Effect of deuteron breakup on (d, p) cross sections

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Deuteron-nucleus breakup wave functions, obtained in a previous study, are applied to calculate the contribution of breakup to zero range stripping matrix elements for the case of 21.6 MeV deuterons incident on ^{40}Ca. The results are compared with the method of Johnson and Soper. The two calculations agree quite well, but the Johnson-Soper method slightly overestimates the effect of breakup for the surface partial waves, and it suppresses too much the stripping amplitudes for the interior partial waves.

[NUCLEAR REACTIONS Effect of deuteron breakup on $\sigma(d, p)$. Comparison with Johnson-Soper theory for 21.6 MeV ^{40}Ca$(d, p)^{41}$Ca ($2p$ and $1f$).]

I. INTRODUCTION

In 1970, Johnson and Soper[1] (JS) proposed a very simple method for including the effect of deuteron breakup into the calculation of stripping and pickup cross sections. As compared to the conventional distorted-wave Born-approximation (DWBA) method of calculation, the JS method gives in most cases an improved fit to the experimental cross sections, and has been widely used for the analysis of data.

The method of Johnson and Soper was reexamined in a recent study of the effect of breakup on the elastic deuteron-nucleus scattering cross section.[2] It was found that the approximations which underlie the method of JS lead to errors which tend to compensate each other, at least as far as the elastic cross section is concerned. The purpose of the present note is to investigate whether such compensatory cancellations also exist for the case of the stripping cross sections. In order to discuss whether such cancellations are or are not to be expected for the stripping cross section, the assumptions of JS will be reviewed below, and a summary of the method of Ref. 2, denoted as the "k-by-k" method will also be given.

The coordinates of the center of mass of the incident deuteron relative to the center of the nucleus is denoted by $\vec{R}$. The coordinates of the neutron and proton relative to the center of the nucleus are $\vec{r}_n$ and $\vec{r}_p$, and the coordinate of the neutron relative to the proton is denoted by $\vec{r}$ i.e., $\vec{r} = \vec{r}_n - \vec{r}_p$, $2\vec{R} = \vec{r}_n + \vec{r}_p$. The nucleus is considered as having no explicit internal degrees of freedom, and antisymmetrization of the nucleons in the deuteron with those of the nucleus is ignored. The interaction of the neutron and proton with the nucleus is described by the phenomenological nucleon-nucleon optical potential for nucleon energies taken at half the incident deuteron energy. These potentials are dencted, respectively, by $U_{n-A}(r_n)$ and $U_{p-A}(r_p)$ and their sum is

$$V_N(\vec{r}, \vec{R}) = U_{n-A}(r_n) + U_{n-p}(r_p) . \tag{1}$$

With these assumptions the wave function which describes the motion of the proton and neutron relative to the nucleus, $\psi(\vec{r}, \vec{R})$, obeys the equation

$$[T_R + H_{np}(\vec{r}) + V_N(\vec{r}, \vec{R}) - E]\psi(\vec{r}, \vec{R}) = 0 , \tag{2}$$

where $H_{np} = T_r + v_{np}(r)$ is the neutron-proton Hamiltonian, v_{np} is the nucleon-nucleon potential, and T_r and T_R are the kinetic energy operators operating on the coordinate $\vec{r}$ and $\vec{R}$, respectively.

In addition to the assumptions already stated, the main assumption of JS consists in replacing all the energies of relative motion ϵ_k of the two nucleons by the binding energy of the deuteron ϵ_b. This is the adiabatic assumption, according to which that part of $\psi(\vec{r}, \vec{R})$ which gives the main contribution to the stripping cross section contains only states of low relative nucleon momenta $\hbar k$. Mathematically this assumption enables one to replace $H_{np}\psi(\vec{r}, \vec{R})$ by $\epsilon_b\psi(\vec{r}, \vec{R})$, and Eq. (2) is replaced by

$$[T_R + V_N(r, R) - E_D]\psi(\vec{r}, \vec{R}) = 0 , \tag{3}$$

where $E_D = E - \epsilon_b$ is the incident deuteron energy.

