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Use of Glauber approximation at low energies

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To describe elastic and inelastic scattering, Glauber theory has been used often at an energy higher than 1 GeV with considerable success. It is also interesting to apply this model to scattering at lower energies to define its limits of validity. Cross sections and polarization values for proton scattering are calculated at 75, 155, 315, and 1040 MeV. The results for optical potentials in an exact solution or in the eikonal approximation are compared. First order corrections due to Wallacè are also included. These improve the fit at small momentum transfers.

[NUCLEAR REACTIONS ^{12}C, ^{208}Pb(p,p), $E=75$–1000 MeV; used Glauber model, calculated $\sigma(\theta)$, P, eikonal approximation and phase additivity effects.]

I. INTRODUCTION

Proton elastic scattering by nuclei has been calculated at high energy (500 MeV $\leq E \leq$ 20 GeV) with considerable success by means of the Glauber multiple scattering theory.[1,2] This method has the great advantage of leading to a straightforward calculation of elastic and inelastic scattering cross sections from a knowledge of free nucleon-nucleon scattering amplitude and densities, without adjustable parameters. It is also very interesting to make use of the same model for proton scattering at lower energies (75 MeV $\leq E \leq$ 320 MeV), i.e., far from the region of known validity of the Glauber model. Such calculations could indicate the range of validity of the Glauber approximation in projectile energy and angle, the effect of varying the input quantities, and perhaps give some idea whether systematic corrections improve the agreement between theoretical and experimental results. Moreover, since at these energies the nucleon-nucleon scattering amplitude is well known, we can take into account the projectile spin and calculate the proton polarization, which is a more sensitive test of the model. The polarization calculation is questionable or impossible at higher energies because of the imprecision of the NN amplitude.

II. SUMMARY OF THE THEORETICAL FORMALISM

In this paper we will study only the even-even nuclei and, thus, the NN scattering amplitude is used in the simplified form $f(q)=A(q)+C(q)\vec{\sigma}\cdot\vec{n}$, the other terms $B(q), \cdots$ not entering in a first order calculation; $\vec{\sigma}$ and $\vec{n}$ are the projectile spin and a vector normal to the projectile nucleon scattering plane. In order to take into account the target nucleus isospin, $A(q)$ is written as $A_\alpha(q)-[(N-Z)/A]A_\beta(q)$; Z, N, and A are, respectively, the number of protons, neutrons, and nucleons in the nucleus. A similar form of the spin term $C(q)$ is also required. The optical limit of Glauber theory is used; this approximation is good for any nucleus with $A\geq 10$. We have from Refs. 1 and 2 that the phase shifts of elastic scattering are

$$\chi(b)=\chi_1(b)+\chi_2(b)\vec{\sigma}\cdot\vec{n}' \tag{1}$$

with the central term

$$\chi_1(b)=(A/k_0)\int F(q)A(q)J_0(qb)q\,dq$$

and the spin dependent term

$$\chi_2(b)=i(A/k_0)\int F(q)C(q)J_1(qb)q\,dq\ ,$$

where $F(q)$ is the nuclear form factor calculated from the spherical matter density, parameters of

TABLE I. Matter density of studied nuclei (Fermi shape). In the case of ^{12}C, density is calculated by means of harmonic oscillator $\rho(r)=(4\nu^3/\pi^{3/2})(1+\frac{4}{3}\nu^2r^2)\times\exp(-\nu^2r^2)$ with $\nu^2=0.4$ fm^{-2}; the corresponding rms radius is $\langle R^2\rangle^{1/2}=2.30$ fm.

	$\langle R^2\rangle^{1/2}$ (fm)	c_p (fm)	a_p (fm)	c_n (fm)	a_n (fm)
^{56}Fe	3.65	4	0.57	4	0.57
^{208}Pb	5.6	6.6	0.5	6.9	0.5

which are in Table I; k_0 is the projectile momentum in the projectile-nucleon center of mass system. We have chosen only the phenomenological densities obtained from other analyses[3,4] and did not try to adjust the density parameters to improve the theoretical results.

J_0 and J_1 are cylindrical Bessel functions and $\vec{n}'$ is the vector defined by $\vec{b}\wedge\vec{k}_0/|\vec{b}\wedge\vec{k}_0|$.

The nucleon-nucleus scattering amplitude is:

$$\begin{aligned} T(q) &= T_1(q) + T_2(q)\cdot\sigma_Z, \\ T_1(q) &= ik\int J_0(qb)\{1-\tfrac{1}{2}[e^{i[\chi_1(b)+\chi_2(b)]}+e^{i[\chi_1(b)-\chi_2(b)]}]\}\,b\,db, \\ T_2(q) &= -\tfrac{1}{2}k\int J_1(qb)\{e^{i[\chi_1(b)+\chi_2(b)]}-e^{i[\chi_1(b)-\chi_2(b)]}\}\,b\,db. \end{aligned} \tag{2}$$

σ_Z is the spin component on the vector normal to the scattering plane. The polarization value is

$$P(q) = 2\mathrm{Re}[T_1^*(q)\cdot T_2(q)]/(d\sigma/d\Omega),$$

where $d\sigma/d\Omega = |T_1(q)|^2 + |T_2(q)|^2$, and k is the proton momentum in the nucleon-nucleus center of mass system. The antisymmetrized on-energy-shell scattering amplitudes are deduced from Yale NN phase shifts. The Coulomb effects are introduced by adding to the nuclear phase shifts $\chi(b)$ the Coulomb phase shifts calculated with the approximation of Ref. 5. The center of mass correction is made by means of the multiplicative factor $R(q)$ in front of $T(q)$, so $R(q) = \exp(q^2R^2/6A)$, R being the nuclear rms radius.

