Classical model for strongly damped collisions in heavy-ion reactions*

J. P. Bondorf
The Niels Bohr Institute, University of Copenhagen, Copenhagen, Denmark

J. R. Huizenga†
Departments of Chemistry and Physics, University of Rochester, Rochester, New York 14627

M. I. Sobel‡
Department of Physics, Brooklyn College, Brooklyn, New York 11210

D. Sperber§
Department of Physics, Rensselaer Polytechnic Institute, Troy, New York 12181
(Received 16 January 1975)

A classical model with nuclear stretching under the influence of a repulsive conservative force and a dissipative radial force is proposed to explain the observed kinetic energy spectra and angular distributions of very heavy-ion reactions. The model is applied to the strongly damped collisions in the ^{209}Bi + ^{84}Kr reaction.

[NUCLEAR REACTIONS Classical model with dissipative forces for very heavy ions.]

I. INTRODUCTION

The purpose of this paper is to suggest a classical model to interpret recent experimental results concerning reactions induced by very heavy ions, and in particular by Kr. Attention is focused on strongly damped collisions, that is collisions in which a large amount of kinetic energy in relative motion is transformed into internal excitation energy. It is assumed that after the target and projectile touch, a neck is formed and the ions remain in contact as they stretch out to the snapping distance. According to the model, strong dissipative forces operate as soon as the nuclei touch and play a significant role as they stretch apart.

Our model is motivated by observing the behavior of inelastic collisions between liquid drops. In particular we consider the asymmetry between the entrance and exit channels in a collision between liquid drops. As soon as the drops touch they start to interact at a distance corresponding approximately to the sum of their radii. As the drops separate, a neck is formed and energy is dissipated until the system snaps at a distance which is considerably larger than the touching distance. Similarly it has been observed that the loss of kinetic energy in strongly damped collisions, or deep inelastic transfer reactions, is so large that the measured kinetic energy corresponds to the Coulomb potential energy for charge centers separated by a distance much larger than the sum of the radii.[1-6] Furthermore, there is a prominent gap in the energy spectrum of the projectile-like particle between strongly damped collisions and elastic collisions or collisions corresponding to ordinary few-nucleon transfer.

Additional evidence for a large snapping distance is found in the size of the relative cross sections for multinucleon transfer reactions. In order to explain the multinucleon transfer cross sections with a statistical model one has to use an effective barrier based on a radial parameter r_0, equal to or greater than 1.5 fm.[7] Although this evidence is indirect and subject to possible modification due to sequential particle decay,[8] it supports the idea that the system stretches before snapping. In addition, the motion described here is very similar to the motion of a fissioning nucleus before scission.

Strongly damped collisions are observed in reactions induced by very heavy ions such as Ar and Kr. The complete fusion or compound nucleus process which is dominant for ions with $A < 40$ is still significant for Ar induced reactions. However, for Kr induced reactions the major fraction of the cross section is accounted for by strongly damped collision processes. The latter process is characterized by (1) nucleon transfer yielding fragments with masses in the vicinity of the target and projectile masses (although if the target mass is large enough the excited heavy fragment will sequentially fission), (2) strong damping of the energy of relative motion of the fragments resulting in final fragment kinetic energies which correspond to Coulomb energies for charge centers of highly deformed fragments, and (3) angular distributions which have features characteristic of a direct reaction. The strongly damped collisions for the Kr induced reactions have a peak in their angular distributions near the grazing angle, whereas there is some evidence for orbiting in the Ar induced reactions.[9-11]