For the calculation of the stripping cross sections in the zero range approximation the full function $\psi(\vec{r}, \vec{R})$ is not needed, but only the projection $\psi_v(\vec{R})$, given by

$$\overline{\psi}(\vec{R}) = \int \psi(\vec{r}, \vec{R}) v_{np}(r) d^3r . \tag{4}$$

Since $v_{np}(r)$ is assumed to be spherically symmetric, i.e., independent of the directions $\hat{r}$ of $\vec{r}$, the projection ψ_v eliminates from $\psi(\vec{r}, \vec{R})$ all the states of angular momenta of relative motion $\hbar\vec{l}$ in the $\vec{r}$ co-

ordinate other than $l=0$. JS arrive at the Schrödinger equation for $\bar{\psi}(\vec{R})$

$$[T_R + \bar{V}(R) - E_D]\bar{\psi}(\vec{R}) = 0 \tag{5}$$

by multiplying Eq. (3) by v_{np} and integrating over d^3r and by replacing the integral involving V_N by

$$\int_0^\infty v_{np}(r) V_N(\vec{r},\vec{R})\psi(\vec{r},\vec{R}) d^3r \to \bar{V}(R)\bar{\psi}(\vec{R}) . \tag{6}$$

The last step involves the further approximation that only the $l=0$ components of $\psi(\vec{r},\vec{R})$ are kept. This can be seen by expanding the $\hat{r}$ dependence of both $\psi(\vec{r},\vec{R})$ and $V_N(\vec{r},\vec{R})$ in a series of spherical harmonics $Y_{l,m}(\hat{r})$, with coefficients $\varphi_{lm}(r,\vec{R})$ and $v_{lm}(r,\vec{R})$, respectively, and noting that the integral in Eq. (6), in addition to the term $\int \varphi_{00}(r,R)v_{00}(r,R)v_{np}(r)r^2dr$ included in Eq. (6), will also contain sums over terms of the form $\int \varphi_{lm}v_{l-m}v_{np}r^2dr$ with $l \neq 0$, neglected in Eq. (6). The latter terms were included explicitly[3] in a generalization of the JS equation based on Eq. (3). This generalization leads to a set of coupled equations, involving $\psi_v(\vec{R})$ as one of the functions, and is obtained by multiplying Eq. (3) by $Y_{lm}(\hat{r})v_{np}(r)$ and integrating over d^3r. Angular momenta $l=0$ and 2 were included in a numerical application of this generalization, and it was found that the effect of the $l=2$ terms on the zero range stripping cross section gave only a small correction to the result based on the original JS procedure.

However, it was also found in this study[3] that very large values of ϵ_k, of 40 MeV and larger, are required to be included in $\psi(\vec{r},\vec{R})$ before the contribution of the $l=2$ term on the zero range stripping cross section becomes small, and hence the validity of the Eq. (5) was put into question. This question is connected with the fact that the calculation of $\bar{\psi}(\vec{R})$ by means of the solution of Eq. (5) does not indicate how large a spectrum of breakup energies is included in $\bar{\psi}(\vec{R})$, and does also not reveal how large a contribution to the stripping cross section arises from the breakup components in $\bar{\psi}(\vec{R})$, as compared to the contribution from the bound (deuteron) component.

The k-by-k procedure,[2] on the other hand, separates explicitly the components of $\psi(\vec{r},\vec{R})$ in the various ranges of momenta k, called momentum bins, and thus lends itself to investigating the extent to which the various momentum bins contribute to the stripping cross section. The results for the crudest version of the presently available k-by-k wave functions—namely in which, in addition to the bound state component, the breakup components of the wave function for only one momentum bin for both $l=0$ and $l=2$ are known for 21.6 MeV deuterons incident on Ca—are presented below and are compared with the stripping results based on the JS method. The comparison shows the extent to which the breakup components contribute to the stripping cross sections—a result not known up to now—and also indicates the non-negligible influence which the $l=2$ components have on the cross section.

