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Quasielastic pion scattering in the (3, 3) resonance region

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After qualitative discussion of what can be expected in quasielastic (π, π') experiments, a semiclassical model is developed. The mean free path for pion absorption in nuclear matter is a more or less unknown ingredient of the model, but the predictions for the effective nucleon number are insensitive to it. The uncertainties of nuclear corrections can be largely avoided by considering ratios of quasielastic cross sections between different nuclei.

[NUCLEAR REACTIONS ^{12}O, ^{63}Cu, ^{208}Pb(π, π'), quasielastic scattering; T_π = 100–400 MeV; calculated $\sigma(\theta)$, semiclassical model; ratios of $\sigma(\theta)$'s.]

I. INTRODUCTION

The imminent availability at meson factories of intense pion beams and their associated high resolution spectrometers has focused attention on the interactions of pions with nuclei. Apart from differential cross sections involving particular final nuclear levels,[1] another kind of pion-nucleus reaction, quasielastic scattering, can be investigated with the new spectrometer facilities. So far we have only very limited information about this process.[2,3] This paper considers what might be expected of quasielastic pion scattering and presents a simple semiclassical model that may provide a reasonable first approach in analyzing the data as these become available.

The structure of the paper is as follows. Section II discusses qualitative aspects of quasielastic scattering, the reaction $(\pi, \pi N)$, and comparisons are often made with the more familiar $(p, 2p)$ reaction. The "one arm" experiment (π, π') is more likely to be done first,[4] however, and we concentrate our attention on that. The semiclassical model for (π, π') is presented in Sec. III, where the effective nucleon number $N_{\rm eff}$ is defined. There is an uncertainty of the mean free path for pion absorption, $\lambda_{\rm abs}$, a parameter in the model; we assume the absorption process to involve two nucleons. Section IV presents and discusses results for $N_{\rm eff}$. Fortunately, they are not very sensitive to the values assumed for $\lambda_{\rm abs}$. The question of various nuclear corrections is considered in Sec. V. These turn out to be large and ambiguous. Hence, to circumvent this difficulty, ratios of quasielastic cross sections are proposed in Sec. VI as the most reliable predictions of the model.

II. QUALITATIVE ASPECTS OF QUASIELASTIC PION SCATTERING

Let us first make a number of remarks about the quasielastic process, in which a projectile scatters incoherently from the individual nucleons of the nucleus. For pions this will be investigated in depth for the first time at meson factories. One of the purposes here is to point out that conditions in the pionic case are sufficiently different from those in the more familiar examples of quasielastic scattering that there may be surprises in store for us when these experiments are done.

The prototype quasielastic reaction is proton knockout by high energy protons $(p, 2p)$. There are several nice reviews of this subject, of which the latest is Ref. 5. The corresponding process for incident pions is $(\pi, \pi N)$. In the following we will often compare parameters for the two knockout processes.

The $(\pi, \pi N)$ reaction occurs with large probability. Chivers *et al.*[6] have found that the total cross sections for ^{12}C$(\pi^-, \pi^- n)^{11}$C and ^{12}C$(\pi^+, \pi N)^{11}$C at 180 MeV are each about 75 mb. Thus more than half the inelastic *scattering* cross section involves nucleon knockout (or evaporation?), a point which is also clear from Refs. 2 and 3. There is a problem with these data, however. The simplest impulse approximation, together with isospin conservation, predicts the ratio of the π^- to π^+ cross sections to be 3; experimentally it is about 1. This implies there must be large distortion effects, e.g., final state interactions,[7] so that the simple argument no longer holds.

A large distortion might well be expected, since the πN interaction near the (3, 3) resonance is so strong. The total cross section per nucleon (i.e., averaged over neutrons and protons) is ≈130 mb at 190 MeV, corresponding to a mean free path for a πN scattering of ≈0.5 fm. In contrast, a 100 MeV proton has a mean free path some 3 times longer. That is, distortion is likely to be much more important for quasifree $(\pi, \pi N)$ reactions than for quasifree $(p, 2p)$ reactions.

Thus a first question to be answered is whether the $(\pi, \pi N)$ reaction can be described as a quasifree process at all. The radio-chemical data of Ref. 6 seem to say "no." On the other hand, the pole-approximation model[8] without distortion effects seems to agree reasonably well with the bubble chamber data of Ref. 2. The question can only be answered by further experiment.

An investigation of the $(\pi, \pi N)$ reaction in the resonance region using counters in coincidence would not be too difficult with pion beams soon to be available at meson factories, but the first extensive look at inelastic pion scattering will likely use a spectrometer.[4] *We thus consider from now on the* (π, π') *reaction,* in which the scattered pion has an energy T'_π appropriate to a quasifree πN scattering. The struck nucleon, presumably knocked out, is not observed in such an experiment. In calculating the quasielastic (π, π') cross section, therefore, one must integrate over the possible final states for this nucleon (as well as those of the recoil nucleus).

In scattering with a free nucleon the pion loses an energy

$$\Delta T_\pi = T_\pi - T'_\pi = (q^2/m)(1 - \cos\theta_{\mathrm{c.m.}}) , \tag{1}$$

where m is the nucleon mass and q and $\theta_{\mathrm{c.m.}}$ are the momentum and scattering angle in the πN center of mass. To distinguish the quasielastically scattered pions from those scattering from the nucleus as a whole, it is necessary that ΔT_π be greater than the energy loss corresponding to the highest nuclear levels excited. Presumably these are the collective giant resonance states, i.e., $\Delta T_\pi \geq 30$ MeV.[9] For such an energy loss at $T_\pi = 190$ MeV the scattering angle must be greater than 60°.

A typical spectrum for the scattered pions at such an angle might look like Fig. 1. The elastic scattering peak is at A, peaks due to inelastic scattering to particular excited nuclear states are at B, and the (expected) scattering which excites the broader giant resonance is at C, some 15 MeV or so below A. The quasielastic peak at D has a position given by Eq. (1), but considerably broadened by Fermi motion and probably shifted somewhat by the binding energy. This peak sits on top of a background of as yet unknown size. If the background does not swamp the signal, then the quasielastic differential cross section $d\sigma_{\mathrm{QE}}/d\Omega$ can be defined as the integral of $d^2\sigma/dT'_\pi d\Omega$ with respect to T'_π over this peak, less the contribution from the background. That is, $d\sigma_{\mathrm{QE}}/d\Omega$ corresponds to the shaded area in the peak at D.

