Letter

Radiative corrections to the beam-normal spin asymmetry in elastic electron scattering from ^{208}Pb

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Estimates for the quantum electrodynamical (QED) and dispersion effects on the spin asymmetry of perpendicularly polarized electrons scattered from a ^{208}Pb nucleus are given. The QED effects are calculated nonperturbatively in terms of their respective potentials. For dispersion, the transient nuclear excitations of ^{208}Pb with low multipolarity and energy below 30 MeV are accounted for. Collision energies up to 1 GeV are considered. The results can help to explain the measured spin asymmetry near 1 GeV at forward angles.

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Measurements of the beam-normal spin asymmetry (also known as Sherman function) in elastic electron-nucleus collisions at energies from 200 MeV to several GeV [1–7] revealed a considerably larger asymmetry than predicted from the phase-shift analysis for potential scattering [8,9]. Such large asymmetries were attributed to dispersive effects resulting from transient excitations of the target during the scattering process. In order to account for hadronic excitations beyond the pion production threshold, two-photon exchange models were developed. For electron-nucleon scattering, these excitations were explicitly taken into account in terms of resonances in inelastic e-p scattering [10] or in terms of one-pion intermediate states [11]. The results were found to be in good agreement with experiment covering scattering angles up to 145° [3,12].

For small-angle electron-nucleus scattering, the hadronic excitations of arbitrary targets were considered with the help of the forward Compton scattering cross section known from experiment [9,13,14]. This hadronic model was able to explain the measurements at forward angles for the lighter nuclei up to ^{90}Zr within the theoretical uncertainty [5–7]. For the doubly magic ^{208}Pb nucleus, however, the small-angle experiments [2,7] could not be reconciled with this theory. It was therefore argued that the consideration of QED corrections might help to explain the discrepancy [7].

In the present work the influence of vacuum polarization and the vertex plus self-energy (vs) effect on the Sherman function is considered nonperturbatively. To this aim, a potential describing the vs effect is generated from the relation between the first-order Born amplitude for the process under consideration and the underlying potential [14,15] (in atomic units, $\hbar = m_e = e = 1$),

$$V_{\rm vs}(r) = -\frac{2Z}{\pi}\int_0^\infty d|\boldsymbol{q}|\; j_0(|\boldsymbol{q}|r)\, F_L(|\boldsymbol{q}|)\, F_1^{\rm vs}(-q^2), \quad (1)$$

where Z is the nuclear charge number, and $q = ((E_i - E_f)/c, \boldsymbol{q})$ is the 4-momentum transfer to the nucleus with $\boldsymbol{q} = \boldsymbol{k}_i - \boldsymbol{k}_f$. The total energy and momentum of the electron in its initial and final state, respectively, are denoted by $E_i, \boldsymbol{k}_i$ and $E_f, \boldsymbol{k}_f$. F_L is the nuclear ground-state charge form factor, j_0 a spherical Bessel function, and $F_1^{\rm vs}$ the electric form factor for the vs process [16] (omitting the infrared-divergent term):

$$F_1^{\rm vs}(-q^2) = \frac{1}{2\pi c}\left\{\frac{v^2+1}{4v}\left(\ln\frac{v+1}{v-1}\right)\left(\ln\frac{v^2-1}{4v^2}\right) + \frac{2v^2+1}{2v}\ln\frac{v+1}{v-1} - 2 + \frac{v^2+1}{2v}\left[{\rm Li}\left(\frac{v+1}{2v}\right) - {\rm Li}\left(\frac{v-1}{2v}\right)\right]\right\}, \quad (2)$$

where ${\rm Li}(x) = -\int_0^x dt\,\frac{\ln|1-t|}{t}$ is the Spence function [17], and $v = \sqrt{1 - 4c^2/q^2}$.

Together with the Uehling potential for vacuum polarization [18,19] and the nuclear potential V_T, generated from a Gaussian ground-state charge distribution [20], $V_{\rm vs}$ is included in the Dirac equation for the electronic scattering states. The resulting scattering amplitude $f_{\rm vac+vs}$, consisting of the direct term A and the spin-flip term B, is obtained from the conventional phase-shift analysis [21]. The soft-bremsstrahlung contribution to the QED effects is disregarded, since it does not influence the Sherman function [15,22]. The latter is given by

$$S_{\rm QED} = \frac{2\,{\rm Re}\,\{AB^*\}}{|A|^2 + |B|^2}. \quad (3)$$

The change of the Sherman function by the QED effects is defined in terms of the difference

$$\Delta S_{\rm QED} = S_{\rm QED} - S_{\rm Coul}, \quad (4)$$

where $S_{\rm Coul}$ relates to potential scattering from the Coulombic field V_T. It is defined according to (3) by inserting for A and B the direct and spin-flip terms which determine the Coulombic scattering amplitude $f_{\rm Coul}$. Figure 1 shows the energy distribution of $\Delta S_{\rm QED}$ at the forward scattering angle ϑ_f of 10°. It is seen that this spin asymmetry change is basically determined by the vs correction, while the contribution from vacuum polarization is in general much smaller and of opposite sign. The QED effects are formidable below 250 MeV, but they are of the order of 10^{-8} at the higher energies. Increasing insta-