Since the stripping cross sections are calculated in zero range, the $l\neq 0$ components in $\psi(\vec{r},\vec{R})$ do not contribute explicitly to the DWBA integrals. Hence, in both the generalization of the JS equation[3] mentioned above and in the calculation discussed below, the $l\neq 0$ terms affect the zero range stripping cross sections only indirectly via the effect which they have on the $l=0$ terms through the coupling potentials in the coupled equations. This situation is somewhat similar to the elastic scattering case. There the elastic component of $\psi(\vec{r},\vec{R})$ is obtained through the $l=0$ projection $\int \phi_b(r)\psi(\vec{r},\vec{R})d^3r$ where ϕ_b is the bound deuteron internal wave function. However, in view of the long radial range of $\phi_b(r)$ as compared to $v_{np}(r)$, a much larger range of values of r is expected to contribute to the elastic cross section than to the stripping cross section. Since the $l\neq 0$ terms in $\psi(\vec{r},\vec{R})$ go to zero as $r\to 0$, the effect of the $l\neq 0$ terms on the elastic component of the wave function is expected to be larger than on the ψ_v component, and thus the assumptions of JS are expected to affect the stripping cross section differently than the elastic cross section.

II. CALCULATION

The k-by-k method has been described in detail[2,3] and only a very brief review is given below. The main idea is to expand ψ in a series of eigenstates $(1/r)u_l(k,r)Y_{lm}(\hat{r})$ of H_{np}, where $\hbar k$ is the nucleon-nucleon relative momentum, $\hbar l$ is the relative angular momentum, $\epsilon_k = \hbar^2k^2/m$ is the energy eigenvalue, and obtain the infinite set of coupled equations for the coefficients of this expansion. In view of the angular momentum algebra involved, it is convenient to expand $\psi(\vec{r},\vec{R})$ in a series of bipolar spherical harmonics $\mathcal{Y}_{(lL)JM}(\hat{r},\hat{R})$ with radial coefficients $(rR)^{-1}F_{(lL)J}(r,R)$ and where $\vec{l}+\vec{L}=\vec{J}$. Upon expanding each $F_{(lL)J}$ in a momentum integral over $u_l(k,r)$, one obtains

$$F_{(lL)J}(r,R) = \delta_{l,0}u_b(r)f_J(b,R) + \int_0^\infty u_l(k,r)f_{(lL)J}(k,R)dk , \tag{7}$$

where $u_b(r)$ is the radial wave function of the deuteron.

The coupled equations involve the functions $f_{(lL)J}(k,R)$ for a continuous range of k and a discrete set of values of l. The next step consists in truncating the series in l to the terms with $l=0$ and 2. For deuteron energies above the Coulomb

barrier of the target nucleus the $l=1$ terms are found to be small,[2] and the $l \geq 3$ terms are also found to be less important than the $l=2$ terms. The continuous range of k is divided into discrete bins each of size Δk, and a new set of discretized functions $f_{(lL)J}(n, R)$ are defined for each bin n.

The projection $\overline{\psi}$ defined in Eq. (4) can be written as

$$\overline{\psi}(\vec{R}) = R^{-1} \sum_J \overline{F}_J(R) Y_{JM_J}(\hat{R}), \tag{8}$$

where in terms of the notation given by Eq. (7)

$$(4\pi)^{-1/2} \overline{F}_J(R) = D_b f_J(b, R) + \sum_n D_0(n) f_{(0J)J}(n, R), \tag{9}$$

and where, as discussed in Eqs. (3.10) and (3.18) of Ref. (3)

$$D_b = \int_0^\infty v_{nb} u_b(r) r \, dr, \tag{10a}$$

$D_l(n)$ = momentum average in bin n of

$$(\Delta k)^{1/2} \int_0^\infty v_{nb}(r) u_l(k, r) r \, dr. \tag{10b}$$

As noted in the Introduction, only the terms with $l=0$ contribute to $\overline{F}_J$.

