±0.15	1.5 ±0.2	327.46±0.42	3.2 ±0.5	539.10±0.87	0.22±0.13
46.18 ±0.05	0.59 ±0.05	167.28 ±0.15	0.28 ±0.05	330.38±0.42	4.3 ±0.7	541.30±0.88	0.15±0.09
48.76 ±0.05	2.6 ±0.3	171.03 ±0.16	0.51 ±0.09	333.40±0.43	1.2 ±0.2	548.63±0.71	2.9 ±0.4
49.56 ±0.03	0.40 ±0.04	174.18 ±0.13	0.68 ±0.08	341.79±0.44	0.97±0.16	557.22±0.92	0.29±0.08
54.71 ±0.04	1.07 ±0.08	176.11 ±0.13	1.7 ±0.3	348.74±0.46	2.5 ±0.4	573.74±0.76	1.3 ±0.4
56.29 ±0.05	0.80 ±0.09	176.70 ±0.13	2.1 ±0.4	354.74±0.47	0.33±0.09	577.40±0.76	3.2 ±0.9
57.00 ±0.05	0.29 ±0.04	178.89 ±0.17	0.028±0.008	357.08±0.47	1.2 ±0.2	581.92±0.98	1.1 ±0.3
59.21 ±0.05	0.22 ±0.03	180.94 ±0.14	0.25 ±0.03	362.33±0.48	0.35±0.07	591.10±1.01	0.37±0.12
62.15 ±0.06	0.099±0.019	184.54 ±0.18	0.048±0.015	367.45±0.50	1.6 ±0.3	596.92±1.02	0.32±0.16
63.42 ±0.07	4.4 ±0.4	188.01 ±0.18	0.024±0.015	370.67±0.50	0.88±0.16	598.97±1.03	0.24±0.14
66.69 ±0.08	2.7 ±0.4	192.66 ±0.19	0.35 ±0.06	375.55±0.51	7.7 ±1.3	604.66±0.81	2.4 ±0.5
69.96 ±0.08	0.043±0.011	194.00 ±0.19	0.28 ±0.04	389.84±0.54	0.71±0.15	610.40±1.06	0.73±0.24
71.29 ±0.08	0.98 ±0.09	199.14 ±0.16	0.89 ±0.09	393.63±0.55	0.17±0.05	612.52±1.06	1.1 ±0.4
72.22 ±0.08	0.13 ±0.02	201.79 ±0.20	0.68 ±0.11	398.82±0.56	0.27±0.07	618.92±1.08	0.52±0.16
75.41 ±0.08	0.078±0.023	205.65 ±0.21	0.054±0.028	406.15±0.57	2.0 ±0.3	625.54±0.86	1.2 ±0.4
75.99 ±0.08	1.1 ±0.1	208.64 ±0.17	1.7 ±0.2	408.80±0.58	0.37±0.10	628.70±0.87	3.4 ±0.8
82.33 ±0.06	0.045±0.011	209.98 ±0.21	0.12 ±0.03	412.88±0.59	0.84±0.30	633.16±1.11	0.13±0.08
84.56 ±0.06	1.6 ±0.1	212.05 ±0.22	0.058±0.021	414.90±0.59	2.7 ±0.5	640.59±0.89	1.7 ±0.4
85.25 ±0.11	0.12 ±0.03	217.03 ±0.23	0.24 ±0.04	418.80±0.60	0.68±0.15	646.40±0.90	2.8 ±0.7
86.73 ±0.06	0.056±0.013	219.51 ±0.23	0.36 ±0.05	426.09±0.62	2.2 ±0.4	653.89±0.92	1.6 ±0.6
88.51 ±0.06	0.20 ±0.03	222.26 ±0.23	0.074±0.040	429.18±0.62	1.0 ±0.2	657.80±0.92	0.97±0.35
93.13 ±0.07	0.26 ±0.03	223.30 ±0.24	0.12 ±0.07	431.70±0.49	0.96±0.19	669.15±1.21	0.89±0.23
97.01 ±0.07	1.1 ±0.1	224.67 ±0.24	4.5 ±0.7	433.56±0.50	1.4 ±0.3	676.47±1.23	1.4 ±0.4
98.98 ±0.07	0.052±0.015	226.56 ±0.24	0.45 ±0.07	434.92±0.50	1.4 ±0.3	684.70±0.98	3.2 ±0.8
103.07 ±0.07	2.7 ±0.3	229.08 ±0.24	0.22 ±0.04	435.91±0.50	1.5 ±0.4	693.11±1.00	2.8 ±0.7
111.50 ±0.13	0.12 ±0.03	232.20 ±0.25	0.039±0.013	443.40±0.51	1.2 ±0.2	696.63±1.01	2.6 ±0.6
111.96 ±0.13	0.25 ±0.05	236.16 ±0.26	0.36 ±0.07				

The detailed analysis of these data is mainly due to Dr. H. I. Liou.

