Precision measurement of the shapes of the first-forbidden unique beta spectra: ^{42}K, ^{142}Pr, ^{86}Rb, and ^{91}Y

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The shapes of the ground state transitions of ^{42}K, ^{142}Pr, ^{86}Rb, and ^{91}Y were carefully measured in an intermediate-image β ray spectrometer. The spectra were analyzed using exact electron radial wave functions of Bhalla and Rose. The measured shapes were compared to the theoretical predictions in terms of a linear fit of the form $(1+aW)$ and a hyperbolic fit of the form $(1+b/W)$. All the spectra were found to show a small order deviation from the simple unique shape. The shape factor term in the analysis of ^{91}Y was found to be $b=+0.027 \pm 0.007$. Some of the existing theoretical approaches in this direction have been discussed.

[RADIOACTIVITY ^{42}K, ^{142}Pr, ^{86}Rb, ^{91}Y; measured E_β, β-spectrum shape.]

I. INTRODUCTION

In the normal approximation of the theory of once-forbidden unique β decay,[1] only the tensor rank-two-matrix element $\langle B_{ij} \rangle$ is involved and hence one expects a unique shape correction factor

$$C_u(W) = q^2 + 9L_1/L_0, \quad (1)$$

where $q = (W_0 - W)$ and the terms L_1 and L_0 can be computed from the tables of Bhalla and Rose,[2] for the β spectrum, unless there are considerable (i) regular third-forbidden contributions and (ii) weak magnetism effects.[3]

As in the case of allowed β ray spectra, the experimental situation in the small order deviations of unique first-forbidden β ray spectra is not satisfactory. Rather, much less work has been reported for the study of unique β ray spectra (see compiliations, Refs. 4 and 5). Even for the most frequently and, perhaps, the more carefully studied decay,[4,5] namely, that of ^{90}Y, the agreement between the reported values for the shape factor is not good. For most of the first-forbidden unique transitions reported in the literature,[4] the deviations from the theoretical unique shape (1) are small and are expressed as an empirical correction factor of the form $(1+aW)$, with a <0. The largest deviation found[6] was for the unique transition in ^{166}Ho.

Considerable attention[7,8] has been directed to finding the possible theoretical reasons for the systematic deviations of unique spectra; but it appears that no simple theoretical explanation exists which satisfactorily explains the observed deviations.

The previous work[9] at our laboratories verified the shapes of the unique spectra of ^{90}Sr and ^{90}Y and the trends of the results are similar to those reported in the literature. The present work reports the results of a careful study of first-forbidden unique spectra in ^{42}K, ^{142}Pr, ^{86}Rb, and ^{91}Y. All these spectra have *ft* values normal for first-forbidden unique transitions.

II. EXPERIMENTAL

A. Apparatus and control experiments

The present measurements were carried out with a Siegbahn-Slatis intermediate image spectrometer (manufactured by LKB Productors, Sweden). The suitability of this spectrometer for precision shape measurements has been discussed elsewhere[10,11] and hence only pertinent points will be mentioned here. The β detector pole piece has been modified for the inclusion of a NE-102 well-type plastic scintillator of suitable geometry coupled to a high gain RCA 6810-A photomultiplier tube outside the pole piece by a short lucite light pipe. Addition of a scintillation detector enables discrimination between electrons of selected momentum and scattered electrons. For the discrimination level used in the present work the back scattering correction amounted to 0.2% at 80 keV.

Scattering in the spectrometer is a serious problem if one attempts to measure small deviations of the allowed unique spectra. By a direct measurement of scattering (viz., scattering due to walls of the chamber, due to the baffles, etc.) and by a study of line shape measurements, we conclude[10,11] that the scattering is negligible in our spectrometer. This finding agrees well with that of Paul,[12,13] who compared the scattering properties of various spectrometers and assigned a

negligible scattering effect to the intermediate-image spectrometer. Source dependent background was taken at each measurement point by closing the central baffle from outside. In most of the reported investigations, the background was calculated from the counting rate at zero current and beyond the end-point energy of the spectrum and hence cannot be relied upon, as the source dependent background varies with the field setting.

