Charge asymmetry effects and the trinucleon binding energy. II. Inclusion of the tensor force and singlet repulsion*

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Calculations of the trinucleon binding energy using separable potentials having a tensor component in the triplet force and strong repulsion in the singlet force are reported. It is shown that the binding energy remains sensitive to the N-N singlet effective range and that $|a_{nn}| < |a_{pp}|$ can be consistent with the ^{3}He Coulomb energy being less than the binding energy difference between ^{3}H and ^{3}He if one has $r_{nn} < r_{pp}$. Sensitivity of the binding energy to other differences in the N-N interactions is explored.

[NUCLEAR STRUCTURE ^{3}H, ^{3}He, separable potential three-body calculation, tensor triplet, repulsive singlet, charge asymmetry.]

I. INTRODUCTION

The binding energy difference for the ^{3}H, ^{3}He isodoublet has long been a subject of interest in few-nucleon physics. Likewise, the sensitive dependence of these binding energies upon the singlet N-N effective range has also been a phenomenon of interest. In this paper we wish to: (1) extend our previous separable potential calculations[1] to include a tensor component in the triplet interaction and short range repulsion in the singlet interaction: (2) demonstrate that the same sensitivity of the trinucleon binding energy to the singlet effective range holds for this more realistic set of potentials as it does for simple, rank one attractive interactions; (3) emphasize that differences arising in the trinucleon binding energy calculated for divers potentials are primarily the result of differing fits to the on-shell N-N scattering data; and (4) reiterate that the experimental binding energy difference $[B_3(^3\mathrm{H}) - B_3(^3\mathrm{He})]$ being larger than the estimated Coulomb energy in ^{3}He does not necessarily imply that $|a_{nn}| > |a_{pp}|$, if one finds $r_{nn} < r_{pp}$.

In Sec. II we describe briefly the separable potential model employed for these calculations. In Sec. III we discuss our numerical results and compare them with previously published results for similar models. Implications for $[B_3(^3\mathrm{H}) - B_3(^3\mathrm{He})]$ are discussed in Sec. IV as are implications for the variation of B_3 with different potentials in Sec. V. Section VI contains a summary of our conclusions.

II. THEORETICAL MODEL

Separable potential models for the triton bound state, which include a tensor component in the triplet force and a singlet interaction that is repulsive so that the $^1S_0$ phase shift passes through zero appropriately, have been discussed by several authors.[2,3] Thus we only outline here the basic elements of our calculation.

We assume a triplet potential of the original Yamaguchi form[4]

$$V_t(k, k') = -\frac{\lambda_t}{2\mu} g_t(k) g_t(k') , \tag{1}$$

where we have

$$\begin{aligned} g_t(k) &= g_c(k) + \frac{S_{ij}(\hat{k})}{\sqrt{8}} g_T(k) , \quad \vec{k} = \vec{k}_{ij} , \\ g_c(k) &= (k^2 + \beta_c^2)^{-1} , \\ g_T(k) &= -\xi_T k^2 (k^2 + \beta_T^2)^{-2} , \\ S_{ij}(\hat{k}) &= 3\vec{\sigma}_i \cdot \hat{k} \vec{\sigma}_j \cdot \hat{k} - \vec{\sigma}_i \cdot \vec{\sigma}_j . \end{aligned} \tag{2}$$

Here μ is the reduced mass of the interacting pair of nucleons, the subscript c refers to the central component and T to the tensor component, and ξ_T is the ratio of tensor to central amplitudes. The singlet potential was assumed to be of a functional form similar to that used by Naqvi[5] and by Gupta[6]

$$V_s(k, k') = -\frac{1}{2\mu} [\lambda_a g_a(k) g_a(k') - \lambda_r g_r(k) g_r(k')] , \tag{3}$$

where we have

$$\begin{aligned} g_a(k) &= (k^2 + \beta_a^2)^{-1} , \\ g_r(k) &= k^2 (k^2 + \beta_r^2)^{-2} . \end{aligned} \tag{4}$$

Here a refers to the attractive term and r to the repulsive term.