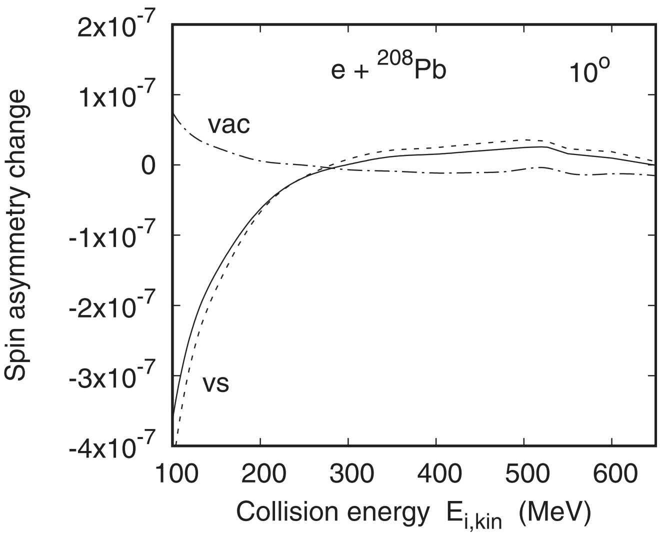

FIG. 1. Energy distribution of the spin asymmetry change by the QED effects (———) in $e + {}^{208}$Pb collisions at $\vartheta_f = 10^\circ$. Also shown are the separate contributions from vacuum polarization (− · − · −) and from the vs correction (− − − − −).

bilities with the number of included partial waves and with the matching point between the inner and outer solutions of the Dirac equation, relying on the Fortran code RADIAL [23], prohibit a precise determination of ΔS_{QED} beyond 500 MeV at this angle.

Dispersion in terms of the two-photon box diagram (within the second-order Born approximation) can be expressed by the transition amplitude [24–26]

$$A_{fi}^{\mathrm{box}} = \frac{\sqrt{E_i E_f}}{\pi^2 c^3} \sum_{L,\omega_L} \sum_{M=-L}^{L} \int d\boldsymbol{p} \times \sum_{\mu,\nu=0}^{3} \frac{1}{(q_2^2 + i\epsilon)(q_1^2 + i\epsilon)} t_{\mu\nu}(p)\, T^{\mu\nu}(LM, \omega_L), \quad (5)$$

where the denominator results from the propagators of the first and second photon with four-momentum $q_1 = k_i - p$ and $q_2 = p - k_f$, respectively, with $p = (E_p/c, \boldsymbol{p})$ and E_p the energy of the intermediate electronic state. The electronic transition matrix element is denoted by $t_{\mu\nu}(p)$. The transient excitation of the nucleus to a state with energy ω_L, angular momentum L, and projection M together with its subsequent decay to the ground state, by means of the transition densities $J = (\varrho, \boldsymbol{J})$, is given by

$$T_{\mu\nu}(LM, \omega_L) = \langle 0|J_\mu(\boldsymbol{q}_2)|LM, \omega_L\rangle \, \langle LM, \omega_L|J_\nu(\boldsymbol{q}_1)|0\rangle. \quad (6)$$

For the ^{208}Pb nucleus, ten dominant excited states with $L \leqslant 3$ and ω_L up to 30 MeV are considered [27]. The required transition densities are calculated within the self-consistent Hartree-Fock (HF) plus random phase approximation (RPA) [28,29], as well as within the RPA-based quasiparticle phonon model [30,31].

The differential scattering cross section of an electron with initial polarization vector $\boldsymbol{\zeta}_i$, including dispersion to lowest

TABLE I. Calculated dispersive spin asymmetry change ΔS_{box} when considering only the dipole states (third column) and when considering all ten states with $L \leqslant 3$ (fourth column). The experiments are by Adhikari *et al.* for 0.95 GeV [7] and by Abrahamyan *et al.* for 1.063 GeV [2].

