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Volume integrals of the absorptive part of the proton optical potentials

D. C. Agrawal and P. C. Sood
Nuclear Physics Section, Physics Department, Banaras Hindu University, Varanasi 221005, India

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The volume integrals of the absorptive part of proton optical potentials are evaluated for various combinations of the form (surface and/or volume), depth, and geometry of the absorptive potential which have been found to give comparable fits to the data for different targets at a particular energy, and also at different energies. It is concluded that the total (sum of the surface and volume absorptions) volume integral per particle of the absorptive part of the proton optical potential is a well-defined quantity in that it is practically independent of (a) the form (surface and/or volume absorption) and the geometrical parameters employed, (b) the mass number of the target nucleus at each energy, and (c) the incident proton energy in the range 10–60 MeV. Examination of the presently available results yields a value 115 ± 15 MeV fm^3 for this quantity. The energy independence of these volume integrals is in contrast with the marked energy dependence observed for the volume integrals of the real part of the optical potentials.

[NUCLEAR REACTIONS Optical model analyses of proton scattering data, studied volume integral of the absorptive potential; $E_p = 9.8\text{–}61.4$ MeV; target $A = 40\text{–}208$.]

The presence of various ambiguities inherent in multiparameter search procedures and interrelationship between various potential parameters does not permit an unambiguous quantitative description of the optical model potentials. In order to overcome this difficulty Feshbach[1] suggested in 1958 employing the volume integrals of the potential

$$J = \int V(r)d\vec{r}$$

which provide a better measure of the strength of the potential because of the inclusion of the contributions from the well depth as well as the geometry. The volume integral per particle (J_S/A) of the real central potential $V_S(r)$ was found[2] to be nearly independent of the mass number of the target nucleus in an earlier analysis of the proton scattering data. Later extensive analysis of the proton elastic scattering data by Greenlees and collaborators[3] at various energies employing the reformulated optical model and more recent examination of their data as well as those of conventional optical model analyses results by present authors[4] have established the well-defined character of J_S/A as also of the volume integrals of the spin-isospin-independent and isospin-dependent parts of the real potential. It was also shown[4] that these volume integrals (except for those of the isospin-dependent part) decrease linearly with the energy of the incident proton, this behavior being very similar to the energy dependence observed for the well depth of the real central potential.[5] However, no investigation of the volume integral of the absorptive part of proton optical potentials has been reported so far. It has been suggested[5] based on phenomenological analyses that the depth of the absorptive part of the optical potential increases with the increase of the energy of the incident particle; this behavior is physically expected from the argument that, with the increase in energy, more inelastic channels get opened. It remains to be seen whether this energy dependence is reflected in a similar dependence of the corresponding volume integrals. Here we report the results of an investigation of these volume integrals.

The absorptive part of the optical potential is found[5] to be surface-peaked for low energy, surface plus volume form for medium energy, and only volume form for high energy incident parti-

TABLE I. Various combinations of the surface and the volume absorptive forms and the respective geometry of the optical potentials which give comparable fits to the 30 MeV proton scattering data from the nucleus ^{60}Ni. The volume integral for each form and the total volume integral (sum of the surface and volume absorptions) per particle are also listed. Units for W are MeV, for r and a (fm) $=10^{-13}$ cm, and for J's MeV fm^3.

W_D	W_v	$r_D=r_v$	$a_D=a_v$	J_{W_D}/A	J_{W_v}/A	J_{W_T}/A $=(J_{W_D}+J_{W_v})/A$
7.50	0.00	1.144	0.767	106	0	106
7.00	0.00	1.25	0.70	105	0	105
6.10	2.60	1.237	0.689	88	25	113
5.30	2.70	1.25	0.70	79	27	106
0.00	5.40	1.659	0.556	0	111	111
0.00	11.76	1.25	0.70	0	116	116

TABLE II. The total volume integral per particle in MeV fm^3 of the absorptive part of the optical potential for individual targets at $E_p = 30$ MeV.