II. CLASSICAL MODEL

According to the classical picture, we assume that nuclei approach each other on a trajectory determined by the conservative and dissipative forces. We assume that for very large initial orbital angular momenta projectile and target fail to reach a critical distance R_i at which they interact strongly and, therefore, elastic scattering or few-nucleon transfer occurs. For initial angular momenta smaller than the limiting value l_{SDC}, projectile and target reach a distance where the motion is highly damped due to strong dissipative forces. For simplicity we assume that as soon as the distance R_i is reached the dissipative forces are so strong that almost all radial kinetic energy is suddenly dissipated. In addition, rotational kinetic energy is dissipated to the maximum allowed by angular momentum conservation corresponding to the sticking condition. Hence, in the following discussion we assume that the radial kinetic energy and angular momentum are lost so quickly that the colliding nuclei do not rotate significantly during the initial impact. At this point a neck develops and the system continues to rotate while stretching under the influence of a repulsive conservative force and a retarding radial friction. The angular velocity decreases slightly as the system stretches because the moment of inertia increases. Finally, as the distance between the charge centers reaches the snapping distance R_f, where $R_f > R_i$, the fragments separate and move apart under the influence of the repulsive conservative force. For a fixed bombarding energy we show schematically in Fig. 1 the potentials and reaction paths for two different initial angular momenta. It can be seen for $l > l_{SDC}$ that the nuclei fail to reach the interaction region characterized by R_i. Hence, for these orbital angular momenta elastic scattering or few-nucleon transfer occurs. As soon as the initial angular momentum is sufficiently small, target and projectile touch and the motion between R_i and R_f is highly dissipative corresponding to considerable energy transport to other degrees of freedom. For even smaller angular momenta, some energy is also lost before reaching the critical distance R_i; the amount depending on the magnitude of R_i or the degree of penetration.

From the value of the angle θ_{SDC} (for the definition of the angles see Fig. 2) at which the angular distribution peaks for the strongly damped collision process we estimate the strength of the radial friction for the region $R_i < r < R_f$. This angle is written as

$$\theta_{SDC} = \pi - \theta_1 - \theta_2 - \theta_3 , \tag{1}$$

where θ_1 and θ_3 are the deflection angles associated with Coulomb orbits for $\infty > r > R_i$ and $R_f < r < \infty$, respectively, and θ_2 is the angle of deflection during the sticking for $R_i < r < R_f$. The deflection an-

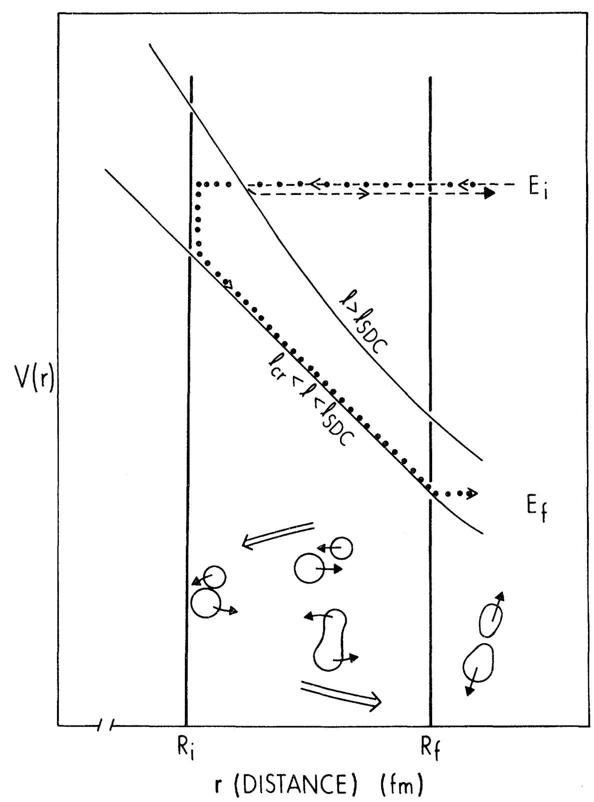

FIG. 1. Schematic representation of the energy for the strongly damped collision process. For $l > l_{SDC}$, the projectile and target fail to reach a distance R_i at which they interact strongly and, therefore, elastic scattering or few-nucleon transfer occurs. For angular momenta $l_{cr} < l < l_{SDC}$, projectile and target reach R_i where the radial and rotational kinetic energies are suddenly dissipated. At this point the system sticks and a neck develops during rotation and stretching under the influence of a repulsive conservative force and a retarding radial friction. The sketches in the lower part of the figure indicate the shape of the system as the strongly damped collision develops in time.

