Elastic electron-deuteron scattering as a probe of the deuteron wave function*

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Relativistic corrections to the electric deuteron form factors are rederived within the framework of particle quantum mechanics. In addition to the kinematic corrections there are necessarily dynamic corrections that are substantially larger. For local potentials the results are identical to those derived by Gross from field theory. The reasons for the agreement of two apparently very different theories are examined in detail. The quantitative aspect of these results are illustrated by several numerical examples. A precise measurement of the polarization cross section for $q^2 < 13$ fm^{-2} could serve to significantly restrict the shape of admissible deuteron wave functions.

[NUCLEAR STRUCTURE Calculation ^{2}H electric form factors, dependence on N-N potential, relativistic effects important.]

I. INTRODUCTION

It is well known that nucleon-nucleon scattering data do not determine the potential.[1] Since different phase shift equivalent potentials may lead to different deuteron wave functions, an experimental probe of the short range part of the deuteron wave function may provide additional constraints on acceptable potentials. High energy electron-deuteron scattering would seem to provide a suitable probe. In the one-photon-exchange approximation the differential cross section for elastic scattering in the laboratory frame is given by the Rosenbluth formula[2,3]

$$d\sigma = d\sigma_{\text{Mott}}[A(q^2) + B(q^2)\tan^2\tfrac{1}{2}\theta] \ , \tag{1.1}$$

where $d\sigma_{\text{Mott}}$ is the Mott cross section. The coefficients A and B are invariant functions of the invariant momentum transfer $q^2 = q^\mu q_\mu$. By definition they are related to matrix elements of the charge density $\rho(x)$ and the current density $\vec{j}(x)$ evaluated in the Breit frame in which $q^2 = |\vec{q}|^2$, that is

$$A = \tfrac{1}{3} \sum_{\mu,\mu'} \{ |\langle \tfrac{1}{2}\vec{q}, \mu' | \rho(0) | -\tfrac{1}{2}\vec{q}, \mu \rangle|^2 + \tfrac{1}{2} |\langle \tfrac{1}{2}\vec{q}, \mu' | \vec{j}(0) | \mu, -\tfrac{1}{2}\vec{q} \rangle|^2 \} \tag{1.2}$$

and

$$B = \tfrac{1}{3}(1+\eta) \sum_{\mu,\mu'} |\langle \tfrac{1}{2}\vec{q}, \mu' | \vec{j}(0) | -\tfrac{1}{2}\vec{q}, \mu \rangle|^2 \ , \tag{1.3}$$

where $\mu = 0, \pm 1$ is the third component of the deuteron spin and η is defined by

$$\eta = q^2/4M_D^2; \tag{1.4}$$

M_D is the deuteron mass.

The charge-density matrix elements can be expressed in terms of two form factors (reduced matrix elements) G_0 and G_2. For $\vec{q} = (0, 0, q)$

$$\langle \tfrac{1}{2}\vec{q}, \mu' | \rho(0) | -\tfrac{1}{2}\vec{q}, \mu \rangle = G_0 \delta_{\mu',\mu} + (-1)^\mu (1\,1\,\mu' - \mu |\, 2\,0)\sqrt{3}\, G_2 \ , \tag{1.5}$$

where $(1\,1\,\mu' - \mu |\, 2\,0)$ is a Clebsch-Gordan coefficient. Similarly, the current density is expressed in terms of the form factor G_1 by

$$\langle \tfrac{1}{2}\vec{q}, \mu' | \vec{j}(0) | -\tfrac{1}{2}\vec{q}, \mu \rangle = i \langle \mu' | \vec{S}\times\vec{q} | \mu \rangle \, G_1/2M_D \ , \tag{1.6}$$

where $\vec{S}$ is the spin of the deuteron. It follows that

$$\langle \tfrac{1}{2}\vec{q}, \mu' | \tfrac{1}{2}(j_1 \pm i j_2) | -\tfrac{1}{2}\vec{q}, \mu \rangle = (1\,1\,\mu' - \mu |\, 1\,\pm 1)\eta^{1/2} G_1 . \tag{1.7}$$

From Eqs. (1.2), (1.3), (1.5), and (1.7) it follows that

$$A = G_0^2 + G_2^2 + \tfrac{2}{3}\eta G_1^2 \tag{1.8}$$

and

$$B = \tfrac{4}{3}\eta(1+\eta) G_1^2 \ . \tag{1.9}$$

Since

$$\langle \tfrac{1}{2}\vec{q}, 0 | \rho(0) | -\tfrac{1}{2}\vec{q}, 0 \rangle = G_0 + \sqrt{2}\, G_2 \tag{1.10}$$

and

$$\langle \tfrac{1}{2}\vec{q}, \pm 1 | \rho(0) | -\tfrac{1}{2}\vec{q}, \pm 1 \rangle = G_0 - G_2/\sqrt{2} \ , \tag{1.11}$$

it follows that the differential cross sections $d\sigma_0$ and $d\sigma_1$ for scattering by helicity 0 and helicity 1 deuterons are given by

$$d\sigma_0 = d\sigma_{\text{Mott}} (G_0 + \sqrt{2}\, G_2)^2 \tag{1.12}$$

and

$$d\sigma_1 = d\sigma_{\text{Mott}}(G_0 - G_2/\sqrt{2})^2 . \quad (1.13)$$

The polarization P has been defined by[4,5]

$$P = \sqrt{2}\,(d\sigma_0 - d\sigma_1)/(d\sigma_0 + 2d\sigma_1)$$
$$= (2G_0G_2 + G_2^2/\sqrt{2})/(G_0^2 + G_2^2) . \quad (1.14)$$

Since the form factors are functionals of the deuteron wave function, measurement of the quantities A, B, and P determines some features of this wave function. Unfortunately, the relation between the form factors and the wave function is not as simple as one might expect at first sight. Even if the wave function is known exactly, the current operator is not. Whenever the potential is nonlocal the continuity equation

$$i[H, \rho(\vec{x})] = -\nabla \cdot \vec{j}(\vec{x}) \quad (1.15)$$

requires the introduction of interaction terms in the current density $\vec{j}$.[6,7] Lorentz covariance of the charge-current density, $\{\rho(x), \vec{j}(x)\} = \{j^\mu(x)\}$, also requires interaction terms. This is easily seen as follows: we must have

$$U^\dagger(\Lambda) j^\mu(x) U(\Lambda) = \Lambda^\mu{}_\nu j^\nu(\Lambda^{-1}x) , \quad (1.16)$$

where $U(\Lambda)$ is the unitary operator representing the Lorentz transformation Λ. For infinitesimal Lorentz transformations it follows from Eq. (1.16) that[8]

$$i[\vec{K}, \rho(\vec{x})] = -\vec{j}(\vec{x}) + i[H, \rho(\vec{x})]\vec{x} \quad (1.17)$$

and

$$i[K_i, j_k(\vec{x})] = -\delta_{ik}\rho(\vec{x}) + i[H, j_k(\vec{x})]x_i . \quad (1.18)$$

In a relativistic theory the interaction modifies both the generator H of time displacements and the generator $\vec{K}$ of proper Lorentz transformations. Therefore, the covariance relations (1.17) and (1.18) require, in general, interaction terms in the charge-current density.

In a strictly phenomenological theory it is possible to satisfy the continuity equation (1.15) by assuming exchange currents that do not contribute to the deuteron form factors.[9] We will make that assumption in the following since the present investigation is restricted to two-nucleon deuteron models without explicit inclusion of meson or antinucleon degrees of freedom. Adler and Drell,[10] Casper,[11] and Blankenbecler and Gunion[12] have calculated meson exchange contributions to the form factors. Arenhövel and Miller[13] found substantial contributions from isobar admixtures. The effect of the Adler-Drell corrections on the coefficient A is negligible for $q^2 < 10$ fm^{-2} and becomes large for $q^2 \gtrsim 16$ fm^{-2}.[14,15] Formulas for the relativistic corrections to the form factors have been derived from field theory by Gross.[16] The effect is not negligible. For $q^2 = 10$ fm^{-2} it decreases A by 15 to 20%.[14] Friar[17] has derived relativistic corrections on a phenomenological basis. His correction terms are identical to some of the terms derived by Gross, but the terms that Friar does not have give the bulk of the correction for $q^2 \lesssim 12$ fm^{-2}. For $q^2 = 10$ fm^{-2} Friar's corrections are typically about $\frac{1}{4}$ of those of Gross. We will show that the charge-current density implied by Friar's formula does not satisfy the covariance conditions (1.17) and (1.18).