The source of the background under the quasielastic peak is (incoherent) multiple scattering. For example, for a single scattering of $\theta_{\mathrm{c.m.}} = 90°$ at 190 MeV, $\Delta T_\pi = 55$ MeV. If, however, the pion undergoes two 45° scatterings, the combined $\Delta T_\pi = 31$ MeV, i.e., on the upper side of the quasielastic peak. On the other hand, if the double scattering consists of a 90° scattering followed by another of 180°, then $\Delta T_\pi = 130$ MeV, below the peak. In view of the free πN angular distributions, these two double scattering cases are roughly equally probable. Higher order scattering is more complex but will likewise contribute to the background under the direct single-scattering quasielastic peak. Finally, we remark that this background may well become more important for larger nuclei and at larger angles.

The cross section $d\sigma_{\mathrm{QE}}/d\Omega$, if it exists, should have an angular dependence like that of the free πN differential cross section, but modified somewhat by distortion effects. This relatively flat angular behavior will contrast sharply with that of the elastic pion-nucleus cross section, which falls off several decades in going away from the forward direction [for nuclei with $A \geq 10$ and pion energies near the (3,3) resonance].

Because of the relative πN and pN total cross sections, pion quasielastic scattering will be more of a surface phenomenon than proton quasielastic scattering. The pion scattering will thus take place more often on nucleons in outer shells.[10] Of course, a (π, π') experiment cannot give information on shell removal energies, as is the case for $(p, 2p)$, $(e, e'p)$, and, to a lesser extent, $(\pi, \pi N)$.

In the analysis of $(p, 2p)$ experiments it is helpful that the pp amplitude has a relatively smooth energy and angle dependence at the energies con-

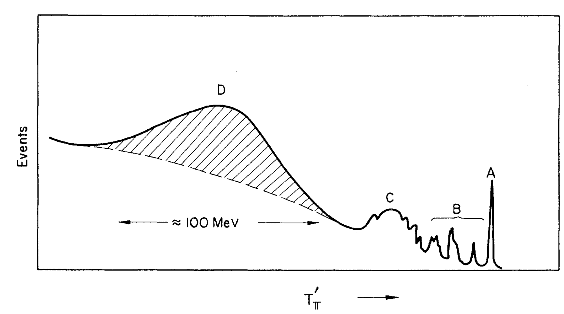

FIG. 1. Possible pion spectrum for scattering at an angle where the quasielastic peak at D is separated from the peaks (at A, B, and C) due to scattering from the nucleus as a whole.

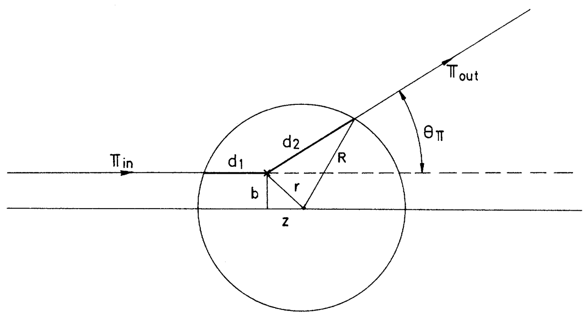

FIG. 2. Model geometry for the case $\phi = 0$. In general the interaction point $\vec{r}$ has cylindrical coordinates (b, z, ϕ).

sidered. This is *not* so for the πN amplitude near resonance and might be a source of trouble. The main effect of the rapid variations of the πN amplitude, however, may be a smearing of the free πN cross section due to Fermi motion. Since this cross section enters as an over-all factor, the smearing effect will tend to cancel in the ratios of quasielastic cross sections which we discuss in Sec. VI.

To conclude this section of qualitative remarks, we repeat that there are a number of uncertainties about what quasielastic pion scattering will be like and, indeed, even if it can be seen. If a quasielastic differential cross section $d\sigma_{QE}/d\Omega$ can be extracted from the (π, π') spectra, however, the semiclassical model to be discussed next may be expected to provide a good first description of such data.

III. SEMICLASSICAL MODEL

The following model for the quasielastic (π, π') reaction is similar to a rather successful model for pion production by medium energy nucleons.[11-14] More details may be found in these references.

A. Quasielastic differential cross section

The picture we have in mind is shown in Fig. 2. The incident pion travels in a straight line within the nucleus to some point $\vec{r}$, where it scatters from a (bound) nucleon and leaves the nucleus at a laboratory angle θ_π, again moving in a straight line. The incoming and outgoing energies, T_π and T_π', differ only by the energy loss in the single quasifree collision, Eq. (1). The recoil nucleon presumably also leaves the nucleus at the same time, but its energy and direction are unobserved. Both the incident and outgoing pion fluxes are attenuated, by πN scattering (including charge exchange) as well as by absorption in the nuclear medium. For simplicity we will ignore the small (≈3%) effects of charge exchange mixing in the pion transport as this necessarily involves two charge exchange reactions.[15]

The quasielastic differential cross section, as defined in the last section, is thus the product of the free πN differential cross section times an effective nucleon number,

$$\frac{d\sigma_{QE}}{d\Omega} = \frac{d\sigma_{free}}{d\Omega} N_{eff}(T_\pi, \theta_\pi; A) , \tag{2}$$

where, for the π^+ case,

$$\frac{d\sigma_{free}}{d\Omega} = \frac{Z}{A}\frac{d\sigma}{d\Omega}(\pi^+ p \to \pi^+ p) + \frac{N}{A}\frac{d\sigma}{d\Omega}(\pi^+ n \to \pi^+ n)$$
$$\approx \frac{Z + N/9}{A}\frac{d\sigma}{d\Omega}(\pi^+ p \to \pi^+ p) . \tag{3}$$

A similar expression holds for the π^- case but with N and Z interchanged. The second, approximate equality in Eq. (3) holds if the πN scattering is dominated by the (3, 3) resonance. Note that the scattering actually takes place on a bound nucleon and that nuclear corrections may be important[13]; we return to these corrections in Sec. V.

In accordance with the model described above, N_{eff} is defined as

$$N_{eff} = \int d^3r \rho(\vec{r}) e^{-d_1(\vec{r})/\lambda} e^{-d_2(\vec{r}, \theta_\pi)/\lambda'} . \tag{4}$$

Here $\rho(\vec{r})$ is the nuclear density, normalized to A. The factor $\exp(-d_1/\lambda)$ represents the probability that the incident pion penetrates to the interaction point $\vec{r}$, having traveled a distance d_1 through nuclear matter; $\exp(d_2/\lambda')$ is similarly the probability the pion escapes in the direction θ_π, traveling a distance d_2. The quantities $\lambda \equiv \lambda(T_\pi)$ and $\lambda' \equiv \lambda(T_\pi')$ are the mean free paths for pions of energy T_π and T_π', respectively, where

$$\lambda^{-1}(T_\pi) = \lambda_{scat}{}^{-1}(T_\pi) + \lambda_{abs}{}^{-1}(T_\pi) . \tag{5}$$

The scattering mean free path λ_{scat} is defined as

$$\lambda_{scat}{}^{-1}(T_\pi) = \rho(\vec{r}_\pi)\bar{\sigma}_{\pi N}(T_\pi) , \tag{6}$$

where $\bar{\sigma}_{\pi N}$ is the total πN cross section, averaged over neutrons and protons. Note that if the density is not constant, then λ_{scat} varies as a function of pion position $\vec{r}_\pi$. Presumably this is also true for the absorption mean free path λ_{abs}, but just how will be discussed below in Secs. III C and III D.