gle for a Coulomb orbit is given by

$$\theta_j = + \text{arc} \cos \frac{1}{\epsilon_j} - \text{arc} \cos \frac{K_j + R_j}{\epsilon_j R_j} \quad (\text{for } j = 1 \text{ and } 3) \tag{2}$$

In Eq. (2) the parameters ϵ and K are determined by

$$\epsilon = \left[1 + \frac{2El^2\hbar^2}{\mu (Z_1 Z_2 e^2)^2} \right]^{1/2} , \tag{3a}$$

$$K = \frac{l^2 \hbar^2}{\mu Z_1 Z_2 e^2} . \tag{3b}$$

The angular velocity during stretching is given by

$$\frac{d\theta_2}{dt} = \frac{l\hbar}{\mathscr{I}} . \tag{4}$$

In Eq. (4) the moment of inertia $\mathscr{I}$ is approximately,

$$\mathscr{I} = \tfrac{2}{5} M_1 R_1{}^2 + \tfrac{2}{5} M_2 R_2{}^2 + \mu r^2 , \tag{5}$$

where M_1 and M_2 and R_1 and R_2 are the masses and radii, respectively, of the two nuclei, μ is the reduced mass of the system, and r the distance between the centers of the two nuclei. The conservative and frictional forces are represented by F_c and F_f. We assume that the frictional force is proportional to the velocity so that

$$F_f = - k_r \frac{dr}{dt} . \tag{6}$$

The conservative force F_c is derived from the potential energy by

$$F_c = - \frac{\partial V}{\partial r} , \tag{7a}$$

where

$$V = V_{\text{Coul}} + V_{\text{nuc}} + V_{\text{rot}} . \tag{7b}$$

A Saxon-Woods form of the nuclear potential with parameters ($r_0 = 1.17$ fm, $d = 0.75$ fm, and $V_0 = -99.9$ MeV) consistent with the liquid drop model[12] is used in the above equation. Such a nuclear potential resembles closely the proximity potential.[13] However, such a potential is for spheres and *does not* include neck degrees of freedom. In this sense the model is inconsistent insofar that the necking is included for the friction in the exit channel but not for the nuclear potential. For simplicity we assume that the friction is sufficiently strong so that the radial velocity reaches its asymptotic value very soon and is given by

$$\frac{dr}{dt} = \frac{F_c}{k_r} . \tag{8}$$

Combination of Eqs. (4) and (8) leads to

$$\theta_2 = k_r l\hbar \int_{R_i}^{R_f} \frac{dr}{\mathscr{I} F_c} . \tag{9}$$

In order to evaluate k_r we associate θ_{SDC} with a range of l values. Let σ_{SDC} be the total cross section for strongly damped collisions, then

$$\sigma_{\text{SDC}} = \pi \lambdabar^2 (l_{\text{SDC}}{}^2 - l_{\text{cr}}{}^2) , \tag{10}$$

where l values in the range $l_{\text{cr}} < l < l_{\text{SDC}}$ contribute to the strongly damped collisions. The value of the critical angular l_{cr} for the reaction ^{209}Bi + ^{84}Kr is small (we assume $l_{\text{cr}} = 75$ for a bombarding energy of 600 MeV). The values of l_{SDC} are approximately 150 and 250 for bombarding energies of 525[2] and 600[3] MeV, respectively. The corresponding values of θ_{SDC} are 95 and 58°, respectively. The value of k_r is determined from Eq. (9) where the integral is evaluated numerically and θ_2 is calculated from Eqs. (1) and (2). In the calculation of θ_2 we assume that θ_{SDC} is independent of l and has a single value given by the peak of the angular distribution. Since the full width at half-maximum of the angular distribution is approximately 25°, this assumption introduces an uncertainty of approximately 50% in k_r. Values of k_r for l values of 240, 200, and 160 are shown in Fig. 3 for the ^{209}Bi + ^{84}Kr reaction[3] at a bombarding energy of 600 MeV. The value of R_f is kept

