Resonating-group study and importance of exchange processes in $N + {}^{16}O$ scattering*

D. R. Thompson and Y. C. Tang
School of Physics, University of Minnesota, Minneapolis, Minnesota 55455
(Received 20 November 1974)

The scattering of nucleons by ^{16}O is studied with the resonating-group method and a generator-coordinate technique. The result shows that, at relatively high energies, the knockout process contributes significantly to the scattering amplitude, while the pickup process is of minor importance.

[NUCLEAR REACTIONS $^{16}O(p,p)$, $E=37.4$ MeV; calculated $\sigma(\theta)$ and $P(\theta)$. Resonating-group method with generator-coordinate technique.]

For elastic scattering involving light nuclei with $A \leq 4$, there is ample evidence that exchange processes, arising as a consequence of the indistinguishability of the nucleons, play an important role. Theoretical studies[1,2] using the resonating-group method have shown that these processes must be properly accounted for, if a satisfactory agreement with experimental data, especially at backward angles, is to be obtained. Because of the presence of these exchange processes (knockout and pickup processes), the effective interaction between two nuclei is necessarily very complicated, being both nonlocal and energy-dependent.[3,4] However, these resonating-group studies did show that, at relatively high energies, the features of the elastic scattering problem can be fairly well reproduced, if the real central part of the effective interaction is represented by a potential of the form

$$V_{\text{eff}} = V_D(r) + V_a(r) + V_b(r)P^r \ , \tag{1}$$

where P^r is a Majorana operator exchanging the position coordinates of the two nuclei, $V_D(r)$ is the direct potential obtained by a folding procedure[5] using a wave function which is not antisymmetrized with respect to nucleons of different clusters, and $V_a(r)$ and $V_b(r)$ are energy-dependent potentials representing knockout and pickup processes, respectively.[1]

Empirically, it has already been found[6] more than 10 years ago that the $p + {}^4He$ effective interaction at 32 MeV contains a significant amount of Majorana component. More recently, potential-model analyses of $p + {}^3He$,[7] $p + {}^4He$,[7] and $^3He + {}^4He$[8] scattering data have also shown that the inclusion of a Majorana term in the effective potential is essential in fitting experimental results. As for the term $V_a(r)$ in Eq. (1), its importance is harder to determine from an empirical viewpoint, because it appears additively to the dominant term $V_D(r)$. However, a resonating-group study of $^3He + {}^4He$ scattering[2] does indicate that its magnitude can also be quite appreciable.

The situation seems to be somewhat different in proton scattering on a heavier nucleus. In $p + {}^{40}Ca$ scattering at 30 MeV, where a resonating-group study is not yet available but a careful optical-model analysis[9] has been made, it was found that $V_b(r)$ has a small magnitude compared to $V_D(r)$. This indicates that the pickup process is not very important in this case, in contrast to the findings in lighter systems. In view of this, it seems, therefore, very interesting to study the intermediate case of $N + {}^{16}O$ scattering using the resonating-group method, in order to achieve a better understanding of the roles played by these exchange processes.

We should mention that, because of computational complexity as the mass number becomes large, previous resonating-group calculations[10] have dealt mainly with scattering and reaction problems involving very light clusters. Recently, however, Sünkel and Wildermuth[11] have proposed a generator-coordinate technique which simplifies greatly the computational problem. In this investigation, we shall therefore apply this technique to derive the integrodifferential equation describing $N + {}^{16}O$ scattering in the one-channel resonating-group approximation. Our experience gained from this calculation does show that this technique is quite useful and can be employed to solve more complicated problems, such as $^{40}Ca(p,p)^{40}Ca$ and $^{16}O(p,\alpha)^{13}N$.

In the one-channel approximation, the wave function for the $p + {}^{16}O$ or $n + {}^{16}O$ system is written as

$$\Psi = \mathcal{A}' \left\{ \phi_{16}\delta(t_{17}, \pm\tfrac{1}{2}) \sum_{J=\frac{1}{2}}^{\infty} \sum_{l=J-\frac{1}{2}}^{J+\frac{1}{2}} \frac{1}{r} f_{Jl}(r) \mathcal{Y}^{\frac{1}{2}}_{Jl\frac{1}{2}} \right\}, \tag{2}$$