The N_{eff} depend on λ and λ' and thus are functions of T_π and θ_π. There is also an implicit dependence on θ_π through d_2 and on A through d_1, d_2, and ρ.

It is worthwhile at this point to make several

comments:
(1) The assumption of straight line trajectories is possible here because multiple scattering events lie in the background.
(2) The physical basis of the model is more or less the same as for the intranuclear cascade (Monte Carlo) calculations.[16] In such calculations some pion absorption mechanism should be included. The advantage of the present approach is that far less computation is involved. On the other hand, the Monte Carlo approach automatically includes the multiple scattering and thus deals with the size of the background under the quasielastic peak in a natural way.
(3) The semiclassical model here involves an assumption of incoherence between the contributions of the individual nucleons. The model is *not* applicable to a coherent process, such as elastic scattering, where the wave properties of the pion are crucial.
(4) There is a difference between this model and that of Refs. 11–14, in that in the latter case there was no damping factor due to *elastic* πN interactions. The desired quantity in the pion production calculations was a total production cross section, so the only attenuation there was due to reactions in which the pion of a given charge disappears, as in charge exchange or absorption processes. In the present case any elastic scattering before or after the one at $\vec{r}$ will throw the event into the multiple scattering background.
(5) There is a more or less well-known connection between this semiclassical model and a more quantum-mechanical approach using the distorted wave impulse approximation (DWIA).[17] The formula for N_{eff}, Eq. (4), can be derived at least heuristically from such a starting point using the closure approximation. Although it is perhaps difficult to assess the validity of the various approximations being made, one can see quite clearly that the attenuation factors have to do with the damping of the incoming and outgoing pion wave functions. The virtue of the present model is that it contains the basic desired features and is simple. A (much more complicated) DWIA calculation may well be in order at some future time when the experimental situation warrants it.

B. Nonuniform nuclear density

The integrals for N_{eff}, Eq. (4), can be easily evaluated numerically, given $\rho(r)$ and an assumption as to the density dependence of λ. The nuclear density we use here is the Woods-Saxon density,

$$\rho(r)=\rho_0[1+\exp(r-r_0)/a]^{-1}\,, \tag{7}$$

with parameters taken from electron-nucleus scattering analyses.[18]

For a nonuniform density the pion mean free path λ varies as the pion moves toward or away from $\vec{r}$. This is already seen in Eq. (6) for λ_{scat}. The absorption mean free path λ_{abs} is also a function of ρ, perhaps different. Thus the exponentials in the definition of N_{eff}, Eq. (4), become

$$\exp(-d_i/\lambda)\rightarrow\exp(-d_i/\lambda_{\text{scat}}-e_i/\lambda_{\text{abs}})\,, \tag{8}$$

where now λ_{scat} and λ_{abs} are *constant* parameters given in terms of the central density ρ_0 instead of $\rho(\vec{r}_\pi)$. The d_i are effective path lengths (instead of geometrical distances) defined by

$$d_1=\rho_0^{-1}\int_{-\infty}^{0}\rho(\vec{r}+s\hat{k})ds\,,$$
$$d_2=\rho_0^{-1}\int_{0}^{\infty}\rho(\vec{r}+s\hat{k}')ds\,, \tag{9}$$

where $\hat{k}$ and $\hat{k}'$ are unit vectors along the incoming and outgoing pion directions. The path lengths e_i appropriate to absorption are analogously defined, depending on the nature of the ρ dependence of $\lambda_{\text{abs}}^{-1}$ (see Sec. III D).

C. Present knowledge of λ_{abs}

The pion absorption process, in which the pion disappears, is a peculiarly nuclear phenomenon. By energy-momentum conservation it cannot take

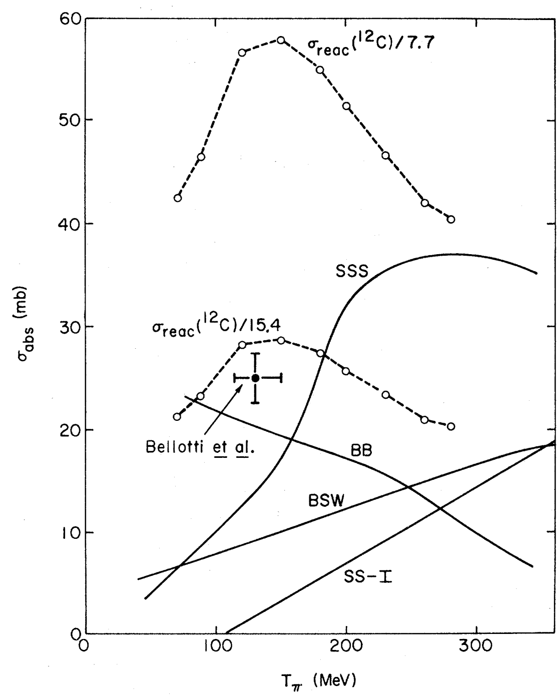

FIG. 3. Cross section per nucleon for pion absorption in nuclear matter. The different curves are described in the text (Sec. III E).

place on a single free nucleon (ignoring radiative capture). The mean free path λ_{abs} may well depend on T_π but nuclear structure effects will presumably give only small variations in λ_{abs} for different nuclei. From λ_{abs} we can define a cross section for pion absorption *per nucleon* according to

$$\lambda_{\text{abs}}^{-1}(T_\pi)=\rho_0\sigma_{\text{abs}}(T_\pi) \ . \tag{10}$$

The cross section σ_{abs} is not very well known over the energy region of the (3, 3) resonance. Figure 3 shows two different curves for $\sigma_{\text{abs}}(T_\pi)$ obtained from fitting data for $\pi^\pm$ production by 730 MeV protons.[19] Curve SS-I is based on a semiclassical model similar to the one presented above[12]; it contains almost no nuclear physics. Taking various nuclear corrections into account[13] gives curve SSS.[14] This latter curve is presumably more correct but nonetheless is not based on an absorption proportional to ρ^2 (see Sec. III D below); the effect of this on the determination of σ_{abs} is being investigated.[20] Finally, we remark that a determination of σ_{abs} based not on the pion spectrum at $\theta_\pi = 15°$ but the angle-integrated spectrum gives a rather flatter σ_{abs} at about 25 mb.[20]

Also shown in Fig. 3 are predictions of two models, curves BSW and BB, for the absorption process,[21, 22] both assuming a two-nucleon absorption mechanism. The model for curve BSW,[21] however, may have an unrealistic energy dependence for the input $pp \to d\pi^+$ cross section.