## II. EXPERIMENTAL DETAILS AND PRELIMINARY RESULTS

The data for  $^{177}\text{Hf}$  were obtained during the same major run as that for the erbium isotopes,<sup>1</sup> the tungsten isotopes,<sup>5</sup> the Yb isotopes,<sup>4</sup> the Sm and Eu isotopes,<sup>2</sup> the In isotopes,<sup>9</sup> and  $^{175}\text{Lu}$ .<sup>17</sup> The experimental details for the cyclotron and time of flight analyzer operation are given in Ref. 1. The 202.05 m transmission measurements covered the energy region above 28 eV in 8192 detection channels. The 39.57 m capture  $\gamma$ -ray measurements, using

the sample at the detector, covered the region above  $\sim 3$  eV. About 250 000 and 400 000 cyclotron bursts, respectively, were devoted to counting with the sample in the beam, yielding  $>4 \times 10^6$  detector counts for each mode of measurement.

The sample of  $\text{HfO}_2$  was prepared as two equal thickness 6.35 cm square samples which made a  $6.35 \times 12.7$  cm area sample for the capture measurements, and was tilted to be equivalent to a single  $3.18 \times 12.7$  cm sample of double thickness for the transmission measurements. The sample had 1.45%, 74.35%, 17.17%, 3.0%, and 4.04% relative abundances of the  $A = 176$ , 177, 178, 179, and 180 Hf isotopes. For the transmission configuration  $(1/n) = 700$  b/atom for all Hf isotopes com-

TABLE II. Cases where  $\Gamma_\gamma$  were obtained for  $^{177}\text{Hf}$ .

$E_0$ (eV)	$\Gamma_\gamma$ (meV)	$E_0$ (eV)	$\Gamma_\gamma$ (meV)
48.76±0.05	61±15	267.57±0.31	74±20
63.42±0.07	57±12	284.69±0.34	80±20
84.56±0.06	75±20	306.96±0.30	70±18
97.01±0.07	62±15	319.85±0.40	77±20
103.07±0.07	56±15	333.40±0.43	60±20
131.59±0.11	65±12	341.79±0.44	60±20
141.12±0.12	70±20	367.45±0.50	75±20
162.98±0.15	68±20	406.15±0.57	61±20
199.14±0.16	72±18	457.36±0.54	58±20
208.64±0.17	65±18	488.60±0.60	68±20
240.71±0.26	58±20	548.63±0.71	60±20
248.74±0.27	57±20	604.66±0.81	54±20
264.48±0.30	51±18		

bined. The sample was obtained as a loan from the Isotope Division of Oak Ridge National Laboratory. The purity of the sample was such that all observed resonances were due to Hf isotopes.

The preliminary resonance analysis for the data was as described in Ref. 1. For each resonance in transmission or capture, an implied relation was obtained between  $g\Gamma_n$  and  $\Gamma$  from an area analysis. A curve  $\Gamma = \langle\Gamma_\gamma\rangle + 2g\Gamma_n$  vs  $g\Gamma_n$  was also constructed.

For most levels the transmission ( $T$ ) and capture ( $C$ ) curves did not form a well enough defined intersection point to permit  $g\Gamma_n$  and  $\Gamma$  to be separately evaluated, but gave consistent intersections with the  $\Gamma = \langle\Gamma_\gamma\rangle + 2g\Gamma_n$  curve, specifying our  $g\Gamma_n$