The resulting system of coupled integral equations that must be solved to obtain the triton bound

state energy and wave function is then

$$u_c(p) = \pi\lambda_t[1-\lambda_t\Lambda_t(p)]^{-1} \times \int p'^2dp'\{I_{cc}(p,p')u_c(p') + I_{cT}(p,p')u_T(p') + 3[I_{ca}(p,p')u_a(p') - I_{tr}(p,p')u_r(p')]\}, \tag{5a}$$

$$u_T(p) = \pi\lambda_t[1-\lambda_t\Lambda_t(p)]^{-1} \times \int p'^2dp'\{I_{Tc}(p,p')u_c(p') + I_{TT}(p,p')u_T(p') + 3[I_{Ta}(p,p')u_a(p') - I_{Tr}(p,p')u_r(p')]\}, \tag{5b}$$

$$u_a(p) = \pi\lambda_a\Delta^{-1}\Big([1+\lambda_r\Lambda_r(p)]\int p'^2dp'\{3[I_{ac}(p,p')u_c(p') + I_{aT}(p,p')u_T(p')] + I_{aa}(p,p')u_a(p') - I_{ar}(p,p')u_r(p')\} - \lambda_r\Lambda_{ar}(p)\int p'^2dp'\{3[I_{rc}(p,p')u_c(p') + I_{rT}(p,p')u_T(p')] + I_{ra}(p,p')u_a(p') - I_{rr}(p,p')u_r(p')\}\Big), \tag{5c}$$

$$u_r(p) = \pi\lambda_r\Delta^{-1}\Big([1-\lambda_a\Lambda_a(p)]\int p'^2dp'\{3[I_{rc}(p,p')u_c(p') + I_{rT}(p,p')u_T(p')] + I_{ra}(p,p')u_a(p') - I_{rr}(p,p')u_r(p')\} + \lambda_a\Lambda_{ar}(p)\int p'^2dp'\{3[I_{ac}(p,p')u_c(p') + I_{aT}(p,p')u_T(p')] + I_{aa}(p,p')u_a(p') - I_{ar}(p,p')u_r(p')\}\Big), \tag{5d}$$

where we have defined

$$\Delta = [1-\lambda_a\Lambda_a(p)][1+\lambda_r\Lambda_r(p)] + \lambda_a\lambda_r\Lambda_{ar}^2(p),$$
$$\Lambda_i(p) = \int \frac{g_i^2(k)d^3k}{k^2+3p^2/4+mB_3}, \quad i=t,a,r, \tag{6}$$
$$\Lambda_{ar}(p) = \int \frac{g_a(k)g_r(k)d^3k}{k^2+3p^2/4+mB_3}.$$

The functions $I_{ij}(p,p')$ are defined explicitly in Appendix A.[7] The lack of symmetry in Eq. (5) arises from the use of the complicated one term potential in the triplet state and a two term potential in the singlet state. The spectator functions $u_i(p)$ are required in the construction of the bound state wave function. The completely symmetric component of that wave function is given by

$$\psi = \psi^{(1)} + \psi^{(2)} + \psi^{(3)},$$
$$\psi^{(1)} = N\frac{g_t(k)[u_c(p) + S_{12}(\hat{p})u_T(p)/\sqrt{8}] + g_a(k)u_a(p) - g_r(k)u_r(p)}{k^2+3p^2/4+mB_3}, \tag{7}$$

where $\psi^{(i)}$ indicates the cyclic permutation of the Jacobi variables

$$\vec{p} = \tfrac{2}{3}[\vec{p}_1 - \tfrac{1}{2}(\vec{p}_2+\vec{p}_3)], \quad \vec{k} = \tfrac{1}{2}(\vec{p}_2-\vec{p}_3).$$

It is clear that for $\xi_T = 0$, $\lambda_r = 0$ the equations reduce to the usual set for simple rank one potentials.

The above equations are easily iterated to obtain both the eigenvalue and the spectator functions. A Gaussian-Gegenbauer integration scheme was used for all numerical integrals.

III. NUMERICAL RESULTS

In Ref. 1 simple rank one potentials were used to represent both the triplet and singlet interactions. It has been long known that such a model overbinds the triton.[8] Consequently, the strength of the triplet potential was reduced by some $5\frac{1}{2}\%$ to yield the approximate experimental triton binding before the effect of varying the neutron-neutron scattering parameters was studied. For the pres-

TABLE I. Potential parameters and scattering length for the triplet potentials considered.

V_t	λ_t (fm^{-3})	β_c (fm^{-1})	ξ_T	β_T(fm^{-1})	a_t (fm)
GS	0.3815	1.406			5.423
Y	0.4145	1.443			5.378
YY	0.2489	1.334	1.784	1.568	5.378

TABLE II. Potential parameters, scattering length, and effective range for singlet potentials considered.

