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Attractive potential with forbidden states for the N-N interaction

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A potential model with forbidden states for describing the nucleon-nucleon interaction in the inner region is suggested. Forbidden states in the N-N system may occur due to the composite structure of nucleons as a consequence of the Pauli exclusion principle. A consistent description of singlet even phase shifts at energies up to 400 MeV has been obtained. The generalization of Levinson's theorem taking forbidden states into account permits a new interesting interpretation of the phase-shift analysis data at higher energies and as a result gives a "smooth matching" of the low-energy phase shifts with the asymptotic region $E \gtrsim 10$ GeV. The loop which the wave function has in the inner region provides a considerable contribution to the deuteron form factor at great momentum transfers. A comparison is made with the repulsive core model. Possible consequences for the three- and many-nucleon systems are discussed.

[NUCLEAR REACTIONS N $(N,N)N$, potential model with forbidden states, $^1S_0$ phase shift to 400 MeV.]

I. INTRODUCTION

Presently a host of data have been accumulated in favor of the composite structure of nucleons. We note here the deeply inelastic scattering of electrons by nucleons,[1] as well as the recent neutrino experiments at CERN with a Gargamelle bubble chamber[2] which provide new strong arguments in favor of the Gell-Mann–Zweig quark model of a nucleon with quarks as Fermi particles.[3]

The composite structure of nucleons should necessarily appear also in the N-N interactions at small distances whereas this is in no way reflected in the currently accepted potential models for the N-N forces. Meanwhile, in nuclear physics experience has been gained in the potential description of the interaction of composite particles (e.g., α particles), which accounts for the Pauli exclusion principle for nucleons.[4,5] Here, as it appeared, there is a striking analogy between the peculiarities of scattering in the N-N and α-α system. Since the N-N interaction is much more complicated than the α-α interaction, it is reasonable to confront the α-α system with only the singlet even partial waves of the N-N system where the interaction is purely central. Analogy between both systems[6] is reflected, for example, in the characteristic behavior of phase shifts. The phase shifts of the lowest partial waves (S and D in the α-α system and $^1S_0$ in the N-N system) at a certain energy become negative while the phase shifts of all the highest partial waves are positive up to very high energies. In order to explain such behavior the α-α phase shifts in accordance with the Pauli principle which, as it seemed earlier, should explicitly forbid α particles to penetrate one another, purely phenomenological cumbersome L-dependent α-α potentials were proposed in the 1950's which contained for the S and D waves a repulsive core of a different radius introduced into a shallow attractive potential of peripheral type.[7]

However, a microscopic analysis of the behavior of the wave function for the relative motion of α particles in their overlap region, performed on the basis of the shell model[8] and especially on the basis of the resonating-group method[9] (RGM), has shown that the Pauli principle in the α-α system leads to oscillations of the wave function different for different partial waves rather than to its dying out in the internal region. The possibility of describing phase shifts in a limited energy range using potentials with a hard core turned out to be associated with the fact that the position of the core coincides with that of the external node of the wave function which is practically immobile at small energies. At high energies these potentials will give an incorrect energy dependence of the phase shifts.

The indicated oscillations have an intimate connection with the inner structure of colliding particles and are very weakly dependent on energy.[9,10] For instance, in terms of the shell model the lowest state of the α-α system in an S wave is the state of the $4S$ type with two nodes in the internal region [the shell-model configuration $(0s)^4\,(1p)^4$ involves four oscillator quanta which, in the cluster representation of the shell-model wave functions, wholly go over to the wave function for the relative motion of two α clusters]. Similarly, in a D wave the lowest state is the $4D$ state with one node while the lowest $4G$ state is free of nodes as are all the states with $L > 4$.

That the lowest state of the α-α system has an oscillating function has been considered quite recently[6] as a clear indication of the fact that in essence the α-α system cannot be described in terms of a potential and that the description of the α-α interaction can be carried out only within the framework of the resonanting-group method, i.e., only by the explicit treatment of the known interaction of nucleons constituting an α particle.

With respect to the N-N system a similar approach was adopted in the known works of Otsuki, Tamagaki, and Wada[11] who suggested the idea of a "structural core" with a microscopic description similar to the RGM calculations in the α-α system. However, in Ref. 4 it has been demonstrated recently that an adequate potential description of the scattering of compound particles of the α-α type is still quite feasible, all the properties of the α-α system known from the shell model and the RGM (e.g., the oscillating character form of the ground state wave function) being preserved. Moreover, on this basis it has become clear which α-α scattering at high energies of hundreds of MeV should be used, namely, deep local attractive potentials with forbidden states introduced.