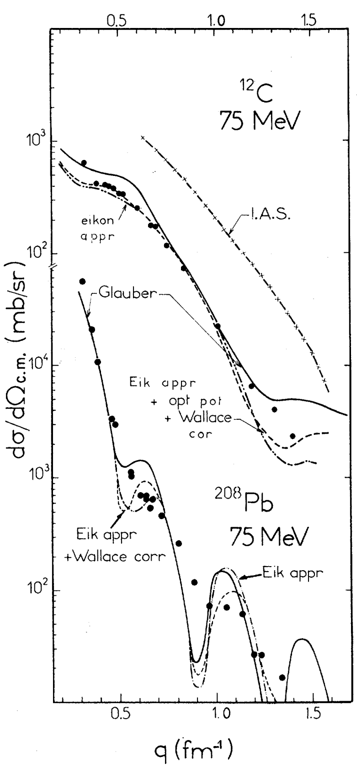

FIG. 1. 75 MeV proton elastic scattering on ^{12}C and ^{208}Pb. Experimental data are from Ref. 6. The curve I.A.S. corresponds to the impulse approximation simple (Ref. 24).

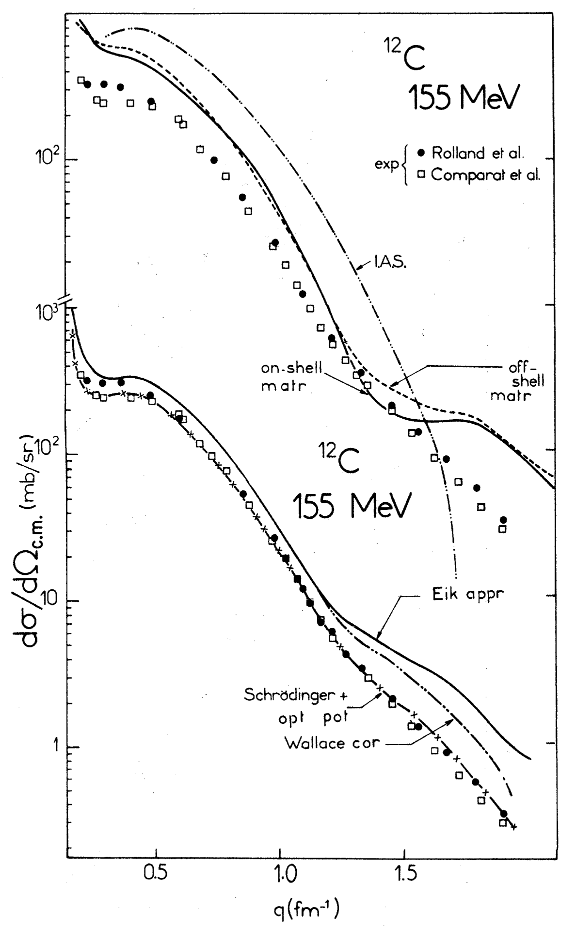

FIG. 2. 155 MeV proton elastic scattering on ^{12}C. Experimental data are: ● ● from Rolland *et al.* (Ref. 6); □□ from Comparat *et al.* (Ref. 7). The curve I.A.S. corresponds to the impulse approximation simple (Ref. 24).

III. COMPARISON OF EXPERIMENTAL DATA WITH THEORETICAL RESULTS

Angular distributions and polarizations are calculated for proton elastic scattering at 75,[6] 155,[6-8] and 315 MeV[9] on the nuclei ^{12}C, ^{56}Fe, and ^{208}Pb. Unfortunately, we are obliged to include ^{56}Fe at 315 MeV because no data on ^{208}Pb are available at this energy. Experimental data and theoretical results are plotted in Figs. 1 to 7.

At 75 MeV the agreement is good for $^{12}C(p,p)$ cross sections up to 1.3 fm^{-1} momentum transfer, i.e., before the first minimum, but the agreement is very bad for ^{208}Pb, for which the theoretical angular distribution is too diffractive. At 155 MeV the theoretical cross sections are systematically too large but are satisfactory. At 315 MeV the agreement is good up to the second minimum. For the polarization, the results are satisfactory only at very small momentum transfers. At 315 MeV the calculations are suitable and, although the cross sections are too large, they reproduce the minimum satisfactorily.

