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Consequences of nuclear dynamics for the nonrelativistic πNN vertex*

J. M. Eisenberg, J. V. Noble,† and H. J. Weber
Department of Physics, University of Virginia, Charlottesville, Virginia 22901
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We examine the form of the πNN vertex in the nonrelativistic limit, under various assumptions on the dynamics of the emitting or absorbing nucleon.

When we consider the absorption or emission of pseudoscalar mesons in medium-energy nuclear reactions, it is convenient to have available the nonrelativistic reduction of the pion-nucleon interaction Lagrangian. For this reason, several authors[1-4] have studied the possible forms which this reduced interaction might take. The question is somewhat ambiguous for the following reason: Clearly we ought to start with a Lorentz-invariant (LI) amplitude for the entire reaction which, upon nonrelativistic reduction, *must* yield a reaction amplitude which is Galilean invariant (GI). However, the requirement that the *over-all* amplitude be GI, i.e. should involve only differences between external velocities, does not require that each piece of the amplitude, and in particular the meson-nucleon vertex, be manifestly GI. That is, the velocity difference which appears in the over-all amplitude could in principle involve any external velocities in the problem, and which ones actually enter is a dynamical question. The reason for this is that the operator γ_5, sandwiched between two spinors in the canonical fashion

$$\bar{\chi}_f\gamma_5\chi_i \equiv \chi_f^\dagger \beta\gamma_5\chi_i \,, \tag{1}$$

gives rise to a cross product of the small and large components: If $\chi_{i,f} = \binom{u_{i,f}}{v_{i,f}}$, with u and v ordinary two-component spinors, Eq. (1) gives

$$\bar{\chi}_f\gamma_5\chi_i = -i(u_f^\dagger v_i - v_f^\dagger u_i) \tag{2}$$

in one particular representation for the Dirac γ matrices.[5] Typically, nuclear physics ignores the small components of the nucleon wave functions, but as Eq. (2) makes clear, meson absorption or emission requires a more careful treatment of this part of the problem. Of course, odd operators also enter the discussion of electromagnetic processes, but in that case gauge invariance requires a definite relationship between the contributions of small and large components, in the sense that an *unambiguous* Foldy-Wouthuysen transformation (and hence an expansion in powers of $1/m$) is possible. (However, we note that the nuclear dynamics *does* affect higher order contributions to electromagnetic processes.[6]) As we shall see, in pseudoscalar-coupled meson absorption or emission, the form of the amplitude in every order depends crucially on the full relativistic nuclear dynamics in a way which is well known to render ambiguous the most straightforward calculations.[4] In other words, there is no such clear distinction between nuclear structure and external probe as can be drawn in, say, electron scattering.

Recently, Bolsterli *et al.*[1] (BGGS) have investigated the emission or absorption of a pseudoscalar meson by a Dirac nucleon interacting with an external potential, and have reached the interesting conclusion that the result obtained after nonrelativistic reduction depends on whether the external potential is a Lorentz scalar (and therefore to be grouped with the mass term in the Dirac equation) or the fourth component of a four-vector (and therefore to be grouped with the energy term in the Dirac equation). The above introductory discussion indicates how it might happen that the use of vector or scalar potentials might give rise to different nonrelativistic limits through effects involving different admixtures of the small components.

In order to try to understand more clearly the result reported by BGGS, we decided to examine the related problem of the nonrelativistic reduction of the diagrams shown in Fig. 1 which describe the emission of a pseudoscalar meson by

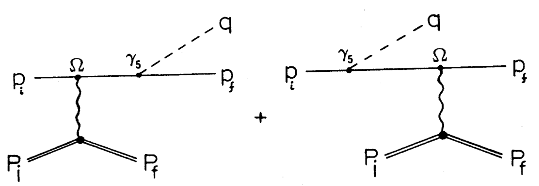

FIG. 1. Feynman diagrams for pion emission from a nucleon in interaction with another particle.

a Dirac nucleon upon scattering of the latter by a third particle. The external interaction Ω was supposed to have either scalar, four-vector, or pseudoscalar coupling with the nucleon, and the amplitude may be represented by the formula

$$M_{fi} \propto \overline{\chi}(p_f)\{\gamma_5[\gamma\cdot(p_f-q)-m]^{-1}\Omega + \Omega[\gamma\cdot(p_i+q)-m]^{-1}\gamma_5\}\chi(p_i), \quad (3)$$

$$\Omega = \begin{cases} 1 & \text{(scalar)} \\ \beta & \text{(four-vector)} \\ \gamma_5 & \text{(pseudoscalar)} \end{cases} .$$

