# Differential cross sections and phase shift analysis of  ${}^{3}He(p, p) {}^{3}He$ between 18.0 and 57 Mev\*

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Measurements of the elastic differential cross section for  ${}^{3}$ He( $\phi$ ,  $\rho$ ) ${}^{3}$ He have been made at laboratory energies of 18.0, 20.0, 22.5, 25.0, 27.5, 30.0, 35.0, 40.0, 42.7, 45.0, 48.5, and 57.<sup>0</sup> MeV, at angles between 14.1 and 165,0' c.m. <sup>A</sup> phase shift analysis was performed in the energy range from 18.0 to 35.0 MeV using these data and polarization and spin correlation data from other sources. Angular momenta up to  $l = 4$  were used in the analysis, including tensor and spin coupling terms, while inelastic channels were treated through use of complex phase shifts. The results of the analysis exhibit smooth variation with energy, extending and clarifying results obtained at lower energies. Comparisons are made to resonating group predictions of the energy dependence of the singlet and triplet phase shifts.

NUCLEAR REACTIONS  ${}^{3}$ He(p, p), E = 18.0, 20.0, 22.5, 25.0, 27.5, 30.0, 35.0, 40.0, 42.7, 45.0, 48.5, and 57.0 MeV; measured  $(E, \theta_{lab})$ ;  $\theta_c = 14$  to 165°; phase shift analysis,  $E = 18.0$ , 20.0, 22.5, 25.0, 27.5, 30.0, and 35.0 MeV.

#### I. INTRODUCTION

The study of the reaction  ${}^{3}He(p, p){}^{3}He$  has greatly aided in the determination of the structure of the  ${}^{4}$ Li nucleus. The unbound ground state and three excited states have been identified from resonant behavior of the phase shifts derived from measurements of differential cross sections, polarizations, recoil  ${}^{3}$ He polarizations, and various spin correlation parameters, for and various spin correlation parameters, for<br>proton energies up to 19.4 MeV.<sup>16</sup> Since the num ber of phase shifts for two nonidentical spin  $\frac{1}{2}$  particles increases as  $10l$  as one passes beyond low energy, 8-state dominated scattering, increasingly more detailed data sets are required to adequately determine a "unique" set of parameters. At 19.4 MeV, Baker et al.<sup>1</sup> required 36 differential cross section values, 31 polarization values, 11 recoil <sup>3</sup>He, 11  $A_{yy}$ , and 8  $A_{xx}$  values to obtain such a solution. At higher energies, far fewer data are available. Thus, even neglecting complex components of the phase shifts, it is most unlikely that any attempt to obtain a unique phase shift solution at one energy can succeed, although shift solution at one energy can succeed, althoug<br>a number of attempts have been made.<sup>13,22</sup> How· ever, there is a possibility that, assuming a valid solution at 19.4 MeV, one can derive solutions that represent minimum deviations from the solution at a lower energy using only differential cross section and polarization data. The discrete

solution sets are generally separated one from another by ridges in  $\chi^2$  space, and thus such a procedure represents an attempt to follow a  $\chi^2$ valley to higher energy solutions.

The importance of extending phase shift solutions to higher energy is indicated by predictions of structure in the 4Li nucleus of energies between 44 MeV (24 MeV protons on  ${}^{3}$ He) and 52 MeV (32 of structure in the <sup>4</sup>Li nucleus of energies betwe<br>44 MeV (24 MeV protons on <sup>3</sup>He) and 52 MeV (32<br>MeV protons on <sup>3</sup>He) in the <sup>4</sup>Li nucleus.<sup>2,3</sup> In addition, recent results<sup>4</sup> using resonating group models have predicted phase shift behavior versus energy in these regions. It is in an attempt to substantiate these predictions that this work was done.

## II. EXPERIMENTAL ARRANGEMENT (REF. 24)

The experiment was performed with the external proton beams of the 192 cm isochronous cyclotron of the Crocker Nuclear Laboratory, using a gas target at the center of the laboratory's 76 cm scattering chamber. Three magnetic quadrupole doublets were used to focus the beam through an entrance collimator of 5 mm diam, placed 50 cm from the center of the target. This collimator was followed by an antiscattering collimator of 6 mm diam, at 30 cm from the center of the target. After passing through the target, the beam was refocused by another quadrupole doublet, and collected in a Faraday cup, located 3.3 m from the target center. Beam currents between

11

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5 and 50 nA mere used. The fraction of the beam stopped by the entrance collimator was monitored continuously, and was typically about  $5\%$ . The beam energy was determined to  $\pm 0.2$  MeV by the crossover method and agreed within  $1\%$  with the value expected from the cyclotron frequency and extraction radius. (See Fig. 1.)