V_S	λ_a (fm^{-3})	β_a (fm^{-1})	λ_r (fm^{-3})	β_r (fm^{-1})	a_s (fm)	r_s (fm)
N	0.7752	1.853	3.865	1.853	−23.69	2.36
G_1	0.2102	1.274	6.537	2.316	−18.04	2.70
G_1'	0.2354	1.320	1.767	1.806	−18.01	2.70
G_1''	0.2494	1.343	1.330	1.691	−17.98	2.70
G_2	0.2803	1.390	1.060	1.575	−18.04	2.70
G_2'	0.2839	1.390	1.115	1.575	−20.09	2.70
G_3	0.3145	1.436	1.038	1.529	−17.98	2.70
N^{eff}	0.2242	1.330			−23.69	2.36
G^{eff}	0.1533	1.183			−18.02	2.69

ent calculations we have used the triplet interactions shown in Table I and the singlet interactions listed in Table II. In Table I the potentials without tensor components were included to verify the reduction in the theoretical binding energy when the Yamaguchi tensor interaction is used and to illustrate the change in the binding when the triplet scattering length is fixed at its modern value.[9] In Table II the rank two singlet potentials of Naqvi[5] and Gupta[6] are listed along with parameters for rank one potentials which have the same scattering length and effective range as N and G_1 (denoted by the superscript "eff").[10]

Our results for the trinucleon binding energy using various combinations of these potentials are given in Tables III and IV. In the first of these we list B_3 for each of the rank two singlet potentials described in Table II calculated with the effective triplet interaction and with the Yamaguchi tensor triplet. It is clear that the decrease in binding due to the presence of the tensor force is in agreement with the results of Ref. 3. It is also apparent that our results disagree with those reported by Mitra and coworkers.[6,11] Except for the potential N, our value for B_3 with the tensor force is consistently about 8.5 MeV. In addition, as noted above, the use of an effective triplet potential (s wave only) increases the binding, which disagrees with the results tabulated in Table V of Ref. 11.

In our Table IV we illustrate for the N and G_1 singlet potentials the dependence of B_3 on the triplet scattering length by comparing results for the Y and GS Yamaguchi type, simple rank one potentials. Altering the triplet scattering length from 5.38 to 5.42 fm increases the binding by some 0.1–0.15 MeV. In this table we also show the effect of the repulsion in the singlet interaction. Comparison of N and G_1 with N^{eff} and G_1^{eff} shows B_3 to be decreased by some $\frac{1}{2}$ to 1 MeV when the repulsion is added. From Table III one sees that adding the tensor component to the triplet interaction decreases the binding by another $1\frac{1}{2}$ MeV. (The larger the binding, the greater the reduction in each case.)

Since the potential G_1 gives one of the best fits to the $^1S_0$ phase shifts (see Ref. 11 for a partial comparison) we have used that particular singlet interaction to study the dependence of B_3 upon the singlet scattering length a_s and the singlet effective range r_s. (In Appendix B we quote expressions for a_s and r_s in terms of the potential parameters.) In Table V(a) we display B_3 for various values of a_s and r_s about the nominal values from G_1 ($a_s = -18$ fm, $r_s = 2.7$ fm). Only the parameters of the attractive part of the potential (λ_a and β_a) were varied. For comparison we show in Table V(b) the variation of B_3 for similar values of a_s and r_s but with only the repulsive parameters (λ_r

TABLE III. Binding energy B_3 (MeV) for each of the repulsive singlet potentials of Table II and triplet potentials with and without the tensor force.

V_s	$B_3(V_t \sim \text{GS})$	$B_3(V_t \sim \text{YY})$
N	10.95	9.37
G_1	9.85	8.51
G_1'	9.86	8.49
G_1''	9.88	8.50
G_2	9.87	8.48
G_2'	9.94	8.55
G_3	9.86	8.47

TABLE IV. Binding energy B_3 for triplet interactions without a tensor component and singlet interactions with and without repulsion.