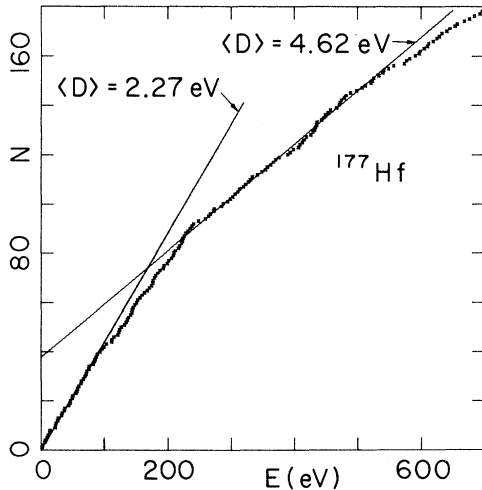


FIG. 1. Plot of the cumulative count of observed levels vs energy for  $^{177}\text{Hf}$  to 700 eV. The straight lines correspond to level slopes in terms of  $\langle D \rangle$ . Our final choice of  $\langle D_0 \rangle$  value for  $l=0$  is given in the text. A detailed analysis shows that about 3, 33, and 137 weak  $s$  levels are missed to 100, 300, and 700 eV, respectively.

TABLE III. Resonance parameters of  $^{178}\text{Hf}$ .

$E_0$ (eV)	$\Gamma_n^0$ (meV)
7.770±0.027	17.6 ±2.5
104.76 ±0.08	0.87±0.20
164.59 ±0.15	1.2 ±0.3
255.17 ±0.23	14 ±3
274.75 ±0.25	13 ±3
351.49 ±0.46	0.75±0.37
382.05 ±0.52	21 ±3
446.47 ±0.66	7.6 ±1.9
502.33 ±0.79	4.9 ±1.8
527.28 ±0.66	5.7 ±1.7
577.52 ±0.76	15 ±3
719.32 ±0.53	39 ±8

value for the level. In 25 cases, however, the  $T$  and  $C$  curve intersection points were well enough defined to give both  $g\Gamma_n$  and  $\Gamma_\gamma$  ( $\equiv \Gamma - 2g\Gamma_n$ ). Table I lists the  $(E_0, g\Gamma_n^0)$  for the observed levels in  $^{177}\text{Hf}$  to 700 eV. Table II lists  $(E_0, \Gamma_\gamma)$  values for the 25 levels where  $\Gamma_\gamma$  was established. A value  $\langle\Gamma_\gamma\rangle = 65$  meV was obtained. Table III lists  $(E_0, \Gamma_n^0)$  values for the 12 levels in  $^{178}\text{Hf}$  to 720 eV.

### III. SYSTEMATICS OF THE RESULTS AND DISCUSSION

Figure 1 is a plot of the cumulative number of levels seen to energy  $E$ , vs  $E$ . The region to  $\sim 100$  eV has a slope corresponding to  $\langle D \rangle = 2.27$  eV, followed by decreasing slope at higher energies where an increasing fraction of weak levels was missed. The indicated slopes do not represent

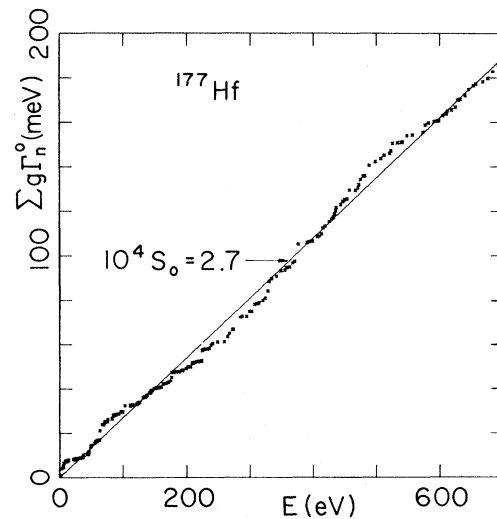


FIG. 2. Plot of  $\sum g\Gamma_n^0$  vs energy for levels in  $^{177}\text{Hf}$ . The slope of the fitted straight line gives the  $s$  strength function. Such a plot is insensitive to missed weak  $s$  levels.

