Interpretation of ^{44}Ti as a soft asymmetric rotor

J. J. Simpson
Department of Physics, University of Guelph, Guelph, Ontario, Canada N1G 2W1
W. R. Dixon and R. S. Storey
Division of Physics, National Research Council, Ottawa, Canada, K1A 0S1
(Received 22 July 1974)

The positive parity levels in ^{44}Ti below 4.2 MeV and many of the $E2$ transitions between them are fairly successfully explained using a soft asymmetric rotor model. A brief discussion of suggested negative parity states is also given.

[NUCLEAR STRUCTURE ^{44}Ti; calculated levels, $B(E2)$'s. Asymmetric rotor model with vibrations.]

Recently, evidence was presented[1] which suggested that the low-energy levels of ^{44}Ti can be arranged in four bands of a rotational-like nature. Three of these bands have positive parity and the evidence favors negative parity for the fourth. In this note we wish to show for the first time that a substantial amount of evidence strongly supports the model of a soft asymmetric rotor[2-5] for ^{44}Ti, and this indicates that phenomenological models can be very useful in understanding the nuclei of the lower f-p shell.

In Fig. 1 the experimentally observed levels of ^{44}Ti below 4.2 MeV are compared with theory. The magnitude of the problem facing a shell model calculation is suggested by the comparison with the $(fp)^4$ model. In this calculation[6] four nucleons are distributed in the full $1f$-$2p$ shell and the two-body residual interaction of Kuo and Brown[7] is used. Although there is good agreement with the "ground-state" band, the shell model in the restricted space specified above seems incapable of accounting for most of the low-energy states. The problem of many extra levels in nuclei at the beginning of the $2p$-$1f$ shell has been recognized for a long time, however, and led to the inclusion of deformed states[8] to explain the excited 0^+ and 2^+ levels in ^{42}Ca. An extension of these ideas to ^{44}Ti would be a difficult problem.

It has been pointed out[9] that the positive parity energy levels give the appearance of a vibrational spectrum of a spherical nucleus, but a difficulty is the occurrence of a strong transition from the second 2^+ state to the excited 0^+ state in conjunction with a weak crossover to the ground state. On the other hand, a low-lying 3^+ state as in the case of ^{44}Ti is often the sign of asymmetry in a deformed system, perhaps as a permanent asymmetric deformation (i.e., three principal moments of inertia unequal) or as an asymmetric vibration of a symmetrically deformed nuclear shape. We have chosen to compare the ^{44}Ti results with the model of a rotor with a permanent asymmetry.[2,3] The parameters of the model specifying the nuclear shape are the usual deformation parameter β and the asymmetry parameter γ. In terms of these parameters the lengths of the three principal nuclear axes ($k=1,2,3$) are[10]

$$R_k = R_0\left[1+\left(\frac{5}{4\pi}\right)^{1/2}\beta\cos\left(\gamma-\frac{2\pi}{3}k\right)\right].$$

For $\gamma=0$, the symmetry axis is the $k=3$ axis, and the nucleus is prolate. For $\gamma=\pi/3$, the symmetry axis is the $k=2$ axis and the nucleus is oblate. The predictions of the model (energy levels and transition strengths) are symmetric about $\gamma=\pi/6$ and we have chosen to confine $\gamma\leq\pi/6$. Therefore our results should not be taken to imply that ^{44}Ti is necessarily prolate-like. A measurement of a static quadrupole moment, for example, would be required to decide this.

If the asymmetric rotor model is extended to include β vibrations,[4,5] it is possible to interpret the excited 0^+ state in ^{44}Ti at 1.90 MeV as the head of a β band. In this extended model an additional stiffness parameter μ is introduced which is essentially a measure of the amplitude of the zero-point β vibrations and is zero in the limiting case of a rigid nucleus.