where $\mathcal{A}'$ is an operator which antisymmetrizes the wave function with respect to nucleons in different clusters, $\delta(t_{17}, \pm\frac{1}{2})$ is a nucleon isospin function, and $\mathcal{Y}^{\frac{1}{2}}_{J l \frac{1}{2}}$ is a normalized spin-angle function.[12] The function ϕ_{16} is a translationally invariant function describing the behavior of the ^{16}O cluster and is chosen to have a $(1s)^4(1p)^{12}$ configuration in a harmonic-oscillator well of width parameter α; it has the form

$$\phi_{16} = \mathcal{A}_{16}\left\{\left[\prod_{i=1}^{16} h_i(\vec{\eta}_i)\right]\exp\left(-\frac{\alpha}{2}\sum_{i=1}^{16}\eta_i^2\right)\xi(s,t)\right\}, \tag{3}$$

where $\mathcal{A}_{16}$ is a 16-particle antisymmetrization operator, $\xi(s,t)$ is an appropriate spin-isospin function, and $\vec{\eta}_i = \vec{r}_i - \vec{R}_{16}$ with $\vec{R}_{16}$ being the center of mass (c.m.) coordinate of the ^{16}O cluster. The functions $h_i(\vec{\eta}_i)$ are polynomials; they are equal to 1, $\eta_i Y_1^1(\hat{\eta}_i)$, $\eta_i Y_1^0(\hat{\eta}_i)$, and $\eta_i Y_1^{-1}(\hat{\eta}_i)$ for i = 1–4, 5–8, 9–12, and 13–16, respectively. The width parameter α is chosen to yield the experimentally determined rms matter radius of ^{16}O; its value is 0.32 fm^{-2}, which corresponds to a rms radius of 2.6 fm.

The relative-motion functions f_{Jl} in Eq. (2) are determined from the projection equation

$$\langle \delta\Psi | H - E' | \Psi \rangle = 0 , \tag{4}$$

where E' is the total energy, composed of the internal energy E_{16} of the ^{16}O cluster and the relative energy E in the c.m. system. The nucleon-nucleon potential V_{ij} employed has the same form as that used in our previous study of $N+\alpha$ scattering[12] and is given by Eqs. (5)–(7) in Ref. 12.

Using Eq. (4), we obtain the following integro-differential equations for $f_{Jl}(r)$:

$$\left\{\frac{\hbar^2}{2\mu}\left[\frac{d^2}{dr^2} - \frac{l(l+1)}{r^2}\right] + E - V_D(r) - V_C(r) - \eta_{Jl} V_{so}(r)\right\} f_{Jl}(r) = \int_0^\infty k_l(r,r') f_{Jl}(r') dr' , \tag{5}$$

where $V_C(r)$ and $V_{so}(r)$ are the direct Coulomb and direct spin-orbit potentials, respectively, and η_{Jl} is given by Eq. (9) of Ref. 12. In computing the spin-orbit and Coulomb contributions, we have for simplicity omitted the exchange part by setting the operator $\mathcal{A}'$ in Eq. (2) as unity. In addition, since the result in the $N+\alpha$ calculation[12] was found to be insensitive to the choice of the spin-orbit range parameter λ, we have computed V_{so} in the zero-range approximation, i.e., by letting λ approach infinity.

The exchange-mixture parameter u in the nucleon-nucleon potential is adjusted to yield the experimental $n+^{16}$O separation energy in the first excited $\frac{1}{2}^+$ state in ^{17}O.[13] The resultant value of u is 0.825, which is reasonably close to the value required in our previous calculation of $\alpha+\alpha$ scattering.[14] With u fixed in this way, the spin-orbit strength is then varied to reproduce the observed value of about 5 MeV for the $d_{5/2}$-$d_{3/2}$ splitting.[13] The value for the quantity $V_\lambda \lambda^{-5/2}$ is found to be 48 MeV fm^5, which is only 33% larger than the corresponding value in $N+\alpha$ scattering obtained by an extrapolation procedure using the (V_λ, λ) values given in Table I of Ref. 12. In addition, the $\frac{5}{2}^+$ state so calculated lies at 0.74 MeV below the $\frac{1}{2}^+$ state mentioned above; this also agrees quite well with the experimental result.