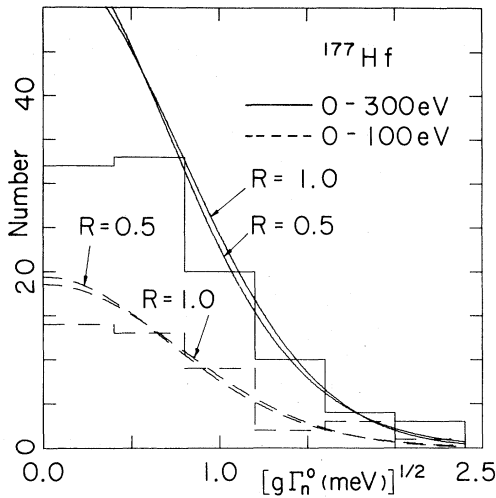


FIG. 3. Histograms of observed  $(g\Gamma_n^0)^{1/2}$  values for  $^{177}\text{Hf}$  to 100 and 300 eV, respectively. Fits are made to the upper parts of the histograms based on two merged single channel Porter-Thomas distributions, since missed weak  $s$  levels will have  $(g\Gamma_n^0)^{1/2}$  values corresponding to the first and second histogram boxes. The curves, normalized to  $10^4 S_0 = 2.70$ , are fitted by 45 levels vs 42 observed levels to 100 eV, and 135 levels vs 102 observed levels to 300 eV, respectively. The level densities for  $J=4$  and 3 are taken proportional to the  $(2J+1)$  values.  $R$  is the ratio of  $S_0$  for  $J=4$  to that for  $J=3$ .  $R=1$  is the usual choice.  $R=0.5$  is an extreme alternative in  $^{177}\text{Hf}$  case.

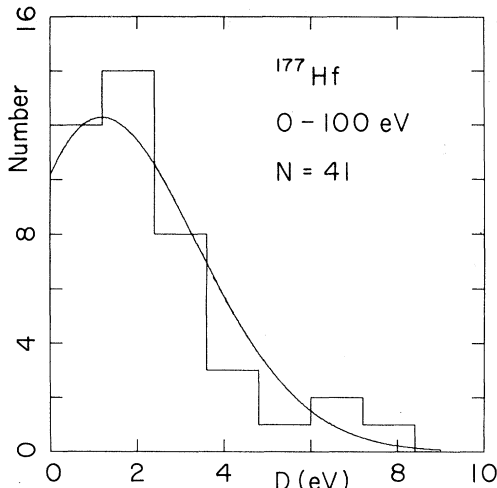


FIG. 4. Histogram of the nearest neighbor  $s$  level spacing distribution for  $^{177}\text{Hf}$  to 100 eV in comparison with two merged Wigner distributions, each having level densities proportional to  $(2J+1)$ . Since we expect three weak  $s$  levels missed in this energy interval, it explains the excess of spacings for the last two histogram boxes.

our final choice for  $\langle D_0 \rangle$  for the complete  $s$  level population.

Figure 2, the plot of  $\sum g\Gamma_n^0$  vs  $E$ , is insensitive to missed weak levels. The mean slope yields the  $s$  wave strength function  $10^4 S_0 = (2.70 \pm 0.25)$ .

Figure 3 shows histograms of the observed  $(g\Gamma_n^0)^{1/2}$  values for the energy intervals to 100 and 300 eV. The theoretical Porter-Thomas single channel distributions are for two merged populations where the total numbers of levels for  $J=4$  and  $J=3$  are taken in a ratio of the  $(2J+1)$  values. The parameter  $R$  represents the assumed ratio of the separate  $J=4$  to  $J=3$  strength functions.  $R=1$  is the usual choice. The ratio  $R = (0.73 \pm 0.20)$  is a value implied by our  $g\Gamma_n^0$  values to 300 eV, using the spin assignments of Coceva *et al.*<sup>22</sup> The curve for  $R=0.5$  is an extreme alternate choice for comparison. The curves are normalized to a net  $s$  wave strength function  $10^4 S_0 = 2.70$  for  $^{177}\text{Hf}$ . The fitted curves are for more than the observed numbers of levels to give best fits to the upper parts of the histograms, implying that three levels were missed to 100 eV and 33 levels to 300 eV.

A threshold sensitivity test for the expected number of missed weak  $s$  levels vs upper energy was made using the parameters indicated for the Porter-Thomas curves in Fig. 3. It suggests that about 3, 17, and 61 weak  $s$  levels were missed to 100, 300, and 700 eV, respectively. It is insensitive to a choice of  $R$  value between 0.5 and 1.0. For our final choice  $\langle D_0 \rangle = 2.22$  eV for  $s$  levels, Fig. 1 shows that we have 3, 33, and 137 fewer levels to 100, 300, and 700 eV, respectively. Since there