Several authors have found that the form factors are rather insensitive to short range variations in the deuteron wave function. Peltola, Laurikainen, and Kouki[18] and Elias *et al.*[14] tested several conventional potentials and found that all were in agreement with available data. Haftel and Tabakin[19] and Vary[20] produced many examples of more violent changes in the wave functions by short range unitary transformations of the Reid soft core wave functions. Only one wave function gave form factors in clear disagreement with experiment. In that case the wave function was modified over a longer range ($r > 2$ fm). Violent variations of the wave functions restricted to $r < 1$ fm produced no appreciable effect.[20] Haftel and Tabakin and Vary did not compute relativistic effects. The conclusions to be drawn from their examples thus remain in doubt.

The electric form factors G_0 and G_2 are proportional to the isoscalar nucleon form factor

$$G_{ES} = \tfrac{1}{2}(G_{Ep} + G_{En}) . \quad (1.19)$$

In view of the insensitivity of the deuteron form factors to reasonable changes in the deuteron wave function, elastic electron-deuteron scattering has been advocated as a means of measuring G_{En}.[21,22] This approach assumes that one has ruled out more exotic wave functions on other grounds. A semiphenomenological determination of G_{En} that is independent of the deuteron wave function[23] could be important in this context.

Moravcsik and Gosh[24] and Brady, Tomusiak, and Levinger[25] have advocated the measurement of the polarization P as a probe of the short-range part of the wave function. The polarization P is a function of the ratio G_2/G_0 and is thus independent of G_{En} except for small relativistic effects. In the region of q^2 where G_0 vanishes the ratio is quite sensitive to short range variations of the wave function. However, Adler's meson exchange effects[10] are also large in that region. Even for lower q^2 these effects are likely to be significant since the corrections to G_0 and G_2 are of opposite sign. Thus they add up in P while they cancel partially in A.

In view of the quantitative importance of the relativistic corrections we have attempted to answer the following question. Does it make sense to apply the field theoretic formulas of Gross to purely phenomenological deuteron wave functions? With this question in mind we give a new phenomenological derivation in Sec. II. The covariance conditions (1.17) and (1.18) require the presence of dynamical corrections that are, for local potentials, in complete agreement with those derived by Gross.[16] In order to illuminate this remarkable agreement we review in Sec. III the field theoretic derivation and its relation to the phenomenological theory. Meson exchange and nucleon isobar effects are outside the scope of the present paper.

In Sec. IV we present the results of representative numerical calculations for the Reid potentials, for the Hamada-Johnston potential, and for wave functions obtained from the Reid wave functions by short range unitary transformations.

II. RELATIVISTIC DEUTERON WAVE FUNCTIONS AND ELECTRIC FORM FACTORS

It is well known[26-30] that Lorentz invariance and relativistic kinematics can be realized for two particles in interaction without the intervention of local fields. The heuristic starting point is the description of one particle by an irreducible unitary representation of the Poincaré group. States are represented by square integrable functions $\chi(\vec{p}, \mu)$ of the momentum $\vec{p}$ and the spin component $-s \leq \mu \leq s$. The normalization condition is

$$\|\chi\|^2 = \int d^3p \sum_{\mu=-s}^{+s} |\chi(\vec{p}, \mu)|^2 = 1 . \tag{2.1}$$

The generators of the infinitesimal Poincaré transformations are space translations $\vec{P}$,

$$\vec{P}\chi(\vec{p}, \mu) = \vec{p}\chi(\vec{p}, \mu); \tag{2.2}$$

time translations H,

$$H\chi(\vec{p}, \mu) = (\vec{p}^2 + \mu^2)^{1/2}\chi(\vec{p}, \mu); \tag{2.3}$$

rotations $\vec{j}$,

$$\vec{J}\chi(\vec{p}, \mu) = \vec{x}\times\vec{p}\chi(\vec{p}, \mu) + \sum_{\mu'} \langle \mu | \vec{s} | \mu' \rangle \chi(\vec{p}, \mu'), \tag{2.4}$$

where $\vec{x} = i\nabla_p$; proper Lorentz transformations $\vec{K}$,

$$\vec{K} = \tfrac{1}{2}(\vec{x}H + H\vec{x}) - (\vec{s}\times\vec{P})/(M + H) . \tag{2.5}$$

For two free particles the Hilbert space $\mathcal{H}$ of the states is, as usual, the tensor product of the spaces $\mathcal{H}_1$ and $\mathcal{H}_2$ of the two particles, i.e.,

$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 . \tag{2.6}$$

The generators are additive in the two particles:

$$\vec{K} = \vec{K}_1 \otimes 1 + 1 \otimes \vec{K}_2, \text{ etc.} \tag{2.7}$$

If an interaction is introduced by modifying the Hamiltonian, the generators $\vec{K}$ must also be modified in order to preserve the commutation relations

$$[H, \vec{K}] = -i\vec{P} \tag{2.8}$$

and

$$[P_i, K_j] = -i\delta_{ij}H . \tag{2.9}$$

In the reducible representation (2.7) such a consistent modification is difficult to construct and has only been given in approximate form.[31-36] On the other hand it is easy to introduce the interaction if we first reduce the reducible representation (2.7) to a direct integral of irreducible representations.[26] The main problem is then to find the correct expression for the charge-current density in the new representation in the presence of a particle interaction.

The reduction is accomplished by a canonical transformation that separates dynamical variables describing the center-of-mass motion, and intrinsic coordinates. The center-of-mass variables are defined as functions of the Poincaré generators as follows: The total momentum is the generator of translations. The center-of-mass position $\vec{X}$ is[37]

$$\vec{X} = \tfrac{1}{2}(H^{-1}\vec{K} + \vec{K}H^{-1}) - [Hh(H+h)]^{-1}\vec{P}\times\vec{W}, \tag{2.10}$$

where

$$h = (H^2 - \vec{P}^2)^{1/2} \tag{2.11}$$

and

$$\vec{W} = H\vec{J} + \vec{P}\times\vec{K} . \tag{2.12}$$

The total spin $\vec{I}$ is defined by

$$\vec{I} = \vec{J} - \vec{X}\times\vec{P} . \tag{2.13}$$

It follows from these definitions that

$$[P_j, X_k] = -i\delta_{jk} , \tag{2.14}$$

and $\vec{I}$ commutes with both $\vec{X}$ and $\vec{P}$. The intrinsic canonical variables, yet to be found, must commute with both $\vec{X}$ and $\vec{P}$.