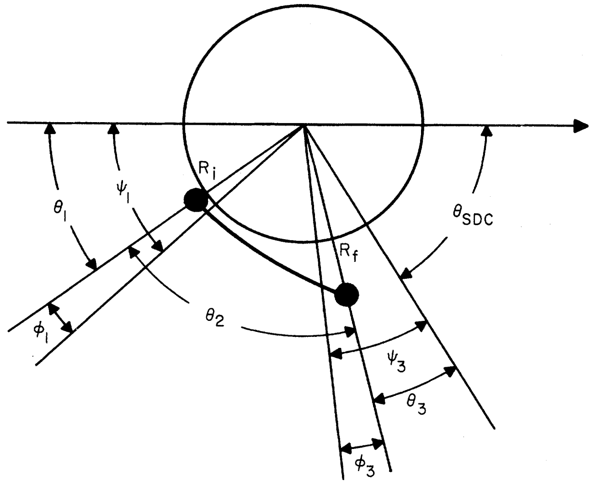

FIG. 2. Schematic diagram of the relevant angles in the strongly damped collision process. The angles θ_1 and θ_3 are the deflection angles associated with Coulomb orbits for $\infty > r > R_i$ and $R_f < r < \infty$, respectively, and θ_2 is the angle of deflection during the sticking for $R_i < r < R_f$. The angles ψ_j and ϕ_j (where $j = 1$ and 3) are given by arcos $1/\epsilon_j$ and arcos $[(K_j + R_j)/\epsilon_j R_j]$, respectively, where $\theta_j = \psi_j - \phi_j$ and ϵ_j and K_j are defined by Eqs. (3a) and (3b). The angle θ_{SDC} is the observed scattering angle for a particular l wave for the strongly damped collision process.

constant at 17 fm. It can be seen from the results plotted in Fig. 3 that if R_i is assumed constant, k_r increases as l decreases. On the other hand a constant value of k_r is consistent with R_i decreasing with decreasing l. The values of k_r plotted in Fig. 3 depend on F_c [see Eq. (9)] and, therefore, are slightly model dependent. This follows because both the conservative and dissipative forces are assumed to act in the whole interval $R_i < r < R_f$. However, for large values of r near R_f, the present model predicts that the dominant part of the integral [Eq. (9)] comes from the Coulomb and centrifugal forces.

Values of k_r similar to those displayed in Fig. 3 are deduced from the $^{209}Bi + ^{84}Kr$ reaction[2] at 525 MeV and the $^{197}Au + ^{63}Cu$ reaction[5] at 365 MeV. It is interesting also to note that the smaller moment of inertia for the $^{232}Th + ^{40}Ar$ system is partially responsible for the orbiting observed[1] at 388 MeV bombarding energy.

In order to describe the complete angular distribution one requires a knowledge of the scattering angles for each l value in the range $l_{cr} < l < l_{SDC}$. It seems reasonable to assume that as l decreases the interaction between the nuclei increases (e.g., the neck is wider and plays a more significant role). Therefore, one expects a higher value of k_r for lower l values. This expectation is consistent with our finding that for a constant value of R_i, k_r increases with a decrease of l. This has

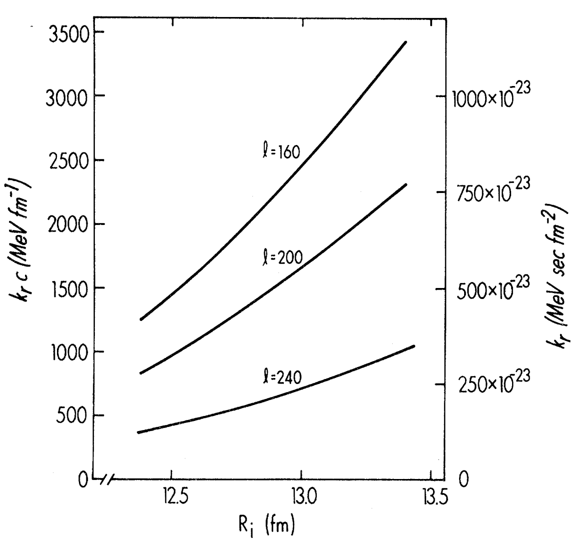

FIG. 3. Experimentally determined values of the frictional constant k_r for l values of 160, 200, and 240 for the reaction $^{209}Bi + ^{84}Kr$ at a bombarding energy of 600 MeV. The values of k_r and $k_r c$ (where c is the velocity of light) are plotted as a function of the assumed initial radial distance R_i. The snapping distance R_f is kept fixed at 17 fm. The values of k_r depend on F_c [see Eq. (9)] and, hence, are slightly model dependent.