It is difficult to define a maximum angle for polarization, beyond which the agreement is poor. But in Fig. 6 it seems that the calculation is acceptable up to $qR = 1.7$, however high the energy of the projectile. That is confirmed also by results of Narboni[10] in which the elastic scattering and

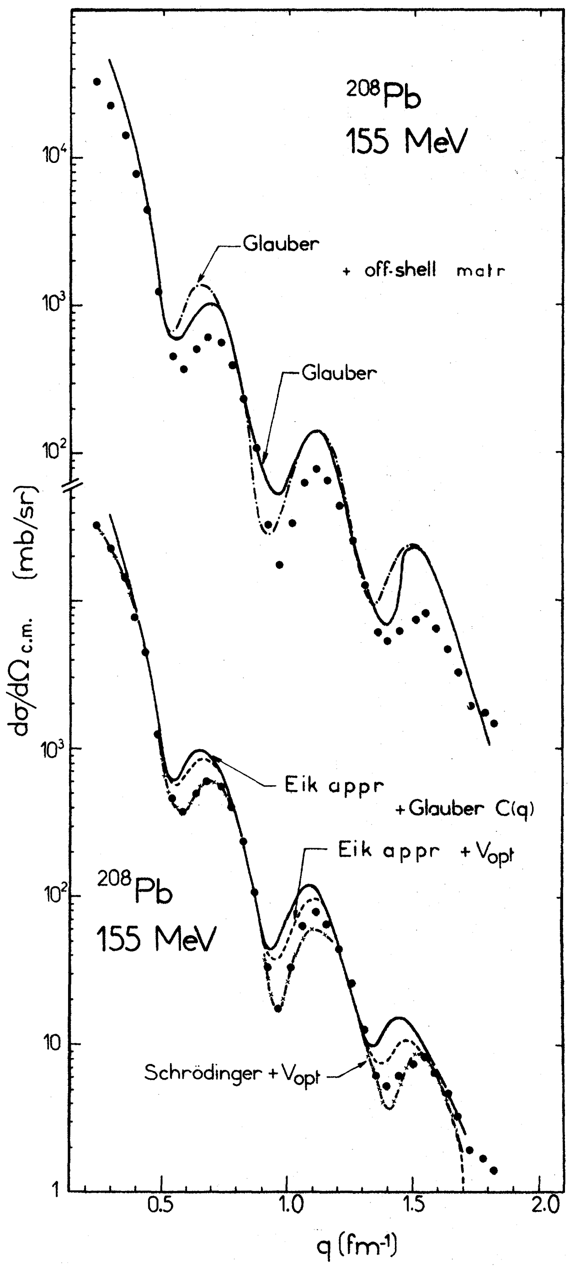

FIG. 3. 155 MeV proton elastic scattering on ^{208}Pb. Experimental data and curve obtained from the Schrödinger equation are from Ref. 7.

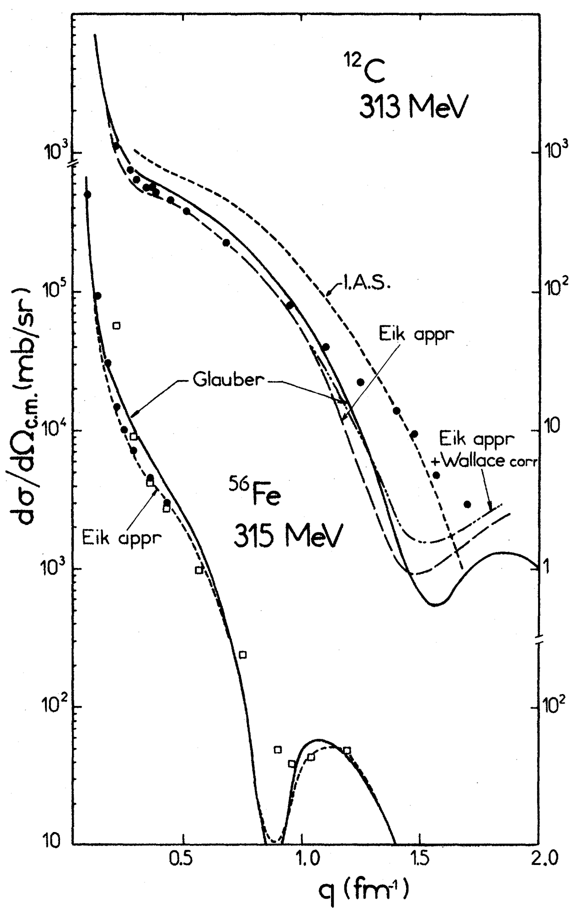

FIG. 4. 313 and 315 MeV proton elastic scattering on ^{12}C and ^{56}Fe. Experimental data are from Ref. 9. The curve I.A.S. corresponds to the impulse approximation simple (Ref. 24).

polarization of 50.6, 100, and 155 MeV protons on a ^{3}He nucleus are analyzed. A good agreement up to $q \leq 2\ \text{fm}^{-1}$ for the differential cross section and up to $q \leq 1.2\ \text{fm}^{-1}$ for the polarization is found by this author.