Actually this means that we subdivide the Hilbert space of states into two orthogonal subspaces, one of which is entirely left out of consideration in the following. In the reduced Hilbert space the system is already incapable of "going down" to the lowest-lying states, although the interaction potential can be used here in the usual manner, as a local operator. On the other hand, if we would want to write that interaction in the whole configurational space, then, in order to prevent the system from going down into the region of forbidden spectrum, we would have to redetermine the Hamiltonian for separate vectors, which would be equivalent to the introduction of a nonlocal pseudopotential yielding a strong repulsion in the subspace of occupied states. The off-mass-shell behavior of the T matrix for such a pseudopotential is quite similar to its behavior for standard potentials with a repulsive core.[12]

In fact, there is a large class of potentials describing the N-N scattering phases by equivalent methods. Local potentials with the repulsive core are far from being the only representatives of the class just mentioned. Furthermore, at small distances one may expect the "true" interaction to be nonlocal and energy-dependent. Forbidden states in some potential, irrespective of whether it is local or nonlocal, should provide a contribution to Levinson's theorem which in this case appears as

$$\delta_L(0) - \delta_L(\infty) = \pi(n_L + m_L) \ , \tag{1}$$

where n_L is the number of allowed states, m_L the number of forbidden states with orbital angular momentum L in a given potential, and where the natural condition $\delta_L(\infty) = 0$ is taken. The theorem (1) was proven by Swan[13] for the very schematic case of exchange potentials of the resonating-group method. In the case of local potentials used in the present paper Eq. (1) merely connects the value of phase shifts at zero with the total number of bound states, $N_L = n_L + m_L$ in the potential well, i.e., it transforms to the standard form of Levinson's theorem[14] with the usual requirements for a local potential.

Proceeding from these considerations, we have just used the generalized Levinson theorem from Ref. 4. Since the theorem has quite a general character it should find wide application. For example, the standard optical model for scattering of nucleons by nuclei is in good agreement with the formula $\delta_L(0) = \pi N_L$.[15,16] The versatile character of theorem (1) can also be well illustrated by the results of calculations of the neutron-deuteron elastic scattering obtained by Sloan[17] in terms of the Faddeev equations with a separable N-N potential, taking into account the inelastic channels and antisymmetrization of the three-particle wave function with respect to permutation of two neutrons. Here the total interval of variation of both the quartet and doublet S phase is equal to π although in the quartet S state there is no physical bound state of three particles whereas in the doublet S state there is such a state (the bound state of tritium).

So, according to Eq. (1) the S phase of the α-α scattering, remaining positive all the time, has a value at zero energy $\delta_0(0) = 3\pi$ (if the threshold $4S$ state is assumed to be bound) and extends to the region of the Born values $\delta_0 \lesssim 1$ at energies $E_{\text{lab}} \gtrsim 300$ MeV. To sum up the foregoing discussions, we can say that the hard core in the α-α and similar systems appears only as a result of a rather crude and essentially inadequate phenomenological description of the complex Fermi structure of such systems connected with the Pauli principle. Basically the potentials with the hard core belong to the class of pseudopotentials well known in the theory of solids.[18] For completeness it should be noted that the pseudopotentials with singularity R^{-2} and with an exponential decrease at infinity[19] (giving the wave function which disagrees with the microscopic treatment at small distances) are only phase equivalent to the potentials under consideration. They do not satisfy the conditions of applicability of Levinson's theorem.

In dealing with the problem of application of deep attractive potentials with forbidden states to the N-N system we ought to consider the arguments in favor of the repulsive core in the N-N interaction.

II. REPULSIVE CORE IN THE N-N INTERACTION - ARGUMENTS IN FAVOR OF AND AGAINST IT

The hypothesis of a repulsive core in the N-N interaction was proposed by Jastrow[20] in 1951 for explaining the U-shaped angular distributions in the n-p scattering. The subsequent phase-shift analysis[21] proved to be consistent with this hypothesis. For example, the $^1S_0$ phases go through zero at $E_{\text{lab}} \cong 300$ MeV, which is considered as a direct indication of the change of sign of the interaction at distances $R \cong 0.5$ fm. However, similar arguments are ambiguous as the phase shifts are determined from experiment within $n\pi$. The repulsive core may be replaced by a node of the wave function whose position is weakly dependent on energy. Then the phase goes not through zero but, in accordance with Levinson's theorem, through $N_0\pi$.