As an over-all check of the spectrometer performance, the allowed spectrum of ^{24}Na ($\log ft = 6.1$) was studied[11] under conditions similar to those used in the actual measurement runs. The resulting Fermi-Kurie (FK) plot showed no deviation from linearity. The measured shape factor for ^{24}Na was found to be $C(W) = 1 - (0.0066 \pm 0.007)\,W$. Our result agrees with those of all other investigators, thus indicating that our spectrometer transmission is energy independent within the statistical accuracy of the measurement.

To verify further that the spectrometer is free from scattering effects, we have measured the shape factor of the 965 keV β component and the K-conversion coefficient of the 412 keV γ transition in ^{198}Au, under experimental conditions similar to those used in the actual measurement runs. The internal conversion coefficient (ICC) was determined by the peak to β spectrum (PBS) method and the reliability of the PBS method depends on how accurately the shape of the β spectrum in question is known. The measured ICC of the 412 keV transition was $300 \pm 3 \times 10^{-4}$, which is in close agreement with the theoretical value 302×10^{-4}. This excellent agreement would not have been possible if scattering were present either in our spectrometer or within the source.

B. Source

The source materials were obtained from Bhabha Atomic Research Center, Trombay. The radioactive materials ^{86}Rb, ^{42}K, and ^{142}Pr are obtained as chlorides in dilute HCl solution. ^{91}Y was obtained as a carrier-free solution. The radioactive materials were evaporated in vacuum on aluminum foils of 180 μg/cm^2 thickness. Sources were also prepared by evaporating a drop of radioactive liquid to dryness on thin conducting foils of Mylar. Insulin was used to help uniform spreading[14] and to define the source area. All the sources were 2 mm in diameter. The source thicknesses range from 130–180 μg/cm^2 for ^{42}K; 80–100 μg/cm^2 for ^{86}Rb; 50–80 μg/cm^2 for ^{142}Pr; and 70–90 μg/cm^2 for ^{91}Y. A thick source of 250 μg/cm^2 was also prepared by evaporating radioactive rubidium chloride on thin (250 μg/cm^2) conducting Mylar film.

C. Experimental procedure and results

Three runs were taken for each isotope. Runs were programmed having total times in the range of 3–8 h, depending on the source strength. In the case of ^{42}K and ^{142}Pr, the total time for each run never exceeded 4 h. As shown by Beekhuis,[6] the shape factor depends on the half-life used for the decay correction if the measurements take a period of time comparable with $T_{1/2}$. In the case of ^{42}K and ^{142}Pr the results were corrected for source decay using $T_{1/2} = 12.36$ h for ^{42}K and $T_{1/2} = 19.3$ h for ^{142}Pr. The spectra were scanned in equal energy steps. No impurities were observed in the isotopes studied with the exception of ^{86}Rb, where a weak contamination of about 0.8% of ^{134}Cs was detected. This did not affect our results, as the ^{86}Rb spectrum was analyzed in the region from 750 keV to the end-point energy E_0, well above the end-point energies of the ^{134}Cs β groups.

The background was taken at each point. A computer program FERMIKURI subtracts the background (corrected for decay), applies decay, backscattering, and resolution corrections, interpolates the electron radial wave functions (ERWF's) from the tables of Bhalla and Rose,[2] calculates the energy and momentum, etc., and draws the FK plot. The mode of analysis of the data was described earlier.[15-17] The Kurie plots were not shown here. Only in the case of ^{91}Y, however, is the FK plot given (see Fig. 1), to show the presence of a weak inner β group of about 0.3%. A second program, BETASHAPE, computes the corrected shape factor