Nucleus	^{40}Ca	^{56}Fe	^{58}Ni	^{59}Co	^{60}Ni	^{63}Cu	^{120}Sn	^{208}Pb
J_{W_T}/A	108	107	100	114	110	107	112	108

cles. The volume and surface-peaked forms are usually taken to be Woods-Saxon and Woods-Saxon derivative types which are, respectively, given by the relations

$$W_v(r) = W_v\left[1+\exp\left(\frac{r-R_v}{a_v}\right)\right]^{-1} \tag{1}$$

and

$$W_D(r) = 4W_D \exp\left(\frac{r-R_D}{a_D}\right)\left[1+\exp\left(\frac{r-R_D}{a_D}\right)\right]^{-2} . \tag{2}$$

Because of the presence of various ambiguities and interrelationship between the various potential parameters, different combinations of the form, depth, and geometry of the absorptive potential have been generally found to give comparable fits to the experimental data in various optical model analyses. Following the suggestion of Feshbach mentioned above we investigate the volume integrals of the absorptive part of proton optical potential.

The volume integrals of the potentials (1) and (2) are, respectively, given by

$$J_{W_v} = \frac{4\pi}{3} R_v{}^3 W_v[1+(\pi a_v/R_v)^2] \tag{3}$$

and

$$J_{W_D} = 16\pi R_D{}^2 a_D W_D[1+\tfrac{1}{3}(\pi a_D/R_D)^2] \ . \tag{4}$$

For our investigation we have included the results of practically every available optical model analysis of proton scattering data wherein one or more parameters of the absorptive part of the potential was searched; the incident proton energy for such cases lies in the range 9.8 to 61.4 MeV and the target mass numbers range from 40 to 208.

First we examine the variations of the volume integrals of the absorptive potential with respect to the various combinations of the form, the depth and the geometry which give comparable fits to the scattering data for a particular nucleus at a specified energy. As a typical illustration we present in Table I the result of the optical model

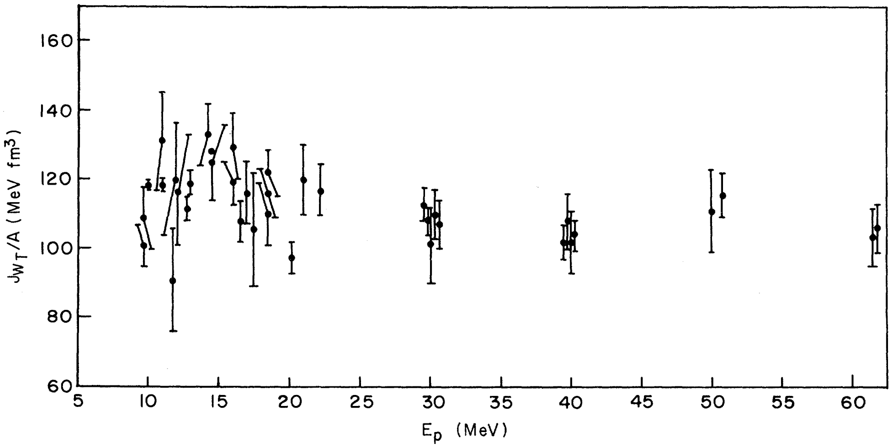

FIG. 1. Plot of the average total volume integral per particle J_{W_T}/A of the absorptive part of proton optical potential as a function of the incident proton energy E_p. The uncertainties shown in the figure correspond to the spread in the average values for that particular optical model analysis.

TABLE III. The average total volume integral per particle and the rms deviation from the average of the absorptive part of proton optical potentials are listed for specified energies based on the parameters given in the cited reference.