The target gas was  $99.8\%$  pure  ${}^{3}$ He. It was held in a cylindrical target of  $26 \text{ cm}$  diam. A Kapton foil of 3 or 8 mg/cm<sup>2</sup> thickness extended for  $360^{\circ}$ around the target, overlapping over an angular range of about  $5^\circ$ . In the overlap region and at  $120^\circ$  to either side, three support posts blocked an angular range of about  $3^\circ$  each. The target temperature was read to 0.<sup>2</sup> 'C accuracy with a mercury thermometer, and the pressure was determined with a mercury manometer to 0.05 cm Hg. Readings were taken whenever the experimental area was entered, and values at the time of each run were obtained by interpolation. The target pressure was on the order of 1 atm and was found to decrease at a rate of about 0.1 cm  $Hg/h$ .

The scattered protons and recoil <sup>3</sup>He particles were detected with a variety of detectors: surface barrier detectors of 0.2, 0.3, and 0.<sup>5</sup> mm thickness, Si(Li) detectors of 2 and 3 mm thickness, and a CsI detector of 1.3 cm diam $\times$ 1.3 cm depth. The CsI detector was used at beam energies above 30 MeV to observe the protons scattered at forward angles, since their range in silicon exceeded the 5 mm obtainable by stacking the  $Si(Li)$  detectors. A 2.5 cm  $\times$  2.5 cm NaI detector was permanently mounted at 20° as a monitor. The ratio of the number of counts in the monitor to the integrated current in the Faraday cup was monitored throughout the experiments, and the small number of runs that deviated by significantly more than the statistical error expected in the ratio were discarded.



FIG. 1. A typical experimental arrangement. The setup shown corresponds to that used at  $E_b = 57$  MeV.

The detector collimators consisted of "infinitely" high front slits about 20 cm from the target center, and circular rear apertures directly in front of the detectors and about 30 cm from the target center. The midth of the front slits and diameter of the rear apertures were chosen between 1.6 and 4.8 mm, depending on the order of magnitude of the cross section over the angular range being investigated. The solid angle subtended was known to  $3\%$  with the smallest slits, and to better accuracy with larger ones. The mean angle subtended in the horizontal plane was about 1.5 to  $4.0^{\circ}$  full width at half-maximum. The central value of the scattering angle was known to  $\pm 0.5^\circ$ .

# III. RESULTS

Differential cross sections were obtained from the data by using the leading term of the expression derived by Silverstein.<sup>5</sup> Corrections due to the higher order terms and to the finite diameter of the beam were found to be less than  $0.1\%$ . Corrections for dead time losses were determined from "real time" and "live time" clocks in the electronics and mere typically on the order of 1.5%.

Corrections for counting losses due to proton induced reactions in the silicon detectors were



FIG. 2.  ${}^{3}$ He $(p, p)$ <sup>3</sup>He elastic scattering cross section at energies of 19.8, 35,0, and 48.5 MeV. Note the change in curvature in the angular region between 30' c.m. and 110' c.m. , and the broadened minimum as energy increases. Both indicate increasingly important  $l = 2$  and  $l = 3$  components, especially in the singlet states, at energies beyond 35 MeV, the cut-off energy of the phase shift analysis.

determined from Makino and <code>co-workers $^{\rm 6.7}$ </code> and Cahill *et al.*<sup>8</sup> No corrections for reaction losses were applied to protons observed with the CsI detector. Losses in NaI are  $3\%$  for 50 MeV protons,<sup>9</sup> and losses in CsI can reasonably be expected to be similar. At present, no measurements for CsI are available.

Other sources of error include a  $1\%$  uncertainty in the integrated current, including quadrupole capture considerations, 0.7% in the gas density, and uncertainties due to counting statistics and 'spectrum analysis. The total error of  $\frac{2}{3}$  of the data points lies between 2 and  $4\%$  and exceeds  $6\%$ for less than  $10\%$  of the data. The results and their errors are available upon request. Figure <sup>2</sup> shows typical experimental data obtained in the energy range studied.