$$C(W) = \frac{N}{pf\,(W_0 - W)^2 (q^2 + 9L_1/L_0)} \qquad (2)$$

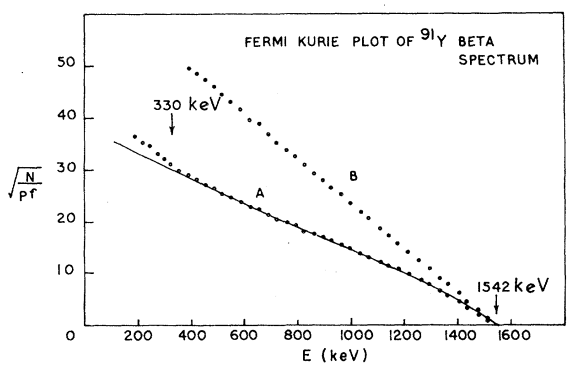

FIG. 1. Fermi-Kurie plot of ^{91}Y beta spectrum. Curve A is the uncorrected Fermi-Kurie plot (total β spectrum). Curve B is the outer β ray linearized with $C(W) = (q^2 + 9L_1/L_0)\,[1 - (0.0088 \pm 0.0035)\,W]$.

and plots the same quantity vs energy for various values of W_0 near end-point energy. We have used the exact Fermi function $f = F_0 L_0 = \frac{1}{2}(f_{+1}^2 + g_{-1}^2)$ (SC), where f_{+1} and g_{-1} are ERWF's taken from the tables of Bhalla and Rose[2] which include finite nuclear size effects and corrections due to finite de Broglie wave lengths, and SC, the screening correction, is taken from the tables of Buhring.[18] The function $9L_1/L_0$ is also corrected for screening from the tables of Buhring. The correct end-point energy was judged, for each run, from the behavior of the shape factor curves in the neighborhood of W_0. Figure (2) shows that the shape factor for ^{86}Rb (thick source 250 μg/cm^2) curves up or down at the high-energy side when W_0 is changed from its true value.

In order to look for deviations from the theoretical shape factor, the experimental shape factor curves [Eq.(2)] for each run were weighted-least-square fitted by a linear function $C_1'(W) = k(1 + aW)$ and by a hyperbolic function $C_2'(W) = k(1 + b/W)$. Figure 3 shows the results of the shape factor measurements. For each isotope the corrected shape factor [Eq.(2)] is plotted against E/E_0, where E and E_0 are the β particle kinetic energies in keV. The coefficients "a" and "b" and the end-point kinetic energy E_0, obtained as a weighted average of three runs, are listed in Table I.

No b/W term was observed in the present measurement for the first-forbidden unique spectra of ^{42}K, ^{142}Pr, and ^{86}Rb. The results of the present measurement are in excellent agreement with the earlier works[19, 20] in the case of ^{42}K and ^{86}Rb. With respect to the nuclide ^{142}Pr, we obtained an end-point energy $E_0 = 2158 \pm 3$ keV, slightly smaller than the value reported by Beekhuis.[6] It may be pointed out, however, that the coefficient "a" obtained by us for the shape factor of ^{142}Pr is in good agreement with that obtained by Beekhuis. It is worth mentioning that the measured[21] longitudinal polarization for ^{86}Rb is unity, whereas Kaminker *et al.*[22] reported appreciable deviation ($6.6 \pm 1.5\%$) from the normal longitudinal polarization for ^{142}Pr, though the coefficient a obtained in the shape factors is of the same order in both cases.

For the case of ^{91}Y, we analyzed the spectrum in two different regions: region A from 200 keV to the end point, i.e., by ignoring the presence of the inner β group, and region B from 400 keV to end point, i.e., the region of the total spectrum beyond the end point energy of the inner beta component [E_0(inner) = 320 keV]. In both cases shown in Table I, we obtained a vanishingly small deviation for ^{91}Y, in obvious disagreement with Langer, Spejewski, and Wortman,[22] who reported a b/W deviation with b value in the limits $0.2 \leq b \leq 0.3$. In fact, a recent measurement[23] of the unique spectrum of ^{89}Sr, on a similar instrument in which scattering[24] has been reduced, did not show such a large hyperbolic term b in the shape factor. The b/W term in the shape factor for ^{91}Y in the present work turned out to be $b = +0.028 \pm 0.008\ mc^2$ when analyzed in region A and $b = +0.05 \pm 0.005\ mc^2$ when analyzed in region B. The spectrum