E_p (MeV)	Mass region	Number of nuclei	J_{W_T}/A	Reference
9.8	112–124	4	109 ± 9	Durisch and Gould, Phys. Rev. 137, B906 (1965)
9.8	54–120	7	101 ± 6	Greenlees *et al.*, Phys. Rev. C 3, 1231 (1971)
10.0	60–65	3	118 ± 1	Marinov *et al.*, Phys. Rev. 145, 852 (1966)
11.0	60–65	3	118 ± 1	Marinov *et al.*, Phys. Rev. 145, 852 (1966)
11.0	48–68	16	131 ± 14	Perey *et al.*, Phys. Rev. 175, 1460 (1968)
11.7	56–64	5	91 ± 15	Benveniste *et al.*, Phys. Rev. 133, B323 (1964)
12.0	48–65	4	117 ± 16	Perey, Phys. Rev. 131, 745 (1963)
12.0	42–65	9	120 ± 16	Marinov *et al.*, Phys. Rev. 145, 852 (1966)
12.7	90–94	3	111 ± 4	Dickens *et al.*, Phys. Rev. 168, 1355 (1968)
13.0	106–108	2	119 ± 4	Robinson *et al.*, Phys. Rev. 146, 816 (1966)
14.3	48–65	5	133 ± 9	Perey, Phys. Rev. 131, 745 (1963)
14.5	70–76	4	129 ± 0	Curtis *et al.*, Phys. Rev. C 1, 1418 (1970)
14.5	54–120	14	125 ± 11	Pyle and Greenlees, Phys. Rev. 181, 1444 (1969)
16.0	110–130	13	129 ± 10	Makofske *et al.*, Phys. Rev. 174, 1429 (1968)
16.0	116–124	4	119 ± 6	Boyd *et al.*, Nucl. Phys. A162, 497 (1971)
16.5	52–62	3	108 ± 6	Kossanyi-Demay *et al.*, Nucl. Phys. A94, 513 (1967)
17.0	56–197	11	116 ± 9	Perey, Phys. Rev. 131, 745 (1963)
17.6	56–115	10	106 ± 16	Baugh *et al.*, Nucl. Phys. 83, 481 (1966)
18.6	48–64	10	110 ± 9	Kossanyi-Demay *et al.*, Nucl. Phys. A94, 513 (1967)
18.6	48–92	10	122 ± 7	Kossanyi-Demay and de Swiniarski, Nucl. Phys. A108, 577 (1968)
18.6	48–64	10	116 ± 7	Glashausser *et al.*, Phys. Rev. 164, 1437 (1967)
20.3	90–92	3	98 ± 4	Glashausser *et al.*, Phys. Rev. 184, 1217 (1969)
21.0	40–116	3	120 ± 10	Baron *et al.*, Phys. Rev. 180, 978 (1969)
22.2	51–197	10	117 ± 7	Perey, Phys. Rev. 131, 745 (1963)
30.0	40–208	8	108 ± 4	Greenlees and Pyle, Phys. Rev. 149, 836 (1966)
30.0	40–208	3	102 ± 11	Watson *et al.*, Nucl. Phys. A92, 193 (1967)
30.0	59–208	4	113 ± 5	Satchler, Nucl. Phys. A92, 273 (1967)

TABLE III *(Continued)*

E_p (MeV)	Mass region	Number of nuclei	J_{W_T}/A	Reference
30.3	58–208	3	107 ± 7	Greenlees *et al.*, Phys. Rev. C 2, 1063 (1970)
30.3	40–59	3	110 ± 7	Hnizdo *et al.*, Phys. Rev. C 3, 1560 (1971)
39.6	58–124	12	102 ± 5	Liers *et al.*, Phys. Rev. C 2, 1399 (1970)
40.0	54–208	6	104 ± 4	Frick and Satchler, Phys. Rev. 139, B567 (1965)
40.0	40–208	4	108 ± 8	Blumberg *et al.*, Phys. Rev. 147, 812 (1966)
40.0	40–208	9	102 ± 9	Frick *et al.*, Phys. Rev. 156, 1207 (1967)
50.0	42–208	19	111 ± 11	Mani *et al.*, Nucl. Phys. A165, 384 (1971)
50.8	144–154	5	115 ± 6	Woollam *et al.*, Nucl. Phys. A179, 657 (1972)
61.4	40–208	6	106 ± 7	Boyd and Greenlees, Phys. Rev. 176, 1394 (1968)
61.4	58–208	5	103 ± 8	Fulmer *et al.*, Phys. Rev. 181, 1565 (1969)

analysis[6] for 30 MeV protons scattered from the ^{60}Ni target. The volume integral per particle separately for each form and also their sum (J_{W_T}/A) are listed in the table. It is seen that the total volume integral J_{W_T}/A is practically constant with a value 110 ± 4 MeV fm^3, independent of the relative composition of the absorptive potential and the variety of the geometrical parameters employed.