# IV. DISCUSSION OF RESULTS

In the energy range covered by this experiment a few similar measurements have been reported. These include works at 19.4 by Vanetsian and These include works at 19.4 by Vanetsian and<br>Fedchenko,<sup>10</sup> at 19.480 MeV by Hutson *et al*.,<sup>11</sup> and  $31$  MeV by Him  ${\it et~al.},^{12}$  at  $30.6$  and  $49.5$  MeV V<br><sup>12</sup> and 31 MeV by Him *et al*.,<sup>12</sup> at 30.6 and 49.5 N<br>by Harbison *et al*.,<sup>13</sup> and at 55 MeV by Hendri  $et~al.^{14}$ 

The early measurement by Vanetsian<sup>10</sup> required large  $l$  values for a good fit and was replaced by our early 19.8 MeV results in the phase shift analysis of Baker  $et al.$ <sup>1</sup> These results are in good ysis of Baker *et al*.<sup>1</sup> These results are in good<br>agreement with the most recent data by Hutson.<sup>11</sup>

Our data at 30.0 MeV show an excellent agreement with early works by Kim at 31 MeV and Harbison at 30.6 MeV.

Harbison's data at 49.<sup>5</sup> MeV appear about 15% larger than ours at 48.5 MeV, especially in the range  $70^{\circ}$  to  $140^{\circ}$ , around the cross section minimum.

Hendrie's 55 MeV data appear consistent with our 57 MeV data in magnitude and shape, with the  $57$  MeV data falling slightly below the  $55$  MeV data, as expected. Drawing a smooth curve through the minimum values of the cross section between 30 and 57 MeV, one finds that our data and Hendrie's are compatible, while Harbison's lies above the curve. Thus, the difference appears to be systematic in nature and not related to a possible displacement of our 48.5 MeV data set.

## V. PHASE SHIFT ANALYSIS

A complete phase shift analysis including S, P, two unsplit *D* wave phases, and one coupling pa-<br>rameter was initially done by Tombrello.<sup>15</sup> In h rameter was initially done by Tombrello.<sup>15</sup> In his work Tombrello used differential cross section and proton polarization data up to 11.<sup>5</sup> MeV. His study resulted in two solutions  $(T1$  and  $T2)$ , one of which was favored on physical grounds. This allowed the determination of four excited  $T=1$ states in the 'Li nucleus.

Other studies have been done in this energy region motivated by the need for clarifying the order and strengths of these  $T=1$  states. A detailed re-<br>view has been given by Fiarman and Meverhof.<sup>16</sup> view has been given by Fiarman and Meyerhof.

At around 20 MeV phase shift sets were generated by Morales and Cahill<sup>17</sup> and by Baker et  $al.$ <sup>1</sup> These phases resemble extrapolations of lower energy solutions by Morrow and Haeberli<sup>18</sup> and<br>Tombrello, <sup>15</sup> and McSherry and Baker.<sup>25</sup> energy solutions by Morrow and Haebe<br>Tombrello,<sup>15</sup> and McSherry and Baker. ombrello,<sup>15</sup> and McSherry and Baker.<sup>25</sup><br>Harbison *et al*.<sup>13</sup> in a phase shift reduction of

differential cross section and proton polarization at only two energies, 30.6 and 49.<sup>5</sup> MeV, produced five different sets. These solutions while giving good fits for the experimental data, present large fluctuations in the phase shift values.

Most recently, the Grenoble group, Darves-'Blanc  ${\it et\ al.},^{19}$  have done a comparative study or the elastic scattering of protons by  ${}^{3}$ He and  ${}^{3}$ H at 19.4 and 30.5 MeV. Differential cross sections and proton polarizations at these energies were phase shift analyzed. Their solutions are very and proton polarizations at these energies were<br>phase shift analyzed. Their solutions are very<br>similar to those reported earlier by us<sup>17,20,21</sup> for these energies, Both of these groups used basically the same code used in this study, though without the complex phases (see below).

A theoretical study by Reichstein et  $al.$ <sup>4</sup> using the resonating-group method, generated a set of unsplit real phase shifts between 0 and 40 MeV. Their predictions allowed a good fit with Kim's 31 MeV data on differential cross section. These phases are similar to some of Harbison's solutions.