The origin of the repulsive core is usually associated with the exchange of heavy vector mesons. Quantitative results can be obtained here within the framework of the model of one-boson exchange (OBE). Due to complications involved in the analysis of many-meson exchanges the OBE potentials are truncated at a small separation (0.6–0.8 fm)[22] and the region of the nucleon core is left for phenomenological treatment, which also accentuates the inadequacy of arguments for solving the repulsive core problem on the basis of meson representations.

One can point out a number of difficulties associated with the OBE models.[22] For example, an attempt to attribute the repulsive core to the ω-meson exchange leads in the $^1S_0$ state to an abnormally great coupling constant up to $g_V^2/4\pi \cong 30-40$ [11] compared with the value $g_V^2/4\pi \cong 11$ [23] or even with $g_V^2/4\pi \cong 1-4$[24] required in higher partial waves. The meson core is too "soft" and all attempts[25] to describe phenomenologically the $^1S_0$ phase up to energies $E_{\text{lab}} \cong 700$ MeV using a repulsive core of the Yukawa type with the radius in the order of a radius of the vector meson exchange prove to be abortive, and it is only a very steep core of the Gaussian type having the height of several GeV that gives the required slope of the curve of the $^1S_0$ phase. Next, if LS forces are ascribed to the exchange of heavy vector mesons, then the required coupling constants become inconsistent with the description of the p-p-scattering phase shifts.[11]

In the high-energy scattering data there are some features which seem to be inconsistent with the idea of a "nontransparent" core. For example, the spin-orbit splitting of the p-p scattering P_J phase at $E_{\text{lab}} \cong 3$ GeV reaches an enormous value of ~160°,[26] which corresponds to great spin-orbit forces located at distances much smaller than the core radius.[27] Furthermore, the reconstruction[28] of the N-N potential by Orear's empirical formula[29] (a relativistic and nonrelativistic calculation with the Born term) leads to a complex attractive potential with the radial dependence at zero of the form $(R^2+\alpha^2)^{-2}$.

Now we shall briefly dwell upon the properties of the ^{3}H-^{3}He three-particle systems. Calculations with "realistic" potentials containing a repulsive core, which have been performed in recent years, lead to a rather general result[30]: the ^{3}H and ^{3}He nuclei turn out to be markedly undercoupled (by 1 or 2 MeV) and the electric form factor of ^{3}He calculated in various models is found to be much smaller than the experimental values at great momentum transfers $q^2 \cong 15-20$ fm^{-2} (in the region of the second maximum). In all calculations the Coulomb energy of ^{3}He is found to be approximately 100 keV smaller than the experimental value, while the calculated r.m.s radii of ^{3}H and ^{3}He are somewhat greater than the experimental ones (from 2 to 15% in various models).

For each of these discrepancies there is its own specific explanation.[31] The common feature of these difficulties, in our opinion, is a too low density of matter (and of charge) in the inner region of the nucleus for potentials with a repulsive core. We note that the N-N potential with forbidden states obviates this difficulty although appropriate calculations are as yet unavailable.[32]

In this context it is interesting to note that strong N-N correlations of attractive type may appear in nuclei at a small separation. For example, in scattering of nucleons from nuclei with great momentum transfer the knockout of rather loosely bound few-nucleon clusters (d,^{3}He, etc.) occurs. This is possible only in the case where several nucleons in the nucleus can be located with a marked probability in a small volume and can interact as an entity in collisions with high-energy protons.[33]

To summarize, we can say that under close examination the concept of a "nontransparent" nucleon core that originally arose from a phenomenological approach proves to be contradictory to some known facts. As the same time the experience gained in the description of scattering of compound systems[4] allows one to give a new interpretation of the data on the N-N scattering without introducing a repulsive core.

III. WAVE FUNCTION NODE IN PLACE OF THE REPULSIVE CORE IN THE N-N POTENTIAL

The possibility of abandoning the repulsive core model was first discussed by Otsuki, Tamagaki, and Wada (the model of a "structural" core).[11] They added to the OBE potential truncated at small distances a very strong exchange term of the resonating group method (RGM) for a system of six ur-fermions with several fitting parameters characterizing their interaction.

Such parametrization enabled Tamagaki *et al.*[11] to describe quite well the E dependence of different N-N scattering phases at energies up to 700 MeV. The radial wave functions for the S and P wave have a node approximately at that point where the repulsive core is located in realistic potentials. Due to a great magnitude of the exchange interaction the position of the node is practically independent of the relative motion energy in a broad interval. The shortcoming of such an approach is the assumption, on the basis of which the RGM calculations are just performed, that the nucleon has exactly "the same" structure as the ^{3}H-^{3}He system: the symmetric orbital part (0S state) and the antisymmetric STY part of the nucleon wave function as a system of three quarks. In fact, as nucleon spectroscopy shows,[34] the STY part of the nucleon wave function is symmetric with respect to permutations. What kind of symmetry the orbital part of the wave function has is unknown now. Besides, use of the interaction between quarks in the explicit form about which nothing is practically known is also arbitrary.