IV. VARIATION OF CROSS SECTIONS VERSUS PROJECTILE ENERGY

In Fig. 5 are shown experimental angular distributions on ^{12}C and ^{208}Pb nuclei and corresponding theoretical curves for 1040 MeV.[11] The theoretical cross sections at 1040 MeV are obtained with the same Glauber model without spin-orbit contributions using the *NN* scattering amplitude from a previous paper.[4] In Table II approximate values of momentum transfers corresponding to first minima are presented for all the nuclei studied and for all energies. For each nucleus these values are the same with good accuracy. Moreover, the corresponding values of qR (R being the rms radius) are constant for all energies and nuclei. This is confirmed by the high energy scattering results. For instance, at $P = 19.3$ GeV/c the differential cross section minima[12] are for $q = 0.6$, 1.0, and 1.4 fm^{-1} for ^{208}Pb; and at $P = 19.1$ GeV/c on ^{12}C, $q = 1.5\ \text{fm}^{-1}$. This qualitative result is reproduced by calculating the interferences of first- to n-order scattering of the projectile on various nucleons of the nucleus. In Ref. 13, Glauber gives an analytic calculation of these scatterings for a very simple form factor $F(q)$. A closely related result has been obtained by Hüfner.[14] This is explained by the relatively weak

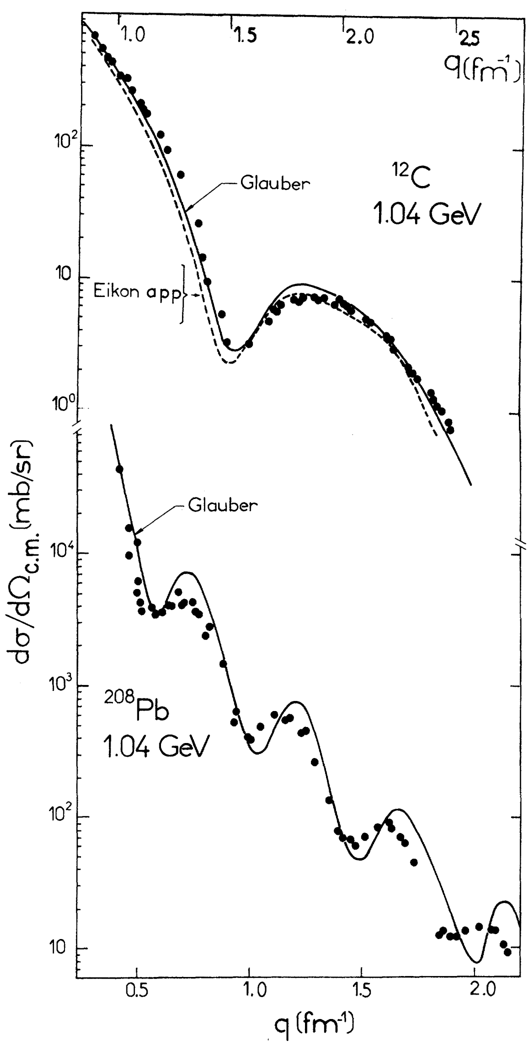

FIG. 5. 1.04 GeV proton elastic scattering on ^{12}C and ^{208}Pb. Experimental data are from Ref. 11.

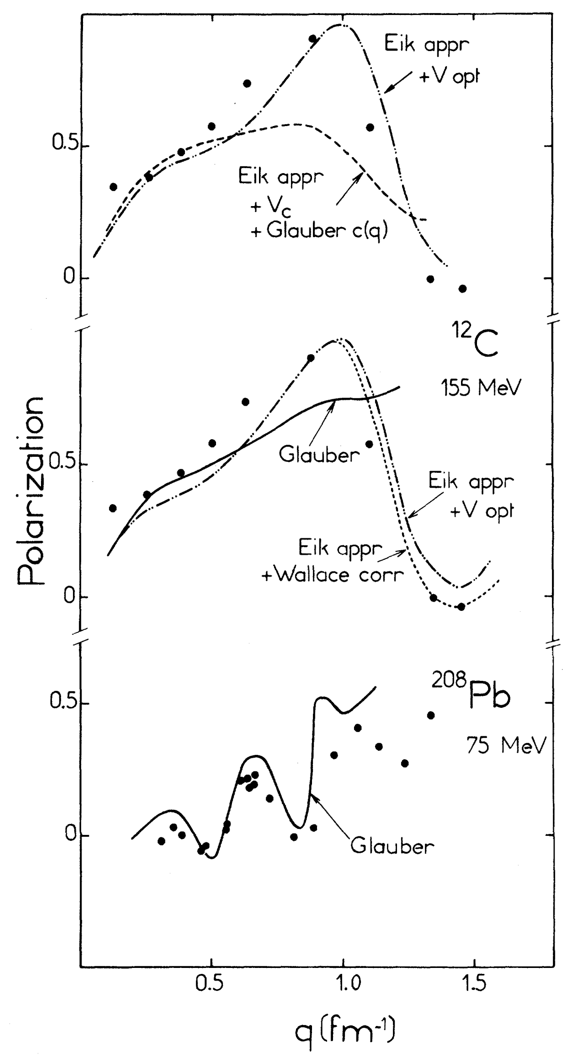

FIG. 6. Polarization results. References of experimental data are defined in corresponding cross sections figures except at 155 MeV for ^{12}C (Ref. 6).