Let $\{P, E\}$ be the total four-momentum of the two particles

$$\vec{P} = \vec{p}_1 + \vec{p}_2 , \tag{2.15}$$

$$E = (\vec{p}_1^{\,2} + M^2)^{1/2} + (\vec{p}_2^{\,2} + M^2)^{1/2} , \tag{2.16}$$

and

$$\omega = (E^2 - \vec{P}^2)^{1/2} . \tag{2.17}$$

The key to the definition of the intrinsic variables is the Lorentz transformation L_P that transforms $\{\vec{P}, E\}$ into $\{0, 0, 0, \omega\}$.[29] The intrinsic relative

momentum $\vec{k}$ is then defined by

$$L_P\{\vec{p}_1, E_1\} = \{\vec{k}, w\} , \tag{2.18}$$

where

$$E_1 = (\vec{p}_1^{\,2} + M^2)^{1/2} \tag{2.19}$$

and

$$w = (\vec{k}^2 + M^2)^{1/2} = \tfrac{1}{2}\omega . \tag{2.20}$$

Two-particle states may be represented by square integrable functions $\chi(\vec{P}, \vec{k}, S, \mu)$, where S and μ are channel spin variables. The channel spin $\vec{S}$ is related[38] to the individual spins $\vec{s}_1$ and $\vec{s}_2$ by Wigner rotations[39] $\mathcal{R}(\vec{p}_1, L_P)$ and $\mathcal{R}(\vec{p}_2, L_P)$:

$$\vec{S} = \mathcal{R}(\vec{p}_1, L_P)\vec{s}_1 + \mathcal{R}(\vec{p}_2, L_P)\vec{s}_2 . \tag{2.21}$$

The Poincaré generators are then given in this representation by

$$H\chi(\vec{P}, \vec{k}, S, \mu) = (\vec{P}^2 + \omega^2)^{1/2}\chi(\vec{P}, \vec{k}, S, \mu) , \tag{2.22}$$

$$\vec{J}\chi(\vec{P}, \vec{k}, S, \mu) = [\vec{X}\times\vec{P} + \vec{r}\times\vec{k}]\chi(\vec{P}, \vec{k}, S, \mu) + \sum_{\mu'}\langle \mu|\vec{S}|\mu'\rangle \chi(\vec{P}, \vec{k}, S, \mu'), \tag{2.23}$$

where $\vec{X} = i\nabla_p$ and $\vec{r} = i\nabla_k$. Finally, we have

$$\vec{K} = \tfrac{1}{2}(\vec{X}H + H\vec{X}) - \vec{I}\times\vec{P}/(H+\omega) , \tag{2.24}$$

where the total spin $\vec{I}$ is

$$\vec{I} = \vec{r}\times\vec{k} + \vec{S} . \tag{2.25}$$

Following Bakamjian and Thomas,[26] an interaction may be introduced without destroying the group structure by the substitution $\omega \to h = \omega + v$, so that

$$h\chi(\vec{P}, \vec{k}, S, \mu) = 2(k^2 + M^2)^{1/2}\chi(\vec{P}, \vec{k}, S, \mu) + \int d^3k' \sum_{S'}\sum_{\mu'} \langle \vec{k}, S, \mu|\, v\,|\, k', S', \mu'\rangle \chi(\vec{P}, \vec{k}', S', \mu') , \tag{2.26}$$

where the interaction v must commute with both $\vec{X}$ and $\vec{P}$ and must be invariant under rotations, i.e.,

$$[\vec{I}, v] = 0 . \tag{2.27}$$

Obviously, the change of representation from $(\vec{p}_1, \vec{p}_2, \mu_1, \mu_2)$ to $(\vec{P}, \vec{k}, S, \mu)$ is given by a unitary matrix $\langle \vec{P}, \vec{k}, S, \mu|\, U_0\,|\, \vec{p}_1, \vec{p}_2, \mu_1, \mu_2\rangle$ that can be inverted to reintroduce formal individual particle coordinates even in the presence of the interaction v. It is important, however, to guard against unstated arbitrary assumptions about the observable significance of the so defined variables.

The relativistic wave function of a deuteron with definite momentum $\vec{P}_0$ and spin component μ_0 has the form

$$\Psi_{\vec{P}_0, \mu_0}(\vec{P}, \vec{k}, \mu) = \delta(\vec{P} - \vec{P}_0)\varphi_{\mu_0}(\vec{k}, \mu) , \tag{2.28}$$

where φ_{μ_0} is an eigenfunction of h, $\vec{I}^2$, and I_3. It is then also an eigenfunction of h^2 and thus satisfies the equation

$$(\vec{k}^2/M + V)\varphi = -\varphi\alpha^2/M , \tag{2.29}$$

where

$$\alpha^2 = M^2 - \tfrac{1}{4}M_D^2 ,$$

and

$$V = (\omega v + v\omega + v^2)/4M . \tag{2.30}$$

We may therefore identify the operator V with the usual phenomenological nucleon-nucleon potential.[40]

The transformation to the coordinate representation and the partial wave decomposition are as usual

$$\psi_{\mu_0}(\vec{r}, \mu) = (2\pi)^{-3/2}\int d^3k\, e^{i\vec{k}\cdot\vec{r}}\,\varphi_{\mu_0}(\vec{k}, \mu) \tag{2.31}$$

and

$$\psi_{\mu_0}(\vec{r}, \mu) = \sum_{L, m} r^{-1}u_L(r)(1\, L\, \mu m|\, 1\, \mu_0)Y_{Lm}(\hat{r}) . \tag{2.32}$$

Since h^2 is by construction a positive self-adjoint operator the equation

$$h\varphi = \varphi M_D \tag{2.33}$$

is a consequence of Eq. (2.29), and the existence of the operator v satisfying Eq. (2.30) follows from the existence of the spectral resolution of h^2.

No approximations have been made so far and nothing can be said about observations of the intrinsic coordinates $\vec{r}$. For deuteron it is reasonable to assume that $(\vec{k}/M)^2$ as well as $(\vec{P}/2M)^2$ is small and we will use this assumption in calculating the matrix elements of the charge density $\rho(\vec{x})$. We should keep in mind, however, that for exotic deuteron wave functions with violent short range variations an expansion in powers of $(\vec{k}/M)^2$ may not be legitimate. To order M^{-2} the trans-

formation matrix U_0 is given by[41]

$$\langle \vec{P},\vec{k},S,\mu|\, U_0|\, \vec{p}_1,\vec{p}_2,\mu_1,\mu_2\rangle \cong \int d^3P' \int d^3k' \sum_{S',\mu'} \langle \vec{P},\vec{k},S,\mu|\, 1+i\Phi_0|\, \vec{P}',\vec{k}',S',\mu'\rangle\, \delta(\vec{P}'-\vec{p}_1-\vec{p}_2) \times \delta[\vec{k}'-\tfrac{1}{2}(\vec{p}_1-\vec{p}_2)]\langle \tfrac{1}{2}\tfrac{1}{2}\,\mu_1\,\mu_2|\, S'\,\mu'\rangle\,, \tag{2.34}$$

where

$$\Phi_0=(\vec{s}_1-\vec{s}_2)\cdot(\vec{P}\times\vec{k})/4M^2-\tfrac{1}{4}\{(\vec{P}\cdot\vec{r}),\ (\vec{P}\cdot\vec{k})\}/4M^2\,. \tag{2.35}$$

In order to determine the electric deuteron form factors we need according to Eq. (1.5) the matrix element

$$\langle \tfrac{1}{2}\vec{q},\mu_0'|\,\rho_D(0)|\,-\tfrac{1}{2}\vec{q},\mu_0\rangle=\sum_{\mu,\mu'}\int d^3k'\int d^3k\,\varphi^*_{\mu_0'}(\vec{k}',\mu')\langle \tfrac{1}{2}\vec{q},\vec{k}',1,\mu'|\,\rho(0)|\,-\tfrac{1}{2}\vec{q},\vec{k},1,\mu'\rangle\,\varphi_{\mu_0}(\vec{k},\mu)\,. \tag{2.36}$$

Thus we must determine the charge-density matrix element that appears in the integral in such a manner that charge and current density satisfy the covariance relations (1.17) and (1.18).

In the Breit frame of the proton the matrix elements of the charge and current density are, to order M^{-2}, related to the conventional form factors G_{Ep} and G_{Mp} by

$$\langle \tfrac{1}{2}\vec{q},\mu_1'|\rho_p(0)|-\tfrac{1}{2}\vec{q},\mu_1\rangle=(1-\vec{q}^2/8M^2)G_{Ep}\,\delta_{\mu_1'\mu_1} \tag{2.37}$$

and

$$\langle \tfrac{1}{2}\vec{q},\mu_1'|\vec{j}_p(0)|-\tfrac{1}{2}\vec{q},\mu_1\rangle=i\langle\mu_1'|\vec{s}_1|\mu_1\rangle\times\vec{q}G_{Mp}/M\,. \tag{2.38}$$

It follows that the charge-density matrix element in an arbitrary frame is in the same approximation

$$\langle \vec{p}_1',\mu_1'|\rho_p(0)|\vec{p}_1,\mu_1\rangle=(1-\vec{q}^2/8M^2)G_{Ep}\,\delta_{\mu_1'\mu_1}+i\langle\mu_1'|\vec{s}_1|\mu_1\rangle\cdot\vec{q}\times(\vec{p}_1'+\vec{p}_1)(G_{Mp}-\tfrac{1}{2}G_{Ep})/M^2\,, \tag{2.39}$$

where $\vec{q}=\vec{p}_1'-\vec{p}_1$. The contributions of proton and neutron to the deuteron form factors are additive. It suffices therefore to obtain the proton contribution and then replace G_p everywhere by G_p+G_n.