V_s	V_t	B_3 (MeV)
N	Y	11.01
N	GS	10.95
N^{eff}	GS	12.01
G_1	Y	9.97
G_1	GS	9.85
G_1^{eff}	GS	10.60

TABLE V. Binding energy B_3 (MeV) as a function of a_s and r_s keeping the repulsive and attractive parameters fixed at the G_1 values and for $V_t \sim$ GS.

a_s (fm) \ r_s (fm)	2.6	2.7	2.8
	(a) Repulsive fixed		
−18	10.11	9.85	9.59
−17		9.80	
−16		9.75	
	(b) Attractive fixed		
−18	10.28	9.85	
−17		9.78	
−16		9.70	

TABLE VI. The binding energy B_3 (MeV) as a function of a_s and r_s keeping the repulsive and attractive parameters fixed at the G_1 values and for $V_t \sim$ YY.

a_s (fm) \ r_s (fm)	2.6	2.7	2.8
	(a) Repulsive fixed		
−18	8.71	8.51	8.31
−17		8.46	
−16		8.42	
	(b) Attractive fixed		
−18	8.83	8.51	
−17		8.46	
−16		8.41	

and β_r) varied. In Table VI we present a similar study of the variation of B_3 with a_s and r_s but for the complete tensor triplet force.

It is clear from the tables that the same qualitative dependence of B_3 upon a_s and r_s as was noted in Ref. 1 for simple, rank one potentials still holds.[12] A 0.1 fm change in r_s produces a change in B_3 of about 0.2 MeV (for variations in the attractive part of V_s), whereas a change in a_s of 1.0 fm produces only about a 0.05 MeV change in the binding. The consequences of this dependence of B_3 on the low-energy singlet scattering parameters will be explored in the following two sections.

IV. IMPLICATIONS FOR CHARGE ASYMMETRY

The measured low-energy proton-proton scattering parameters are[13-16]

$$a_{pp}^{C} = -7.823 \pm 0.01 \text{ fm}, \qquad r_{pp}^{C} = 2.794 \pm 0.015 \text{ fm}. \tag{8}$$

Removal of the pure Coulomb scattering effects adds measurably to the uncertainty in the "Coulomb corrected" scattering parameters, although Arnold[17] has pointed out that for the assumption of a local N-N interaction, the p-p scattering data do not permit much model dependence. Current estimates of the Coulomb corrected parameters are[13, 16, 18]

$$a_{pp} = -17.1 \pm 0.3 \text{ fm}, \qquad r_{pp} = 2.84 \pm 0.03 \text{ fm}. \tag{9}$$

This is to be compared with the low-energy neutron-neutron scattering parameters[13]

$$a_{nn} = -16.4 \pm 0.9 \text{ fm}, \qquad r_{nn} = 2.8 \pm 0.5 \text{ fm}. \tag{10}$$

The experimental binding energy difference between the trinucleons is

$$\Delta B_3 = B_3(^3\text{H}) - B_3(^3\text{He}) = 0.76 \text{ MeV}. \tag{11}$$

In perturbation calculations, wave functions determined from realistic potentials yield a value for the Coulomb energy due to the Coulomb interaction of the two protons in ^{3}He of only 0.60–0.66 MeV.[19, 20] More recently estimates of the Coulomb energy have been obtained from the charge form factor of ^{3}He; such procedures yield a value for E_C of 0.64–0.66 MeV.[21, 22] Thus the difference between E_C and ΔB_3 is nonzero, and it has been attributed to a slight charge asymmetry in the N-N interaction (a difference in the n-n and non-Coulomb part of the p-p interaction).

Because ^{3}H contains an n-n pair in contrast to the p-p pair in ^{3}He, one could account for the difference between E_C and ΔB_3 if the matrix element of V_{nn} in the three-body problem were greater than the matrix element of V_{pp} by about 0.1 MeV. But $|a_{nn}| < |a_{pp}|$ pushes the theoretical estimate of this difference in the wrong direction. However, as was pointed out in Ref. 1 B_3 is much more sensitive to r_{nn} than to a_{nn}, so that one can still have $|a_{nn}| < |a_{pp}|$ and account for the discrepancy between E_C and ΔB_3 if $r_{nn} < r_{pp}$.

The same conclusion can be drawn from the more realistic calculations described in Sec. III. There we dealt with V_s as if V_{np}^s and V_{nn} were identical. In fact they are not, and the singlet interaction is approximately $(V_{nn} + \frac{1}{2}V_{np}^s)$. Thus varying just r_{nn} by 0.1 fm (or a_{nn} by 1.0 fm) implies a change in B_3 that is just $\frac{2}{3}$ of that listed in Tables V and VI. If one has $|a_{nn}| - |a_{pp}| = -1.5$ fm, then $B_3 - E_C = 0.1$ MeV would imply that $r_{nn} - r_{pp} = -0.11$ fm, according to Table VI(a).

