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Aspects of π-d scattering in a relativistic three-body model*

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A relativistic three-body model of the π-d system is employed to calculate elastic scattering in the (3,3) resonance region. The results demonstrate that this effect is not related to a resonance pole in the π-d amplitude, and may be simply understood in terms of well known rescattering singularities. Related questions concerning convergence of the multiple scattering series and pion absorption are discussed.

[NUCLEAR REACTIONS ^{2,2}H(π, π); calculated scattering amplitudes in three-body system.]

I. INTRODUCTION

In order for pions to realize their full potential as nuclear probes, one must resolve two major theoretical questions. The first is fundamental, concerning the nature of the basic π-N interaction in the presence of other nucleons. This involves both a knowledge of the off-shell π-N scattering amplitude and an understanding of how to deal with virtual pion production in the nuclear interaction. In this connection one must observe that even the *on shell* π-N parameters involve large experimental uncertainties. On the other hand, since a practical theoretical treatment must necessarily be approximate, it is essential to test the characteristic assumptions which have been employed in handling this special n-body problem. In particular, one might question the adequacy of the "frozen" approximation (treating the nucleons as fixed scattering centers) and the neglect of all but the simplest relativistic corrections.

A three-body treatment of the π-d system affords a unique opportunity to study both of these questions, since one can generate for comparison both exact and approximate results corresponding to the same dynamical assumptions. In turn, one may test those assumptions by comparing the exact results with experiment. These considerations have prompted several recent calculations based on the (nonrelativistic) Faddeev equations. Thus, assuming separable interactions, Afnan and Thomas used this approach to study pion production and absorption for energies near the π-d threshold,[1] while Myhrer and Koltun investigated scattering near the (3, 3) resonance.[2] However, there are drawbacks to this procedure inasmuch as one may criticize both the separability assumption and the use of a nonrelativistic description. In addition, Myhrer and Koltun appear to have missed the main point to be learned concerning the nature of the (3, 3) "resonance" in nuclei.

In the present work we employ a relativistic generalization of the author's boundary condition formalism (BCF),[3] which has been applied previously to calculate the ω as a 3π resonance,[4] and to give a dynamical description of the basic N-N interaction.[5] Specializing to an initial p-wave state, we calculate elastic scattering for laboratory energies T_L ranging from threshold to 260 MeV. The model enables us to focus on the nature of the (3, 3) resonance, convergence of the multiple scattering series (MSS), and the effect of pion absorption in this energy region. In particular, our results demonstrate that the π-d system does *not* possess a resonance in the usual sense of the word, and that a description of the enhancement in terms of Breit-Wigner resonance parameters is both incorrect and misleading.[6] On the contrary, the effect is a manifestation of the rescattering singularities well known in the literature, and previously discussed within the context of the three-body problem by the author and Peierls.[7]

We begin in Sec. II with a description of our model and the required two-particle input parameters. The numerical results are presented in Sec. III and interpreted in the context of the general program outlined above. Our conclusions and their implications are discussed in Sec. IV.

II. INTERACTION MODEL

In the present application, we deal with a special case of the BCF which is analogous to the Feshbach-Lomon boundary condition model (BCM).[8] This corresponds to an interaction which is compressed to the surface of an impenetrable core of radius r_b; for $r > r_b$ the wave function takes its asymptotic form. However, in contrast to the

BCM we introduce an *energy dependent* logarithmic derivative to characterize the behavior at the boundary; this permits us to retain the exact experimental phase shifts as our input. In addition, of course, the model is applied in the three-particle sector. The result is a set of one-dimensional integral equations comparable in simplicity to the Faddeev equations (for a rank one potential), although no separability assumption is required. The derivation of these equations, their physical content, and their precise mathematical form have been discussed in a number of previous articles, and we shall not consider them here.[9] It is worth pointing out, however, that the nonrelativistic formalism produces results which are identical to those obtained in Faddeev calculations (for comparable input).[10]