The present phase shift analysis started with the earlier solution at  $19.4 \text{ MeV}$ .<sup>1</sup> It is felt that at this energy, a reliable solution was found due to the availability of an extensive data set. Any choice of initial real phases always searched to a unique solution, widely separated in  $\chi^2$  from all alternative solutions. In addition to the measured differential cross sections for these energies, proton polarization data were included in the analysis. polarization data were included in the analysis.<br>Tivol's<sup>22</sup> and Harbison's<sup>23</sup> proton polarization measurements permitted interpolation of data at many angles between energies of 20 and 30 MeV, with extrapolations up to 35 MeV, These extrapolated data are of larger uncertainty than the interpolated ones.

No attempt was made to extend the phase shifts beyond 35 MeV because of uncertainty in polarization values and the increasing number of partial waves.

A computer program initially written by one of us  $(T.A.C.)$  was extended to include up to  $G$  waves, both singlet and triplet, including all spin coupling

$\delta_{s l j}$ $E_p$ (MeV)	18.0	20.0	22.5	25.0	27.5	30.0	35.0
$l = 0: \delta_{000}$ $\delta_{101}$	$-93.9$ $-108.7$	$-117.5$ $-110.5$	$-121.2$ $-113.8$	$-131.7$	$-140.7$	$-138.2$	$-155.4$
				$-115.8$	$-118.0$	$-116.8$	$-118.2$
$l = 1: \delta_{011}$	68.3	46.2	53.6	50.3	30.2	35.2	27.7
$\delta_{110}$	40.8	42.8	44.7	35.9	39.7	38.9	38.9
$\delta_{111}$	32.4	28.1	27.0	25.6	25.8	29.2	26.2
$\delta_{112}$	63.1	64.1	60.5	59.1	60.4	60.6	56.2
$\epsilon_1$	$-30.2$	$-37.4$	$-46.1$	$-41.9$	$-26.8$	$-55.8$	$-23.2$
$\delta_{022}$ $l=2$ :	$-1.0$	$-3.5$	0.4	0.2	4.0	5.8	15.7
$\delta_{121}$	$-0.3$	3.6	2.2	1.0	$-0.9$	$-1.1$	$-2.5$
$\delta_{122}$	3.6	3.0	3.9	3.7	4.3	10.9	9.6
$\delta_{123}$	3.9	6.1	5.8	5.5	6.4	6.7	9.8
$\epsilon$ <sub>2</sub>	$-0.3$	$-10.6$	$-28.7$	$-29.2$	$-37.7$	$-41.6$	$-43.1$
$T_{02}$	1.8	$-4.2$	$-6.0$	$-2.5$	$-5.9$	$-7.0$	$-6.8$
$l = 3: \delta_{033}$	1.9	3.1	6.4	4.2	1.6	6.7	11.9
$\delta_{132}$	3.3	2.3	2.0	1.9	2.4	1.1	$-1.8$
$\delta_{133}$	1.1	0.3	1.4	2.3	4.3	6.0	9.4
$\delta_{134}$	3.2	3.5	3.5	3.9	4.4	5.1	4.4
$\epsilon_3$	fixed at $0^{\circ}$						
$T_{13}$	$-6.4$	$-5.4$	$-1.0$	1.0	$-8.3$	$-2,1$	$-9.8$
$x^2/pt \sigma(\theta)$	1.87	1.21	1.45	1.08	1.69	1.50	1.30
$p(\theta)$	1.75	1.22	1,18	1.48	0.37	0.40	1.30
Total	1.82	1.21	1.39	1.17	1.37	1.10	1.30

TABLE I. Phase shift solutions for  ${}^{3}$ He(p, p)<sup>3</sup>He between 18.0 and 35.0 MeV. The <sup>1</sup>P<sub>1</sub> and  ${}^{3}P_{1}$  phase shifts are ordered so that the coupling constant is zero at  $E_{p}=0$ .

terms, and complex  $S$ ,  $P$ , and  $D$  waves. These extensions were judged necessary because of the energy range covered. However, the number of parameters increased to 34. Since it turned out that  $l = 4$  waves were small at most of these energies, it was decided to consider only a singlet G wave term.