Experimentally, the only direct information available on σ_{abs} comes from a propane bubble chamber experiment[23] with a 130 MeV π^+ beam. Taking into account the shadowing effect in a crude way gives an effective number of absorbing nucleons of 7.7 instead of 12. The single datum plotted in Fig. 3 is this total ^{12}C absorption cross section divided by this number.

There is also an "upper limit" for σ_{abs} that can be obtained from the total *reaction* cross sections measured in the CERN $\pi^- + {}^{12}$C experiment.[24] This is plotted as the upper dashed curve in Fig. 3, again correcting for shadowing. If we were to further assume that about half the reaction cross section is due to pion absorption, as seems to be the case at 130 MeV[2,23] and at 260 MeV,[24] then we can draw the lower dashed curve in Fig. 3 as an experimental estimate of σ_{abs}. It is hard to assign any sort of error to this procedure, however.

Probably the only conclusion to be drawn from Fig. 3 is that, as yet, we have only meager knowledge of σ_{abs}. We can presume with some confidence, however, that $\sigma_{\text{abs}} \leq 40$ mb.

Connected with the uncertain value of $\sigma_{\text{abs}}(T_\pi)$, or more properly, of $\lambda_{\text{abs}}(T_\pi)$, is an uncertainty concerning the nature of the absorption mechanism itself.[25] Presumably this is a two-nucleon process (π, NN), since the single-nucleon reaction (π, N) requires a large momentum transfer to the residual nucleus. Certainly (π, N) cross sections are much smaller than those for (π, NN) (e.g., Ref. 26). For our purposes the contribution to λ_{abs} from (π, N) reactions can be safely ignored. There are, however, recent indications that at least some of the time pions are absorbed on more than two nucleons, such as α clusters.[25,23,27] This is a complication that we will ignore here.

D. Treatment of λ_{abs}

The assumption of two-nucleon absorption, made here, suggests that

$$\lambda_{\text{abs}}^{-1} \propto \rho^2(r_\pi) \ , \tag{11}$$

in which case

$$e_1 = \rho_0^{-2} \int_{-\infty}^{0} \rho^2(\vec{r} + s\hat{k})ds \ , \tag{12}$$

and likewise for e_2. Equation (12) assumes that the two absorbing nucleons are exactly at the same location at the time of the absorption. This is not quite right; for one thing it might violate the Pauli principle. To do the problem correctly with a correlation length, however, is more ambitious than is warranted by the insensitivity to λ_{abs} that will be shown below.

In the two-nucleon model the probability for absorption on like and unlike nucleons can be different. Accordingly we will assume that, for the π^+ case,

$$\lambda_{\text{abs}}^{-1} = \rho_0^2(NZ + \xi N^2)\alpha_{\text{abs}}(T_\pi)/A^2 \ , \tag{13}$$

with the π^- expression obtained as usual by interchanging N and Z. Here α_{abs} is an unknown function of T_π. The parameter ξ has the value $\frac{1}{10}$ in the simplest isobar model mentioned (but not used) in Ref. 12. A more realistic isobar model[28] or the production data[22] suggest that $\xi \approx \frac{1}{6}$. In any case, "like" absorption is probably small compared with "unlike" absorption. The uncertainty in ξ gives very little variation in results for N_{eff}.

It is convenient to define an absorption cross section per nucleon, σ_{abs}, in terms of the unknown α_{abs} by

$$\sigma_{\text{abs}}(T_\pi) = \tfrac{1}{4}(1+\xi)\rho_0^2\alpha_{\text{abs}}(T_\pi) \ . \tag{14}$$

For an $N=Z$ nucleus, then, Eq. (13) reduces to Eq. (10). [This, incidently, is why we wrote Eq. (13) with N^2 instead of $N(N-1)$].

At this point two remarks are in order. First, in Refs. 13 and 14 $\lambda_{\text{abs}}^{-1}$ was assumed proportional to ρ, like $\lambda_{\text{scat}}^{-1}$, rather than to ρ^2. Thus the exponentials there involved d_i/λ, d_i given by Eq. (9), rather than the more complicated situation of Eq.

(8) etc. Second, for a uniform sphere density, assumed in Ref. 12, there is no distinction between absorption proportional to ρ^2 or ρ. This is because d_i and e_i in this case are both equal to the geometrical path length.

E. Energy dependence of λ_{scat}

In the calculations to be presented in the next section, it is necessary to take account of the energy dependence of λ_{scat} entering through the total cross section $\bar{\sigma}_{\pi N}$. We will assume dominance of the (3, 3) phase shift, the form and parameters of which are taken as the "Breit-Wigner shape with two radii."[29] All results for N_{eff} at a given T_π can then be given in terms of two parameters λ_{abs} and λ'_{abs}.

IV. RESULTS AND DISCUSSION

A. Angular dependence of N_{eff}

Figure 4 shows the calculated values of $N_{eff}(\theta_\pi)$ for $\pi^\pm$ quasielastic scattering from ^{12}C at four incident energies. The bands labeled "$\sigma_{abs}=5$ mb" refer to a mean free path parameter for absorption for the incoming pion λ_{abs} that corresponds to that absorption cross section [see Eqs. (13) and (14)]. The top edge of the band refers to the outgoing mean free path parameter $\lambda'_{abs}=2\lambda_{abs}$ (i.e., half as large an absorption cross section at the outgoing energy) and the lower edge to $\lambda'_{abs}=\frac{1}{2}\lambda_{abs}$. The bands labeled "$\sigma_{abs}=40$ mb" are defined similarly. These choices span the expected range of $\lambda_{abs}(T_\pi)$. Figure 5 shows similar results for π^- scattering from ^{208}Pb (note change of scale) and Fig. 6 compares the $N_{eff}(\theta_\pi)$ for π^+ and π^- scattering from ^{208}Pb for $\lambda_{abs}=\lambda'_{abs}$ corresponding to $\sigma_{abs}=20$ mb.

The following remarks apply:

(1) The $\pi^-+{}^{208}Pb$ case has smaller N_{eff} than the π^+ case because the larger number of neutrons means that, because of the dominant $I=\frac{3}{2}$ interaction, π^- is more likely to suffer attenuation due to scattering.