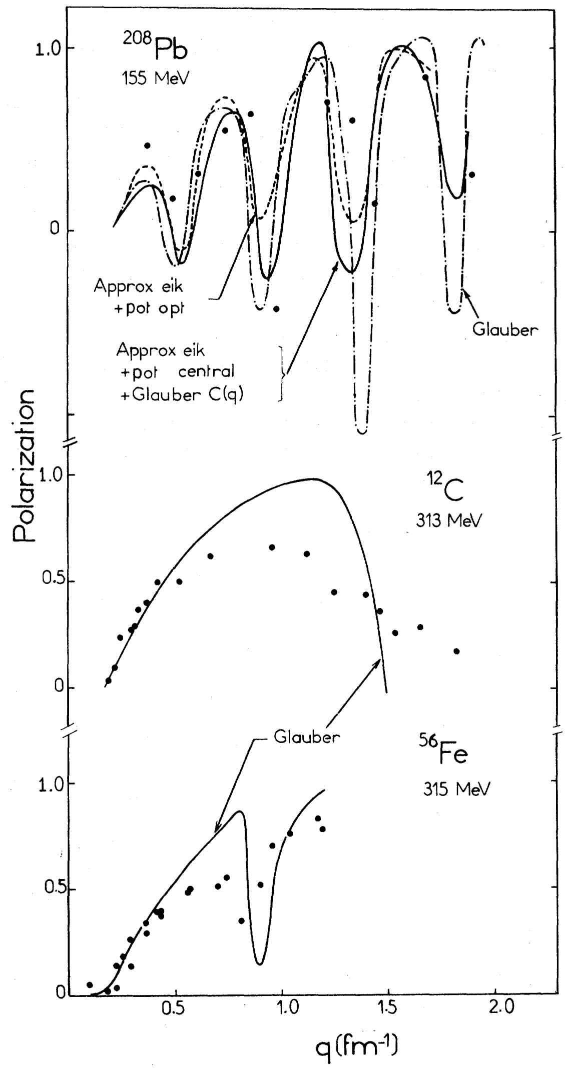

FIG. 7. Polarization results. References of experimental data are defined in corresponding cross section figures except at 155 MeV for ^{208}Pb (Ref. 8).

TABLE II. q and qR values corresponding to minima of angular distribution for different nuclei and energies. For Fe at 315 MeV these values are 0.95 and 3.45.

	^{12}C R=2.3 fm		^{208}Pb R=5.6 fm			
Energy MeV	q_1 (fm)	Rq_1	q_1 (fm)	Rq_1	q_2 (fm)	Rq_2
75	1.4	3.2	0.6	3.35	0.9	5.1
155	1.45	3.3	0.6	3.35	1	5.6
315	1.45	3.3				
1040	1.5	3.45	0.6	3.35	1	5.6

energy dependence, between 100 and 1000 MeV, of the NN total cross sections and NN interaction range. This range remains always much smaller than the nuclear radius R. This means that the position of the diffraction minimum is only defined by R (Ref. 15) and that the reaction always occurs on the nuclear surface. That is corroborated by the diffractional shape of experimental angular distribution whatever the energy of the projectile is. Such a characteristic of diffraction scattering is also observed at low energies, for example in the scattering of α particles.

V. DISCUSSION ON THE MULTIPLE DIFFRACTION MODEL

The hypotheses inherent in the Glauber model are discussed in many papers (principally in Ref. 1). They can be summarized as following:
(a) The scattering in the nucleus occurs by series of free collisions, i.e., supposing that the nucleons in the nucleus are sufficiently distant so that a projectile interacts only with one nucleon at a time. The principal consequence is the additivity of phase shifts.
(b) The use of the eikonal approximation to calculate the phase shifts; it assumes only forward scattering at small angles and a momentum transfer with no longitudinal component. This is corroborated by the fact that at high energy (for example 1 GeV) the NN scattering amplitude $A(q)$ is very concentrated at small angles, whereas at smaller energies (155 MeV for example) this amplitude is more isotropic.

A. Use of eikonal approximation

The use of the eikonal approximation is supposed to be sufficient up to a momentum transfer $q_L \ll (k/R)^{1/2}$. For a ^{12}C target at 75, 155, and 315 MeV, the values of $q_{\rm lab}$ are 0.9, 1.1, and 1.3 fm^{-1}; and for ^{208}Pb they are 0.6, 0.7, and 0.9 fm^{-1}. Consequently the calculation is expected to be valid for only very small momentum transfers q. The different figures confirm that, but also show that the agreement may extend to a larger angular distribution, principally for heavy nuclei. A simple explanation of this extension is that only the tail of the nuclear potential V is acting on the projectile because the reaction is peripherical; that means that the condition[1] $V/E \ll 1$ is true even for small energies E. However, these figures show that the agreement becomes very bad at the first minimum in the angular distributions for each nucleus and each energy. In the Glauber theory the minima are the interferences between single and double scatterings, double and triple scatterings, etc. It is normal that at this inter-

TABLE III. Parameters of Woods-Saxon optical potentials:

$$V_{\rm opt}(r)=\frac{V_R}{1+\exp[(r-r_R A^{1/3})/a_R]}+i\frac{W_I}{1+\exp[(r-r_I A^{1/3})/a_I]}+(V_{LS}+iW_{LS})\left(\frac{\hbar}{m_\pi}\right)^2\frac{d}{r\,dr}\left\{\frac{1}{1+\exp[(r-r_{LS}A^{1/3})/a_{LS}]}\right\}.$$

At 75 MeV the imaginary part is of surface shape.