In order to speed up the program, since it makes use of a grid search method, a parabolic interpolation was introduced. Each phase is searched in turn, moving the initial value in a fixed amount in the direction of decreasing  $\chi^2$ . When a point is reached that makes  $\chi^2$  increase, a parabola is traced and its minimum is quoted as the partial phase value. During this search the other phases remain unchanged. After this minimum is found, the search continues with the next phase until all the parameters have been scanned. A new complete search is started again, this time with a smaller search interval. This method allows a relatively fast way of approaching the real minimum in refined steps. This program, in an earlirelatively fast way of approaching the real min<br>mum in refined steps. This program, in an ea<br>erform, was used in several earlier studies.<sup>1,13</sup>'

Once a solution had been obtained at one energy, it was then used as the starting point for the next higher energy. In this way, the solution at 19.4 MeV was extended to 35.0 MeV. The solutions thus obtained were stable against small arbitrary

displacements of the real phase shifts. The coupling parameters and imaginary parts of the phase shifts were not stable against such displacements, and many similar families of solutions exist for these variables in the vicinity of the preferred solution.

In Table I, the phase shifts and  $\chi^2$  values obtained for the energy range studied have been summarized. In Figs. 3 to 8, the phase shifts are plotted as a function of energy. The results of other analyses and the predictions of Reichstein et  $al.$ <sup>4</sup> are included for comparison. Solution I of Haeberli and Morrow<sup>18</sup> was also close to the others below 12 MeV.

# VI. DISCUSSION OF THE PHASE SHIFTS

In this section, the results of the phase shift analysis are examined in terms of the consistency of each result with values at higher and lower energies, the correlation of a least squares fit extrapolation to 12 MeV with earlier results at that energy, and the relationship of the values to Reichstein's predictions.

## A, S-wave phase shifts

S-wave phase shifts by and large continue to exhibit the hard sphere behavior found at lower en-



FIG. 3. The singlet  $l = 1$ ,  $j = 1$  phase shift versus energy. The notations used are the spectroscopic notation and  $\delta_{s}^{j}$ . The open circles are the results of Tombrello (Ref. 15) solution 1, the triangles that of McSherry and Baker (Ref. 25), and the solid square that of Baker et al. (Ref. 1). The open squares are the present work. The dashed line is a least squares fit to a straight line to the present data, while the solid line is the prediction of Reichstein et  $al$ . (Ref. 4). These notations are the same for all succeeding figures.

ergies. However, beyond a, proton energy of 20 MeV, smooth deviations from hard sphere behavior are seen, especially in the  ${}^{3}S_{1}$  phase shift, in a direction Such as to make the 8-wave interaction less repulsive than that predicted by the hard sphere assumption. This may be due to the increasing importance of inelastic channels that draw upon  $S$ -wave strength, a view that is qualitatively confirmed by the fact that the most important complex terms found in the search were in the S waves.

### B. P-wave phase shifts

The four known "states" of <sup>4</sup>Li are exhibited by resonant structure in all four P-wave phase shifts



FIG. 5. The triplet  $l=1$ ,  $j=1$  phase shift versus energy. The notations are as in Fig. 3.



FIG. 4. The triplet  $l=1$ ,  $j=0$  phase shift versus energy. The notations are as in Fig. 3.

at proton energies below 12 MeV. No further resonant structure was found in this study up to incident proton energies of 35 MeV, and probably up to 48.5 MeV.

 ${}^{1}P$ . The  $l = 1$ ,  $j = 1$ , singlet phase shift decreases in this energy region, but the standard deviation of a. least squares fit to a straight line is about  $6^\circ$ , or 15% of the value of the phase shift. The extrapolation to  $E_b = 12$  MeV lies considerably above most solutions, and the slope is much greater than that of other P-wave phase shifts. The solution of Baker *et al.* at  $19.8 \text{ MeV}$ ,<sup>1</sup> the starting point for the fits described in this work, lies on a reasonable extrapolation from lower energy solutions. Heichstein's prediction fits the trend of the lower energy solutions as well as the value of Baker et al. If the present  ${}^{1}P_{1}$  solution is indeed erroneous, as these other indicators imply, a possible reason might lie in the insensitivity of this phase shift to polarization data and the coupling of  ${}^{1}P_1$  and  ${}^{3}P_1$  via the constant  $\epsilon_1$ . This constant was highly insensitive to the existing data



FIG. 6. The triplet  $l=1$ ,  $j=2$  phase shift versus energy. The notations are as in Fig. 3.



FIG. 7. The singlet  $l=2$ ,  $j=2$  phase shift versus energy. The notations are as in Fig. 3.

set, and in fact the large spin-correlation parameter set of Baker  $et al$ , was barely adequate to constrain this parameter. The existence of polarized 'He experiments at about 25 to 30 MeV may improve this problem, but probably polarized beam-polarized target data will be required.