The calculation of the stripping cross section consists in solving the coupled equations for the $f_{(lL)J}(n, R)$ described above and in detail in Ref. 2, then evaluating $\overline{F}_J$ according to Eq. (9), and then utilizing the result in evaluating the conventional DWBA stripping overlap matrix elements. The functions $u_l(k, r)$ are evaluated by solving numerically the Schrödinger equation for various values of k in bin n, with v_{nb} given by a Gaussian potential.[4] The resulting values of D_b and $D_0(k)$ are listed in Table A.I of Ref. 3, where they are denoted as $D_b^{(0)}$ and $D_0^{(0)}(k)$, respectively. Inserting these results in Eq. (10b), one obtains

$$(4\pi)^{1/2} D_b = 126.9 \text{ MeV fm}^{3/2}, \tag{11a}$$

$$R_0(1) \equiv D_0(1)/D_b = 0.59, \tag{11b}$$

where bin 1 ($n=1$) extends from $0 \leq k < 0.5$ fm^{-1}, $0 \leq \epsilon_k < 10$ MeV.

The difference between the conventional optical model DWBA, the JS, and the k-by-k methods for calculating DWBA stripping cross sections consists in the choice of the function $\overline{F}_J(R)$ defined in Eq. (9). In the conventional optical model method the breakup terms in the sum over n in Eq. (9) are absent, and the f_J's (the elastic deuteron distorted waves) are obtained from the requirement that they are the solutions of a Schrödinger equation with local complex potentials such as to give rise to the experimental elastic deuteron-nucleus scattering cross section. At the same time the value of $\sqrt{4\pi} D_b$ is taken as equal to $(8\pi\alpha)^{1/2}(\hbar^2/m) \sim 101$ MeV fm$^{3/2}$ times a factor such as 1.5, where $\alpha = (m\epsilon_b/\hbar^2)^{1/2}$. However, in the calculation reported below, the value of D_b is taken from Eq. (11a) in all three cases. In the JS method, $\overline{F}_J(R)$ is calculated directly from the solution of Eq. (5), with $\overline{V}$ calculated according to Eq. (20b) of Ref. 1 from the phenomenological nucleon-nucleus optical potentials.[1,3] In the k-by-k method the breakup contributions $\Sigma_n R_0(n) f_{(0J)J}(n, R)$, as well as the bound state component $D_b f_J(b, R)$, are calculated by solving the coupled equations for the f's. The potentials which enter the coupled equations are calculated from the nucleon-nucleus optical potentials,[2] without any adjustable parameters.

The k-by-k method gives different results for $\overline{F}_J$ than the JS method because the two methods of calculation are different. It is, however, possible to change the k-by-k method so that it gives nearly the same answer as the JS method. This change, denoted as the JS simulation,[2] is as follows. The terms with $l \neq 0$ in the coupled equations for the functions $f_{(lL)J}(n, R)$, given by Eq. (4.3) in Ref. 3, are deleted, since, as discussed in the Introduction, the JS equations do not allow for the presence of these terms. Further, denoting the breakup function $f_{(0L)L}(1, R)$ by $f_1(R)$, the equations simplify to

$$[\langle T_R \rangle_L - E_D + V_{bb}] f_b(R) + V_{b1} f_1(R) = 0, \tag{12a}$$

$$[\langle T_R \rangle_L - E_1 + V_{11}] f_1(R) + V_{1b} f_b(R) = 0. \tag{12b}$$

Plots of the potentials V_{bb}, the Watanabe potential, V_{11} and $V_{b1} = V_{1b}$ are given in Ref. 2. If furthermore ϵ_k is set equal to ϵ_b, such that $E_1 = E_D$, and if V_{11} is replaced by V_{bb}, then by adding Eqs. (12a) and (12b) one obtains