	V_R	r_R	a_R	W_I	r_I	a_I	V_{LS}	W_{LS}	r_{LS}	a_{LS}
^{12}C $E=1.04$ GeV	40.79	0.948	0.536	126.50	0.988	0.531				
^{12}C $E=313$ MeV	15	1.05	0.5	18	1.05	0.5				
^{12}C $E=155$ MeV	10.9	1.49	0.43	14	1.12	0.6	3	−1.30	0.915	0.48
^{208}Pb $E=155$ MeV	21	1.15	0.66	14	1.30	0.55	3	−1.30	1.15	0.84
^{12}C $E=75$ MeV	25.44	1.13	0.49	8.51	1.45	0.45	0.72		1.86	0.40

ference the deficiency of the model is more apparent. This effect was previously estimated by Hüfner[14] and recently by Lombard,[16] who showed that the eikonal approximation at 1 GeV and for ^{4}He produces an angular distribution which is too diffractive. Yet for ^{12}C at 1 GeV, Lesniak and Wolek[17] obtain a smaller effect. This may indicate that the eikonal approximation is better for heavy nuclei.

We have done two tests to evaluate the importance of this approximation:

(a) Equations (2) are used but the nuclear phases $\chi_1(b)$ and $\chi_2(b)$ are calculated with the simple eikonal approximation from phenomenological optical potentials of Woods-Saxon shape. The parameters of these potentials are obtained[6,7,18] from an optical model code, like JIB by Perey. The phase shifts are

$$\chi_1(b)=-\frac{1}{\hbar v}\int_{-\infty}^{+\infty}V_c(b,z)dz \tag{3}$$

and

$$\chi_2(b)=-\frac{1}{\hbar v}\int_{-\infty}^{+\infty}V_{LS}(b,z)kb\,dz\ , \tag{4}$$

where $V_C(r)$ and $V_{LS}(r)$ are, respectively, the central and spin-orbit parts of the potentials. The corresponding parameters are presented in Table III.

(b) In order to study separately the effects of χ_1 and χ_2, only $\chi_1(b)$ is calculated by Eq. (3), and $\chi_2(b)$ is deduced from the Glauber expression (1) with $C(q)$. The cross sections and polarization values obtained are plotted in Figs. 1–3, 6, and 7. The angular distributions corresponding to an "exact" calculation, i.e., by the resolution of the Schrödinger equation, are also drawn on these figures. In this last case the agreement with data is, of course, excellent.

Therefore, the approximation (a) is found to be better chiefly at small momenta than the method (b), especially for ^{208}Pb.

However, the exact calculation by the Schrödinger equation is always more satisfactory near diffraction minima. For ^{12}C, the polarization values are well reproduced by method (a) and poorly by method (b). For ^{208}Pb, the results are of medium quality with either approximation. We have calculated the elastic scattering differential cross section on ^{12}C at 1 GeV using equation (3) with optical potential parameters which are presented in Table III. These parameters are extracted from exact phenomenological analysis by Willis.[19] At this energy the agreement is very good, emphasizing that this approximation is perfectly valid. More recently Wallace[20] has proposed an improvement on the simple eikonal approximation by adding some corrections. We have calculated the first correction term to evaluate its importance and to confirm the role of the eikonal approximation in the observed disagreements with experimental data. This correction consists in adding to $\chi_1(b)$ [calculated by Eq. (3)] a phase shift

$$\tau_1(b)=-\frac{1}{k\hbar^2v^2}\int_{-\infty}^{+\infty}\left(2+r\frac{\partial}{\partial r}\right)V_{\rm opt}{}^2(b,z)dz\ , \tag{5}$$

v being the projectile velocity in the nucleon-nucleus center of mass system.

The angular distributions at 75, 155, and 313 MeV for ^{12}C are presented in Figs. 1, 2, and 4. The results of Wallace are confirmed: this first correction slightly improves at these energies the cross section for small momentum transfers and is as efficient at 75 MeV as at 155 MeV. In order to improve the cross section, it is necessary to introduce higher order terms in the eikonal expansion but the calculation becomes very tedious. The same conclusion is obtained for Pb but the disagreement remains always important. Moreover, the polarization is only slightly improved by taking $\tau_1(b)$ into account (Fig. 6). The result of these tests is that the eikonal approximation itself is responsible for a part of the disagreement. The error is more important at 155 MeV (and at 315 MeV) than at 1 GeV.[12,16] Gillespie, Gustafson, and Lombard[21] have studied the importance of the three first Wallace corrections and confirmed these present results showing that Wallace series converges more rapidly at 1 GeV than at 100 MeV.