A comparison between calculated (solid curves) and experimental[15] $p+^{16}$O differential cross sections and polarizations at E = 37.4 MeV is shown in Fig. 1. To take reaction effects into account, we have replaced $V_D(r)$ in Eq. (5) by $V_D(r)+iW(r)$,

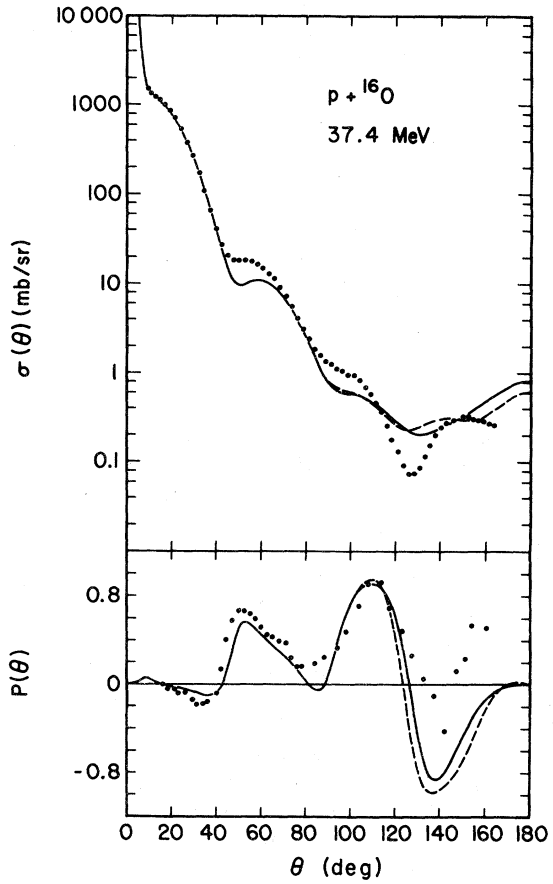

FIG. 1. Comparison of calculated and experimental results for $p+^{16}$O scattering at 37.4 MeV. The experimental data are those quoted in Ref. 15.

with $W(r)$ being the imaginary potential obtained by van Oers and Cameron[15] in an optical-model analysis of the $p+^{16}O$ scattering data. As is seen, the agreement is quite satisfactory at angles up to about 130°. Beyond 130°, our calculation does correctly predict a rise in the cross section, but a detailed agreement is not obtained.[16]

In $p+^{40}Ca$ scattering,[9] it was found that the incorporation of an odd-even l dependence into the imaginary potential can significantly improve the agreement with experiment at large angles. To see if a similar feature might also be required here, we have made a calculation with $W(r)$ multiplied by a factor $[1+C_I(-1)^l]$. The result obtained with $C_I=0.03$ is shown by the dashed curves in Fig. 1. Here one sees that with a value of C_I similar in magnitude to that used in $p+^{40}Ca$ scattering, there is indeed a noticeable improvement in the shape of the cross-section curve at backward angles.

To see the effect of the antisymmetrization operator $\mathcal{A}'$, we have also made a calculation in which the kernel k_l and the spin-orbit potential V_{so} in Eq. (5) are set as zero but the direct potential V_D is multiplied by a factor C_a. The results for $p+^{16}O$ scattering at 30 MeV are shown in Fig. 2, where the dashed and dot-dashed curves represent results for unantisymmetrized calculations with $C_a=1.0$ and 1.4, respectively, while the solid curve represents the result of the fully antisymmetrized calculation. Here it is seen that there is an over-all agreement between the antisymmetrized calculation and the unantisymmetrized calculation using $C_a=1.4$, which indicates that the operator $\mathcal{A}'$ does have an important effect. At extreme backward angles, there is a discrepancy of a factor of about 2.5. Even though this discrepancy may seem quite large, one should note that in $^3He+^4He$ scattering at 44.5 MeV[2] where a similar comparison has been made and where $V_b/(V_D+V_a)$ was found to be -0.17, the cross sections at backward angles differ by a factor of almost 1000.[17] Therefore, we conclude that, for $p+^{16}O$ scattering at relatively high energies, the pickup amplitude is rather small and V_b in Eq. (1) is small compared to (V_D+V_a).

Together with our previous findings in light nuclear systems and in $p+^{40}Ca$ scattering, the present finding in $p+^{16}O$ scattering leads us to speculate that, while it is relatively easy for a heavy cluster to pick up a lighter cluster, it is quite difficult for a light cluster to pick up another cluster which is many times heavier. If this is generally so, then one expects that in proton scattering on a medium-heavy or heavy nucleus, the use of an optical potential without a Majorana component in the real central part should yield an adequate description, over a large angular region, of the experimental data. On the other hand, for heavy-ion scattering such as $^{12}C+^{13}C$, the presence of a Majorana term in the potential should be quite important, as has indeed been found particularly by von Oertzen.[18]

In conclusion, we feel that the present calculation is an important step towards an understanding of the importance of exchange processes in scattering and reaction problems, and similar calculations on more complicated problems such as $^{40}Ca(p,p)^{40}Ca$ and $^{16}O(p,\alpha)^{13}N$ should be performed in the future in order to understand even better the roles played by these processes.