$$[\langle T_R \rangle_L - E_D + V_{bb} + V_{b1}](f_b + f_1) = 0. \tag{13}$$

The simulation of the $\overline{F}_J$ as obtained in the JS method by $f_b + f_1$ now is possible in view of the approximate validity of the relation[3]

$$\overline{V} \sim V_{bb} + V_{b1} \tag{14}$$

provided that one also set $R_0(1) \equiv D_0(1)/D_b$ equal to unity, so as to make Eq. (9) compatible with $\overline{F}_J \sim f_b + f_1$. To the extent that Eq. (14) is not exactly valid, the JS simulation provides only an approximation to the JS results. In passing, it can be noted that Eq. (14) is an approximation to Eq. (4.8) of Ref. 3 which, when written in the present notation, reads $\overline{V}(R) = V_{bb}(R) + \Sigma_{n=1}^\infty R_0(n) V_{bn}(R)$. According to Table I in Ref. 3, the contribution from the

first two terms in the sum over n already gives a very good description of $\overline{V}$ for the case of d-Ca. According to that table, the first term in the sum is more than three times larger than the second, thus justifying the approximate validity of Eq. (14). Further, changing the value of $R_0(1)$ from 0.58 to unity suggests that the JS method exaggerates the contribution from the breakup component to the stripping cross section.

III. RESULTS

The numerical results presented below refer to 21.6 MeV deuterons incident on the nucleus of ^{40}Ca. This case has been chosen because the numerical wave functions were already available from the study of Ref. 2. The values of $l=0$ and 2 are included, and only momentum bin 1 from $0 \leq k < 0.5$ fm^{-1} and $0 \leq \epsilon_k < 10$ MeV is included. The bound states in ^{41}Ca are described by a neutron bound to the ^{40}Ca core in a $l=3$ $(1f)$ and $l=1$ $(2p)$ states, with separation energies equal to -8.58 and -6.42 MeV, respectively. These states are bound in a Woods-Saxon potential of radius 4.14 fm, diffuseness 0.65 fm, and a spin-orbit potential of the same geometry and strength of 6.927 MeV$\times(\hbar/m_\pi c)^2 r^{-1} d/dr$. No j values are indicated since the spins of the nucleons are ignored through the remainder of the calculation. The outgoing proton waves are distorted by a Woods-Saxon potential given by potential H of Hnizdo *et al.*[5] The deuteron-nucleus potential $\overline{V}$ required for obtaining the JS results is obtained from the nucleon-nucleus potential as given by Eq. (20b) of Ref. 1. The respective nucleon-nucleus potentials are the same as those described in Ref. 2. For the optical model calculations denoted in the figures by the letters OM, the deuteron-Ca elastic optical potential is taken from Table II of Perey and Perey.[6]

Stripping cross sections are illustrated in Fig. 1. The long dashed and the solid lines illustrate the JS and JS simulation results, respectively. The good agreement shows that the simulation works well, as was also the case[2] for the elastic phase shifts. The dash-dot curves, obtained for $R_0=0$, are considerably different from the $R_0=1$ result (the JS simulation), which illustrates the large contribution which the breakup components make to the stripping cross sections. For both the p and f transitions the breakup contribution markedly increases the cross section in the forward peak, and deepens the first minimum. In the absence of the breakup contribution, the cross sections illustrated by the dash-dot curves resemble the OM result (short dashes) for the p transitions. For the f transitions a similarity can be seen in the backward region, and in this case the breakup contribution has a coherently destructive effect markedly reducing the cross section. In part c of Fig. 1 the stripping cross section obtained with the k-by-k method is shown by the solid and dash-dot lines. For the former, the average value of the energy ϵ_k in bin 1 is set equal to 5 MeV, in the latter it is set equal to ϵ_b. The effect due to the change in energy is quite small. Both curves lie much closer to the JS result than to the OM result, thus supporting, on the whole, the JS method. Nevertheless, differences are apparent, such as a slightly smaller cross section in the forward stripping peak (also present for the p case) and a less rapid fall off with angle, at the larger angles. The circles represent experimental points,[7] but comparison with experiment is not as yet too meaningful since the calculation is in zero rather than finite range, and the effect due to coupling to