Preserving the idea about the replacement of the repulsive core by the wave function node, we use the description of the N-N interaction in terms of a deep attractive potential with forbidden states. In spite of the simplicity of such a description we obtain new results. For example, application of generalized Levinson's theorem as a natural consequence of the potential description allows one to describe on a unified basis the scattering phases at low ($E_{\rm lab} \lesssim 500$ MeV) and high ($E_{\rm lab} \gtrsim 1$–3 GeV) energies. The change of the character of the wave function in the inner region affects also the form factors at relatively great momentum transfers. We shall discuss singlet even states. Here first, most clearly exhibited are the problems associated with the repulsive core and, second, there are no calculational complexities involved in spin (phase splitting, tensor forces, etc).

Certainly, the form of the potential discussed in the intermediate and especially in the outer region has to be similar to the form of the usually employed OBE potentials. The character of the potential in the intermediate and inner region, as the following discussion shows, is very closely connected with the features of the E dependence of the lowest scattering phases. We shall see that the experimental data unambiguously point to the presence of one forbidden state for $L=0$ and to the absence of forbidden states for $L \geqslant 2$. First let us evaluate the radius R_0 of the inner region where the relative motion wave function $u(R)$ is fully determined by structural effects and the boundary condition at a point $R=R_0$

$$\frac{1}{u(R)}\frac{du}{dR}\bigg|_{R=R_0}=\eta(E)\cong\text{const} \tag{2}$$

is practically independent of energy.

For an S wave this gives the following additive contribution to the phase shift

$$\Delta^1\delta_0(k)=-kR_0+\arctan\frac{k}{\eta}+n\pi \ . \tag{3}$$

At fairly great energies the first term on the right-hand side determines the slope of the curve of the S phase shift

$$\frac{d^1\delta_0}{dk}=-R_0 \ . \tag{4}$$

As is known from the phase-shift analysis,[21,26] this slope is constant up to energies 2–3 GeV and is equal to

$$R_0\cong 0.6\ \text{fm}. \tag{5}$$

Since the intermediate region also provides some contribution to the slope of the curve of the $^1S_0$ phase shift, it is reasonable to take a somewhat smaller value

$$R_0\cong 0.4\text{–}0.5\ \text{fm} \ . \tag{6}$$

The quantity (6) is used in "realistic potentials" as the radius of the repulsive core[35–37] and the quantity (5) was taken by Feshbach and Lomon in their model with the boundary condition[38] as a radius of truncation of the interaction potential part (the boundary of the "black box").

We shall use the quantity (6) as a dimension of the region where the forbidden S state is localized. Then in the vicinity of the point R_0 a node of the wave function of the allowed state which is almost immobile at $E \lesssim U_0$ will occur, which just ensures the fulfillment of the condition (2) [Here U_0 is the characteristic depth of the potential $V(R)$.]

IV. CHOICE OF A POTENTIAL AND RESULTS OF CALCULATIONS

As the investigated variants of N-N potentials have shown, a consistent description of the singlet-even phase shifts S, D, and G at energies $E_{lab} \lesssim 500$ MeV by a deep attractive potential of interest can be obtained but the potential must be sufficiently singular as the $^1D_2$ phase shift is small. The singularity of the potential (weaker, of course, than R^{-2}) ensures strong attraction for S waves in that region which D waves cannot reach due to the centrifugal barrier.

So, we discuss the potential

$$V(R) = -V_0 \frac{e^{-\beta R}}{(\beta R)^{\epsilon}} + V_{\rm OPEP}(R) , \tag{7}$$

where $1 < \epsilon < 2$ and $\beta^{-1} \sim 0.5$ fm, and by way of comparison, to illustrate the dependence of the lowest phase shifts on the potential character, we give the results for the Woods-Saxon potential

$$V(R) = -V_0 \frac{1}{1+\exp[(R-R_w)/a]} . \tag{8}$$

The phase shifts for the potential (7) are represented in Fig. 1 by solid curves versus the experimental data at $E_{lab} \lesssim 500$ MeV at the optimum values of parameters

$$V_0 = 470 \text{ MeV}, \quad \beta^{-1} = 0.64 \text{ fm}, \quad \epsilon = 1.225 , \tag{9}$$

the quantity $V_{\rm OPEP}$ being, as usual, of the form

$$V_{\rm OPEP}(R) = -10.464 \text{ MeV} \frac{e^{-0.7R}}{0.7R} ,$$

where R is expressed in fm.