If we obtain an almost equivalent polarization with a Glauber calculation or with a phenomenological spin-orbit optical potential, it means that the knowledge of this potential $V_{LS}(r)$ is not sure and also that the calculation of the polarization with multiple scattering theory is insufficiently accurate. Thus the angular range where the agreement is satisfactory between experimental and theoretical results is smaller for the polarization than for the cross section. That was also observed by Narboni[10] for 155 MeV proton scattering on ^3He.

B. Phase additivity

For the energies studied ($E < 500$ MeV), the first hypothesis on the phase additivity is doubtful. The interaction range r_p calculated from $A(q)$ is about 1.5 fm at 155 MeV and 1.4 fm at 315 MeV while the mean distance between nucleons in nuclei is about 1.3 fm. On the contrary, for higher energies r_p is much smaller, about 0.8 fm for 1 GeV projectiles,[3,4] and the hypothesis is more reliable.

If the projectile interacts simultaneously with two nucleons of the target, the effect connected to the error associated with this hypothesis would be proportional to A (the number of nucleons in the nuclei) and not to $A(A-1)$, because only the neighboring nucleons are involved. But the disagreement which subsists between the angular distribution calculated by the Glauber model and by the eikonal approximation seems similar for ^{12}C and ^{208}Pb.

Moreover, we need to emphasize that the reaction occurs at the surface of the nucleus where the nuclear density is small and therefore the probability that the projectile has to interact simultaneously with two nucleons is reduced.

In Sec. V A the eikonal approximation using a phenomenological potential has been tested. But such potentials include many effects neglected in the Glauber model: additivity of phases, intermediate states transitions, etc. To separate these effects we have calculated an optical potential $V_{opt}(r)$ directly from the Glauber phase shifts $\chi_1(b)$ by the well known relation [Ref. 1, Eq. (201)]

$$V_{opt}(r) = \frac{\hbar v}{\pi} \frac{1}{r} \frac{\partial}{\partial r} \int_0^\infty \frac{\chi_1(b)\, b\, db}{(b^2 - r^2)^{1/2}} .$$

The result obtained for ^{208}Pb at 155 MeV is plotted in Fig. 8 together with the Woods-Saxon phenomenological potentials. It is shown that the calculated potential seems specially reliable on

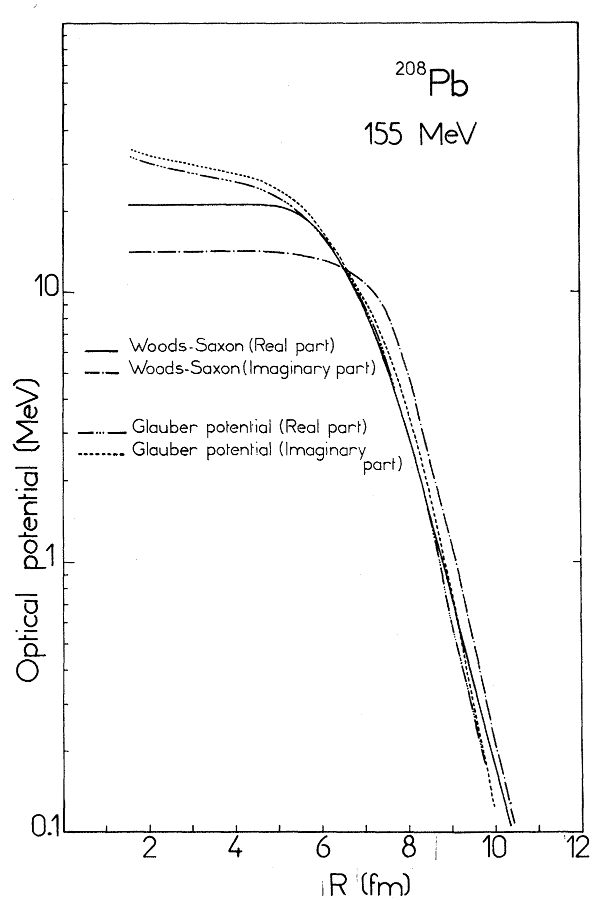

FIG. 8. Optical potentials calculated from Glauber model. Comparison is done with phenomenological potentials (Table III).

the nuclear surface. The agreement for the nuclear interior is poor, but it is known that this region does not participate in the reaction mechanism. The angular distributions corresponding to these calculated potentials are obtained by using the code JIB3. These results are presented in Fig. 9 at different energies only for Pb and Fe in order not to increase the number of figures.