The predictions of the asymmetric rotor model for the positive parity levels are shown in Fig. 1. For the rigid case ($\mu=0$), the levels are labeled by JN, where J is the angular momentum and N is the ordinal of those levels with angular momentum J. For soft nuclei ($\mu\neq0$), the levels are labeled by JNn, where n is an ordinal number labeling the vibrational quantum number which is not

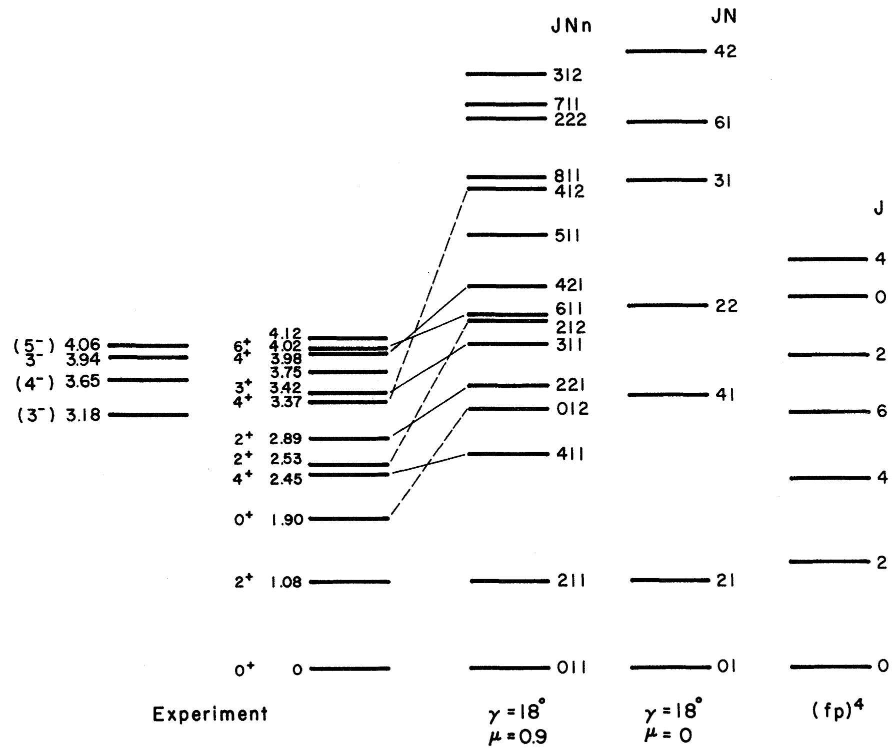

FIG. 1. Experimental and theoretical energy levels of ^{44}Ti. The two left-most diagrams show all known energy levels of ^{44}Ti below about 4.2 MeV (Refs. 1, 11, and 15). The three diagrams to the right are the calculated energy levels for the soft asymmetric rotor ($\gamma = 18°$, $\mu = 0.9$), the rigid asymmetric rotor ($\gamma = 18°$, $\mu = 0$), and the $(fp)^4$ shell model.

in general an integer ($n = 1$ is the ground-state band and $n = 2$ is the first excited β band). The calculations have been normalized to fit the first 2^+ state to the experimental value.

The value of $\gamma = 18°$ was somewhat arbitrarily chosen because the transition strengths were best fitted here in the rigid rotor case ($\mu = 0$). There is little real improvement in the energy levels by allowing a larger value of γ. There is, however, appreciable improvement in the energy level fit when β vibrations are introduced. The value of $\mu = 0.9$ was found to give the lowest rms relative deviation of the energies.

One sees that the model provides a ready explanation for the experimental positive parity levels. (A level which is not completely identified,[11] at 3.75 MeV, is not inconsistent with an interpretation as the second 2^+ level of the β band, i.e., $JNn = 222$.) Besides an over-all expansion in the spectrum with $\gamma = 18°$, $\mu = 0.9$, the main problem in the energy level fits is the position and expansion of the β band, the $JN2$ levels. The β band cannot be pulled down further even by a drastic increase in μ. Increasing μ has primarily the effect of compressing the β band but raising the bandhead energy. Davidson[12] has suggested that this results from the β^2 dependence of the moments of inertia in the hydrodynamic model which causes an increase in the moments of inertia as the vibration amplitude increases and thus a lowering of the energies of the ground-state band. This prevents the 012 level coming below an energy of about three times the 211 level energy.