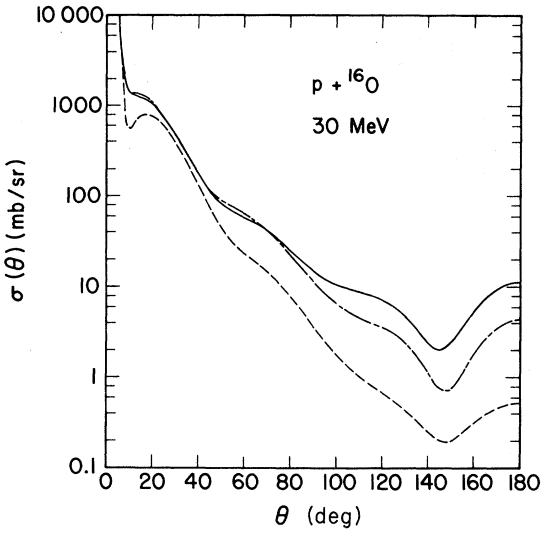

FIG. 2. Comparison of antisymmetrized (solid curve) and unantisymmetrized (dashed and dot-dashed curves) calculations. The unantisymmetrized calculation refers to the case where an equivalent local potential ($1.0V_D$ or $1.4V_D$) is used as the nuclear interaction between the proton and the ^{16}O nucleus.

*Work supported in part by the U.S. Atomic Energy Commission.

[1]D. R. Thompson and Y. C. Tang, Phys. Rev. C 4, 306 (1971).

[2]Y. C. Tang and R. E. Brown, Phys. Rev. C 4, 1979 (1971).

[3]H. C. Benöhr and K. Wildermuth, Nucl. Phys. A128, 1 (1969).

[4]H. Feshbach, Ann. Phys. (N. Y.) 5, 337 (1958); 19, 287 (1962); 43, 410 (1967).
[5]G. W. Greenlees, G. J. Pyle, and Y. C. Tang, Phys. Rev. 171, 1115 (1968).
[6]J. L. Gammel and R. M. Thaler, Phys. Rev. 109, 2041 (1958); C. C. Giamati, V. A. Madsen, and R. M. Thaler, Phys. Rev. Lett. 11, 163 (1963).
[7]L. G. Votta, P. G. Roos, N. S. Chant, and R. Woody, III, Phys. Rev. C 10, 520 (1974).
[8]P. E. Frisbee, Ph.D. thesis, University of Maryland, 1972 (unpublished).
[9]G. W. Greenlees, W. Makofske, Y. C. Tang, and D. R. Thompson, Phys. Rev. C 6, 2057 (1972).
[10]F. S. Chwieroth, Y. C. Tang, and D. R. Thompson, Phys. Rev. C 9, 56 (1974); and references contained therein.
[11]W. Sünkel and K. Wildermuth, Phys. Lett. 41B, 439 (1972).
[12]I. Reichstein and Y. C. Tang, Nucl. Phys. A158, 529 (1970).
[13]F. Ajzenberg-Selove, Nucl. Phys. A166, 1 (1971).
[14]R. E. Brown and Y. C. Tang, Nucl. Phys. A170, 225 (1971).
[15]W. T. H. van Oers and J. M. Cameron, Phys. Rev. 184, 1061 (1969).
[16]Our calculated value for σ_R is 452 mb, which agrees well with the experimental value of 440 ± 35 mb [J. M. Cameron, UCLA Technical Report No. P-80, 1967 (unpublished)].
[17]An examination of the phase-shift behavior shows that, if a term $C_b V_D P^r$ is also included in the equivalent local potential, a small value of C_b equal to about 0.03 is required in order to yield a better agreement with the resonating-group result. This value is substantially smaller in magnitude than the corresponding value of about −0.2 found in ^{3}He + ^{4}He scattering (Ref. 2).
[18]W. von Oertzen, Nucl. Phys. A148, 529 (1970).