In a recent publication Mehdi and Gupta[23] have argued against such a conclusion using a model in which the singlet potentials N and G_1' were com-

bined with the triplet potential Y (no tensor). However, they use the procedure of Ref. 11 which disagrees both quantitatively and qualitatively with our results as well as those of Ref. 3 and their calculated value of E_C is in disagreement with the values obtained in Refs. 19–22.

V. IMPLICATIONS FOR MODEL BINDING CALCULATIONS

One can ascertain from Tables II and III that the differences in B_3 among the various Gupta singlet potentials are due almost entirely to the slight differences in the values of a_s and r_s to which they were fitted. Also, the binding energy for the Naqvi potential differs from that of the Gupta potentials just as one would expect from the differing values of a_s and r_s. Thus our calculations contradict Mitra's assertion[6, 11, 24] that the sensitivity of B_3 to the singlet effective range vanishes when repulsion is introduced into the singlet potential.

Indeed, the trinucleon binding energy is sensitive to the low-energy N-N scattering data to which the different potentials are fitted. On the basis of the results in Sec. III one would expect a variation in B_3 as follows: a model in which V_s is fitted to the n-p singlet data when compared to a model in which V_s is fitted to the p-p singlet data would yield a binding energy difference of something like 0.4–0.5 MeV (assuming that the triplet interactions were identical).

Thus it is clear that for potential models in which both the underlying triplet and singlet scattering data differ, "discrepancies" in the triton binding energy of as much as an MeV or so are quite possible. Furthermore, one can see by combining the effects of (1) the omission of the higher partial waves,[25–27] (2) the apparent charge asymmetry of the singlet interaction, and (3) the known charge dependence of the singlet interaction that a theoretical estimate of the triton binding energy using an ideal triplet potential and a singlet potential fitted perfectly to the p-p scattering data should underbind by about 0.5 MeV.

VI. CONCLUSIONS

First, we note the good fit to the experimental binding energy obtained when the Gupta singlet potentials are used in combination with the tensor triplet force. The representation of the $^1S_0$ phase shifts obtained by Gupta appears to be better than one might be led to expect from such a simple analytic form.

Second, we point out that even when (1) the tensor component of the triplet interaction is included and (2) strong repulsion is introduced into the singlet interaction, the trinucleon binding energy remains very sensitive to the singlet effective range. Because of this sensitivity, $|a_{nn}| < |a_{pp}|$ is compatible with $E_C < [B_3(^3\mathrm{H}) - B_3(^3\mathrm{He})]$ if one finds that $r_{nn} < r_{pp}$ as was shown in Ref. 1.

Last, we emphasize that small differences in the fits to the low-energy N-N scattering data can produce sizeable differences in B_3. In particular, model calculations in which the singlet potential is fitted to different sets of the scattering data (e.g., p-p versus n-p singlet) can produce differences in the calculated value of B_3 of the order of 0.5 MeV; other model differences could increase the "discrepancies" in B_3 to an MeV or more.

APPENDIX A

The integrals $I_{ij}(p, p')$ required in Sec. II of the text are given as follows:

$$I_{cc}(p,p') = \int_{-1}^{1} \frac{dx}{D(x)} [g_c(q)g_c(q') - 2g_T(q)g_T(q')P_2(\hat{q}\cdot\hat{q}')],$$

$$I_{cT}(p,p') = \int_{-1}^{1} \frac{dx}{D(x)} \{g_c(q)g_T(q')P_2(\hat{p}'\cdot\hat{q}') - 2g_T(q)g_c(q')P_2(\hat{p}'\cdot\hat{q}) + \sqrt{\tfrac{1}{2}}\, g_T(q)g_T(q')[P_2(\hat{p}'\cdot\hat{q}') + P_2(\hat{p}'\cdot\hat{q}) + P_2(\hat{q}\cdot\hat{q}') - 1]\},$$

