$E6$ transition in ^{53}Fe†

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The $\frac{19}{2}^- \rightarrow \frac{7}{2}^-$ $E6$ transition in ^{53}Fe is calculated in a model including all basis states of the configurations $(f_{7/2})^{13}_{IT}$ and $[(f_{7/2})^{12}_{I'T'} \times f_{5/2}]_{IT}$. Several interactions, which yield the observed amount of $f_{5/2}$ configuration mixing, have been considered. In all cases an effective charge $\delta \cong -0.4$ is needed to fit experiment (where $e_p = 1 + \delta$, $e_n = \delta$).

[NUCLEAR STRUCTURE ^{53}Fe; calculated $B(E6)$. Model space $(f_{7/2})^{13}$ and $(f_{7/2})^{12} f_{5/2}$.]

In this note we present theoretical results concerning the effect of weak configuration mixing on the observed[1] $E6$ transition in ^{53}Fe. The most recent measurement[2] of the mean lifetime of the $\frac{19}{2}^-$ state at 3.0406 MeV for decay to the $\frac{7}{2}^-$ ground state is

$$\tau_m = 3.7 \times 10^5 \text{ sec.}$$

The Moszkowski estimate[3] for the transition (if r_0 is taken to be 1.2 fm) is

$$\tau_{sp} = 0.92 \times 10^5 \text{ sec}$$

indicating the transition is inhibited by a factor of 4. With the particular $f_{7/2}$ interaction chosen in Ref. 2 the $(f_{7/2})^{13}$ calculation, with bare nucleon charge, gives

$$\tau_{f_{7/2}} = 3.9 \times 10^4 \text{ sec.}$$

Thus the many-particle $f_{7/2}$ result yields a lifetime an order of magnitude shorter than observed. Since Bertsch[4] has concluded that the effect of core excitation on the effective proton charge for an $E6$ transition should be small, it is of interest to see if this lifetime can be explained by weak configuration mixing within the $(0f, 1p)$ shell itself. If the states involved are taken to be dominantly $(f_{7/2})^{13}$ we show that the inclusion of terms linear in the admixed $(0f, 1p)$ shell configuration does not yield an explanation of the observed retardation when the correction to the free nucleon charge is assumed to be isoscalar and small.

Because of the high multipolarity of the transition, only mixing of the $0f_{5/2}$ state can contribute linearly in the admixture coefficients to the transition rate if we restrict ourselves to the $(0f, 1p)$ shell. Instead of carrying out the standard perturbation calculation, we have diagonalized the shell model Hamiltonian for 13 nucleons outside an inert ^{40}Ca core with the restriction that at most one nucleon can be in the $0f_{5/2}$ level. The two body matrix elements used in the diagonalization are listed in Table I. The $(f_{7/2})^2$ energies were taken from the spectrum[5] of ^{42}Sc. Since the $(0f_{7/2}0f_{5/2})$ interaction mediates the mixing, these matrix elements were chosen in two different ways:

First, we considered a δ-function potential[6] with interaction strengths chosen to give a best fit to the empirical $(f_{7/2})^2$ matrix elements. If we write the interaction in the isospin state T as

$$V_T = -4\pi A_T \delta(\Omega_{12}) ,$$

the best fit to the $(f_{7/2})^2$ matrix elements, given in columns 2 and 3 of Table I, leads to the values

$$A_0 = 1.239 \text{ MeV},$$

$$A_1 = 0.719 \text{ MeV}.$$

These values of A_0 and A_1 were used to compute the matrix elements listed in the columns headed "surface δ" in Table I.

Second, we used the Kuo-Brown[7] interaction. These matrix elements result when the G matrix (including three-particle–one-hole corrections) is calculated using the "realistic" Hamada-Johnston potential.

The Hamiltonian matrix was diagonalized using the matrix elements in Table I for various values of the single-particle energy difference. All basis states that arise from the configuration $(f_{7/2})^{13}$ and $[(f_{7/2})^{12}_{J'T'} \times f_{5/2}]_{IT}$ were used. The matrix elements of the $E6$ operator

$$T^6_\mu = \sum_i e_i r_i^6 Y^6_\mu(\theta_i \phi_i)$$

were evaluated using the eigenfunctions from these diagonalizations. The radial integrals that come into the transition rate,

$$S = \int R_j, r^6 R_j r^2 dr \qquad (1)$$

were determined by use of harmonic oscillator wave functions with the oscillator constant $\hbar\omega$

TABLE I. Matrix elements of the residual two-body force in MeV. The diagonal $f_{7/2}$ energies were taken directly from experiment, Ref. 5. The surface-δ values were computed by taking the interaction in the isospin state T to be $V_T = -4\pi A_T \delta(\Omega_{12})$, with $A_0 = 1.239$ MeV and $A_1 = 0.719$ MeV. The Kuo-Brown energies were taken from Ref. 7.