To form a three-body state of total angular momentum J, we couple particles β and γ to a state with total $l_{\beta\gamma}$ and couple this to l_α of the third particle (relative to the $\beta\gamma$ c.m.); thus $\vec{J}=\vec{l}_{\beta\gamma}+\vec{l}_\alpha$. The corresponding relation for the orbital angular momentum is $\vec{L}=\vec{l}_{\beta\gamma}+\vec{l}_\alpha$, and the parity is equal to $(-)^{l_{\beta\gamma}+l_\alpha}$ times the intrinsic parities of the three particles. For a given choice of $l_{\beta\gamma}$ the equations couple states in which $l_\alpha - l'_\alpha$ is *even*, since parity is conserved. Furthermore, as a simple consequence of angular momentum barriers, a force which acts in a given $l_{\beta\gamma}$ state is strongest in a three-body state for which $l_{\beta\gamma}+l_\alpha$ is minimal, i.e., the smallest value of l_α consistent with L and the parity. Typically, since $l_\alpha - l_\alpha^{\min} \geq 2$, this means that in practice one can retain only $l_\alpha^{\min}$ to an excellent approximation. On the other hand, for a given total $l_{\beta\gamma}+l_\alpha$, one may attempt to neglect those $l_{\beta\gamma}$ states for which the two-particle scattering is known to be small empirically. This approximation is less well founded, but is usually justified if the disparity between the two-particle states is very large (e.g., one state is resonant).

In the present case these considerations lead us to choose the π-d state in which $L=1$, $J^\pi=2^+$ for our investigation. The reason is simply that the "force" generated by the P_{33} π-N interaction will be greatest if $(l_{\beta\gamma}=1)$ $l_\alpha=0$. One would thus expect convergence of the MSS to be poorest in this state, which is also the one in which a true π-d resonance is most likely to develop. The next most important π-N state should be the P_{11}, which contains the nucleon pole, and coupling to this state is eliminated (for $l_\alpha=0$) by choosing $J=2$. Conversely, when we study the absorption channel below, we choose $J^\pi=0^+$ to eliminate the P_{33} state. If one considers only centrifugal barriers, one should also include S_{11}, S_{31} states coupled to a p-wave nucleon, and P_{13} coupled to an s-wave nucleon (P_{31} in the 0^+ case), but we shall neglect these states in view of their small phases.

With regard to the N-N states, we shall clearly need to include the deuteron, which means that we should take $^3S_1$, $^3D_1$ coupled to a p-wave pion. However, the d state should not be vital for our purposes, and hence we shall drop it to simplify the calculation. We also neglect the $^3P_2$ ($^3P_0$) coupled to an s-wave pion, since these phases are very small. As a result of the fact that our $NN\pi$ system contains two identical particles, both our 2^+ and 0^+ calculations reduce to a set of two coupled integral equations, written in terms of a continuous variable q corresponding to the momentum of the spectator particle. Our choices of J, L require that $S=1$, and hence j-j and L-S couplings are identical (and trivial). The only modifications of our basic equations arise from isospin recoupling coefficients and the symmetrization, which are straightforward.[11]

The input to these equations consists of the $^3S_1$ and P_{33} (P_{11}) phases, which determine the corresponding functions $\lambda_l(K^2)$ via the logarithmic derivative

$$(\psi_l'/\psi_l)_{r=r_b}=\lambda_l(K^2)\ , \tag{1}$$

and the asymptotic representation

$$\psi_l(r)=j_l(Kr)+ie^{i\delta_l}\sin\delta_l\, h_l(Kr)\ . \tag{2}$$

For this calculation we have parametrized λ_l in the form

$$r_b\lambda_l(K^2)=\alpha+\gamma/(\delta K^4+\epsilon K^2+\beta)\ ; \tag{3}$$

the parameters corresponding to the various channels are given in Table I. The $^3S_1$ phase shift employed is that of MacGregor, Arndt, and Wright,[12] while the P_{33} phase was taken from Ball *et al.*[13] These phases are of course well determined. For the P_{11} channel we have explicitly imposed the nucleon pole below threshold and varied the parameters to fit the phase shift given by Carter, Bugg, and Carter.[14] The fit obtained in that case is illustrated in Table II, in part to point out the sizable experimental errors. The $^3S_1$ (P_{33}) phases which result from our parametrization are virtually indistinguishable from the experimental values

TABLE I. Parameters for the $^3S_1$, P_{11}, and P_{33} phases corresponding to the parametrization defined in the text.