the effect of bending the trajectories for lower l values more forward, and focusing them at the same scattering angle as for the higher l values. A more detailed study of the angular distribution, in which not only the position of the peak but also the width of the peak is determined, requires the inclusion of charge and mass transfer which is specially significant for the lower l values.

Having determined the value of the frictional constant which reproduces the angle of the peak in the angular distribution, one may ask if this friction is consistent with the observed energy loss. According to the present model, the system moves apart with a very small kinetic energy in the regions $R_i < r < R_f$. This condition is satisfied if the friction is strong enough so that

$$\tfrac{1}{2}\mu\left(\frac{dr}{dt}\right)^2 \ll V(R_f) \tag{11}$$

or

$$\left[\frac{1}{2}\,\frac{\mu F_c^2}{V(R_f)}\right] \ll k_r^{\,2}\ .$$

For the $^{209}Bi + ^{84}Kr$ reaction at 600 MeV and $l = 240$, the above condition implies that $k_r c > 200$ MeV/fm, a limiting value consistent with the values deduced for the friction from angular distributions (see Fig. 3). One may ask also if the friction is strong enough to insure that the velocity reaches the asymptotic condition early in the stretching process. The requirement for this condition is similar to the above condition and is fulfilled. If the sudden impact approximation at $r = R_i$ (which is assumed in the present model) is relaxed, the value of k_r calculated from Eq. (9) is decreased because some rotation of the system occurs before R_i is reached. However, the limiting value of k_r from Eq. (11) is not altered. According to the present model, the loss of kinetic energy in the exit channel accounts for only approximately 50% of the total energy loss. Hence, a considerable energy loss occurs during the sudden impact in the entrance channel. This feature exhibits the incompleteness of the sudden impact approximation and the importance of finding a more detailed description of the energy loss in the entrance channel. In this connection, however, degrees of freedom corresponding to deformations, which play a role in the kinetic energy loss, are neglected in the exit channel of the present simple model. As already mentioned, k_r may increase due to the deformation dependence of F_c in the exit channel and additional loss in kinetic energy appears at scission as deformation energy.

A magnitude of approximately 10^{-23} MeV sec fm^{-3} for the nuclear viscosity, discussed recently by several authors,[14-18] has been estimated from the damping of the motion from saddle to scission, widths of β vibrations and giant dipole resonances[14] as well as from the effect of viscosity on fission fragment kinetic energies.[15] In the present model we have assumed that the frictional force is proportional to the velocity in the direction r as defined by Eq. (6). The quantity k_r is identified as the parameter in the Rayleigh dissipation function,[19,20] where we assume that the rate of energy loss is due to friction alone. Comparison of the values of k_r shown in Fig. 3 with the above estimates of the nuclear viscosity can be made through the Rayleigh dissipation function. However, such a comparison requires a model for the dissipation of energy by viscosity.

The reaction time[12] for the strongly damped collision process is given by Eq. (4). This time depends on the angular momentum and is derived from measurements of the sticking angle θ_2. Reaction times of approximately 0.5×10^{-21}, 1×10^{-21}, and 2×10^{-21} sec are deduced for l values of 240, 200, and 160, respectively, from the $^{209}Bi + ^{84}Kr$ reaction at a bombarding energy of 600 MeV. These times are comparable to or shorter than those estimated for the descent from saddle to scission in nuclear fission[21] depending on the assumptions (e.g. about the nuclear viscosity) made for the fission process.

In summary, the present classical model is consistent with the main experimental results obtained with Kr[2,3] and Ar[1] projectiles and explains (1) the gap in the energy spectra, (2) the angles of the peak in the angular distributions for the strongly damped collisions, and (3) the major part of the energy losses in such collisions.