For two free nucleons the proton contribution to the charge density is given by

$$\rho(\vec{x})=U_0\rho_p(\vec{x})\otimes 1U_0^\dagger\,. \tag{2.40}$$

The covariance relations are satisfied because they are satisfied for a single proton and

$$U_0^\dagger\vec{K}U_0=\vec{K}_1\otimes 1+1\otimes\vec{K}_2\,. \tag{2.41}$$

When the nucleons interact we replace U_0 by a modified transformation matrix U. In the approximate form (2.34) we replace Φ_0 by $\Phi=\Phi_0+\Phi'$. To order M^{-2} we have then

$$U^\dagger\vec{K}U=\vec{K}_1\otimes 1+1\otimes\vec{K}_2+\tfrac{1}{2}\{\vec{X},V\}+2iM[\vec{X},\Phi'], \tag{2.42}$$

where $\vec{X}=\frac{1}{2}(\vec{x}_1+\vec{x}_2)$. The covariance relations are not satisfied for $\Phi'=0$. In order to satisfy them we need

$$U^\dagger\vec{K}U=\vec{K}_1\otimes 1+1\otimes\vec{K}_2+\tfrac{1}{2}\{\vec{x}_1,V\}, \tag{2.43}$$

and hence

$$\Phi'=-\tfrac{1}{4}\{(\vec{P}\cdot\vec{r}),V\}/2M\,, \tag{2.44}$$

where $\vec{r}=\vec{x}_1-\vec{x}_2$.

The deuteron charge density

$$\rho_D=(\psi,U\rho_p\otimes 1U^\dagger\psi)+(\psi,U\rho_n\otimes 1U^\dagger\psi) \tag{2.45}$$

is then conveniently written as a sum of five terms,

$$\rho_D=\rho_{D0}+\rho_{D\mathrm{so}}+\rho_{D1}+\rho_{D2}+\rho_{D3}\,, \tag{2.46}$$

where ρ_{D0} is the nonrelativistic limit, $\rho_{D\mathrm{so}}$ is the contribution arising from the spin-orbit term in Eq. (2.39), ρ_{D1} includes all other kinematic relativistic corrections to order M^{-2}, and the last two terms give the dynamic corrections involving Φ'. Explicit expressions for the terms in Eq. (2.46) are as follows:

$$\langle \tfrac{1}{2}\vec{q},\mu_0'|\rho_{D0}(0)|-\tfrac{1}{2}\vec{q},\mu_0\rangle=2G_{ES}\sum_\mu\int d^3r\,\psi^*_{\mu_0'}(\vec{r},\mu)\,e^{i\vec{q}\cdot\vec{r}/2}\psi_{\mu_0}(\vec{r},\mu)\,, \tag{2.47}$$

where $2G_{ES} = G_{Ep} + G_{En}$;

$$\langle \tfrac{1}{2}\vec{q}, \mu_0' | \rho_{D\mathrm{so}}(0) | -\tfrac{1}{2}\vec{q}, \mu_0 \rangle = (2G_{MS} - G_{ES})/M^2 \sum_{\mu,\mu'} \langle 1, \mu' | \vec{s}_1 \otimes 1 | 1, \mu \rangle \times \vec{q} \cdot \int d^3r \{ \psi^*_{\mu_0'}(\vec{r}, \mu') \nabla \psi_{\mu_0}(\vec{r}, \mu) - \psi_{\mu_0}(\vec{r}, \mu) \nabla \psi^*_{\mu_0'}(\vec{r}, \mu') \}, \tag{2.48}$$

where $2G_{MS} = G_{Mp} + G_{Mn}$;

$$\rho_{D1} = -2\eta \rho_{D0} + i(\psi, [\Phi_0, (\rho_p + \rho_n) \otimes 1] \psi), \tag{2.49}$$

$$\langle \tfrac{1}{2}\vec{q}, \mu_0' | \rho_{D1}(0) | -\tfrac{1}{2}\vec{q}, \mu_0 \rangle = -2G_{ES}\eta \sum_{\mu} \int d^3r \, \psi^*_{\mu_0'}(\vec{r}, \mu) e^{i\vec{q}\cdot\vec{r}/2} (2 - \tfrac{1}{4} i \vec{q} \cdot \vec{r}) \psi_{\mu_0}(\vec{r}, \mu), \tag{2.50}$$

$$\langle \tfrac{1}{2}\vec{q}, \mu_0' | \rho_{D2}(0) | -\tfrac{1}{2}\vec{q}, \mu_0 \rangle = -i(G_{ES}/4M)(\psi_{\mu_0'}, \{V, \tfrac{1}{2}\vec{q}\cdot\vec{r}\, e^{i\vec{q}\cdot\vec{r}/2}\} \psi_{\mu_0}), \tag{2.51}$$

$$\langle \tfrac{1}{2}\vec{q}, \mu_0' | \rho_{D3}(0) | -\tfrac{1}{2}\vec{q}, \mu_0 \rangle = -i(G_{ES}/4M)(\psi_{\mu_0'}, [\tfrac{1}{2}\vec{q}\cdot\vec{r} V e^{i\vec{q}\cdot\vec{r}/2} + e^{i\vec{q}\cdot\vec{r}/2} V \tfrac{1}{2}\vec{q}\cdot\vec{r}] \psi_{\mu_0}). \tag{2.52}$$

In the expression for ρ_{D2} we may replace $V\psi$ by $(\nabla^2/M + E)\psi$. For ρ_{D3} this is possible only if the nonrelativistic limit of the potential is strictly local, i.e., it commutes with the vector $\vec{r}$. In that case $\rho_{D3} = \rho_{D2}$. For Serber potentials, i.e., potentials that vanish for odd partial waves, ρ_{D3} vanishes.

It follows from Eq. (1.5) that the form factors G_0 and G_2 are given by the expressions

$$G_0 = 2G_{ES} C_E(q^2) + 2(2G_{MS} - G_{ES}) J_C(q^2) \tag{2.53}$$

and

$$G_2 = 2G_{ES} C_Q(q^2) + 2(2G_{MS} - G_{ES}) J_Q(q^2), \tag{2.54}$$

where J_C and J_Q are the contributions of $\rho_{D\mathrm{so}}$ and

$$C_E = C_{E0} + C_{E1} + C_{E2} + C_{E3}, \tag{2.55}$$

$$C_Q = C_{Q0} + C_{Q1} + C_{Q2} + C_{Q3}. \tag{2.56}$$

The nonrelativistic limits C_{E0} and C_{Q0} are given by the familiar expressions

$$C_{E0} = \int dr (u^2 + w^2) j_0(\tfrac{1}{2} qr) \tag{2.57}$$

and

$$C_{Q0} = \int dr \, w(u - w/\sqrt{8}) j_2(\tfrac{1}{2} qr). \tag{2.58}$$

The kinematic corrections C_{E1} and C_{Q1} are in agreement with both Gross[16] and Friar[17]:

$$C_{E1} = -2\eta C_{E0} - \eta q^2 \frac{d^2}{dq^2} C_{E0} = -\eta \int dr (u^2 + w^2)[2 j_0(\tfrac{1}{2} qr) - \tfrac{1}{4} qr \, j_1(\tfrac{1}{2} qr)] \tag{2.59}$$

and

$$C_{Q1} = -\eta C_{Q0} - \eta q^2 \frac{d^2}{dq^2} C_{Q0} = -\eta \int dr \, w(u - w/\sqrt{8})[j_2(\tfrac{1}{2} qr) + \tfrac{1}{2} qr \, j_1(\tfrac{1}{2} qr)]. \tag{2.60}$$