Phase	r_b (fm)	α	β	γ	δ (fm^4)	ϵ (fm^2)
$^3S_1$	0.860	−1.079	0.276	−0.298	1.0	0.58
P_{11}	0.240	−1.892	−0.968	0.194	0.0	1.00
P_{33}	0.695	−0.551	−3.841	2.770	0.0	1.00

TABLE II. Comparison of the P_{11} phase resulting from our fit to the experimental analysis of Ref. 14.

T_L (MeV)	δ^0 (expt.)	δ^0 (fit)
88.5	-1.03 ± 0.44	-0.92
119.3	-0.85 ± 0.39	-0.73
114.2	-0.54 ± 0.44	-0.26
161.9	0.57 ± 0.67	0.27
191.9	1.78	1.66
219.6	3.40 ± 0.48	3.63
237.9	5.43 ± 0.35	5.40
263.7	8.78 ± 0.70	8.74
291.6	13.53 ± 0.30	13.84
310.0	20.27 ± 0.59	18.34

for c.m. energies ranging from 0 to 300 (600) MeV, respectively.

For reference below we observe that iteration of our integral equation generates a multiple scattering series comparable to the Faddeev-Watson series. This is illustrated in Fig. 1, in which open or solid blobs represent off-shell pairwise amplitudes appropriate to our model, and not elementary vertices. Note that elastic π-d scattering requires that the nucleons interact first and last. Diagrams (a), (b), and (c) correspond to pion rescattering, whereas (d) involves a virtual scattering of the N-N pair in an intermediate state, and hence represents an effect which is neglected in the frozen approximation. Due to the inclusion of nucleon recoil, the former diagrams also contribute corrections to that picture.

III. NUMERICAL RESULTS

Via the three-body equations, the model described in the last section can be applied to treat simultaneously all competing channels relevant to the π-d system in an explicitly unitary fashion.

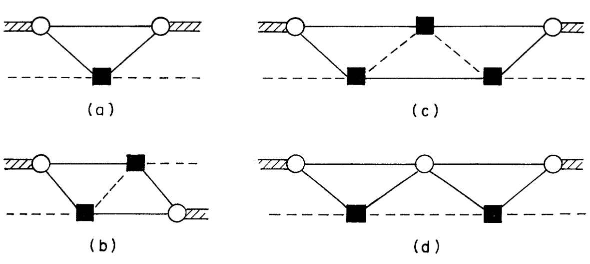

FIG. 1. (a)–(d) Lowest order diagrams in the multiple scattering series corresponding to our three-body model. The open and solid blobs represent off-shell two-particle amplitudes. The cross-hatched pair of lines represent the deuteron; solid and dashed single lines correspond to the nucleon and pion, respectively. Diagram (d) is the first term containing nucleon rescatterings.

Thus, elastic scattering, deuteron breakup, and pion absorption are automatically and uniquely linked in the calculation; this is an important advantage of the three-body approach. All follow from the same input (we discuss the absorption mechanism below). There is also no mystery in identifying the appropriate amplitudes; one has only to pick off the residue of the appropriate pole in the complete 3-to-3 amplitude. In practice this just changes the driving term in the basic integral equation. Therefore, although we only quote numerical results for the elastic amplitude, our calculation "knows" about the other channels, and this is reflected in the inelasticity parameter η.