In the region $R \gtrsim 0.5$ fm, the potential (7) is very close to the Gammel-Thaler potential,[36] for which $\epsilon = 1$, $V_0 = 425.5$ MeV, $\beta^{-1} = 0.69$ fm. The essential difference, however, is that at $R = 0.5$ fm the Gammel-Thaler potential has a hard core, whereas the potential (2) has a deep-lying forbidden S state with binding energy about 730 MeV.

We see that at an energy of $E_{lab} \cong 300$ MeV the S phase shift passes through π and should extend to the Born region $\delta_0 \lesssim 1$ at great energies of tens of GeV. Figure 1 also shows (dotted curves) the $^1S_0$ and $^1D_2$ phase shifts for the potential (8) with the parameters

$$V_0 = 3281 \text{ MeV}, \quad R_w = 0.3028 \text{ fm}, \quad a = 0.2019 \text{ fm} \tag{10}$$

which give an optimum description of the E dependence of the S phase shift. These curves clearly illustrate the need for a singular potential.

Finally, for comparison we give similar phase shifts for the L-independent "realistic" potential obtained by Reid (dashed curves).[37] It is clear that the efficiency of both approaches is approximately equal.

In Figs. 2 and 3 the wave functions of the poten-

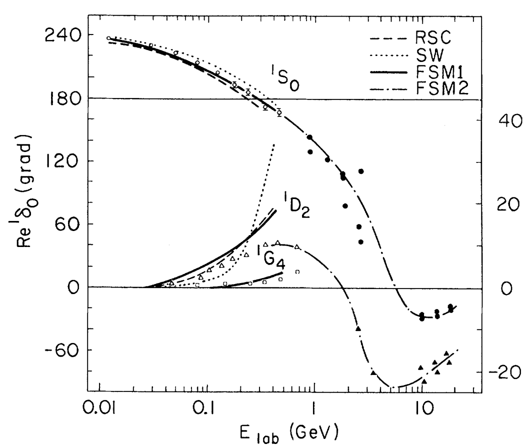

FIG. 1. The phase shifts in the model with a forbidden $0S$ state. The solid lines: potentials (7), (9); the dotted lines: potentials (8), (10); the dashed lines: the Reid soft core potential (Ref. 37) (the potential with no L dependence). For ease of comparison the $^1S_0$ phases in the Reid potential are displaced by the value of π. The dot-dash line shows the qualitative idea of the behavior of the phase shifts in the model with a forbidden $0S$ state in the high-energy region. The experimental data: at energies $E_{lab} < 750$ MeV the np phase shifts are from Ref. 21; at $E_{lab} > 750$ MeV and pp phase shifts are from Refs. 26 and 39. (The $^1S_0$ experimental pp phase shifts at $E_{lab} < 320$ MeV are practically identical with the $^1S_0$ phase shifts in the Reid potential.)

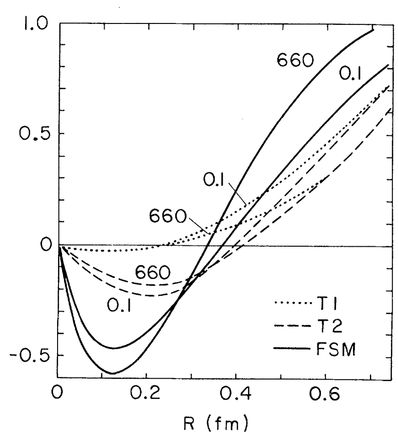

FIG. 2. The continuum wave functions. The numbers denote energy in the laboratory system (MeV). $T1$ is the "structural core" model of Tamagaki (Ref. 11) with nonlocal "repulsion" at small distances. $T2$ is the same but the nonlocal "attraction." FSM denotes the potentials (7) and (9). (The normalization is arbitrary: All functions become unity at the point $R \cong 0.85$ fm).

tial (7) are compared to the wave functions of the model of the "structural core"[11] and the wave functions for the "realistic" potential with the hard core.[35] One can see that the wave function of the S state has one node in the overlap region of nucleons at $R \cong 0.4$ fm, i.e., there is one forbidden state. This fact is also in good qualitative agreement with the data on scattering phases at high energies $E_{\text{lab}} \gtrsim 1$ GeV[26,39]: Increasing the value of the S phase shift at zero from $\delta(0)=0$ to $\delta_0(0)=\pi$ in accordance with the generalized Levinson's theorem Eq. (1) we obtain natural "matching" of the data at low and high energies to form a unified picture (the dot-dash line in Fig. 1). In analogy with the nuclear optical potential,[41] the transition of the S phase shift from zero at an energy of $E_{\text{lab}} \cong 5$ GeV to small negative values seems to be caused by strong absorption and "depression" of the real part of the potential with increasing E.