(2) The energy dependence and dip structure in these curves reflects the (3, 3) resonance. The 200 MeV curves are relatively more backward peaked than the 100 MeV curves. This is because at 200 MeV there is relatively less attenuation due to scattering for a backward-going pion, $\bar{\sigma}_{\pi N}$ becoming smaller as T'_π falls more and more below the resonance energy. At energies above the resonance a dip appears in the angular distribution which moves to larger angles as the incident energy increases. The angle at which the dip occurs is that for which $T'_\pi=T_{res}$. This dip structure is a qualitative prediction of the model coming from the incoherence of the quasielastic πN scattering leading to the flux reduction factor $\exp(-\rho\bar{\sigma}_{\pi N}d)$. Its position and displacement with T_π should be independent of A or Z.

(3) Below resonance the spread and widths of the 5 and 40 mb bands is fairly large. At resonance and above these bands are generally much narrower and closer together. This is somewhat more

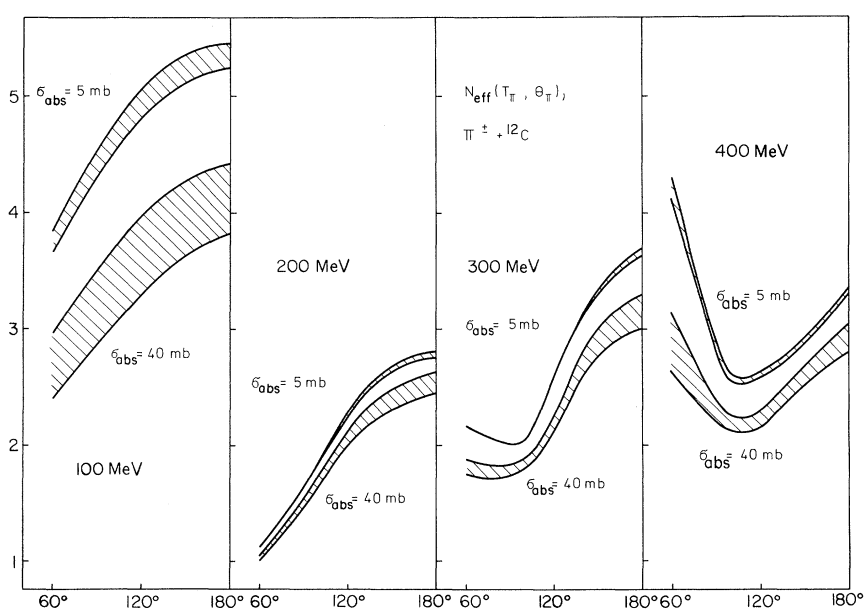

FIG. 4. $N_{eff}(T_\pi, \theta_\pi)$ for $\pi^\pm$ quasielastic scattering from ^{12}C. The bands correspond to $\sigma_{abs}(T_\pi)=5$ and 40 mb, with $\sigma_{abs}(T'_\pi)$ from one-half to twice as large as $\sigma_{abs}(T_\pi)$.

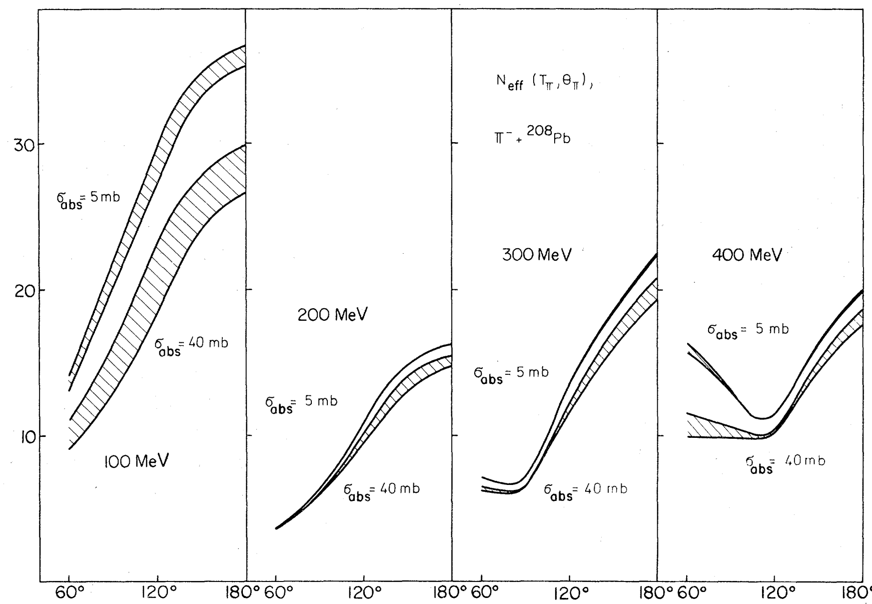

FIG. 5. Same as Fig. 4, but for $\pi^- + {}^{208}$Pb.

pronounced for ^{208}Pb than for ^{12}C. In fact, for 200 and 300 MeV, the difference between $\sigma_{abs} = 5$ and 40 mb gives only 20% differences in the predicted N_{eff}.

B. Comparison with earlier related work

The pion production calculations of Ref. 12 assumed a uniform sphere density. To check the importance of using a more realistic nuclear density we compared results for $N_{eff}(\theta_\pi)$ for the Woods-Saxon density (WSD) used above and the uniform sphere density (USD). It was found that the N_{eff} for the WSD are larger than those for the USD. This point has been noted before in other contexts (see, e.g., Ref. 30). Also, the WSD angular distributions are flatter than those of the USD. Both these differences are more pronounced when λ is small. Since $\lambda < \lambda_{scat} \leq 1$ fm in the resonance region, it is quite possible that the USD can give values for $N_{eff}(\theta_\pi)$, which are wrong by as much as a factor of 2 in the case of quasielastic