 ${}^{3}P_{0}$ . The triplet  $l = 1$ ,  $j = 0$ , phase shift decreases smoothly with energy. The least squares fit to a, straight line gives a standard deviation of  $\pm 2.3^{\circ}$  or about  $\pm 6\%$  of the value of the phase shift. Extrapolation of the fit to  $E_b = 12$  MeV gives a result that appears to agree with solutions at lower energy, confirming the location of the  $0<sup>-</sup>$  state at the higher of its two possible energies. The solution of Reichstein lies about 10' above the extrapolation to 12 MeV, but decreases smoothly to a, good match at 35 MeV.

 ${}^{3}P_{1}$ . The triplet  $l = 1$ ,  $j = 1$ , phase shift decrease smoothly from 18 to 35 MeV. A least squares fit to the points by a straight line results in a standard deviation of  $\pm 2^{\circ}$  or about  $\pm 7\%$  of the value of the phase shift. Extrapolating the fit to  $E_b = 12$ MeV gives a value of  $+30^{\circ}$  for this phase shift in good agreement with the value obtained in the solutions Tombrello, Haeberli and Morrow, and McSherry and Baker. Beichstein's prediction lies about  $20\%$  above the fit at that energy but is very similar in trend.

 ${}^{3}P_{2}$ . The  $l$  = 1,  $j$  = 2 phase shift decreases smooth ly with energy, and the standard deviation of a least squares fit to a straight line is  $\pm 1.3^{\circ}$ , or only about  $\pm 2\%$  of the value of the phase shift. The extrapolation to  $E_p = 12$  MeV fits very well values at lower energies. Reichstein's solution is similar in shape but lies low by about  $17^\circ$  at all energies.

#### C. D-wave phase shifts

The  $l = 2$  phase shifts obtained in earlier studies up to  $E_p = 12$  MeV indicated a strongly negative



FIG. 8. The singlet  $l=3$ ,  $j=3$  phase shift versus energy. The notations are as in Fig. 3.

singlet  $D$  phase shift and small triplet  $D$  phase shifts. No resonant behavior is inferred from the present results.

 $L_{D_2}$ . The  $l = 2$ ,  $j = 2$  singlet phase shift rises sharply from a slightly negative value at 18 MeV to a positive value of  $+16^{\circ}$  at 35 MeV. The standard deviation of the least squares fit to a straight line was about  $\pm 1^{\circ}$ . Extrapolation to  $E_p = 12$  MeV results in a value of about  $-11^{\circ}$ , which does not appear to allow a smooth fit to the earlier solutions which lie at about  $-16^\circ$ . The latter solutions are considerably more repulsive than a hard sphere value. The prediction of Reichstein agrees qualitatively with both sets of results, going negative between 2 and 20 MeV and then becoming positive around 20 MeV. Reichstein's solution multiplied by a constant factor of about 2.5 would provide a reasonable fit for energies up to 35 MeV.

 $b^3D_1$ .  $l = 2$ ,  $j = 1$  triplet phase shift is everywhere small. and becomes negative about 25 MeV. It bears little resemblance to Reichstein's predictions, but its extrapolation to 12 MeV could be consistent with the solution of McSherry and Baker.

 ${}^3D_2$ . The  $l = 2$ ,  $j = 2$  triplet phase shift is positive and steadily increasing between 18 and 35 MeV. Extrapolation to 12 MeV gives a phase shift close to 0', which is in reasonable agreement with the solution of McSherry and Baker, and the unsplit phases of Tombrello 1. It also bears striking similarity to the solution of Reichstein.

 ${}^3D_3$ . The  $l = 2$ ,  $j = 3$  triplet phase shift is very similar to the  $l = 2$ ,  $j = 2$  triplet phase shift, and all previous comments apply to it also.

# D. F-wave phase shifts

No results are available at lower energies.  $F_3$ . The singlet  $l = 3$ ,  $j = 3$  phase shift is positive and rises from about  $+2^{\circ}$  at 18 MeV to  $+12^{\circ}$  at 35 MeV. It is in good agreement with Beichstein's

solution.