The numerical procedures are straightforward and are identical to those employed extensively in the three-nucleon problem.[15] One thus deforms the integration contour to avoid singularities of the integral, maps onto a finite domain, and introduces appropriate Gaussian quadrature points and weights to approximate the integral. The result is a finite matrix equation which can be inverted by standard techniques. Introducing the S matrix $S_{J,l}=\eta e^{2i\delta}$ to describe the uncoupled 2^+ and 0^+ p waves, we obtain values for the 2^+ parameters which are illustrated in Fig. 2. These results demonstrate an important distinction between the underlying π-N (3, 3) resonance and the strong enhancement observed in this reaction. In the former case, the scattering is elastic ($\eta=1$) and the phase increases through 90°, corresponding to a pole on the second sheet with the usual

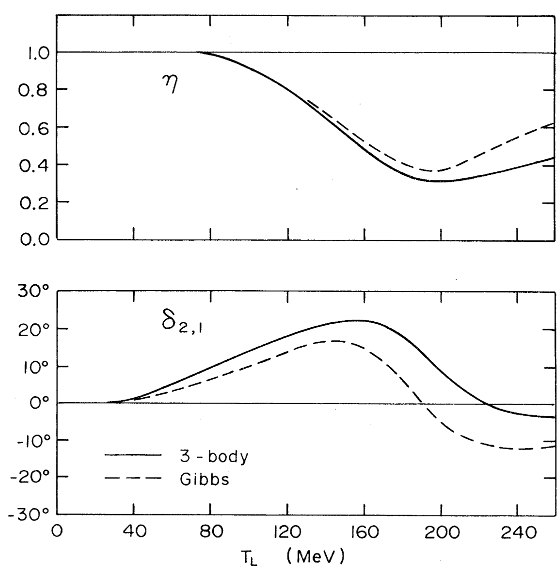

FIG. 2. Elastic π-d scattering in the 2^+ state (δ and η parameters). The solid lines are the results of our model; the dashed were obtained by Kaufmann according to the method of Gibbs (Refs. 16–17).

interpretation as an unstable state. In contrast, the peak in the π-d total cross section is due to the rapid decrease in η associated with deuteron breakup; the real phase is small and there is no nearby pole (as verified explicitly in this calculation).

This result is not a special consequence of our model, as indicated by the dashed curves shown in Fig. 2. The latter were obtained by Kaufmann for the purposes of this comparison,[16] using a code developed by W. Gibbs and the same P_{33} phase as input. Gibbs's approach provides a means of summing up the pion rescattering in the MSS, assuming fixed nucleons; in its present manifestation a separable amplitude is employed to describe the basic π-N interaction.[17] It is clear that the two results are in excellent qualitative agreement, and that the quantitative discrepancies are relatively minor through the resonance region. In view of Gibbs's π-d results using a more realistic model (including the deuteron d state) and summing all partial waves, one would anticipate that a more complete calculation along these lines would be in excellent agreement with experiment. On the other hand, the discrepancies are a measure of the frozen approximation, and the results suggest that significant corrections may be necessary even in elastic scattering if one excludes the immediate vicinity of the (3, 3).

Our results thus indicate that the introduction of Breit-Wigner parameters to characterize pion-nucleus scattering in the (3, 3) region is both incorrect and misleading. This may be illustrated by considering Fig. 3, in which we have plotted the p-wave contribution to σ_t, σ_r assuming spin independence (strictly speaking, we have plotted

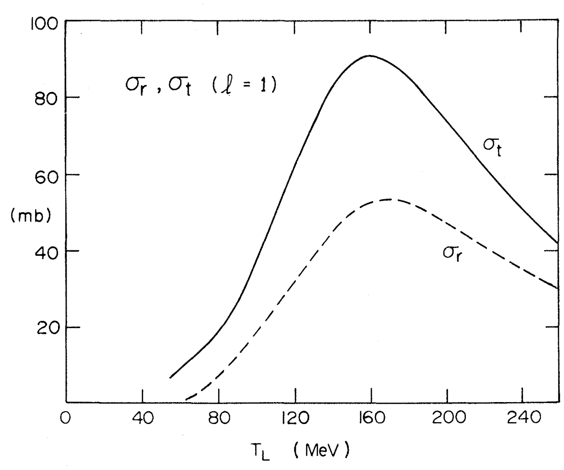

FIG. 3. Contribution of the p wave to the total (σ_t) and reaction (σ_r) cross sections, assuming spin independence. These curves correspond to the δ and η parameters obtained for our model and plotted in Fig. 2.