The dynamical corrections are absent in Friar's work since he assumes implicitly that $\Phi' = 0$. For local potentials, when $C_{E2} = C_{E3}$ and $C_{Q2} = C_{Q3}$, the dynamic corrections are in complete agreement with the field theoretic result of Gross.[16] We have

$$C_{E2} = \tfrac{1}{3}\eta \int dr (u\mathcal{K}u + w\mathcal{K}w - 6w^2)[j_0(\tfrac{1}{2} qr) + j_2(\tfrac{1}{2} qr)] \tag{2.61}$$

and

$$C_{Q2} = -\tfrac{1}{3}\eta \int dr [(u - w/\sqrt{2})\mathcal{K}w + w\mathcal{K}u - 6(u - w/\sqrt{2})w] \times [\tfrac{2}{5} j_0(\tfrac{1}{2} qr) + \tfrac{1}{7} j_2(\tfrac{1}{2} qr) - \tfrac{9}{35} j_4(\tfrac{1}{2} qr)], \tag{2.62}$$

where

$$\mathcal{K} = r^2 \left(\frac{d^2}{dr^2} - \alpha^2 \right).$$

If the potential does not commute with $\vec{r}$ then it cannot be eliminated from the remaining terms. Expressions for C_{E3} and C_{Q3} are obtained from Eqs. (2.52) and (2.32). A straightforward evaluation of the angle intervals gives

$$C_{E3} = \sum_{L_1, L_2} \sum_{L_1', L_2'} \sum_{J} F(L_1, L_2, L_1', L_2', J, 1) \tag{2.63}$$

and

$$C_{Q3} = \sum_{L_1, L_2} \sum_{L_1', L_2'} \sum_{J, l} F(L_1, L_2, L_1', L_2', J, l)[3(2l+1)]^{1/2} (1\, l\, 0\, 0 |\, 2\, 0) W(2\, 1\, 1\, J; 1\, l), \tag{2.64}$$

where

$$F=(1/4M)\int dr\int dr' u_{L_2}(r')\tfrac{1}{2}qr'\langle r', L_2'|V|r, L_1'\rangle j_l(\tfrac{1}{2}qr)u_{L_1(r)}i^{l-1}(2J+1)(2L_1+1)^{1/2}(2L_2+1)^{1/2}$$

$$\times(l\,L_1\,0\,0|\,L_1'\,0)(1\,L_2\,0\,0|\,L_2'\,0)W(l\,L_1J\,1;L_1'\,1)W(1\,L_2J\,1;L_2'\,1)\,. \tag{2.65}$$

The W's are Racah coefficients, $u_0=u$ and $u_2=w$. We get nonvanishing contributions only if l, L_1', and L_2' are odd.

The coefficients J_C and J_Q have been previously obtained by both Gross[16] and Friar.[17] We have

$$J_C=2\eta\int dr w^2[j_0(\tfrac{1}{2}qr)+j_2(\tfrac{1}{2}qr)] \tag{2.66}$$

and

$$J_Q=-\tfrac{9}{5}\int dr\,(ruw'-rwu')\frac{[j_1(\frac{1}{2}qr)+j_3(\frac{1}{2}qr)]}{\sqrt{2}\frac{1}{2}qr}+uw\frac{[3j_1(\frac{1}{2}qr)-2j_3(\frac{1}{2}qr)]}{\sqrt{2}\frac{1}{2}qr}-w^2\frac{[\frac{1}{2}j_1(\frac{1}{2}qr)-2j_3(\frac{1}{2}qr)]}{\frac{1}{2}qr}\,. \tag{2.67}$$

The leading term in the form factor G_1 is of order M^{-1}. To order M^{-2} there are no relativistic corrections.

III. DEUTERON FORM FACTORS DERIVED FROM FIELD THEORY

In order to relate the results of the last section to field theory we define nucleon creation operators $\hat{a}^\dagger(\vec{p})$ that create physical one-nucleon states when operating on the physical vacuum $|0\rangle$. (In this section we suppress the spin variables for the sake of simplifying the notation.) Let us consider the vertex function

$$\Lambda_{\vec{P}}^{\dagger}(\vec{p}_1,\vec{p}_2)=\langle 0|\hat{a}(\vec{p}_2)\hat{a}(\vec{p}_1)(H-E_1-E_2)|\vec{P}\rangle_D$$
$$=\lambda_{\vec{P}}^{\dagger}(\vec{p})\delta(\vec{p}_1+\vec{p}_2-\vec{P})\,, \tag{3.1}$$

where $|\vec{P}\rangle_D$ is the deuteron state with momentum $\vec{P}$,

$$\vec{p}=\tfrac{1}{2}(\vec{p}_1-\vec{p}_2)\,, \tag{3.2}$$

and

$$E_i=(\vec{p}_i^{\,2}+M^2)^{1/2}\,. \tag{3.3}$$

The definition (3.1) suggests the identification

$$\lambda_0(\vec{k})=v\varphi(\vec{k})\,, \tag{3.4}$$

and hence

$$\varphi(\vec{k})=[M_D-2w(\vec{k})]^{-1}\lambda_0(\vec{k})\,, \tag{3.5}$$

where

$$w(\vec{k})=(\vec{k}^2+M^2)^{1/2}\,. \tag{3.6}$$

Equation (3.5) implies the assumption that the state vector of the deuteron at rest can be adequately approximated by its projection on the two-nucleon subspace.

The charge-density matrix element for the deuteron may be written in the form

$${}_D\langle\tfrac{1}{2}\vec{q}|\rho(0)|-\tfrac{1}{2}\vec{q}\rangle_D=\int d^3p_2\int d^3p_1'\int d^3p_1\Psi_{1/2\,\vec{q}}^{*}(\vec{p}_1',\vec{p}_2)\langle\vec{p}_1'|\rho_p+\rho_n|\vec{p}_1\rangle\Psi_{-1/2\,\vec{q}}^{\dagger}(\vec{p}_1,\vec{p}_2)\,. \tag{3.7}$$

The wave function $\Psi_{\vec{P}}^{\dagger}(\vec{p}_1,\vec{p}_1)$ is related to $\varphi(\vec{k})$ by

$$\Psi_{\vec{P}}^{\dagger}(\vec{p}_1,\vec{p}_2)=[\partial(\vec{k},\vec{P})/\partial(\vec{p}_1,\vec{p}_2)]^{1/2}\varphi[\vec{k}(\vec{p}_1,\vec{p}_2)]\delta(\vec{P}-\vec{p}_1-\vec{p}_2)\,, \tag{3.8}$$

where $\partial(\vec{k},\vec{P})/\partial(\vec{p}_1,\vec{p}_2)$ is the Jacobian of the transformation

$$\vec{k}=\vec{k}(\vec{p}_1,\vec{p}_2)\,,$$
$$\vec{P}=\vec{p}_1+\vec{p}_2\,. \tag{3.9}$$

If we define $\vec{k}(\vec{p}_1,\vec{p}_2)$ by

$$L_P\{\vec{p}_2,E_2\}=\{-\vec{k},w(\vec{k})\}\,, \tag{3.10}$$

where L_P is the rotationless Lorentz transformation defined by

$$L_P\{\vec{P},E_D\}=\{0,M_D\}\,, \tag{3.11}$$

then the form factors derived from Eq. (3.7) are precisely the same as those obtained for local potentials in the last section. After expansion in powers of $\vec{P}$ to second order it follows from Eq.

(3.10) that

$$\vec{k}=\vec{p}+\tfrac{1}{4}(\vec{p}^2+\alpha^2)\vec{P}/M^2-\vec{P}(\vec{P}\cdot\vec{p})/2M_D{}^2\,. \tag{3.12}$$

The dynamic corrections arise from the second term in Eq. (3.12).