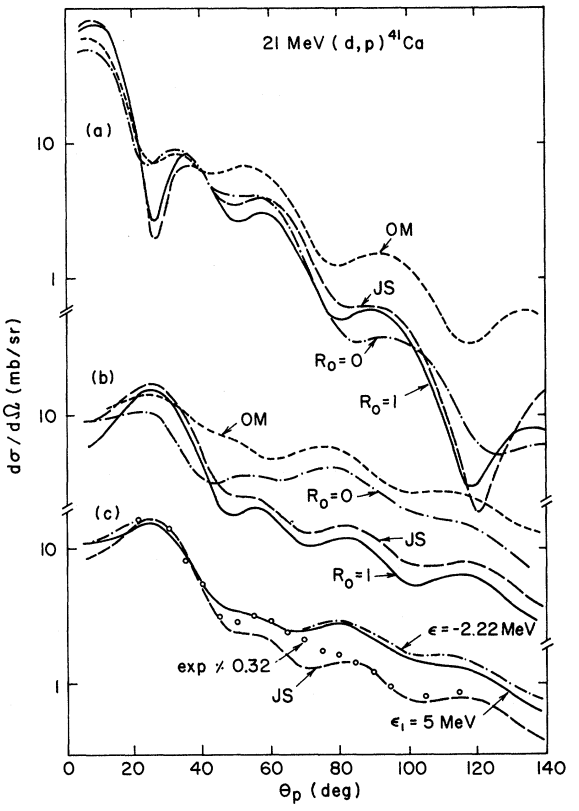

FIG. 1. (a)–(c) Differential stripping cross sections versus proton angle for 21.6 MeV deuterons incident on ^{40}Ca. The curves in parts (a) and (b), whose detailed meaning is explained in the text, illustrate stripping to the $2p$ and $1f$ neutron states in ^{41}Ca, respectively. In part (c) the $1f$ stripping cross sections, calculated by the JS and k-by-k methods are compared to each other and to experiment. The latter is for 21.0 MeV deuterons rather than 21.6 MeV.

stripping channels, possibly of importance in this case,[8] is not included.

A more detailed comparison between the various methods is shown in Fig. 2, which illustrates, by means of an Argand diagram, the amplitudes A_J for the stripping to the $1f$ state in ^{41}Ca. The values of J are given by the numbers in the figure, points of equal J are connected by lines, and the angular momenta of the distorted proton waves, L_p, are $L_p = J - 1$. The k-by-k results are indicated by squares. The value of $R_0(1)$ was taken equal to 0.51 (rather than 0.58, inadvertently). The JS simulation, for which $R_0(1)$, is shown by crosses (×), and the JS simulation results in the absence of the breakup contribution is shown by (+). One sees that the breakup terms contribute significantly to the stripping amplitudes, particularly for the angular momenta $5 \leq J \leq 9$, for which the magnitude of the stripping amplitude (the distance to the origin) is increased. These partial waves apparently contribute to the forward stripping maximum, which gets enhanced by the breakup contributions. The enhancement of the stripping amplitudes for the partial waves $J = 7-9$ is not as pronounced for the k-by-k results, for which the stripping amplitudes (solid squares) lie approximately half way in between the (×)'s and (+)'s, which explains why the k-by-k forward stripping peak is slightly smaller than the JS result. On the other hand, for the smaller value of J (the "interior" partial waves) the k-by-k stripping amplitudes are larger than the JS results, but not as large as the OM results which fall outside of the figure. If one interprets the contribution from the interior partial waves as responsible for the steepness of the falloff of the cross section with proton angle, one can understand why the k-by-k results fall off less steeply than the JS results, and more steeply than the OM results. Both results (a smaller peak cross section and a slower falloff with angle than the JS results) appear to be in the right direction for one to expect an improved agreement with experiment, as compared with the fits based on the JS method, for the case of ^{16}O$(d, p)^{17}$O, as discussed by Cooper, Hornyak, and Roos.[9] These authors find that although the JS method gives better agreement with experiment than the conventional optical model, the JS result falls off too steeply with angle and appears to give too small a spectroscopic factor, particularly in the case of the stripping to the unbound $d_{3/2}$ level, where a basic inconsistency with the elastic neutron-^{16}O scattering is found.[10]