It is natural to analyze also the electron form factors for a deuteron in order to find out how these are influenced by the fact that at small distances the wave function does not die out, but oscillates, has a node, and has a loop.

In our preliminary calculations, we have used a modification of the Hulthen function obtained by its orthogonalization to the $0S$ oscillator function of small radius a:

$$u_d(R)=\frac{1}{N_1}\left(e^{-\alpha R}-e^{-\gamma R}\right)-\frac{R}{N_2}e^{-R^2/2a^2} \quad . \tag{11}$$

The parameters γ and a have been chosen from the requirement of correct description of the deuteron range characteristics [the r.m.s and effective radii are, as is known, $(\langle R^2\rangle)^{1/2}=1.96$ fm and $\rho(-\epsilon,-\epsilon)=1.82$ fm, respectively], with the additional requirement that the node of the wave function (11) be approximately where the "realistic" potential is in the repulsive core (0.4–0.5 fm). It has turned out that all these requirements may be met simultaneously only provided that the position of the node is close to the point $R=0.36$ fm, at which the node of the wave function for the potential (7) is located. At the parameter values $\alpha=0.232$ fm^{-1}, $\gamma=1.4$ fm^{-1}, and $a=0.28$ fm we obtain $(\langle R^2\rangle)^{1/2}=1.972$ fm, $\rho(-\epsilon,-\epsilon)=1.840$ fm, and $R_{\text{node}}=0.393$ fm. For normalization taking into account that the contribution from the D wave is 7% $[\int_0^\infty u_d^2(R)dR=0.93]$ the normalizing constants are $N_1=1.151217$ fm$^{1/2}$ and $N_2=0.490306$ fm$^{3/2}$. As a wave function for the deuteron $^3D_1$ wave, $w_d(R)$, we have used the wave function in the Hamada-Johnston potential.[40] At distances $R \gtrsim 0.5$ fm, the wave function (11) is close to that in the realistic potential (Fig. 3). At small distances it has a loop whose contribution to the deuteron form factors is easy to estimate: It is equal approximately to a one-half of the Fourier transform of the squared Gaussian term in formula (11)[42]

$$F_{\text{loop}}(q^2)\cong\frac{1}{2}\sqrt{\tfrac{1}{2}\pi}\;\frac{(a\sqrt{2}\,)^3}{8N_2^{\,2}}\,e^{-q^2a^2/16} \cong 0.02e^{-0.0049q^2}\ \text{fm}^{3/2} \quad . \tag{12}$$

It is very essential that this term damps but weakly at large q^2, whereas the contribution to the form factors from the remaining part of the wave function for the deuteron $F_0(q^2)=\int_0^\infty(u_d^{\,2}+w_d^{\,2})j_0(\tfrac{1}{2}qR)\times dR$ dies out much faster (it practically coincides

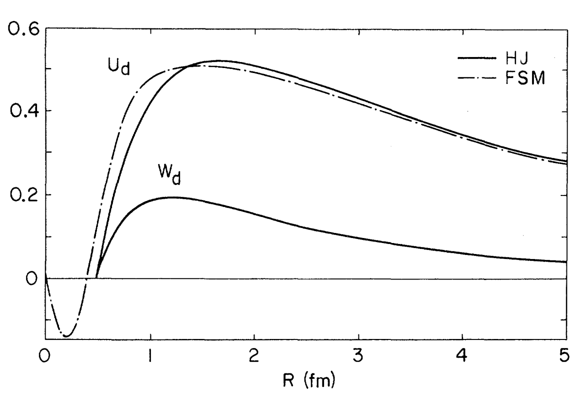

FIG. 3. The wave functions of the deuteron in the model with a forbidden state (FSM). HJ is the function for the Hamada-Johnston potential (Ref. 40).