$$I_{ca}(p,p') = \int_{-1}^{1} \frac{dx}{D(x)} [g_c(q)g_a(q')],$$

$$I_{cr}(p,p') = \int_{-1}^{1} \frac{dx}{D(x)} [g_c(q)g_r(q')],$$

$$I_{TT}(p,p') = \int_{-1}^{1} \frac{dx}{D(x)} \{-2g_c(q)g_c(q')P_2(\hat{p}\cdot\hat{p}') + \sqrt{\tfrac{1}{2}}\, g_c(q)g_T(q')[P_2(\hat{p}\cdot\hat{p}') + P_2(\hat{p}'\cdot\hat{q}') + P_2(\hat{p}\cdot\hat{q}') - 1] + \sqrt{\tfrac{1}{2}}\, g_T(q)g_c(q')[P_2(\hat{p}\cdot\hat{p}') + P_2(\hat{p}\cdot\hat{q}) + P_2(\hat{p}'\cdot\hat{q}) - 1] + \tfrac{1}{4} g_T(q)g_T(q') \times [2P_2(\hat{p}\cdot\hat{q}) + 2P_2(\hat{p}'\cdot\hat{q}') - P_2(\hat{p}\cdot\hat{p}') - P_2(\hat{q}\cdot\hat{q}') - P_2(\hat{p}\cdot\hat{q}') - P_2(\hat{p}'\cdot\hat{q})]\},$$

$$I_{Ta}(p,p') = \int_{-1}^{1} \frac{dx}{D(x)} [g_T(q) g_a(q') P_2(\hat{p}\cdot\hat{q})],$$

$$I_{Tr}(p,p') = \int_{-1}^{1} \frac{dx}{D(x)} [g_T(q) g_r(q') P_2(\hat{p}\cdot\hat{q})],$$

$$I_{aa}(p,p') = \int_{-1}^{1} \frac{dx}{D(x)} g_a(q) g_a(q'),$$

$$I_{ar}(p,p') = \int_{-1}^{1} \frac{dx}{D(x)} g_a(q) g_r(q'),$$

$$I_{rr}(p,p') = \int_{-1}^{1} \frac{dx}{D(x)} g_r(q) g_r(q'),$$

where we have defined

$$\vec{q} = \tfrac{1}{2}\vec{p} + \vec{p}',$$

$$\vec{q}' = \vec{p} + \tfrac{1}{2}\vec{p}',$$

$$x = \hat{p}\cdot\hat{p}',$$

$$P_2(y) = \tfrac{1}{2}(3y^2 - 1),$$

$$D(x) = p^2 + p'^2 + pp'x + mB_3,$$

and the symmetry relation $I_{ij}(p,p') = I_{ji}(p,p')$ is valid.

APPENDIX B

For the rank two separable potential of the form described in Eqs. (3) and (4) of the main text, one can express the scattering length and effective range as follows:

$$\frac{1}{a} = -\left\{\frac{\beta_a^4}{2\pi^2\lambda_a} - \frac{\beta_a}{2}\left[1 - \frac{\lambda_r}{\lambda_a}\left(\frac{\beta_a}{2\beta_r}\right)^3 + \frac{\lambda_r\pi^2}{(2\beta_r)^3}\right] + \frac{\lambda_r\pi^2}{2\beta_r^2(1+\beta_r/\beta_a)^4}\right\} \Big/ \left[1 + \frac{\lambda_r\pi^2}{(2\beta_r)^3}\right],$$

$$r = \left\{\frac{3}{\beta_a} + \frac{1}{8\beta_r}\frac{\lambda_r}{\lambda_a}\left(\frac{\beta_a}{\beta_r}\right)^4 + \frac{\lambda_r\pi^2}{(2\beta_r)^3}\frac{[3-(\beta_a/\beta_r)^2]}{\beta_a} + \frac{2\lambda_r\pi^2}{(\beta_a+\beta_r)^4}\left(\frac{\beta_a}{\beta_r}\right)^4\left(1+\frac{2\beta_r}{\beta_a}\right)\right.$$

$$\left. - \frac{1}{a}\left[\frac{4}{\beta_a^2} - \frac{\lambda_r\pi^2(1-2\beta_r^2/\beta_a^2)}{4\beta_r^5} + \frac{4\lambda_r\pi^2}{\beta_r^5(1+\beta_r/\beta_a)^2}\right]\right\} \Big/ \left[1 + \frac{\lambda_r\pi^2}{(2\beta_r)^3}\right].$$

These expressions agree with those given by Naqvi in Ref. 5.

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