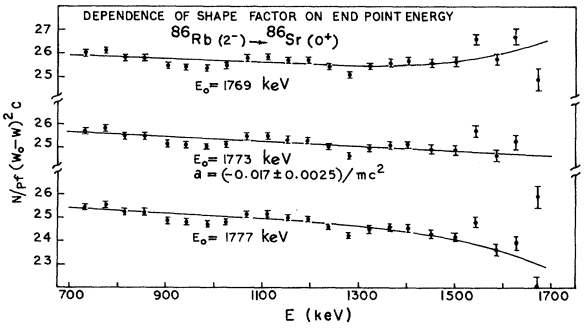

FIG. 2. The shape factor of the 1773 keV β component of ^{86}Rb. The figure shows the behavior of the shape-factor curves near W_0, when W_0 is varied from its true value. Source thickness is 250 μg/cm^2, $C = q^2 + 9L_1/L_0$.

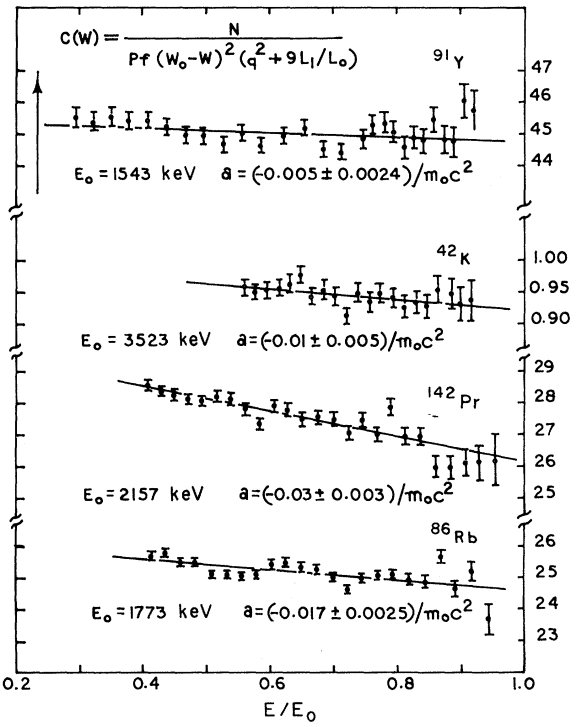

FIG. 3. Experimental results of the shape factor measurements. The reduced shape factor $C(W) = N/[pf(W_0 - W)^2(q^2 + 9L_1/L_0)]$ for each isotope is plotted against E/E_0. The solid lines are least-square fits according to the equation $C(W) = k(1 + aW)$.

TABLE I. Results of the shape factor analysis.

Isotope	log*ft*	E_0 (MeV)	a $(mc^2)^{-1}$	b (mc^2)
^{42}K	8.4	3.524± 0.006	$-14\pm 5.0\times10^{-3}$	
^{86}Rb	8.4	1.775± 0.003	$-19\pm 2.5\times10^{-3}$	
^{142}Pr	7.8	2.158± 0.003	$-33\pm 3.2\times10^{-3}$	
^{91}Y	8.5	1.543± 0.002	$-8\pm 3.5\times10^{-3}$ [a]	$+28\pm 7\times10^{-3}$
			$-10\pm 4.2\times10^{-3}$ [b]	$+52\pm 5\times10^{-3}$

[a] Value obtained when analyzed in region B.
[b] Value obtained when analyzed in region A.

(region B) was also analyzed by fitting the data to a linear shape factor of the form $C(W)=k(1+aW)$ with $a=-0.010\pm 0.004$. In order to arrive at the best fit of the experimental data, a χ^2 test was performed. For 24 degrees of freedom, the χ^2 values ($\chi^2=0.06$ for linear fit and $\chi^2=0.2$ for hyperbolic fit) obtained are very much less than the value (45.56) given in statistical tables[25] at a 5% significance level and are of nearly the same order. As such, both the linear and hyperbolic fits are consistent with the present experimental data of ^{91}Y.