ACKNOWLEDGMENTS

Three of us (JRH, MIS, and DS) acknowledge the hospitality of the Niels Bohr Institute and NORDITA in Copenhagen where the initial part of this work was performed. One of the authors (JRH) wishes to thank J. R. Nix for stimulating discussions on nuclear viscosity.

---

*Work supported in part by the U.S. Atomic Energy Commission.

†Guggenheim Fellow, 1973-74.

‡NATO Fellow, 1973–74.

§NORDITA Professor in Copenhagen, 1973-74.

[1]A. G. Artukh, G. F. Gridnev, V. L. Mikheev, V. V. Volkov, and J. Wilczynski, Nucl. Phys. A215, 91 (1973).

[2]F. Hanappe, M. Lefort, C. Ngô, J. Pêter, and B. Tamain, Phys. Rev. Lett. 32, 738 (1974).

[3]K. L. Wolf, J. P. Unik, J. R. Huizenga, V. E. Viola, J. R. Birkelund, and H. Freiesleben, Phys. Rev. Lett. 33, 1105 (1974).

[4]S. G. Thompson, L. G. Moretto, R. C. Jared, R. P. Babinet, J. Galin, M. M. Fowler, R. C. Gatti, and J. B. Hunter, Lawrence Berkeley Laboratory Report No. LBL-2940, 1974 (unpublished).

[5]F. Hanappe, C. Ngô, J. Pêter, and B. Tamain, in *Proceedings of the International Conference on Reactions Between Complex Nuclei, Nashville, Tennessee, June 10–14, 1974,* edited by R. L. Robinson, F. K. McGowan, J. B. Ball, and J. H. Hamilton (North-Holland, Amsterdam/American Elsevier, New York, 1974), Vol. I, p. 116; J. Pêter, C. Ngô, and B. Tamain, J. Phys. Lett. (to be published).

[6]R. Vandenbosch, M. P. Webb, and T. D. Thomas, in Proceedings of the Symposium on Classical and Quantum Mechanical Aspects of Heavy Ion Collisions, Heidelberg, Germany, October, 1974 (unpublished).

[7]J. P. Bondorf, F. Dickmann, D. H. E. Gross, and P. J. Siemens, J. Phys. (Paris), 32, 145 (1971).

[8]J. P. Bondorf and W. Nörenberg, Phys. Lett. 44B, 487 (1973).

[9]J. Wilczynski, Phys. Lett. 47B, 484 (1973).

[10]J. P. Bondorf, M. I. Sobel, and D. Sperber, Phys. Rep. 15C, 83 (1974).

[11]D. H. E. Gross, H. Kalinowski, and J. H. De, to be published.

[12]J. R. Huizenga, University of Rochester Report No. UR-NSRL-90, 1974 (unpublished); Nukleonika (to be published).

[13]W. J. Swiatecki, J. Randrup, and C. F. Tsang, to be published; Phys. Scr. (to be published).

[14]R. Wieczorek, R. W. Hasse, and G. Süssmann, in *Proceedings of the Third International Conference on the Physics and Chemistry of Fission, Rochester, 1973* (IAEA, Vienna, 1974), Vol. 1, p. 523.

[15]J. R. Nix and A. J. Sierk, Los Alamos Report No. LA-UR-74-880 (unpublished); Phys. Scr. (to be published); private communication from J. R. Nix.

[16]R. H. Davis, Phys. Rev. C 9, 2411 (1974).

[17]C. T. Alonso, Lawrence Berkeley Laboratory Report No. LBL-2993, 1974 (unpublished).

[18]B. Giraud, J. Le Tourneux, and E. Osnes, Phys. Rev. C 11, 82 (1975).

[19]H. Lamb, *Hydrodynamics* (Dover, New York, 1945), p. 581.

[20]H. Goldstein, *Classical Mechanics* (Addison-Wesley, Reading, 1965), p. 21.

[21]J. R. Nix, Nucl. Phys. A130, 241 (1969).