	Surface δ						Kuo-Brown			
Configuration	$\frac{7}{2}\frac{7}{2}$; $\frac{7}{2}\frac{7}{2}$		$\frac{7}{2}\frac{7}{2}$; $\frac{7}{2}\frac{5}{2}$		$\frac{7}{2}\frac{5}{2}$; $\frac{7}{2}\frac{5}{2}$		$\frac{7}{2}\frac{7}{2}$; $\frac{7}{2}\frac{5}{2}$		$\frac{7}{2}\frac{5}{2}$; $\frac{7}{2}\frac{5}{2}$	
Spin \ Isospin	0	1	0	1	0	1	0	1	0	1
0		−3.115								
1	−2.500		2.891		−6.374	0	1.894		−3.621	−0.287
2		−1.522		−0.335	−2.314	−0.164		0.0	−2.731	−0.096
3	−1.617		0.872		−1.535	0	1.005		−0.985	0.022
4		−0.315		−0.354	−1.577	−0.373		−0.406	−1.886	0.031
5	−1.597		0.619		−0.665	0	0.901		−0.112	0.156
6		0.085		−0.410	−2.022	−1.005		−0.716	−2.217	−0.894
7	−2.490									

$= 41A^{-1/3}$ MeV. Because of this procedure our results differ from a standard perturbation calculation in two respects:

(1) Our eigenfunctions are normalized including all components in the wave functions. Although there are only three $I = \frac{7}{2}^-$ and one $I = \frac{19}{2}^-$ $T = \frac{1}{2}$ states within the $(f_{7/2})^{13}$ configuration there are 42 $I = \frac{7}{2}^-$ $T = \frac{1}{2}$ and 19 $I = \frac{19}{2}^-$ $T = \frac{1}{2}$ states of the configuration $[(f_{7/2})^{12}{}_{J'T'} \times f_{5/2}]_{IT}$. Thus even a small component of each admixed state can appreciably change the normalization of the $(f_{7/2})^{13}$ contribution to the wave function. In Table II we list the probability for both the lowest $\frac{7}{2}^-$ and $\frac{19}{2}^-$ eigenfunctions that the $(f_{7/2})^{13}$ configuration is realized.

(2) By calculating matrix elements of T^6_μ between diagonalized states we also take into account small contributions that can arise from

$$\delta M = \langle [(f_{7/2})^{12}{}_{J'T'} \times f_{5/2}]_{7/2\ 1/2} | T^6_\mu | [(f_{7/2})^{12}{}_{J''T''} \times f_{5/2}]_{19/2\ 1/2} \rangle \ .$$

These would, of course, be absent in perturbation theory.

In Table II we list the values of $[B_p(E6)]^{1/2}$ and $[B_n(E6)]^{1/2}$, the square root of the reduced transition probability $B(E6)$ for the protons and neutrons, respectively. These values are tabulated (with the appropriate phase) for unit charge and various values of the single-particle energy difference. Thus

$$B(E6; \tfrac{19}{2}^- \to \tfrac{7}{2}^-) = |e_p[B_p(E6)]^{1/2} + e_n[B_n(E6)]^{1/2}|^2 . \quad (2)$$

TABLE II. Theoretical values of $[B(E6)]^{1/2}$ in e fm^6 as a function of the $f_{7/2}$-$f_{5/2}$ single particle energy splitting. The probability that the lowest $I = \frac{7}{2}^-$ $T = \frac{1}{2}$ and $I = \frac{19}{2}^-$ $T = \frac{1}{2}$ is $(f_{7/2})^{13}$ is also tabulated. To be consistent with our model $(\epsilon_{5/2} - \epsilon_{7/2}) = 4.969$ MeV for the surface-δ interaction and 2.463 MeV for the Kuo-Brown force. The experimental value of $[B(E6)]^{1/2}$ is 524 e fm^6.