 $11\,$ 

 ${}^3F_2$ ,  ${}^3F_3$ ,  ${}^3F_4$ . The triplet  $l=3$ ,  $j=2$  phase shift behaves very similarly to the triplet  $l = 2$ ,  $j = 1$ phase shift, in that it is generally small and tends to negative values at 35 MeV. This behavior is at variance with the solution of Reichstein, and quite different from the triplet  $j = 3$  and  $j = 4$  phase shifts which generally agree well with the predictions. Also it should be noted that by multiplying the  $l = 2$ phase shifts by about 0.8, one arrives at a reasonable fit to the  $l = 3$  phase shifts. At 35 MeV, Reichstein predicts a factor of about 0.85 for  $unsplit$  D and  $F$  waves, but the agreement state by state is remarkable. One possibility might involve some artifact of the search code, since the phases were always searched in the order: (1) singlet; (2) triplet,  $j = l - 1$ ; (3)  $j = l$ ; and (4)  $j = l + 1$ . However, displacement of the phase to an arbitrary value displacement of the phase to an arbitrary value<br>(i.e., the  $J = l - 1$  phase to the  $J = l$  value), followe by a search in which the  $J = l - 1$  phase was not allowed to vary while all other parameters were varied, resulted in significantly poorer  $\chi^2$  values (often by a factor of 2). When released and researched, the  $J=l-1$  phase soon returned to its original value. The  $J = l$  coupling parameter for  $l = 2$  and  $l = 3$  states had no significant effect on the  $\chi^2$  values, and thus were not fixed by these searches (as expected).

The conclusion could be drawn that these searches isolated the case of antiparallel spins as a. special problem in a resonating group formulation for the 4Li nucleus.

#### VII. CONCLUSIONS

This work is an attempt to generate information on <sup>4</sup>Li despite severe limitations in the <sup>3</sup>He(p, p)- ${}^{3}$ He data set above 20 MeV. Thus, an obvious conclusion is that improvement in the data set would be very beneficial, especially if the improvement involves data generated from polarized 'He nuclei. Otherwise, little can be done to fix the singlettriplet coupling parameter.

Regarding the phase shifts themselves, no surprises are revealed in the  $S$  and  $P$  waves. These phases appear to be quite consistent with solutions at lower energy, with the exception of the  ${}^{1}P_{1}$ phase shift which, along its coupling parameter  $\epsilon_1$ , appears to wander away from the well constrained solution at 19.4 MeV. No hint of further resonant behavior is contained in the P waves, as a straight line provides a good fit to their variations with energy. In the case of the S waves, the deviations from predictions based upon the hard sphere assumption are greatest for the  ${}^{3}S$ , phase shift. If the deviations are due to increasing inelasticity, then information on the reaction mechanism is contained therein. The  $D$  waves generally bear a striking resemblance to the resonating group predictions of Reichstein, Thompson, and Tang.<sup>4</sup> The negative values between  $E_b = 2$  and 20 MeV followed by positive values beyond 20 MeV are in qualitative agreement with the  ${}^{1}D_{2}$  phase shift as determined between 0 and 12 MeV and beyond 18 MeV. The magnitude is, however, only about  $40\%$  of what would be required for a reasonable fit, and the trend of the low energy solutions does not appear to be in good accord with the values beyond 18 MeV. The  ${}^3D_2$  and  ${}^3D_3$  phase shifts are in good agreement, while the  ${}^{3}D_1$  phase shift is not. This same pattern holds for the  $F$  waves, with the  ${}^{3}F_{2}$  phase shift trending slightly negative.

In no case is there strong or unambiguous evidence for resonant structure in the  $D$  or  $F$  waves. Several phase shifts, especially the  ${}^1D_2$  are going quite positive by 35 MeV, more so than predictions, but we do not wish to use such minimal evidence to make a statement about high-lying structure in 4Li. From the trend of the data at energies above 35 MeV, including the polarization data, one can predict and qualitatively confirm via phase shift analyses that the  ${}^{1}D_{2}$  and  ${}^{1}F_{3}$ phase shifts become increasingly important, more so than the triplet  $l=2$  and  $l=3$  phase shifts. Reichstein's predictions and the present work both indicate that little strength is contained in  $l = 4$  and higher phase shifts, even at  $E<sub>b</sub> = 50$  MeV.

#### ACKNOWLEDGMENTS

The authors wish to express their thanks to the Crocker Nuclear Laboratory staff for providing continual support for this program. J. R. M. and H. W. wish to express their appreciation of hospitality and assistance offered by members of the Crocker NUclear Laboratory and the Physics Department of the University of California at Davis during their visits. Financial support is acknowledged to the Associated Western Universities (H. Willmes) and to Ford Foundation's Universidad de Chile-University of California Cooperative Program and to Facultad de Ciencias, Universidad de Chile (J. R. Morales).