TABLE III. Results for the 2^+ calculation in terms of the δ and η parameters. In our notation SS signifies single pion scattering, DS means single plus double pion scattering, and Tot. corresponds to the exact result.

T_L	Phase (δ^0)			Elasticity (η)		
(MeV)	SS	DS	Tot.	SS	DS	Tot.
61	5.54	5.61	6.03	1.00	1.00	1.00
85	8.96	9.08	9.80	1.00	1.00	0.97
142	22.4	22.3	21.4	0.84	0.82	0.65
160	27.2	27.2	22.4	0.69	0.68	0.49
180	29.3	31.4	17.9	0.49	0.51	0.35
190	27.6	32.6	13.2	0.40	0.42	0.32
200	23.3	32.8	8.4	0.34	0.34	0.31
225	8.3	24.3	0.02	0.35	0.18	0.36
256	2.3	−7.4	−3.4	0.53	0.20	0.44

$\frac{9}{5}$ times the 2^+ contribution). It is then clear that the peak in the cross section has very little to do with a zero in the real part of the elastic amplitude, as is implied by the resonance parametrization. In our case the zero occurs near 225 MeV, whereas the total and reaction cross sections peak at 165 and 180 MeV, respectively. This point went unnoticed by Myhrer and Koltun,[2] who identified the location of the zero with the shift in the position of the resonance.

In order to investigate the convergence of the MSS, one may iterate the integral equation and sum selected terms. The results of this procedure are illustrated in Table III, in which SS denotes single pion scattering [Fig. 1(a)], and DS denotes single plus double scattering [Figs. 1(a) and 1(b)]. It is evident that the SS term dominates, which is to be expected on the basis of the (3, 3) mechanism discussed in the next section. However, the addition of the next term constitutes no improvement, and one has to sum the whole series (or at least some good approximation to it) to approach the exact result. Furthermore, if one considers the difference between SS and the total, terms such as Fig. 1(d) which involve nucleon correlations are at least as important as the higher pion scattering terms (this was also observed by Myhrer and Koltun). These effects become rapidly more pronounced at energies beyond the resonance, which suggest that significant corrections to the frozen approximation will be necessary.

As noted above, by choosing $J=0$ we eliminate the P_{33} channel in favor of the P_{11} (whereas $J=1$ would required a coupled channel calculation involving both of these phases). The significance of the P_{11} state is that its inclusion represents the possibility of pion absorption, as discussed by Afnan and Thomas.[1] This is equivalent to the assertion that the pion-emission-absorption process should be described by analytically continuing the P_{11} amplitude below threshold to the nucleon pole. This idea is not new, of course, and has been invoked in N/D approaches to this phase shift.[18] From the standpoint of the $NN\pi$ system, however, it has the effect of placing π-d scattering and the basic N-N interaction on the same footing; in fact, both processes are described by the same analytic function (in different energy regimes). One can thus test this description by using the three-body equations to calculate N-N scattering as a two-body channel of $NN\pi$. This was tried by Afnan and Thomas in their nonrelativistic calculation, and they reported rather impressive agreement with the one-pion exchange results for the f and g waves (simple estimates suggest that a relativistic calculation would substantially improve that agreement). Furthermore, using the same relativistic model described above, the present author and H. P. Noyes recently demonstrated that the primary characteristics of the N-N s waves could be obtained *without the adjustment of empirical parameters*.[5] There is thus considerable justification for regarding the P_{11} state as the correct absorption mechanism in a few-body treatment. Formally, this is equivalent to treating the nucleon as an N-π bound state.