	Surface δ				Kuo-Brown			
$\epsilon_{5/2} - \epsilon_{7/2}$	$(f_{7/2})^{13}$ (%)				$(f_{7/2})^{13}$ (%)			
(MeV)	$\frac{7}{2}^-$	$\frac{19}{2}^-$	$[B_p(E6)]^{1/2}$	$[B_n(E6)]^{1/2}$	$\frac{7}{2}^-$	$\frac{19}{2}^-$	$[B_p(E6)]^{1/2}$	$[B_n(E6)]^{1/2}$
∞	100	100	1.32×10^3	83.4	100	100	1.32×10^3	83.4
6.0	92.9	89.5	1.13×10^3	172	94.8	95.9	1.09×10^3	139
5.0	91.6	85.7	1.09×10^3	163	94.0	95.1	1.07×10^3	137
4.0	89.8	79.9	1.04×10^3	146	93.1	93.9	1.04×10^3	133
3.0	87.5	71.0	954	115	91.9	92.3	1.01×10^3	126
2.0	84.4	58.1	833	67.9	90.5	89.9	961	112
1.0	79.8	42.5	682	14.4	88.6	86.1	896	85.8

With the units used in Table II ($e\,\mathrm{fm}^6$) the relationship between $B(E6)$ and the mean lifetime is

$$\frac{1}{\tau_m} = 5.11\times10^{-18}E_\gamma^{13}B(E6)\ . \tag{3}$$

We have carried out the calculations for various values of the single-particle splitting $(\epsilon_{5/2}-\epsilon_{7/2})$. If we make the assumption that ^{56}Ni is a closed $(f_{7/2})^{16}$ shell, ^{55}Ni is a single neutron hole in this shell and that the $\frac{5}{2}^-$ state in ^{57}Ni at 760 keV [8] can be described as a single neutron in the $f_{5/2}$ orbit outside a closed $f_{7/2}$ shell, then there is only one value of this parameter that is consistent with experiment. To find this value, define $E_B(^{56+n}\mathrm{Ni})$ as the binding energy of the states discussed above. With the assumed model it follows that

$$E_B(^{56}\mathrm{Ni}) - E_B(^{55}\mathrm{Ni}) = \epsilon_{7/2} + \tfrac{1}{8}\sum_{JT}(2J+1)(2T+1)E_{JT}(\tfrac{7}{2}\,\tfrac{7}{2};\,\tfrac{7}{2}\,\tfrac{7}{2}) \tag{4}$$

and

$$E_B(^{57}\mathrm{Ni}) - E_B(^{56}\mathrm{Ni}) = \epsilon_{5/2} + \frac{1}{12}\sum_{JT}(2J+1)(2T+1)E_{JT}(\tfrac{7}{2}\,\tfrac{5}{2};\,\tfrac{7}{2}\,\tfrac{5}{2})\ , \tag{5}$$

where $E_{JT}(jj_1;\,jj_1)$ is the diagonal matrix element of the residual two-body force (listed in Table I) when the two nucleons are in the single-particle orbits (jj_1) and couple to spin J and isospin T. The binding energies involved are [9] $E_B(^{55}\mathrm{Ni}) = -467.387$ MeV, $E_B(^{56}\mathrm{Ni}) = -484.003$ MeV and $E_B(^{57}\mathrm{Ni}, I=\frac{5}{2}^-) = -494.269 + 0.760 = -493.509$ MeV. When these values are combined with the matrix elements in Table I, Eqs. (4) and (5) lead to

$$(\epsilon_{5/2}-\epsilon_{7/2}) = 4.969 \text{ MeV}$$

for the surface-δ interaction,

$$= 2.463 \text{ MeV}$$

for the Kuo-Brown interaction.

The reason the splitting is smaller for the Kuo-Brown interaction is that their matrix elements involving an $f_{5/2}$ nucleon are smaller than those computed with the δ-function force.