In order to relate the probability amplitude $\Psi_{\vec{P}}(\vec{p}_1,\vec{p}_2)$ to the vertex function $\Lambda_{\vec{P}}(\vec{p}_1,\vec{p}_2)$ we introduce covariant fields as follows. Let $\{f_n(x)\}$ be a complete set of positive-energy solutions of the free Dirac equation

$$\left\{\frac{\partial}{\partial t}+\vec{\alpha}\cdot\nabla+i\beta M\right\}f_n(x)=0\,, \tag{3.13}$$

i.e.,

$$f_n(x)=(2\pi)^{-3/2}\int d^3p\,u(p)e^{ipx}C_n(\vec{p})\,, \tag{3.14}$$

where

$$px=\vec{p}\cdot\vec{x}-\omega t\,, \tag{3.15}$$

and $\{C_n(\vec{p})\}$ is a complete set of square integrable functions. The spinor matrix $u(\vec{p})$ is defined by

$$u(\vec{p})=[\vec{\alpha}\cdot\vec{p}+\omega+M][2\omega(\omega+M)]^{-1}\tfrac{1}{2}(1+\beta)\,. \tag{3.16}$$

Let $\psi(x)$ be any local covariant spinor field such that the annihilation operator

$$a_n(t)=\int d^3x f_n^\dagger(x)\psi(x) \tag{3.17}$$

satisfies the relation

$$\langle 0\,|\,a_n(t)\,|\,n'\rangle=\delta_{nn'}\,. \tag{3.18}$$

If $\psi(x)$ is not a free field then $a_n^\dagger$ does not create a physical one-nucleon state, i.e., $a_n^\dagger\neq\hat{a}_n^\dagger$.[42] The creation operator $\hat{a}_n^\dagger$ may be obtained from a covariant quasilocal spinor field $\hat{\psi}(x)$ as follows[43]:

$$\hat{a}_n(t)=\int d^3x f_n^\dagger(x)\hat{\psi}(x)\,, \tag{3.19}$$

where

$$\hat{\psi}(x)=(2\pi)^{-2}\int d^4p\,e^{ipx}g(-p^2)\tilde{\psi}(p)\,, \tag{3.20}$$

$$\tilde{\psi}(p)=(2\pi)^{-2}\int d^4x\,e^{-ipx}\psi(x)\,, \tag{3.21}$$

and $g(\kappa)$ is a test function such that $g(M^2)=1$, $0\leq g(\kappa)\leq 1$, and $g(\kappa)=0$ for $|\kappa-M^2|>\frac{1}{2}M^2$.

The vertex function $\Lambda_{\vec{P}}$ may be expressed in the form

$$\Lambda_{\vec{P}}(\vec{p}_1,\vec{p}_2)=\langle\vec{p}_2\,|\,[\hat{a}(\vec{p}_1),H]-\hat{a}(\vec{p}_1)\omega_1\,|\,\vec{P}\rangle$$

$$=i\frac{\partial}{\partial t}\langle\vec{p}_2\,|\,\hat{a}(p_1,t)\,|\,\vec{P}\rangle\,. \tag{3.22}$$

From Eq. (3.19) it follows that

$$i\frac{d\hat{a}_n(t)}{dt}=\int d^3x\bar{f}_n(x)\mathfrak{D}_x\hat{\psi}(x)\,, \tag{3.23}$$

where

$$\bar{f}_n(x)=if_n^\dagger(x)\beta\,, \tag{3.24}$$

$$\mathfrak{D}_x=\gamma^\mu\partial_\mu+M\,, \tag{3.25}$$

and

$$\gamma^\mu=-i\beta\alpha^\mu,\quad \alpha^0=1\,. \tag{3.26}$$

We have thus

$$\lambda_{\vec{P}}(\vec{p})=\bar{u}(\vec{p}_1)\langle\vec{p}_2\,|\,\mathfrak{D}_x\hat{\psi}(0)\,|\,\vec{P}\rangle_D\,. \tag{3.27}$$

In this form it is easy to relate the function $\lambda_{\vec{P}}(\vec{p})$ to $\lambda_0(\vec{k})$ by virtue of the transformation properties

$$U(L_P{}^{-1})\,|\,0\rangle_D=|\,\vec{P}\rangle_D(E_D/M_D)^{1/2}\,, \tag{3.28}$$

$$U(L_P{}^{-1})\,|\,\vec{k},\mu\rangle=\sum_{\mu'=-1/2}^{+1/2}|\,\vec{p}_2,\mu'\rangle\mathfrak{R}_{\mu'\mu}(L_P{}^{-1},\vec{p}_2)\times[E_2/w(\vec{k})]^{1/2}\,, \tag{3.29}$$

and

$$U^\dagger(L_P)\mathfrak{D}_x\hat{\psi}(0)U(L_P)=S(L_P)\mathfrak{D}_x\hat{\psi}(0)\,, \tag{3.30}$$

where $S(L_P)$ is the appropriate unimodular representation of the Lorentz transformation L_P, i.e.,

$$S(L_P)=[E_D+M_D+\vec{\alpha}\cdot\vec{P}]/[2M_D(E_D+M_D)]^{1/2}\,. \tag{3.31}$$

The Wigner rotations $\mathfrak{R}$ cancel out in the end and we shall therefore ignore them in the following. To second order in M^{-1} we have

$$\bar{u}(\vec{k})S(L_P)=\bar{u}(\vec{p}_1)[E_1/w(\vec{k})]^{1/2}\,. \tag{3.32}$$

Since

$$P\cdot p_2=\vec{P}\cdot\vec{p}_2-E_DE_2=-M_Dw(\vec{k}) \tag{3.33}$$

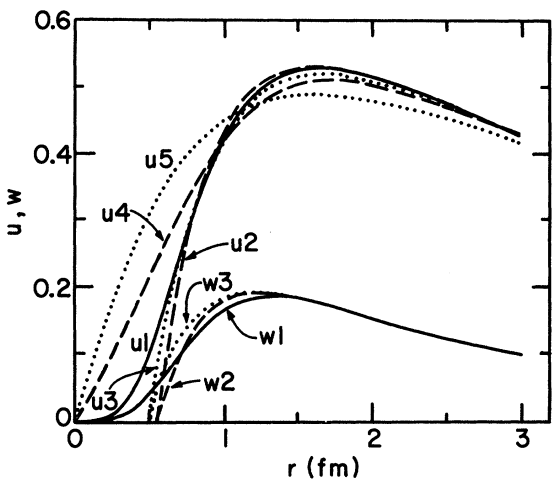

FIG. 1. "Reasonable" deuteron wave functions listed in Table I. Both S waves $u(r')$ and D waves $w(r')$ are shown.

it follows that

$$\Psi_{\vec{P}}(\vec{p}_1, \vec{p}_2) = -\{M_D/[(P-p_2)^2+M^2]\} \times [E_1 E_D/(-P\cdot p_2)]^{1/2} \lambda_{\vec{P}}(\vec{p}) \,. \quad (3.34)$$

The wave function (3.34) is equivalent to the one derived by Gross[44] to order M^{-2}.

In order to establish the relation of the wave function (3.34) to the Bethe-Salpeter formalism[45] it is necessary to use the Lehmann-Symanzik-Zimmerman (LSZ) reduction formulas[46] which require that the fields be local. It is therefore necessary to assume that $\hat{\psi}(x)$ may be approximated by a local field $\psi(x)$. This means that in the wave function the physical nucleons are approximated by point particles.

IV. NUMERICAL RESULTS

We have shown in Sec. II that the deuteron electric form factors depend, in general, not only on the deuteron wave function but also explicitly on the ^3P and $^3F_2$ potentials. The wave functions alone are sufficient to calculate the form factors if either the static limit ($M \to \infty$) of the potential is strictly local, i.e., the same in all partial waves, or if it vanishes in odd partial waves (Serber exchange). Conventional potentials such as the Reid potentials[47] and the Hamada-Johnston potential[48] are not strictly local; neither are they Serber potentials. However, the P-wave potentials are relatively small (compare Ref. 47, Figs. 3 and 4, with Fig. 11). We have computed C_{E3} and C_{Q3} for the Reid potentials and the Hamada-Johnston potential and found their contribution to the observable quantities A and P to be small.