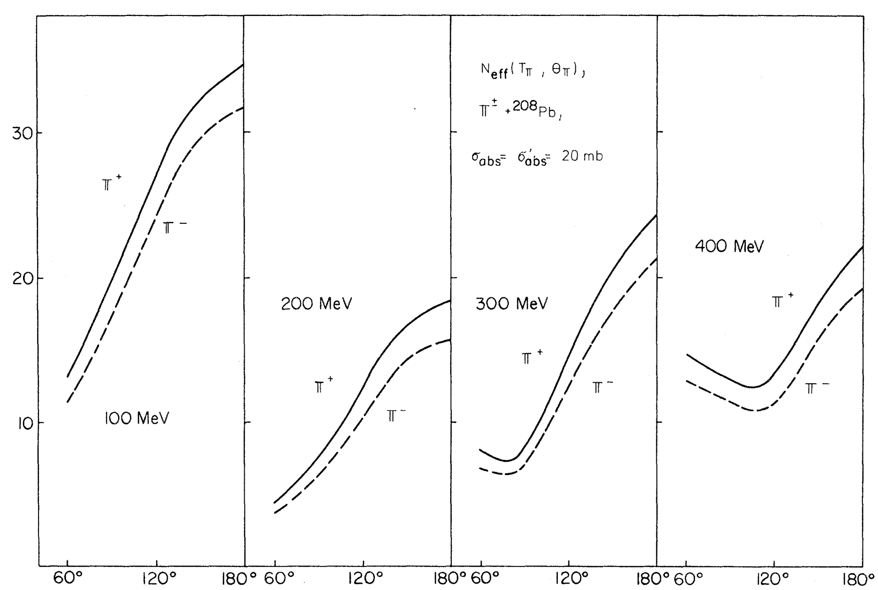

FIG. 6. $N_{eff}(T_\pi, \theta_\pi)$ for π^+ and π^- quasielastic scattering from ^{208}Pb; $\sigma_{abs}(T_\pi) = \sigma_{abs}(T'_\pi) = 20$ mb.

pion scattering. This difference is not so pronounced, however, in the pion production case.

As mentioned at the end of the last section, it is also the case that the pion production calculations of Refs. 11, 13, and 14 implicitly assumed $\lambda_{abs}^{-1} \propto \rho(r)$. In comparing this assumption with the two-nucleon assumption $\lambda_{abs}^{-1} \propto \rho^2(r)$, we found values of N_{eff} some 25% smaller than those presented above. This can be understood by noting that, for the WSD, $\rho(r)/\rho_0 \leq 1$, whence the $e_i \leq d_i$.

V. NUCLEAR CORRECTIONS

As was the case in Ref. 13, nuclear corrections to the input cross sections might be expected to have large effects here. In this section we will consider two different ways in which these corrections lead to a great deal of uncertainty in the model's predictions for N_{eff}.

A. Corrections to $\bar{\sigma}_{\pi N}$

The total πN cross section enters into the scattering mean free path through Eq. (6). This is the total cross section within nuclear matter, however, and might well be affected by such things as the Fermi motion of the struck nucleon, the Pauli principle, and the pion optical potential. The effects of these three things on the total charge exchange cross section was discussed in some detail in Ref. 13. In particular, three curves for $\sigma_{CEX}(T_\pi)$ were shown in Fig. 3 of that reference:
(A) The (3, 3) dominated free πN charge exchange cross section, which peaks at 180 MeV at a value of 45 mb.
(B) The same cross section as corrected for Fermi motion and Pauli principle, which has a broader and smaller peak of 22 mb at 220 MeV. The apparent upwards displacement comes from the fact that the Pauli principle suppression is most effective for low energies.
(C) The same cross section as B but including the effect of the pion optical potential on the pion's wave number. This curve again peaks near 220 MeV with an enhanced height of 38 mb.
For the total πN cross section in nuclear matter we can consider exactly the same corrections. Indeed, one can read off the values for σ_{CEX} from Fig. 3 of Ref. 13 and get $\bar{\sigma}_{\pi N}$ by simply multiplying by the appropriate factor.

There is a reasonable doubt, however, that it makes any sense at all to correct for pion optical potential (as was done in Ref. 13 and by many other authors).[31] The wave number of the pion inside the nucleus is indeed changed, *on the average*, according to the dispersion relation which gives the index of refraction. But it may be that, at the time of a particular πN collision, especially if the nucleons are usually farther apart than the range of the πN interaction, only the free wave number is what enters into the collision.[32] In such a case only on-shell πN amplitudes are to be used. In the following we will show the effect of this optical

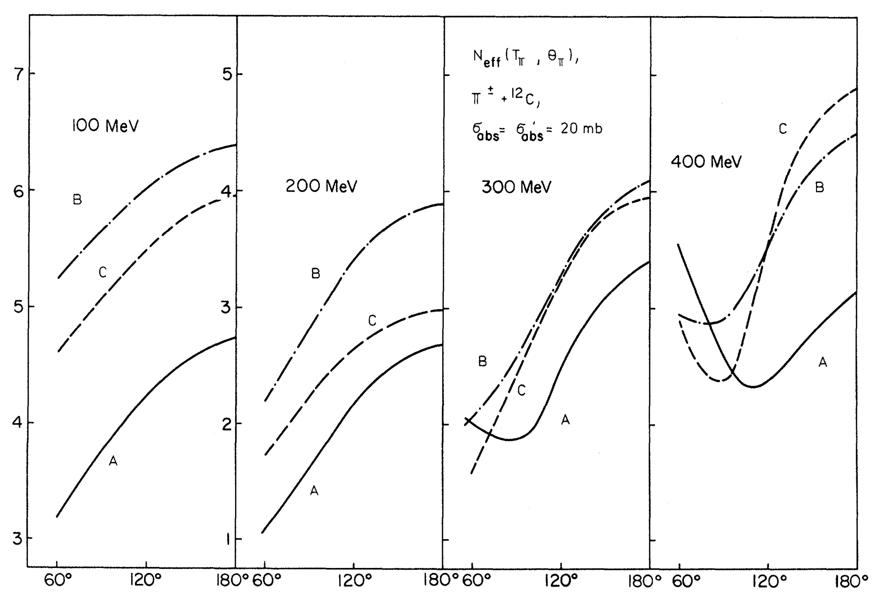

FIG. 7. Comparison of $N_{eff}(T_\pi, \theta_\pi)$ for ${}^{12}C$ for three choices of input cross section $\sigma_{\pi N}$: A, no nuclear corrections; B, corrections for Fermi motion and Pauli principle; and C corrections for Fermi motion, Pauli principle, and pion optical potential.

potential correction separately and let the reader decide if it ought to be included or not.

Figure 7 shows the results of calculations of $N_{\rm eff}$ for ^{12}C using these three different choices of $\bar{\sigma}_{\pi N}$, for the special case $\lambda_{\rm abs} = \lambda'_{\rm abs}$ corresponding to $\sigma_{\rm abs} = 20$ mb. The following conclusions can be drawn:

(1) There are very large effects (up to 100% or so) on $N_{\rm eff}$ due to these nuclear corrections.

(2) The inclusion of the pion optical potential, although questionable, is very important, as can be seen from comparing curves B and C. Unfortunately, even if this correction should be made, the prescription for handling this optical potential is not unambiguous. Some other approach (and one can imagine many) would likely give curves rather different from the C curves in Fig. 7.