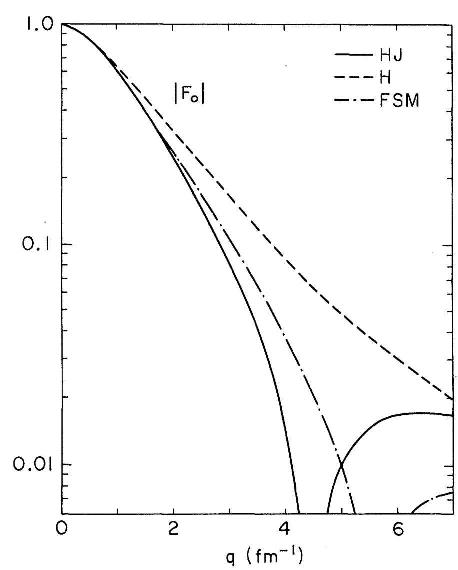

FIG. 4. The integral $|F_0|$ determining the deuteron charge monopole form factor. H stands for the Hulthen function without a repulsive core; HJ for the Hamada-Johnston potential; and FSM for the model with forbidden states.

with the deuteron form factor for the "realistic" potential). At momentum transfer $q^2 \cong 8$–$10\ \mathrm{fm}^{-2}$ the contribution from (12) is as high as 20 or 30% of the whole matrix element of the deuteron $^3S_1$ wave (see Figs 4 and 5).

Unfortunately, the available experimental results on the elastic scattering do not allow one to establish whether or not the term (12) is present in the deuteron charge monopole form factor $G_0(q^2) = G_{ES}F_0$, since the cross section depends on the form factor $G_0(q^2)$ only at small momenta $q^2 \lesssim 6\ \mathrm{fm}^{-2}$, when one may distinquish only cruder details of the deuteron (if models with or without a repulsive core lead in this region to cross sections different by 10 to 20%, the finer effects that distinguish models with forbidden states from those with a core do not exceed here 5%). At large q^2, the cross section is determined by the quadrupole form factor $G_2(q^2)$ (i.e., by the contribution from the deuteron D wave), which dies out only at $q^2 \gtrsim 60\ \mathrm{fm}^{-2}$. At the present time, many authors[43] propose more delicate polarization experiments, which would allow the contribution from the monopole form factor $G_0(q^2)$ to be separated in the region of interest $q^2 \cong 20\ \mathrm{fm}^{-2}$, where it changes its sign. Figure 4 demonstrates that the position of the corresponding minimum $|F_0|$ in the model with forbidden states differs from that obtained for the "realistic" potential. Here one should bear in mind, however, that at large momentum transfer, the nonrelativistic treatment of the deuteron is inadequate. Further, one should take into account the nonadditive effects from the exchange meson currents. Accordingly, the shift to the right of the minimum $|F_0|$ obtained in the model with forbidden states may be considered merely as a qualitative prediction.

In experiments on the ed inelastic backward scattering near the threshold of deutron breakup ($E^* = 1$–3 MeV), a pure $M1$ transition[44,45] into the S states of the continuum has been observed. The structural functions for magnetic transition ("nuclear" matrix elements) may be expressed, in the nonrelativistic impulse approximation, in terms of the wave function for the continuum $z_s(E^*, R)$ and the deuteron wave functions $u_d(R)$ and $w_d(R)$[44,45]:

$$H_S(q^2) = \int_0^\infty u_d(R) z_S(E^*, R) j_0(\tfrac{1}{2}qR)\,dR\ ,$$
$$J_S(q^2) = \int_0^\infty w_d(R) z_S(E^*, R) j_0(\tfrac{1}{2}qR)\,dR\ . \tag{13}$$

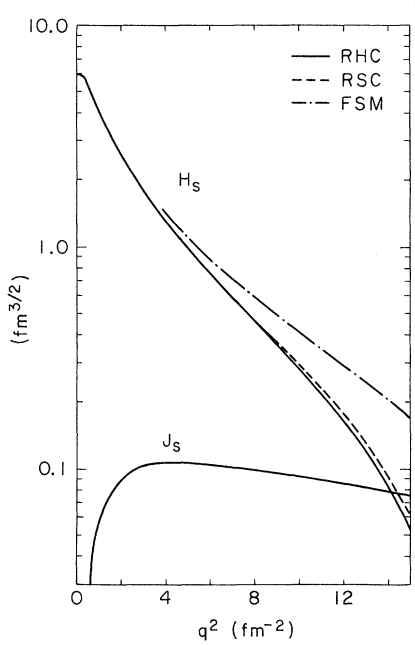

FIG. 5. The structural functions for the dipole magnetic transition. RHC and RSC denote the Reid potential with a hard and soft core, respectively (Ref. 45). FSM denotes the model with forbidden states.