It should be pointed out that the employment of this mechanism in our three-body $NN\pi$ formalism does not lead to double counting, since the pion cannot become confused with those associated with the nuclear force. The reason for this is illustrated in Fig. 4, in which (a) corresponds to the fact that two-nucleon intermediate states will be included in our MSS as a result of the P_{11} interaction (we have stretched out the blob corresponding to the π-N off-shell amplitude). If diagram (b) could occur, one would indeed have to worry about disentangling the two kinds of pions, since both would disappear into the same nucleon before subsequent emission. However, the rules of our series (and all Faddeev-type series) do not allow this to happen, since the "other" nucleon is forbidden to interact until the bound pion is again explicitly present. More specifically, the series

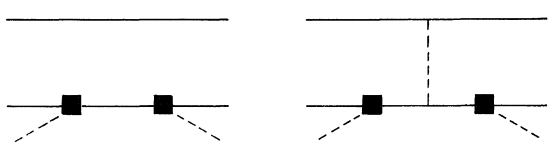

FIG. 4. Diagrams corresponding to a pion-nucleon pair binding through the P_{11} interaction to form a two-nucleon intermediate state. Our equations require the upper nucleon to remain a noninteracting spectator until the pion is reemitted, and hence diagram (b) cannot occur.

TABLE IV. Results for the 0^+ calculation; the notation is the same as in Table III.

T_L (MeV)	Phase (δ^0) SS	DS	Tot.	Elasticity (η) SS	DS	Tot.
61	0.07	−2.85	−0.83	0.935	0.900	0.987
85	0.58	−9.97	−0.92	0.864	0.770	0.989
142	5.45	28.7	−0.27	0.636	1.41	0.997
160	9.50	18.2	0.07	0.573	2.49	0.999
180	15.6	10.7	0.51	0.527	4.47	1.000
200	24.0	3.90	1.06	0.532	8.27	1.000
225	32.6	−2.73	1.75	0.612	17.4	0.998
256	38.4	−10.2	2.79	0.753	52.2	0.992

sums those diagrams in which a pair interacts in the presence of a noninteracting spectator, which is equivalent to retaining only two-body forces. Diagrams such as Fig. 4(b) require all three particles to be close together, and hence correspond to a three-body force. Naturally, there is no formal proof that such terms are unimportant, but that is the implicit hope in every three-body treatment.

The results for the elastic 0^+ amplitude are presented in Table IV, in which the columns have the same interpretation as in Table III. It is evident that individual terms in the MSS are totally misleading in estimating the exact result; in fact, a remarkable cancellation occurs in summing the series. Thus the SS term predicts a large inelastic cross section, whereas the predicted net effect is that the deuteron is virtually transparent to such pions. This is true despite the divergence of the MSS, which is reflected in the large values for η given under DS. In view of the delicacies of the cancellations involved, it appears unlikely that a partial summation including only pion rescatterings would prove adequate in estimating the total. In fact, except for the very special (3, 3) region, our results do not support the contention that nucleon rescatterings are unimportant, and the former exception merely reflects dominance of the SS term *in that case*.

In concluding this section, we point out that η in the 0^+ calculation is not uniquely linked to pion absorption, since deuteron breakup ($NN\pi$ final state) remains an important channel. It is therefore not possible to infer the absorption cross section directly from the elastic amplitude. In this case, of course, the total of breakup and absorption effects are very small, and the value hardly matters. Nevertheless, one can estimate the absorption by comparing Tables III and IV at T_L $=61$ MeV. If one assumes that deuteron breakup is no more likely in the P_{11} channel than in the P_{33} at this energy, the value of η (0.987) is a rea-

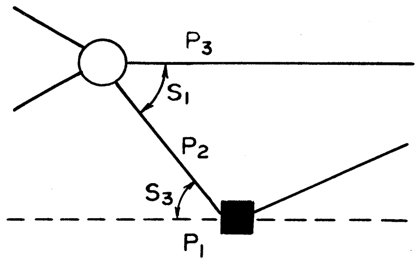

FIG. 5. The lowest order diagram contributing to the breakup process. The label P_α denotes the four-momentum of particle α, while $S_\alpha^{1/2}$ is the invariant energy of the $\beta\gamma$ pair.