For purposes of illustration we assume in the following that the deuteron wave functions under consideration are produced either by a strictly local potential or by a Serber potential. They will be in two classes: wave functions with reasonable shapes and wave functions with exotic short-range variations. In the first class we have the Reid[47] and Hamada-Johnston[48] (HJ) wave functions as well as wave functions that are obtained from the Reid hard-core (RHC) wave functions by a Baker transformation[49] that removes the S-wave hard core. These wave functions are shown in Fig. 1. The Baker transformation is given by

$$\tilde{u}(r) = \left(\frac{dR}{dr}\right)^{1/2} u(R(r)) \,, \quad (4.1)$$

where

$$R = r + a + 2\beta \ln\left[\frac{1+(1+\rho e^{-r/\beta})^{1/2}}{1+(1+S)^{1/2}}\right] \quad (4.2)$$

TABLE I. Summary of deuteron wave functions, D-state probabilities, and quadrupole moments.

Wave function number	Description	P_D (%)	Q
1	RSC	6.47	0.280
2	RHC	6.50	0.277
3	HJ	6.95	0.284
4	RHC + Baker transf. of $u(r)$, $\beta = 0.3$	6.50	0.276
5	RHC + Baker transf. of $u(r)$, $\beta = 0.7$	6.50	0.272
6	RSC + u-w twist $\rho = 0.8$	5.27	0.276
7	RSC + u-w twist $\rho = 1$	4.35	0.268
8	RSC + UT101	6.47	0.279

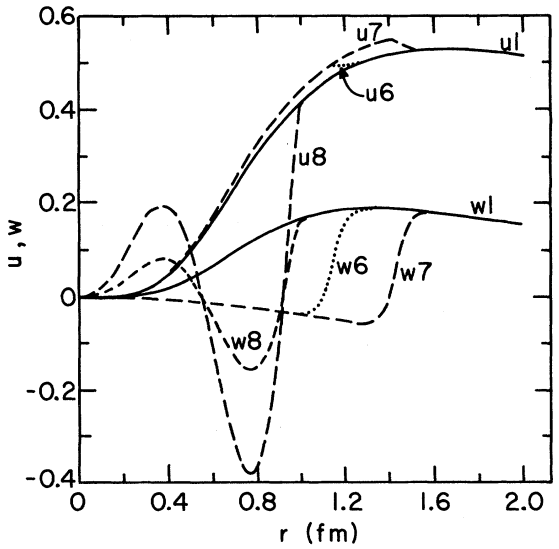

FIG. 2. Exotic deuteron wave functions listed in Table I. The solid lines show the RSC wave functions for comparison.

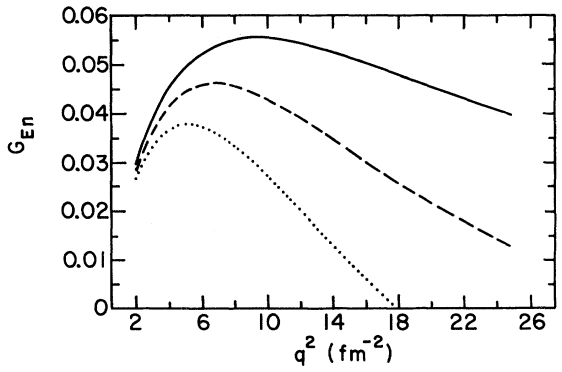

FIG. 3. Neutron form factor $G_{En}(q^2)$ according to Eq. (4.8) (solid line) and two five-parameter fits of Ref. 23. The dashed line is the dipole fit and the dotted line is the monopole fit.

and a is the hard-core radius. The parameter S is determined by the asymptotic condition

$$\lim_{r\to\infty}[R(r)-r]=0. \tag{4.3}$$

Exotic shapes are represented by the wave function UT101 given by Vary[20] and two wave functions obtained from the Reid soft-core (RSC) wave functions by a unitary transformation designed to lower the D-state probability, i.e.,

$$\tilde{u}(r)=C(r)u(r)+S(r)w(r), \tag{4.4}$$

$$\tilde{w}(r)=-S(r)u(r)+C(r)w(r), \tag{4.5}$$

where

$$S(r)=A_{\rm tr}\tanh(r/\gamma)e^{-(r-\rho)/\tau}(1+e^{-(r-\rho)/\tau})^{-1} \tag{4.6}$$

and

$$C(r)=[1-S^2(r)]^{1/2}. \tag{4.7}$$

The parameters chosen are $A_{\rm tr}=0.4472$, $\gamma=0.02$ fm, $\tau=0.02$ fm, $\rho=0.8$ fm, and $\rho=1$ fm. The wave functions are shown in Fig. 2. The D-state probabilities and quadrupole moments are summarized in Table I.

The coefficient A in the Rosenbluth cross section depends on the neutron form factor G_{En}. Figure 3 shows the q^2 dependence of G_{En} proposed by Bertozzi *et al.*,[22]

$$G_{En}=(1+0.58q^2/12)^{-2}-(1+0.7q^2/12)^{-2}, \tag{4.8}$$

and by Iachello, Jackson, and Lande.[23] In Fig. 4 we show the values of A computed from the Reid soft-core wave function for $G_{En}=0$ and for G_{En} given by Eq. (4.8). The corresponding curves for the Reid hard-core and Hamada-Johnston wave functions differ by $\frac{1}{2}\%$ or less. Figure 5 exhibits the dependence of A on different wave functions for $G_{En}=0$. A positive neutron form factor would merely shift the whole pattern in the manner shown in Fig. 4. The experimental data shown in Figs. 4 and 5 are taken from Galster *et al.*[21] and from

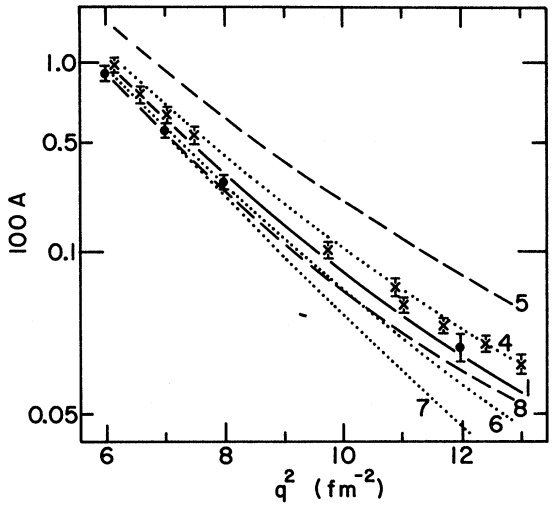

FIG. 5. Plot of $A(q^2)$ for wave functions listed in Table I. $G_{En}=0$ and Serber exchange is assumed.

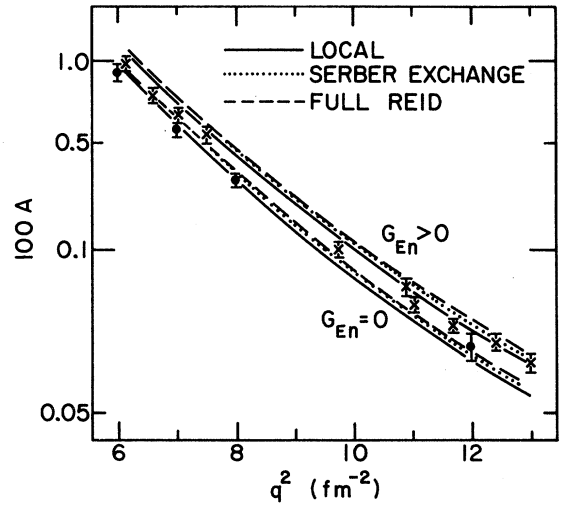

FIG. 4. Plot of $A(q^2)$ for the RSC wave function with $G_{En}=0$ and G_{En} given by Eq. (4.8) (see solid line in Fig. 3). The dashed lines are computed with the full RSC potential using Eqs. (2.63) and (2.64). The solid lines obtain if the wave function is produced by a local potential. The experimental data are taken from Refs. 21 (crosses) and 50 (dots).