The present calculation also reveals that the effect of the $l = 2$ breakup channel on the stripping cross section is quite considerable. It is found

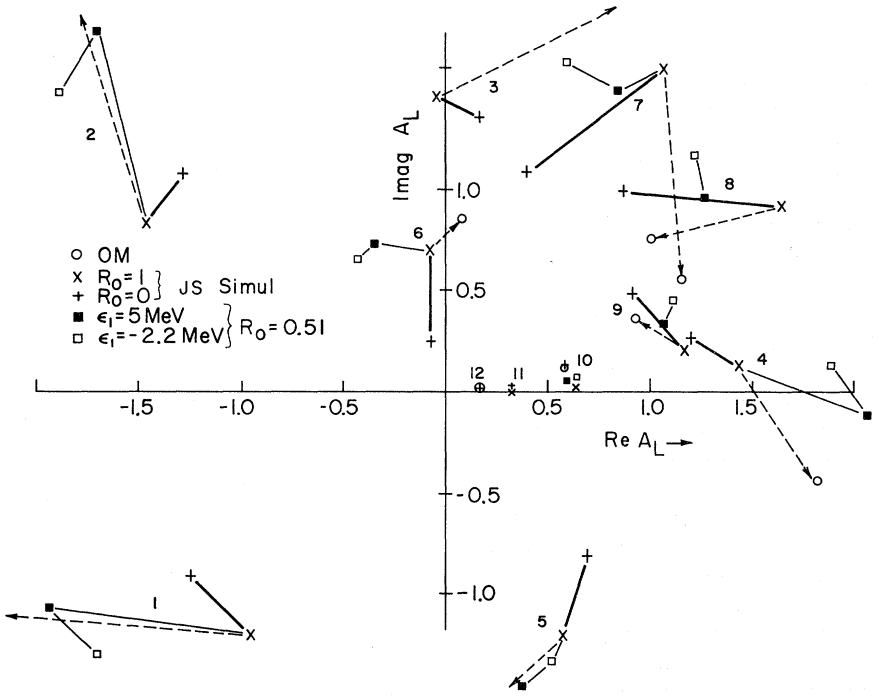

FIG. 2. Argand diagram for the $1f$ stripping amplitudes whose corresponding cross sections are illustrated in Fig. 1. The real and imaginary parts are plotted along the horizontal and vertical axes, respectively. Points of equal J values are connected by lines, $L_p = J - 1$, and the meaning of the points is described in the text.

that leaving out coupling to the $l=2$ channel has the effect of decreasing the f-stripping cross section, shown by the solid line in Fig. 2(c), to roughly the JS result, illustrated by the dashed line. In view of the much smaller effect resulting from replacing ϵ_k by ϵ_b, these two effects do not cancel in the stripping case nearly as well as they do in the elastic case. It is possible that the effect on the stripping cross section, due to the indirect coupling of the $l=2$ with the $l=0$ waves found in the present study, could be reduced once contributions from higher momentum bins are included. This conclusion is based on the fact that in Ref. 3 the effect of the $l=2$ channel on the stripping cross section was found to be much smaller than that found here, presumably because contributions from larger momenta k are included in that study. However, in this case the replacement[3] of ϵ_b by ϵ_k will certainly not be justified.