In Fig. 1 we identified experimental levels with the model levels. This identification was based partly on the energy location and partly on the $B(E2)$ values,[1] the reduced $E2$ transition strengths, which are presented in Table I in W.u. (Weisskopf units).[13] One W.u. is 9.2 $e^2\,\mathrm{fm}^4$. The transition strengths require one further parameter, the intrinsic quadrupole moment. This parameter is effectively determined by normalizing to the experimental $B(E2; 211 \rightarrow 011)$ value. The rms value of β determined on the basis of the hydrodynamic model is 0.23 (using $r_0 = 1.3$ fm) for $\gamma = 18°$, $\mu = 0$.

In contrast to the energy levels, the rigid model with $\gamma = 18°$ is quite successful for those $E2$ transition strengths which it does predict. The soft rotor model however is still qualitatively quite good, predicting weak transitions to be weak and strong ones strong even though there is a span of more than two orders of magnitude. It also predicts

TABLE I. Transition strengths in ^{44}Ti.

Transition				$B[E2;\ (JNn)_i \rightarrow (JNn)_f]$ W.u.		
					Theoretical	
E_i (keV)	E_f (keV)	$(JNn)_i$	$(JNn)_f$	Experimental	$\gamma = 18°$, $\mu = 0$	$\gamma = 18°$, $\mu = 0.9$
1083	0	211	011	13 ± 3	13	13
2454	1083	411	211	30 ± 6	19	38
4015	2454	611	411	17 ± 3	22	86
2886	0	221	011	$0.65^{+0.33}_{-0.18}$	0.9	0.9
	1083	221	211	$<3.8^{+2.1}_{-1.3}$ [a]	3.5	6.9
	1904	221	012	<4.7 [b,c]		0.8
3415	1083	311	211	1.7 ± 0.3 or 0.24 ± 0.13	1.7	3.1
	2454	311	411	<2.5 [b]	4	14
	2886	311	221	<67 ± 14 [a]	23	113
	2531	311	212	<3 [b]		1.8
3980	1083	421	211	$0.24^{+0.13}_{-0.08}$	0.02	0.03
	2886	421	221	19^{+11}_{-7}	7	35
	2454	421	411	<1.8 ± 1.1 [a]	3.5	14
	3415	421	311	<85 ± 70 [a]	14	82
2531	0	212	011	0.17 ± 0.03		0.26
	1083	212	211	6.7 ± 1.0		3.0
	1904	212	012	24 ± 6		17
3365	1083	412	211	2.6 ± 0.5		0.02
	2531	412	212	29 ± 16		38

[a] Transition observed, but mixing ratio unknown. Value calculated assuming pure $E2$.
[b] Transition unobserved. Upper limit is 2 standard deviations and assumes pure $E2$.
[c] Because of new data, this value supersedes that quoted in Ref. 1.

quite successfully transitions involving states of the β band, and in particular the strong $212 \rightarrow 012$ transition compared to the $212 \rightarrow 011$ transition. These latter transitions are strong evidence against a spherical vibrational model, even including anharmonicities. The chief inadequacy seems to be the tendency to overenhance the transition strengths as μ increases. This is very evident in the case of the $6^+ \rightarrow 4^+$ transition which is predicted very well in the rigid model case, but is about a factor of 5 too large when $\mu = 0.9$.

In conclusion, it appears that the asymmetric rotor model with β vibrations gives a satisfactory qualitative fit to both the energy levels and reduced $E2$ transition strengths in ^{44}Ti. The main problem is that, on the assumption of hydrodynamic moments of inertia, it is not possible to bring the β-band energy as low as experimentally observed even allowing a very large μ and that simultaneously the large μ decreases the over-all quantitative agreement for the reduced $E2$ transition strengths in the ground-state band achieved by the rigid asymmetric rotor model. The transitions from the β band are however fairly successfully explained.