(3) The position of the dip at higher energies is correlated with the position of the peak in $\bar{\sigma}_{\pi N}$. This gives hope that quasielastic pion scattering may be able to check whether the nuclear corrections to $\bar{\sigma}_{\pi N}$ (or to $\sigma_{\rm CEX}$ of Ref. 13) are actually present or not.

B. Corrections to $d\sigma_{\rm free}/d\Omega$

One might also hope to correct for the fact that the pion scatters off a bound nucleon by introducing a factor $F_{\rm corr}$ in Eq. (3),

$$\frac{d\sigma_{\rm QE}}{d\Omega} = \frac{d\sigma_{\rm free}}{d\Omega} F_{\rm corr} N_{\rm eff} , \tag{15}$$

where

$$F_{\rm corr} = \left(\frac{d\sigma_{\pi N}}{d\Omega}\right)_{\rm ave} \Bigg/ \left(\frac{d\sigma_{\pi N}}{d\Omega}\right)_{\rm free} . \tag{16}$$

The cross section in the numerator is the result of averaging the corrected cross section over the nucleus.

We have calculated $F_{\rm corr}$ using a simple Fermi gas model, taking into account Fermi motion, the Pauli principle, and the pion optical potential. We forego giving the details of this calculation here; the results are shown in Fig. 8, which shows also the effects of the Pauli principle and pion potential taken separately. We make the following remarks:

(1) The peaks around 80° (where there is a minimum in the free πN cross section, i.e., $\cos\theta_{\rm c.m.} = 0$) largely reflect the Fermi averaging process, which smears out the scattering angle. Away from this minimum the smearing does not affect things much.

(2) The Pauli principle suppresses the near forward scattering and at low energies it even reorders the 75, 100, and 150 MeV curves in the backward direction.

(3) The effect of the pion optical potential is large. E.g., $F_{\rm corr}$ at 200 MeV is twice what it would be otherwise. It even restores something of the original low-energy ordering [compare Figs. 8(b) and 8(d)]. Again we have the question of whether this correction is to be made.

The conclusion to be drawn from Fig. 8 (as well as Fig. 7) is a warning. We have corrected for the pion optical potential and in only one of many possible ways. The magnitude of $F_{\rm corr}$ (as well as $N_{\rm eff}$) is quite sensitive to this potential. We must conclude that $F_{\rm corr}$ is at best ambiguous (as is $N_{\rm eff}$) and that *direct* use of Eq. (15) to describe quasielastic (π, π') scattering would be inadvisable.

VI. RATIOS BETWEEN DIFFERENT NUCLEI

The uncertainty due the nuclear correction factor $F_{\rm corr}$ discussed in Sec. V B can be avoided by taking ratios of quasielastic cross sections between different nuclei. This is because $F_{\rm corr}$ is presumably independent of A and Z (it certainly is so in the Fermi gas model), and the factor cancels out. One might hope that the other uncertainty discussed in Sec. V A might also largely disappear, though in this case there is no direct cancellation.

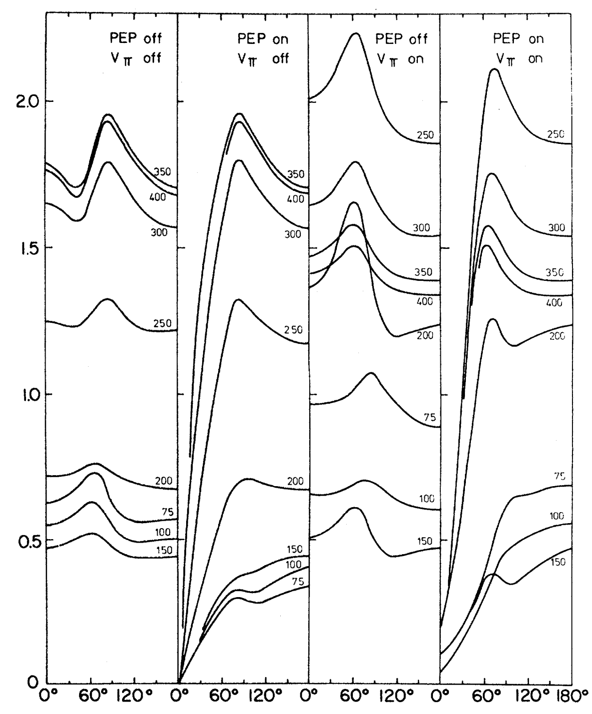

FIG. 8. Nuclear correction factor $F_{\rm corr}(T_\pi, \theta_\pi)$. The numbers attached to the curves give T_π in MeV. "PEP" means Pauli exclusion principle and "V_π" means pion optical potential.

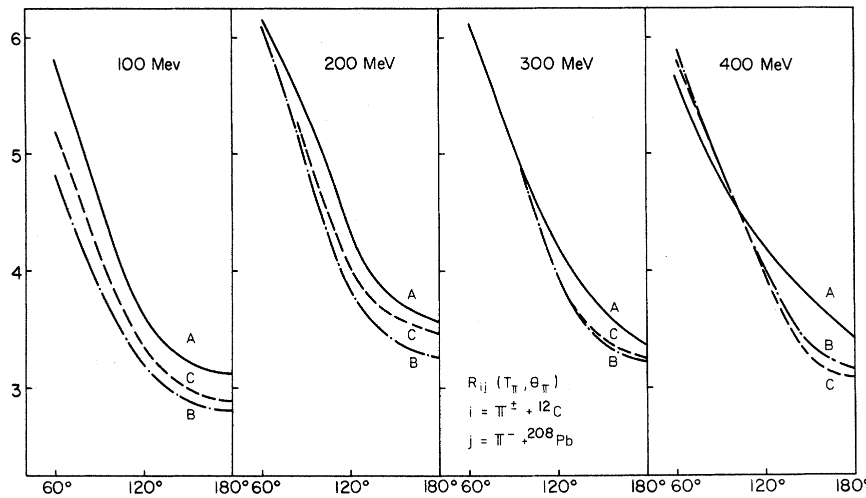

FIG. 9. The ratio R_{ij}, Eq. (18) for π^- case, comparing ^{12}C to ^{208}Pb. The three curves A, B, and C correspond to the same three choices for $\sigma_{\pi N}$ described in caption for Fig. 7.