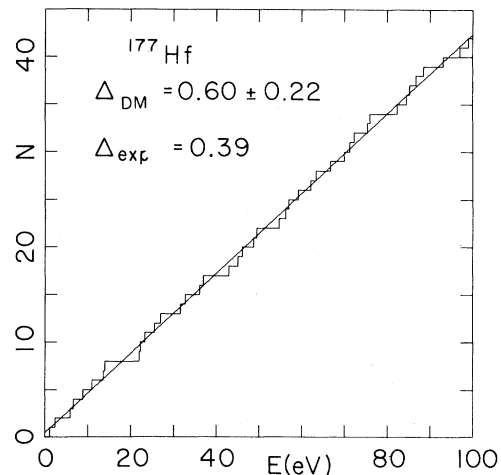


FIG. 5. Comparison of the staircase plot of number of observed levels  $N$  vs  $E$  to 100 eV with the best fit straight line. Dyson-Mehta  $\Delta$  is the mean square deviation. The excellence of the fit is made less significant by the fact that three weak  $s$  levels were probably missed in this energy interval.

is no level repulsion between  $s$  levels of different  $J$ , many weak  $s$  levels above 100 eV, which could have been detected if no other levels were too close, were not resolved from their stronger overlap  $s$  levels (mainly other  $J$ ).

An attempt to compare the  $J=4$  and  $3$   $s$  strength functions is confused by some aspects of  $J$  assignments of Coceva *et al.*<sup>22</sup> to 300 eV. Their plot of the numbers of  $J=4$  and  $J=3$  levels observed vs energy gives good straight lines to 100 eV, with indications of missed levels above 100 eV. To 100 eV, the ratio of the observed level densities agrees well with the ratio of  $(2J+1)$  values, and using our  $g\Gamma_n^0$  values, a value of  $R=1.5\pm 0.7$  is obtained which suggests a possible larger  $S_0$  for  $J=4$  than  $J=3$ . However, their plot seems to miss only 4  $J=3$  levels to 300 eV, but 20  $J=4$  levels. The favored value of  $R$  is thus strongly energy dependent, so  $R=1$  should probably be recommended until more conclusive evidence is obtained.

Figure 4 shows a comparison of the observed histogram for the nearest neighbor spacing distribution to 100 eV with the Wigner distribution for two merged populations, each having level densities proportional to  $(2J+1)$ . If three levels were missed, as described above, this helps to explain the excess of large spacings for the last two histogram boxes. Otherwise, the agreement is fair.

Figure 5 shows a best fit of a straight line to the

ladder plot of observed number of levels to 100 eV. The mean square deviation is the Dyson-Mehta  $\Delta$  statistic.<sup>24</sup> The predicted value is  $\Delta_{DM}=(0.60\pm 0.22)$  compared with the observed value  $\Delta=0.39$ . We obtain  $\rho=-0.41$  for the correlation coefficient of adjacent nearest neighbor spacings, vs a predicted value<sup>15</sup>  $\rho_{OE}=(-0.26\pm 0.13)$  for two merged orthogonal ensemble populations having the expected ratio of level densities. If extra levels are added at the centers of the two largest nearest neighbor spacing intervals, at 18 and 79.2 eV, and another at 90.8 eV, the fit to the nearest neighbor spacing histogram is improved [ $\Delta=0.40$  vs  $\Delta_{DM}=(0.62\pm 0.22)$ ] and  $\rho=-0.36$ . The addition of a level at 90.8 eV helps to improve Dyson  $F$  statistic test.<sup>25,26</sup> Our final choice of  $\langle D_0 \rangle$  for  $^{177}\text{Hf}$  is  $(2.22\pm 0.13)$  eV by considering three levels probably missed to 100 eV.

Our  $\Gamma_n^0$  for the 12 levels in  $^{178}\text{Hf}$  to 720 eV gives  $10^4 S_0=(1.9\pm 0.8)$  in reasonable agreement with previous determinations.<sup>19</sup> Our value  $10^4 S_0=(2.7\pm 0.25)$  for  $^{177}\text{Hf}$  compares with a value 2.5 evaluated to 300 eV, and 1.7 to 900 eV in Ref. 19.

The authors wish to thank Dr. H. Ceulemans, Dr. H. S. Camarda, Dr. M. Slagowitz, and Dr. S. Wynchank for their involvement in the measurements, and Dr. S. F. Mughabghab for helpful discussions.

<sup>†</sup>Research supported by the U. S. Atomic Energy Commission.

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