III. DISCUSSION

The results of the present measurements confirm that the first-forbidden unique spectra of ^{42}K, ^{142}Pr, ^{86}Rb, and ^{91}Y do not show the previously expected shape [Eq.(1)], even when screening and finite size effects are taken into account. The spectra ^{42}K, ^{142}Pr, and ^{86}Rb require an additional shape factor of the form $C(W)=k(1+aW)$ for their complete description, while such a small order shape factor nearly vanishes for ^{91}Y.

Beekhuis[6] attempted to explain such an anomalously large deviation in the case of ^{166}Ho in terms of finite size effects[26] and third-forbidden[27] matrix elements, but arrived at improbably large values of higher order matrix elements. Also, such an explanation predicted an energy dependence of the β-γ correlation coefficients $\epsilon(W)$, which contradicted the earlier measurements.[28,29]

An exhaustive analysis of the unique first-forbidden β transitions can be found in the works of Eman, Krmpotic, and Tadic[7] and Abacasis and Krmpotic.[8] They attempted to explain the observed deviations by considering (i) higher order terms in the usual multipole expansion of the weak Hamiltonian, (ii) terms induced by strong interaction, and (iii) the variation of the lepton wave functions over the nuclei. While Eman *et al.* considered only the effect (ii), all three effects were taken into account by Abacasis and Krmpotic. The latter also presented theoretical expressions which included all the above three effects for other β decay observables. Eman *et al.* did not consider higher-order effects in the usual multipole expansion of the $V-A$ theory, saying that the uniformity of the experimental results speaks against such an explanation.

They could satisfactorily explain the spectrum shape data of Ref. 19 for ^{42}K, ^{86}Rb, ^{90}Y, and ^{90}Sr with induced terms alone, but the required signs for the induced terms disagreed with the other experimental information. Hence, they did not regard their analysis with induced terms as satisfactory. Abacasis and Krmpotic were also able to produce the shape correction factors that agreed with experimental results of Refs. 6 and 19 for the unique spectra of ^{42}K, ^{142}Pr, ^{166}Ho, ^{86}Rb, ^{90}Y, and ^{90}Sr by considering the nuclear structure effects and terms induced by the strong interaction separately. The measured[30] deviation from the normal longitudinal polarization, for the ^{142}Pr nuclide, was explained by them only by pseudoscaler interaction, and the value of the parameter adopted was $d=+30\pm 7$. It may be pointed out here that Kaminker *et al.*[30] also found appreciable deviation for the allowed decay of ^{114}In^m and in order to explain their results Abacasis and Krmpotic[8] had to take the parameter d as $d=-40\pm 20$. With respect to the nuclide ^{166}Ho, the experimental results for the shape factor[6] and the directional correlation[28] could not be simultaneously explained by them.[8] They attributed the above discrepancies to the inconsistencies in the experimental results and concluded that precise experiments determining shape factors, together with longitudinal polarization and/or β-γ directional correlation, combined with better knowledge of the nuclear structure, are required in order to draw definite conclusions.

The recent investigations on the nonunique first-forbidden β transitions in both ^{86}Rb and ^{42}K have shown[31,32] that the spectrum shape and the magnitudes of the β nuclear matrix elements associated with these transitions could not be explained by both shell and core-excitation models unless both

the ground states of the parent and the daughter nuclei were considered as admixtures of particle and particle-phonon configurations. Perhaps such an approach would throw light on the present situation of the small order deviations in unique transitions.

The present measurements were carried out with the same instrumental arrangement for which no deviation was observed for the allowed spectrum of ^{24}Na. The experimental results on other β decay observables for this transition are mutually consistent.[33,34] Hence, the observed deviations cannot be attributed to the instrumental effects. In conclusion, it appears that no simple theoretical explanation exists which can explain the measured deviations satisfactorily.

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