Comparing these curves to those from Figs. 1,

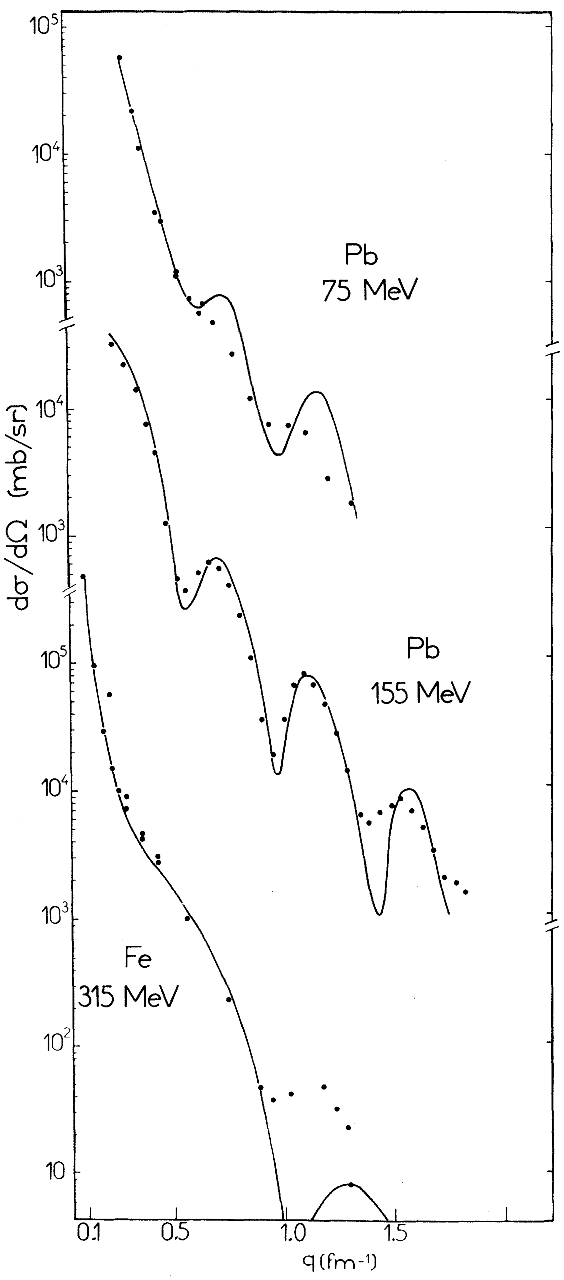

FIG. 9. Angular distributions calculated by the exact Schrödinger equation and Glauber optical potentials.

3, and 4, we see the agreement is better at small momenta chiefly at 155 and 315 MeV; at 75 MeV, the pattern is still too diffractional. But from this comparison, we establish that the phase additivity, the principal hypothesis, is fairly reliable at 155 and 315 MeV. On the contrary, the eikonal approximation is certainly the most unacceptable hypothesis except at 315 MeV. At this energy, these two hypotheses seem equivalent. The angular distributions calculated by eikonal approximation from phenomenological approximation and the ones calculated by Glauber potentials and the Schrödinger equation are not failing at the same momenta q. That means that sometimes there can be compensation of the different approximations in the Glauber model calculations, and it is difficult to distinguish the effects and the importance of each one.

C. Two other approximations in Glauber theory

(i) The two particle scattering amplitude is taken on-shell as in the impulse approximation. At 155 MeV, Ballot and L'Huillier[22] have calculated off-shell NN amplitudes. These amplitudes have been introduced in the present calculations using the Glauber theory for the ^{12}C and ^{208}Pb cross sections. The difference is very small. This can be explained by the small momentum transfers involved in these calculations, leading to kinematics very near to the energy shell. For the polarization the disagreement is more important, emphasizing a greater sensitivity to the off-energy shell amplitude.

(ii) In the previous calculations the Glauber theory was used, neglecting two step mechanisms via an intermediate excited state. So we have neglected many terms of the NN amplitude like $B(q)$; these terms are important and are not negligible in such transitions by states different from the ground state. However, the corrections are certainly not important at small momentum transfer and just limit the range of validity. Let us note that these corrections are in part included in the phenomenological potentials and the improvement obtained by using these potentials instead of Glauber potentials is partly due to these corrections.

Wilkin[23] suggested a new improvement by omitting in the Glauber calculations the values of $A(q)$ which increase at high momentum transfers and correspond to the backward nucleon-nucleon scattering cross section, and which have no sense in this model. But with this ansatz the Glauber model gives the same result, because in Eq. (1) the form factor is very small at high momenta.

VI. CONCLUSION

In this paper we have used the Glauber approximation at low energies to test the limits of its validity, and to better understand the satisfactory results at 1 GeV. For cross sections the agreement is adequate up to the second maximum for light nuclei and further for heavy nuclei. Therefore, a semiclassical approach is acceptable in the angular range connected with nuclear surface. Thus the diffractional structure of angular distributions is defined only by the nuclear radius. However, the comparison with the simple impulse approximation shows the great importance of multiple scattering, mainly when the momentum transfer increases. The important disagreement in the diffraction minima is largely due to the eikonal and additivity approximation. Many improvements are possible, for instance the Wallace corrections. However, the calculations quickly become very tedious. On the other hand, these calculations do not take into account the rescattering with intermediate excited states different from the ground state. These processes contribute chiefly to large momentum transfers, especially at the diffraction minima and for nuclei such as ^{12}C having strongly excited collective levels.

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