The dependence of the total volume integral of the absorptive potential on the mass number of the target nucleus at a given energy is illustrated in Table II from the same 30 MeV proton data analysis. An average value of 108 ± 4 MeV fm^3 is obtained for the eight target nuclei in the mass range $A = 40$ to 208. Similar characteristics are observed from the data analyses at other energies as well leading to the conclusion that J_{W_T}/A is a well-defined quantity independent of the mass number of the target nucleus as well.

We examine the variation of the total volume integral of the absorptive potential with the energy of the incident protons. The average total volume integral and the root mean square deviation from the average value are calculated for each specific incident energy using the optical model analyses results available in the literature; these quantities are listed in Table III, and are plotted in Fig. 1 as a function of the incident proton energy. We find that, over the energy range investigated, J_{W_T}/A comes out to be 115 ± 15 MeV fm^3 and is thus practically independent of the energy of the incident proton also. This energy independence for the volume integrals of the absorptive part is entirely unexpected in view of the general belief, mentioned earlier, that due to opening up of more inelastic channels the absorptive potential should get deeper with the increase in incident energy. Further, these conclusions are quite in contrast with the results obtained earlier[4] for the refractive part of the optical potential. Whereas the volume integrals for the latter were seen to exhibit approximately the same relative energy dependence as observed for the real well depths, here we find the volume integrals of the absorptive part to hardly vary with energy.

It is concluded that the total (sum of the surface and volume absorptions) volume integral per particle of the absorptive part of the proton optical potential is a well-defined quantity in that it is practically independent of (a) the form (surface and/or volume absorption) and the geometrical parameters employed, (b) the mass number of the target nucleus at each energy, and (c) the incident proton energy in the range 10–60 MeV.

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[1] H. Feshbach, Annu. Rev. Nucl. Sci. 8, 49 (1958).

[2] P. G. McManigal, R. D. Eandi, S. N. Kaplan, and B. J. Moyer, Phys. Rev. 137, B620 (1965).

[3] G. W. Greenlees, G. J. Pyle, and Y. C. Tang, Phys. Rev. 171, 1115 (1968); R. N. Boyd and G. W. Greenlees, *ibid.* 176, 1394 (1968); G. J. Pyle and G. W. Greenlees, *ibid.* 181, 1444 (1969); G. W. Greenlees, W. Makofske, and G. J. Pyle, Phys. Rev. C 1, 1145 (1970); G. W. Greenlees, V. Hnizdo, O. Karbon, J. Lowe, and W. Makofske, *ibid.* 2, 1063 (1970); H. S. Liers, R. N. Boyd, C. H. Poppe, J. A. Sievers, and D. L. Watson, *ibid.* 2, 1399 (1970); G. W. Greenlees, C. H. Poppe, J. A. Sievers, and D. L. Watson, *ibid.* 3, 1231 (1971); V. Hnizdo, O. Karbon, J. Lowe, G. W. Greenlees, and W. Makofske, *ibid.* 3, 1560 (1971).

[4] P. C. Sood and D. C. Agrawal, Indian J. Pure and Appl. Phys. 11, 571 (1971); D. C. Agrawal and P. C. Sood, Nucl. Phys. Solid State Phys. (India) 15B, 167 (1973); P. C. Sood and D. C. Agrawal, in *Nuclear Structure Study with Neutrons*, edited by J. Erö and J. Szücs (Plenum, New York, 1974); D. C. Agrawal, Ph.D. thesis, Banaras Hindu University, 1974 (unpublished).

[5] A. E. S. Green, T. Sawada, and D. S. Saxon, *The Nuclear Independent Particle Model* (Academic, New York, 1968).

[6] G. W. Greenlees and G. J. Pyle, Phys. Rev. 149, 836 (1966).