- \*Work supported in part by the U. S. Atomic Energy Commission and the National Science Foundation.
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- <sup>1</sup>S. D. Baker, T. A. Cahill, P. Catillon, J. Durand, and D. Garretta, Nucl. Phys. A160, 428 (1971).
- ${}^{2}P$ . Kramer and M. Moshinsky, Phys. Lett. 23, 574 (1966).
- ${}^{3}P.$  P. Szydlik, Phys. Rev. C  $1$ , 146 (1970).
- <sup>4</sup>I. Reichstein, D. R. Thompson, and Y. C. Tang, Phys. Rev. C 3, 2139 (1971).
- ${}^{5}E.$  A. Silverstein, Nucl. Instrum. Methods 4, 53 (1959).
- ${}^6$ M. Makino, C. N. Waddell, and R. M. Eisberg, Nucl. Instrum. Methods 60, 109 (1968).
- ${}^{7}$ M. Makino, R. Eisberg, K. Richie, R. Carlson, and C. N. Waddell, Nuel. Instrum. Methods 80, 299 (1970).
- T. A. Cahill, F. P. Brady, S. Corbett, W. Hammontree, K. Isaacs, and E. Young, Nucl. Instrum. Methods 87,  $151(1970)$ .
- $^{9}D.$  F. Measday and C. Richard-Serre, Nucl. Instrum. Methods 76, 1 (1969).
- $^{10}$ R. A. Vanetsian and E. D. Fedchenko, At. Energ.
- (USSR) 2, <sup>123</sup> (1857) [Sev. J. At, Energy 2, 141 (1957)].  ${}^{11}$ R. L. Hutson, N. Jarmie, J. L. Detch, Jr., and J. H.
- Jett, Phys. Rev. C 4, 17 (1871). <sup>12</sup>C. C. Kim, S. D. Bunch, D. W. Devins, and H. Forster,
- Nucl. Phys. 58, 32 (1964).
- <sup>13</sup>S. A. Harbison, R. J. Griffiths, N. M. Stewart, A. R.

Johnston, and G. T. A. Squier, Nucl. Phys. A150, 570 (1970).

- $^{14}$ D. L. Hendrie, B. G. Harvey, O. N. Jarvis, J. Mahoney, and W. Tivol, UCRL Report No. 17299, 1969 (unpublished) .
- $^{15}$ T. A. Tombrello, Phys. Rev. 138, B40 (1965).
- $16S$ . Fiarman and W. E. Meyerhof, Nucl. Phys. A206, 1  $(1973)$ .
- $^{17}$ J. R. Morales and T. A. Cahill, Bull. Am. Phys. Soc. 14, 554 (1e69).
- $^{18}$ L. W. Morrow and W. Haeberli, Nucl. Phys. A126, 225 (1969); in Proceedings of the Symposium on Few Body Problems, Light Nuclei and Nuclear Interactions (Gordon and Breach, New York, 1969), p. 459.
- <sup>19</sup>R. Darves-Blanc, N. Van Sen, J. Arvieux, J. C. Condrand, A. Fiore, and C. Perrin, in Proceedings of the International Conference on Few Particle Problems in the Nuclear Interaction, edited by I. Slaus, S. Moszkowski, R. Haddock, and W. Van Oers (North-Holland. , Amsterdam, 1972), p. 617.
- $^{20}$ J. R. Morales, Ph.D. thesis, University of California, Davis, 1970.
- $^{21}$ J. R. Morales and T. A. Cahill, Bull. Am. Phys. Soc. 16, 541 (1971).
- $22\overline{W}$ . F. Tivol, Ph.D. thesis, University of California, Berkeley (unpublished) [UCRL Report No. 18137, 1968 (unpublished)].
- 3S. A. Harbison, F. G. Kingston, R. J. Griffiths, A. R. Johnston, and J. H. P. C. Megraw, Nucl. Phys. A112, 137 (1968).
- $24$ For more detail, see T. A. Cahill, J. Greenwood, H. Willmes, and D. J. Shadoan, Phys. Rev. C 4, 1499  $(1971).$
- $^{25}D.$  H. McSherry and S. D. Baker, Phys. Rev. C 1, 888  $(1970)$ .