Shorn of coupling constants, of the propagator for the exchanged particle, as well as of the normalization factors for the external particles, we found[7] the nonrelativistic expressions (μ is the meson mass, m the nucleon mass, M the mass of the external particle, and b and a are constant two-component spinors)

$$\frac{i}{2m^2}b^\dagger\vec{\sigma}\cdot\left[\vec{q}-\frac{\mu}{2m}(\vec{p}_f+\vec{p}_i)\right]a \quad \text{(scalar case)}, \quad (4a)$$

$$\frac{i}{2m^2}b^\dagger\vec{\sigma}\cdot[\vec{q}+\vec{p}_f-\vec{p}_i]a \quad \text{(vector case)}, \quad (4b)$$

$$\frac{1}{2m}b^\dagger a \quad \text{(pseudoscalar case)}. \quad (4c)$$

As is immediately clear, each of these forms is manifestly GI, since $\vec{q}+\vec{p}_f-\vec{p}_i=\vec{P}_i-\vec{P}_f=M(\vec{V}_i-\vec{V}_f)$, with $\vec{V}_f-\vec{V}_i$ the velocity increment of the recoiling external particle. Yet, if they were to be constructed from an inherently nonrelativistic theory, each would require a different πNN coupling.

In trying to make contact between the above results and those of BGGS, we followed their approach and calculated the matrix element

$$M_{fi}=\int d^3x\,\overline{\chi}_f(\vec{x})\gamma_5\chi_i(\vec{x})e^{-i\vec{q}\cdot\vec{x}}, \quad (5)$$

using spinors which satisfy the equation

$$[-i\vec{\alpha}\cdot\vec{\nabla}+\beta m+(\beta)^\lambda V]\chi=E\chi, \quad (6)$$

where $\lambda=1$ corresponds to a scalar interaction and $\lambda=0$ to the timelike component of a four-vector. In Eq. (5), $e^{-i\vec{q}\cdot\vec{x}}$ is the wave function of the emitted meson. We now compare results based on Eqs. (5) and (6) with those of Eq. (3). We apply Eq. (6) to Eq. (5), keeping only terms linear in V, and use the Lippmann-Schwinger equation for $u_{i,f}$ also to that order. Our results obtained in this way are

$$\frac{i}{(2m)^2}b^\dagger\vec{\sigma}\cdot\left[\vec{q}-\frac{\mu}{2m}(\vec{p}_i+\vec{p}_f)\right]a \quad \text{(scalar case)}, \quad (7a)$$

$$\frac{i}{(2m)^2}b^\dagger\vec{\sigma}\cdot[(\vec{p}_i-\vec{p}_f)+\vec{P}_f-\vec{P}_i]a \quad \text{(vector case)}. \quad (7b)$$

Both Eqs. (7a) and (7b) are manifestly GI, since the transformation $\vec{v}\to\vec{v}-\vec{V}$ leaves them invariant.

To recapitulate, what we have done up to this point is to follow the mathematical procedure of BGGS; but instead of using it to deduce the form of an operator which could then be sandwiched between appropriate *nonrelativistic* wave functions, we have extracted the actual amplitude for meson emission, up to first order in the external interaction. We are now in a position to compare the results of the BGGS method with those of relativistic perturbation theory. Clearly Eq. (7a) agrees with Eq. (4a), but Eq. (7b) has an extra term relative to Eq. (4b). The reason for this is that two limiting processes have to be applied in order to get the small velocity limit of the meson production amplitude in the static-potential model of BGGS: First, one assumes the mass of the recoiling nucleus is infinite; and then one takes the remaining velocity to be small. Evidently the order of performing these operations is important because particle production (and hence a change in a mass) is not a concept which can be expressed naturally in Galilean-invariant terms.[8] As a result of the above comparison, we feel the BGGS approach is unreliable. This conclusion is further supported by the fact that if we take the effective operator calculated by BGGS and sandwich it between nonrelativistic wave functions including distortion to first order in the external potential, we obtain results different from either Eq. (4) or Eq. (7).

We conclude by remarking that although we disagree with the details of the calculation of BGGS we do support the correctness of their claim, that the nonrelativistic reduction of pseudoscalar absorption or emission depends sensitively on the dynamics of the nucleon which absorbs or emits the meson. As a consequence, (p,π) or (π,p) reactions may be sensitive to unusual aspects of nuclear structure involving small components of the wave functions, in addition to all the usual uncertainties which have been noted previously.[4]

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[6]J. L. Friar, Phys. Rev. C 10, 955 (1974).

[7]We note here that had we begun with the conventional Galilean-invariant operator

$$H_{\pi NN} = -i\vec{\sigma} \cdot [\nabla_\pi - (\mu/2m)(\nabla_i - \nabla_f)],$$

and sandwiched it between nonrelativistic single-particle wave functions, we would have obtained Eq. (4a), to first order in the external potential.

[8]The reason that the electromagnetic interaction has a nonrelativistic limit which is unambiguous to lowest order is just the masslessness of the photon, i.e. gauge invariance, and the theory is therefore not plagued by mass changes.