sonable estimate of this effect. It clearly decreases with increasing T_L as one would expect, and appears unlikely to be significant except at low energies (and hence in the π-d s wave). The latter case has been investigated by Afnan and Thomas, who demonstrate a sizable (25%) correction to the π-d scattering length.[1]

IV. DISCUSSION

As noted above, a three-body treatment of π-d scattering provides a theoretical laboratory in which to test specific aspects of the general pion-nucleus problem. In this article we have focused primarily on the nature of the strong enhancement associated with the (3, 3) resonance. Via an explicit calculation, it has been demonstrated that the π-d system does not have a resonance pole, and that the effect arises from the rapid energy dependence of the breakup cross section. In turn, this may be traced to a weak (logarithmic) singularity in the breakup amplitude for states of definite J, L communicating with the P_{33} π-N channel. The origin of this behavior is well known in the literature and was the subject of an earlier investigation by the author and Peierls.[7] The essential point is that singularities in n-body amplitudes arise at energies such that a particular sequence of rescatterings can occur *on shell*, as a real physical process. This situation is illustrated in Fig. 5, in which P_1, P_2, and P_3 are the four-momenta of the three particles and $S_\alpha = (P_\beta + P_\gamma)^2$. Thus, in the present example, $S_1 = M_d^2$; $S_3 = M_\Delta^2$. The rescattering singularity occurs at a total c.m. energy $\sqrt{S}$ such that one can satisfy these conditions on S_1, S_3 *simultaneously* with each particle on its mass shell ($P_\alpha^2 = m_\alpha^2$). The value of S turns out to depend on the angle θ_2 between $\vec{P}_1$ and $\vec{P}_3$, and hence this requirement leads to a cut in the S variable, with branch points $S\pm$ corresponding to $\theta_2 = 0$, $\theta_2 = \pi$. With a little algebra, one can work out an explicit formula for the values of $S\pm$. Due to the small mass ratio μ/M, the result is a short cut ($S+ \simeq S-$) centered at a value $\sqrt{S}$ slightly less than $M + M_\Delta$.

The presence of this singularity is reflected in the rapid energy dependence exhibited by the breakup amplitude, of which Fig. 5 represents the lowest order term. Thus, the peak in the reaction cross section is correlated with the position of the Δ in free π-N scattering. The enhancement of this inelastic channel shows up in the dip in η observed in the SS term [Fig. 1(a)], which is of the same order and competes with this process. The same reasoning can be applied to the π-nucleus problem considered as a three-body system (πN + core); this results in the same prediction for the reaction cross section. On the other hand, the total cross section is proportional to $(1 - \eta \cos 2\delta)$, and hence the peak is shifted to the left since δ is decreasing. One would expect this effect to increase in heavier nuclei since δ should be larger in the presence of more scattering centers; this is in accord with the experimental facts. The basic mechanism is thus quite distinct from that responsible for an elementary object such as the Δ, and a resonance parametrization is inappropriate.

Except for the SS term in the (3, 3) region, individual terms in the MSS were shown to provide poor estimates to the exact result. This was particularly dramatic in the 0^+ calculation driven by the P_{11} interaction, which is the important channel if one is to account for pion absorption. However, the more relevant question concerns the effectiveness of approximate schemes for partially summing the MSS (e.g., optical potentials); this point is still unclear. In view of the discussion above, one might argue that the (3, 3) region is not the place to look in order to answer this question. This means that one must consider either lower energies (where absorption will be important), or higher energies (where nucleon rescatterings and relativistic corrections will be significant). Unfortunately, the fixed scatterer approximation is ill equipped to deal with these complications.

One possible solution to this problem would be to treat the most critical channels as a three-body problem involving π, N plus residual core, and to treat the remainder via conventional techniques. For example, one could include $(\pi N)P_{33}$ coupled to the core with angular momentum $l_\alpha^{\min}$, and similarly for the P_{11} channel. This would enable one to treat nucleon knockout and charge exchange as well in the same calculation. The technique employed in this article can be trivially extended to this general situation and provides a practical means of incorporating relativistic effects.

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