The magnitudes of the $l=2$ wave functions $f_{(lL)J}(1,R)$ in bin 1 are comparable to those of the $l=0$ breakup wave functions $f_{(0J)J}(1,R)$, and both have magnitudes between 10 and 20% of the elastic wave functions $f_{(0J)J}(b,R)$ as is illustrated in Fig. 3. The magnitude of the $l=2$ functions is expected to[2] remain as large for the second momentum bin ($10 \leq \epsilon_k < 40$ MeV) not yet included in the calculation. It is conceivable that for these large momenta the direct contribution of the $l \neq 0$ waves to the stripping amplitudes which will occur in finite range calculations could have a non-negligible effect, particularly on the vector and tensor analyzing powers.[11] Further work along these lines would be desirable, particularly an attempt to apply the k-by-k method to the analysis of experimental data such as $^{16}O(d,p)$ discussed above, in view of the fact that the corrections obtained by the present k-by-k results appear to be in the right direction so as to improve the fits based on the JS method.

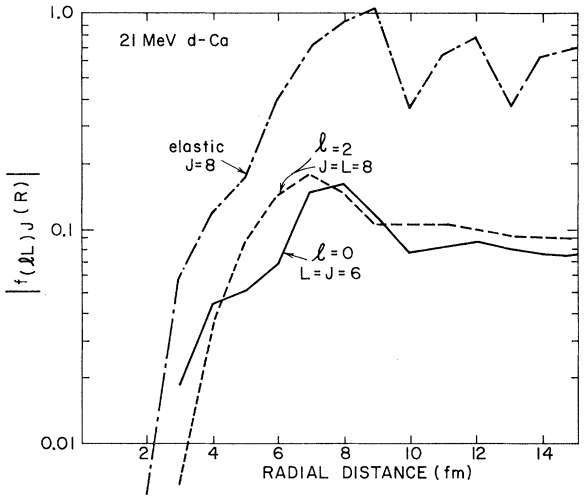

FIG. 3. Absolute values of the radial wave functions $f_{(lL)J}$ obtained by solving the k-by-k coupled equations as described in the text. Plotted are the values of these functions at discrete radial intervals of 1 fm, and the resulting points are connected by straight lines so as to guide the eye. The functions are normalized such that the elastic omponent $f_{(0J)J}(b,R)$ is asymptotically equal to $F_J^C + (1/2i)[\exp(2iK_J) - 1](G_J^C + iF_J^C)$ where F_J^C and G_J^C are the regular and irregular point Coulomb wave functions, respectively, for an orbital angular momentum $L=J$, and where K_J is the corresponding nuclear phase shift.

IV. SUMMARY AND CONCLUSION

The k-by-k method was employed to calculate stripping cross sections for the d-Ca transitions to $1f$ and $2p$ states in ^{41}Ca. By employing a modification of this method it was possible to examine the JS method of calculation, and determine separately the contributions from the breakup components which, in the JS method, are lumped together with the elastic deuteron component. It is found that the breakup contribution raises the JS stripping cross section in the forward direction by approximately a factor of 1.5 for both the $2p$ and $1f$ stripping transition, and it tends to decrease the cross section for the $1f$ transition at the larger angles. The k-by-k method confirms the JS results in the forward stripping peak, but the increase is found to be slightly smaller, which suggests that the JS method tends to overemphasize the breakup contributions to the stripping peak. The large effect of the breakup amplitude in the forward stripping peak also suggests that fits with the conventional optical model–DWBA method, which does not include such breakup effects, may need reinterpretation.[12] At larger angles the stripping cross section for the f transition falls off less steeply with angle in the k-by-k method than in the JS method, which is due to less of a suppression of the contribution of the "interior" partial waves to the stripping amplitude, as compared to the JS calculation. The presence of the coupling of the $l=0$ parts of the wave function to the $l=2$ breakup amplitude is found to have a nonnegligible effect on the f stripping cross sections. Omission of the $l=2$ channels is not compensated by the neglect of breakup energies, as had been the case to some extent for the elastic cross section.[2] Additional work with the k-by-k method, in particular the removal of the zero range approximation, would be very desirable.

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