It is to be expected that the model of a symmetric rotor with rotation-vibration interaction would give results similar to those of the asymmetric rotor. (See for example Eisenberg and Greiner.[14]) We have not investigated this point beyond comparing certain experimental ratios of $B(E2)$ values with the values predicted for no rotation-vibration interaction. The intraband ratios are in good agreement, but for transitions from the β band to the ground-state band we find the experimental ratio $B(E2; 212 \rightarrow 211)/B(E2; 212 \rightarrow 011)$ to be 39 ± 9, while the predicted value is only 1.43. For the latter transitions it is clearly necessary to include the rotation-vibration interaction.

Negative parity levels. In Fig. 1 we also show suggested negative parity levels[1, 15] of ^{44}Ti. In Ref. 1, it was suggested that the lowest 3^-, the 4^- and 5^- states form a rotational band because of their interconnection by γ-ray transitions. If this is a $K = 3^-$ band, its origin can perhaps be understood (for $\beta = +0.23$) as the promotion of a particle from the filled Nilsson $d_{3/2}, \Omega = \frac{3}{2}$ level to the unfilled $f_{7/2}, \Omega = \frac{3}{2}$ level, or (for $\beta = -0.23$) from the filled $d_{3/2}, \Omega = \frac{1}{2}$ level to the unfilled $f_{7/2}, \Omega = \frac{5}{2}$ level. If the nucleus is asymmetric, K is no longer a good quantum number but the above configurations are still dominant components in the particle-hole wave functions.[16] It is then tempting to suggest that the second 3^- state is also a member of this rotational band, perhaps as a β-vibration mode or simply as the second spin 3 state expected in an odd K band.[17] Thus we have demonstrated that it might be possible to explain the known energy levels of ^{44}Ti below 4.2 MeV with a phenomenological rotational model.

We should like to acknowledge gratefully the generous help of Professor J. P. Davidson who calculated the soft asymmetric rotor results for us and enlightened us on several aspects of the model.

[1]J. J. Simpson, W. R. Dixon, and R. S. Storey, Phys. Rev. Lett. 31, 946 (1973).
[2]A. S. Davydov and G. F. Filippov, Nucl. Phys. 8, 237 (1958).
[3]A. S. Davydov and V. S. Rostovsky, Nucl. Phys. 12, 58 (1959).
[4]A. S. Davydov and A. A. Chaban, Nucl. Phys. 20, 499 (1960).
[5]J. P. Davidson, U. S. Naval Radiological Defense Laboratory Report No. USNRDL-TR-901, 1965 (unpublished).
[6]J. J. Simpson, W. R. Dixon, and R. S. Storey, Phys. Lett. 30B, 478 (1969).
[7]T. T. S. Kuo and G. E. Brown, Nucl. Phys. A114, 241 (1968).
[8]W. J. Gerace and A. M. Green, Nucl. Phys. A93, 110 (1967); B. H. Flowers and L. D. Skouras, Nucl. Phys. A116, 529 (1968); A136, 353 (1969).
[9]J. J. Simpson, W. R. Dixon, and R. S. Storey, Phys. Rev. C 4, 443 (1971).
[10]M. A. Preston, *Physics of the Nucleus* (Addison-Wesley, Reading, Massachusetts, 1962), Chap. 10.
[11]W. R. Dixon, R. S. Storey, and J. J. Simpson, unpublished.
[12]J. P. Davidson, private communication.
[13]D. H. Wilkinson, in *Nuclear Spectroscopy, Part B*, edited by F. Ajzenberg-Selove (Academic, New York, 1960), p. 852.
[14]J. M. Eisenberg and W. Greiner, *Nuclear Theory* (North-Holland, Amsterdam, 1970), Vol. 1, Chap. 6.
[15]J. Rapaport, J. B. Ball, R. L. Auble, T. A. Belote, and W. E. Dorenbusch, Phys. Rev. C 5, 453 (1972).
[16]T. D. Newton, Atomic Energy of Canada Limited Report No. CRT-886, 1960 (unpublished).
[17]J. P. Davidson, *Collective Models of the Nucleus* (Academic, New York, 1968), p. 27.