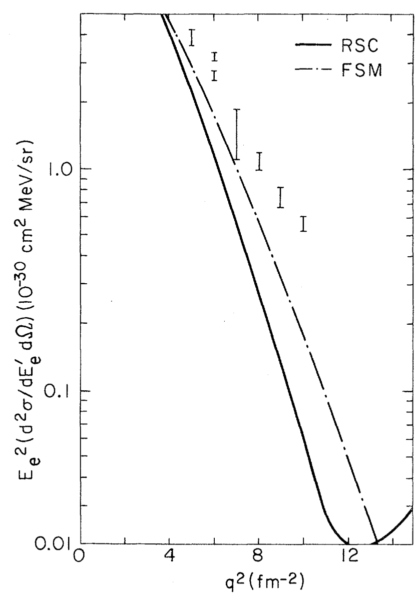

FIG. 6. The differential cross section $E_e^2(d^2\sigma/dE_e'\,d\Omega_e)$ for the deuteron breakup in electron backward scattering (the excitation energy $E^* = 3$ MeV is above the threshold). The values shown at experimental points in Ref. 46 are divided here by the factor 0.92, which takes into account the contribution from radiative processes. The curves show the total "nuclear" cross section for the transition to the $^1S_0$ and $^3S_1$ states. Designations are the same as in Fig. 5.

Figure 5 compares the functions H_S and J_S for the model with forbidden states for the transition to the ${}^1S_0$ state with the corresponding functions for the "realistic" potentials. Figure 6 shows the respective contributions to the cross section calculated by the formula of impulse approximation[45]

$$E_e{}^2 \frac{d^2\sigma}{dE_e' d\Omega_e} = \frac{\alpha^2}{6\pi}\left(\frac{M}{E}\right)(pM)\left(\frac{q}{M}\right)^2 G_{MV}{}^2(q^2) \times [H_S(q^2) - 2J_S(q^2)]^2 , \tag{14}$$

where $\alpha = 1/137$, $p = (E^*M)^{1/2}$, M is the nucleon mass, $G_{MV}(q^2)$ is the magnetic isovector form factor for the nucleon,[47] E_e, E_e' are the initial and final electron energies $E = (M^2 + q^2)^{1/2}$, $E^*_{\rm cms} = 3$ MeV. For momenta $q^2 \gtrsim 8$ fm^{-2}, the term (12) turns out to be very significant in comparison with the value $(H_S - 2J_S)$ standing in the expression for the cross section. As a result, the cross section (14) increases by a factor of 2 or 3, as compared with that obtained for the "realistic" potential. However, in the region $q^2 \gtrsim 8$ fm^{-2} the "nuclear" cross section (6) in any case turns out to be several times lower than does the experimental one which is accounted for by a large contribution from the exchange meson currents to the magnetic transition. The contribution from meson currents may be calculated only fairly approximately; therefore, against such a background one cannot uniquely determine the (relatively great) contribution to the "nuclear" cross section, predicted by the model with forbidden states. According to the most recent calculations by Hockert *et al.*,[45] taking into account the exchange currents only in the peripheral π-meson region of the deuteron gives the breakup cross sections close to the experimental ones. The additional term in the "nuclear" cross section arising in the model with forbidden states increases the total cross section only by about 20% if $q^2 \cong 10$ fm^{-2}. This serves to improve our preliminary results[48] based on an earlier paper by Adler.[44]

V. CONCLUSION

In conclusion, just a few words as to further possibilities. It is known that as the nucleon energy $E_{\rm lab}$ approaches 2 or 3 GeV, the exchange effects associated with the operators P_x, P_σ, etc., became insignificant.[49] Therefore, it may be thought that at energies of 2–3 GeV all partial waves will be described approximately by a single potential possessing a strong spin-orbital interaction (the gradient term of the singular potential), together with a strong absorption. Indeed, at $E_{\rm lab} \cong 3$ GeV, the spin-orbital splitting of the ${}^3P_J$ phases reaches a very high value of ~160°.[26] Obviously, at such high energies it is necessary to use the relativistic quasipotential equation[50]: Here one should remember that the role of the imaginary part of the quasipotential, which grows proportionally to the energy,[51] becomes dominant at energies $E_{\rm lab} \cong 5$–10 GeV. However, at energies of several GeV the quasipotential approach, combined with consideration of the LS forces obtained by differentiating the potential (7), appears to provide us with the interesting possibility of interpreting polarization in the N-N scattering. Moreover, the approach suggested here may be applied to describe the interaction between any two hadrons. Comparison between the N-N and N-$\bar{N}$ interactions in the region beyond the node of the wave function may give an idea of the contribution from the vector mesons and so on.

Of course, our treatment is rather tentative and should be critically analyzed further, but if the concept of a deep potential with forbidden states turns out to be correct, it may bring important information on the statistics of quarks, their interactions, and the character of symmetry of the orbital part of the nucleon wave function.

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