We define the ratio between nuclei i and j as

$$R_{ij} = (A_j/A_i)(d\sigma_{QE}/d\Omega)_i/(d\sigma_{QE}/d\Omega)_j \ , \tag{17}$$

normalized so that $R_{ij} \approx 1$. The free πN cross section in Eq. (15), given by Eq. (3), does not cancel out if $N \neq Z$. We will assume dominance of the (3, 3) resonance. Thus, for π^+ scattering, the semiclassical model gives

$$R_{ij} = \frac{A_j^2}{A_i^2} \frac{(Z_i + N_i/9)}{(Z_j + N_j/9)} \frac{N_{\text{eff}}(Z_i, A_i)}{N_{\text{eff}}(Z_j, A_j)} \ , \tag{18}$$

and similarly for π^- scattering. Note that R_{ij} is a function of T_π and θ_π still.

We first examine the dependence of the R_{ij} on the various nuclear corrections to $\bar{\sigma}_{\pi N}$. Figure 9 shows the ratio for π^- scattering on ^{12}C to ^{208}Pb for the same three $\bar{\sigma}_{\pi N}(T_\pi)$ considered in Fig. 7. One now sees rather smaller (<17%) differences between the curves. The distinctive dip positions seen in Fig. 7 no longer appear. Thus we conclude the ratios R_{ij} are indeed less sensitive to this nuclear correction.

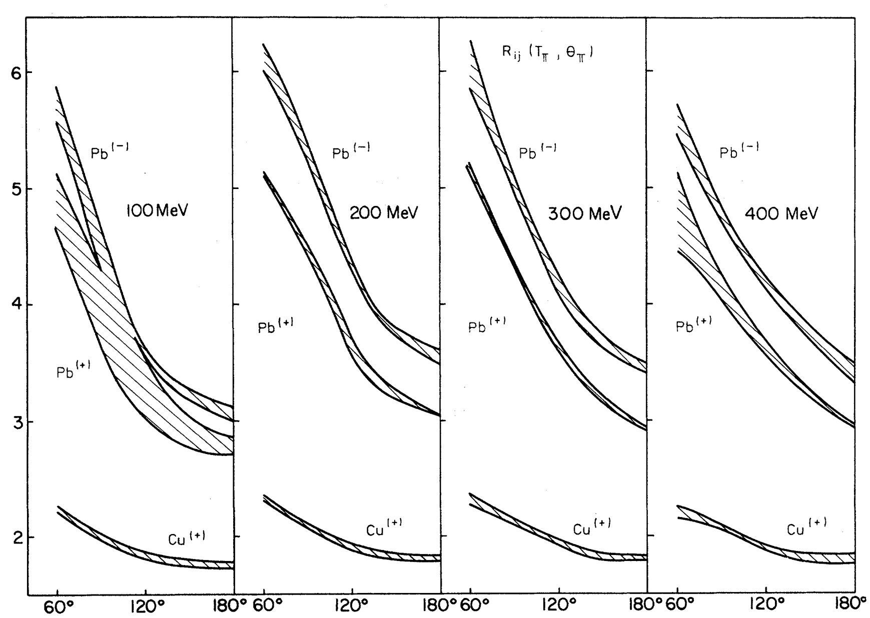

FIG. 10. The ratios comparing $\pi^\pm + ^{12}$C to $\pi^+ + ^{63}$Cu and $\pi^\pm + ^{208}$Pb scattering. The bands represent all values of $\sigma_{abs}(T_\pi)$ and $\sigma_{abs}(T_\pi)$ between 5 and 40 mb.

Figure 10 shows the predictions for the ratios of ^{12}C to ^{63}Cu and to ^{208}Pb, using the uncorrected $\bar{\sigma}_{\pi N}$ for values of $\lambda_{abs} = \lambda_{abs}(T_\pi)$ corresponding to σ_{abs} between 5 and 40 mb and for values of $\lambda'_{abs} = \lambda_{abs}(T'_\pi)$ from one-half to twice λ_{abs}. The following comments can be made:

(1) The R_{ij} are almost independent of T_π.

(2) There is a relatively sharp angular dependence. The falloff at backwards angles reflects the fact that as the size of the nucleus gets larger, the $N_{eff}(\theta_\pi)$ are relatively more backward peaked.

(3) As seen by the narrowness of the bands, most of the dependence on σ_{abs} in the R_{ij} has disappeared, except at the lowest and highest energies. Even in the best case, π^+ scattering from Pb, the variations of the R_{ij} for all reasonable σ_{abs} are less than 20%, and usually of the order of 4% or less.

VII. CONCLUSIONS

As we have seen, quasielastic pion scattering differs in a number of aspects from the more familiar $(p, 2p)$ and $(e, e'p)$ cases. It may therefore be unobservable in a (π, π') experiment because of a possibly large multiple scattering background. If a quasielastic differential cross section can be extracted from the inelastic scattering data, however, then the semiclassical model developed above suggests that:

(1) In contrast with pion production calculations, the effect of the uncertain pion absorption on the predicted effective nucleon numbers $N_{eff}(T_\pi, \theta_\pi; A)$ is small. The high probability for a πN scattering dilutes the effect of the absorption.

(2) The nuclear corrections are rather important. In this respect, the position of the dip in the angular dependence of the $N_{eff}(\theta_\pi)$ (see Fig. 7) may give clues as to the nature of these corrections.

(3) The ratios R_{ij} are rather less sensitive to these corrections (and still less sensitive to λ_{abs}). These are the most reliable predictions of the model and are basically statements of geometry. It will be interesting to see if experiment agrees with them.

In a sense it is unfortunate that this process is so insensitive to $\lambda_{abs}(T_\pi)$. As emphasized in Sec. III C, this quantity is only very roughly known. It is nonetheless an interesting and useful quantity. An example of its practical utility is seen in a recent calculation of pion production from nuclei by neutrinos.[33] Here the hope is to establish (or set upper limits for) the existence of weak neutral currents. In this case the uncertainty in λ_{abs} (as seen by comparing predictions using the fits of Refs. 12 and 13 as input) leads to $\approx$17% uncertainties in the predicted ratios of π° production with and without change of lepton charge.[33] It would be very useful for our understanding of weak interactions to eliminate this uncertainty.

More fundamentally for nuclear physics, λ_{abs} undoubtedly plays a significant role in understanding how pions interact with nuclei. For example, the imaginary part of the pion-nucleus optical potential has a term proportional to λ_{abs}. At low pion energies this contribution is clearly quite important.[34] At energies near the (3, 3) resonance the contribution also appears to be important, but its effect has not yet been seriously investigated.[1,35]

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[10] It might turn out that the quasielastic peak is therefore narrower for pions than protons. This effect might, moreover, depend upon the pion energy, being biggest at the resonance. There is also the possibility that the p-wave nature of the πN interaction gives, through the pion-nucleus optical potential, other surface effects depending upon derivatives of the nuclear density. In this regard, however, see

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