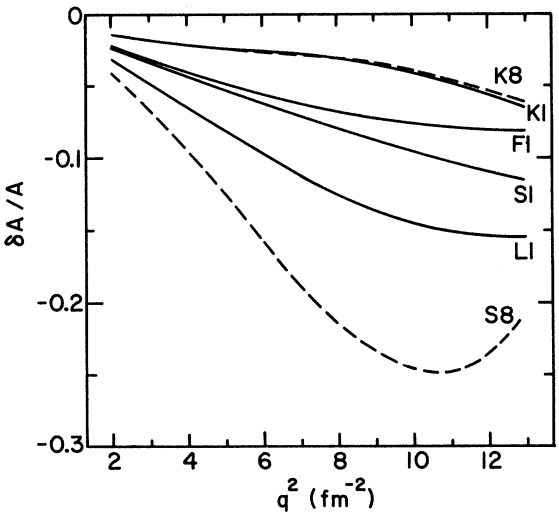

FIG. 6. Relativistic corrections to $A(q^2)$ for the wave functions 1 and 8 of Table I. Curves labeled K show the kinematic corrections only. Curves labeled S show the full correction assuming Serber exchange. The curve labeled L is obtained assuming a local potential. The curve labeled F is obtained with the full RSC potential.

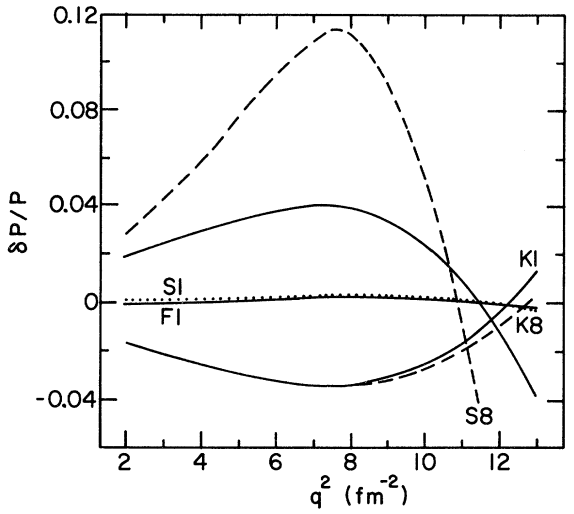

FIG. 7. Relativistic corrections to $P(q^2)$ for the wave functions 1 and 8. The curves are labeled as in Fig. 6.

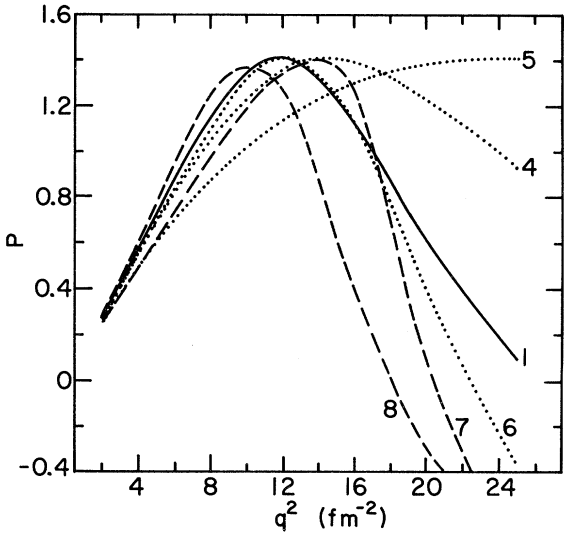

FIG. 9. Plot of $P(q^2)$ for wave functions listed in Table I and Serber exchange.

Buchanan and Yearian.[50]

The relativistic corrections with and without inclusion of the dynamic effects are illustrated in Fig. 6 by a typical example, the RSC wave function, and an extreme case, the UT101 wave function.

The polarization P depends on the neutron form factor G_{En} only through a term proportional to G_{MS}/G_{ES} in the kinematic relativistic corrections. That effect is negligible (about 0.4% for $q^2 < 13$ fm^{-2}). For $q^2 < 13$ fm^{-2} the polarization is also much less sensitive than A to the relativistic corrections. The corrections shown in Fig. 7 for the Reid soft core wave functions are typical for all cases except the wave function UT101 for which the kinematic corrections are the same but the dynamic corrections are much larger. For $q^2 < 13$ fm^{-2} there is no visible difference between the polarizations obtained from the wave functions 1–3 (Fig. 8). The polarization predicted by the other wave functions is significantly different even for $q^2 < 13$ fm^{-2} (Fig. 9). The dependence of P on the exchange character of the potential is illustrated

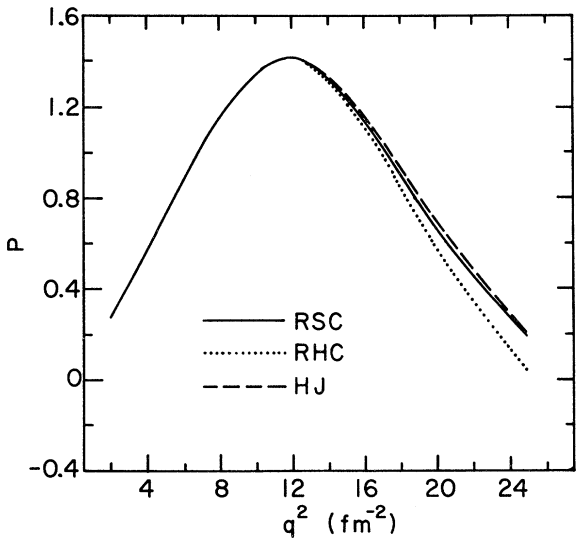

FIG. 8. Plot of $P(q^2)$ for the full potentials RSC, RHC, and HJ.

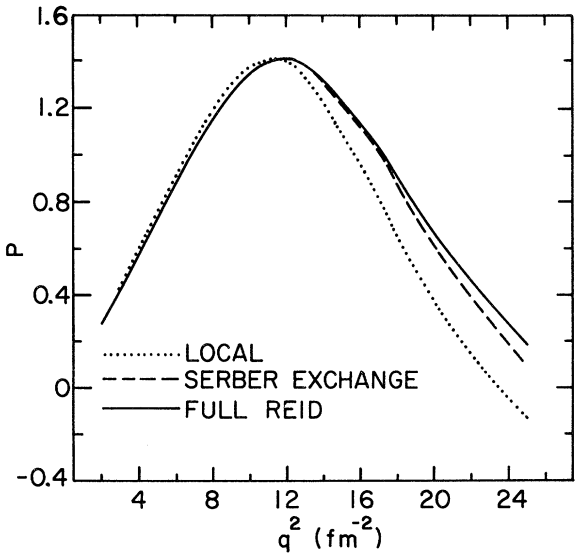

FIG. 10. Dependence of $P(q^2)$ on the exchange character of the potential.

in Fig. 10. The difference between local potentials and Serber exchange is visible even for $q^2 < 10$ fm^{-2}.

V. SUMMARY AND CONCLUSIONS

What can be learned from elastic electron-deuteron scattering about the deuteron wave function and the nucleon-nucleon interaction? The question implies the model we have assumed here: The description of the deuteron as a two-nucleon system with no other degrees of freedom. Covariance of the current density four-vector requires certain well-defined interaction terms in the charge and current density. Within the framework of a phenomenological theory we cannot rule out arbitrary additional interaction currents that are separately covariant and conserved. Such interaction currents may represent the effects of field degrees of freedom that have been eliminated. For the present study we have assumed that such effects are absent as long as the momentum transfer is not too large.

For nonlocal potentials the form factors depend not only on the deuteron wave functions but also explicitly on the ^3P and $^3F_2$ interactions. For potentials that are local in the nonrelativistic limit we have obtained the same results as those derived from field theory by assuming that the state vector of the deuteron at rest can be approximated by its projection on the two-nucleon subspace.

A precise measurement of the deuteron electric form factor can be used to determine the neutron form factor only if the shape of the deuteron wave function is sufficiently restricted by theoretical requirements. Conversely, the shape of the deuteron wave function can be restricted by such a measurement if the neutron form factor is determined by other data. The polarization cross section is practically independent of the neutron form factor. A precise measurement of P for $q^2 < 13$ fm^{-2} could serve to limit admissible deuteron wave functions considerably.

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