The experimental value of $B(E6)$ may be deduced from the lifetime data by use of Eq. (3),

$$B_{\mathrm{exp}}(E6;\,\tfrac{19}{2}^-\to\tfrac{7}{2}^-) = 2.75\times10^5\ e^2\mathrm{fm}^{12}$$

or

$$[B_{\mathrm{exp}}(E6;\,\tfrac{19}{2}^-\to\tfrac{7}{2}^-)]^{1/2} = 524\ e\,\mathrm{fm}^6\ .$$

The theoretical predictions for $[B(E6)]^{1/2}$ with the single-particle splittings deduced above may be interpolated using the values given in Table II. The results are nearly the same for the surface-δ and Kuo-Brown interaction:

$$[B_p(E6)]^{1/2} \sim 1000\ e\,\mathrm{fm}^6$$

and

$$[B_n(E6)]^{1/2} \sim 130\text{–}160\ e\,\mathrm{fm}^6.$$

In order to extract values for the effective proton and neutron charges we have assumed the isoscalar form $e_p = 1+\delta$ and $e_n = \delta$. With this assumption our result is only consistent with the experimental value of 524 $e\,\mathrm{fm}^6$ if δ is about -0.4.

These conclusions are not sensitive to the assumed $(f_{7/2})^2$ matrix elements provided the single-particle splitting is chosen to be consistent with Eqs. (4) and (5). For example, if the Kuo-Brown matrix elements are used for the $(f_{7/2})^2$ interaction (Table III of Ref. 7) the single-particle splitting would have to be $(\epsilon_{5/2}-\epsilon_{7/2}) = 9.817$ MeV [much larger than the above values since the Kuo-Brown $(f_{7/2})^2$ matrix elements are weaker than those of Table I]. For this interaction it follows that

$$[B_p(E6)]^{1/2} = 1.04\times10^3\ e\,\mathrm{fm}^6$$

and

$$[B_n(E6)]^{1/2} = 109\ e\,\mathrm{fm}^6,$$

values very similar to those noted above.

One other question remains. How sensitive is the single-particle radial matrix element, S of Eq. (1), to the choice of single-particle potential? To answer this, we have evaluated S using Woods-Saxon wave functions. If the Woods-Saxon well is chosen to fit the average $f_{7/2}$-nucleon binding energy in the ^{53}Fe ground state, i.e.,

$$[E_B(^{53}\mathrm{Fe}) - E_B(^{40}\mathrm{Ca})]/13 = -8.95 \text{ MeV},$$

the matrix element is

$$S = 1.022\times10^4\ \mathrm{fm}^6$$

when the spin-orbit force is neglected and the well radius is taken to be $1.25A^{1/3}$ fm. This choice of well radius leads to

$$\langle r\rangle = \int R_f r R_f r^2 dr = 4.03 \text{ fm}.$$

For an oscillator with $\hbar\omega = 41A^{-1/3}$ MeV one finds

$$\langle r\rangle = 4.02 \text{ fm}$$

and

$$S = 0.88\times10^4\ \mathrm{fm}^6.$$

Thus, as might have been expected, the matrix element of r^6 is larger with Woods-Saxon eigenfunctions and hence in this case the calculated $[B(E6)]^{1/2}$ of Table II must be multiplied by (1.022/0.88). Therefore, the theoretical value is

even farther from the experimental result.

The $f_{5/2}$ admixture into either the $\frac{7}{2}^-$ or $\frac{19}{2}^-$ states is calculated to be about 10% for the appropriate single-particle splitting. This value is consistent with the mixing required to explain the ^{48}Ti(d, t) ^{47}Ti data.[10] With this admixture and with the assumption of an isoscalar effective $E6$ charge one must take $\delta \approx -0.4$ to fit experiment when the single-particle energies are chosen to be consistent with experimental binding energies. However, Bertsch[4] has concluded that excitations out of the ^{40}Ca core can account for only a small effective charge δ because of destructive interference between the central and two-body spin orbit interactions. Since $p_{3/2}$ excitations contribute to the $E6$ decay only through a renormalization of the $(f_{7/2})^{13}$ and $(f_{7/2})^{12}$ parts of the wave functions, very large $p_{3/2}$ admixtures would be required to fit experiment with $\delta \cong 0$. Although no data are directly available on the $p_{3/2}$ contaminants in ^{53}Fe, the ^{54}Fe(p, d) ^{53}Fe $l=1$ pickup strength (Ref. 8) $\sum C^2S$ is $\lesssim 0.25$. This indicates at most a 12% admixture of $(p_{3/2})^2{}_0$ in the ^{54}Fe ground state. Thus large $p_{3/2}$ mixing in the ^{53}Fe states seems unlikely and, consequently, unless $\delta \approx -0.4$ the $E6$ transition in ^{53}Fe cannot be explained.

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