$E_{\mathrm{i,kin}}$ (GeV)	ϑ_f	$(\Delta S_{\mathrm{box}})_{\Sigma_{L=1}}$	$(\Delta S_{\mathrm{box}})_{\Sigma_{L\leqslant 3}}$	ΔS_{exp}
0.950	4.7°	6.69×10^{-7}	6.96×10^{-7}	$(5.8 \pm 2.0) \times 10^{-7}$
1.063	5°	1.36×10^{-6}	1.25×10^{-6}	$(4.9 \pm 2.5) \times 10^{-7}$

order, is calculated from [21,26]

$$\frac{d\sigma_{\mathrm{box}}}{d\Omega_f}(\boldsymbol{\zeta}_i) = \frac{|\boldsymbol{k}_f|}{|\boldsymbol{k}_i|} \frac{1}{f_{\mathrm{rec}}} \sum_{\sigma_f} \left[|f_{\mathrm{Coul}}|^2 + 2\,\mathrm{Re}\left\{ f_{\mathrm{Coul}}^* A_{fi}^{\mathrm{box}} \right\} \right], \quad (7)$$

where f_{rec} is the recoil factor, and the sum runs over the two polarization directions σ_f of the electron in its final state. For $\boldsymbol{\zeta}_i$ perpendicular to the scattering plane, the Sherman function is obtained from the relative cross section difference when $\boldsymbol{\zeta}_i$ is flipped,

$$S_{\mathrm{box}} = \frac{d\sigma_{\mathrm{box}}/d\Omega_f(\boldsymbol{\zeta}_i) - d\sigma_{\mathrm{box}}/d\Omega_f(-\boldsymbol{\zeta}_i)}{d\sigma_{\mathrm{box}}/d\Omega_f(\boldsymbol{\zeta}_i) + d\sigma_{\mathrm{box}}/d\Omega_f(-\boldsymbol{\zeta}_i)}, \quad (8)$$

and its dispersive correction is obtained by means of

$$\Delta S_{\mathrm{box}} = S_{\mathrm{box}} - S_{\mathrm{Coul}}. \quad (9)$$

Table I displays the results for ΔS_{box} in comparison with the two experimental data points [2,7]. Since the spin asymmetry from Coulombic scattering is included in the measurements, S_{Coul} was subtracted from the data.

In accordance with Fig. 1, the spin asymmetry changes by the QED effects are negligibly small in the GeV region (an

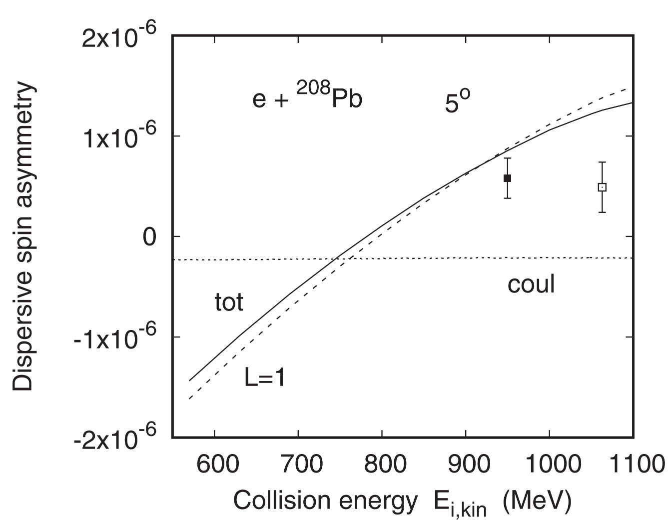

FIG. 2. Energy distribution of the dispersive correction ΔS_{box} (———) for $e + {}^{208}$Pb collisions at $\vartheta_f = 5^\circ$. Included are its dipole contribution (− − −−) and S_{Coul} (· · · · · ·), as well as the dispersive experimental results ΔS_{exp} from Adhikari *et al.* (■, [7]) and from Abrahamyan *et al.* (□, [2]).

estimate gives $\Delta S_{\mathrm{QED}} \approx 5 \times 10^{-9}$ for 0.95 GeV at 4.7° to an accuracy of a factor of 2) as compared to the dispersive corrections. The accuracy of the latter is estimated to be about 50%. The main source of uncertainty is due to the choice of the nuclear models for calculating the transition densities, followed by the incomplete consideration of all $L \geqslant 2$ excitations. Some 5% are attributed to numerics. A more detailed analysis is provided in [32].

Figure 2 displays the energy dependence of the dispersive correction to the Sherman function at an angle of 5°. At such a small angle, diffraction structures are still absent and ΔS_{box} increases monotonically with kinetic energy $E_{\mathrm{i,kin}}$. It is also demonstrated that ΔS_{box} results basically from the nuclear dipole excitations, whereas higher multipoles gain importance at the larger scattering angles [27].

In conclusion, it has been shown that the dispersive effects lead to a considerable increase of the beam-normal spin asymmetry beyond its value from Coulombic potential scattering which, amounting to -2×10^{-7} at high energies and a scattering angle of 5°, has, however, still to be considered. On the other hand, QED corrections seem to be negligible at collision energies beyond 300 MeV.

The calculated spin asymmetry $S_{\mathrm{box}} = 5.19 \times 10^{-7}$ at 950 MeV and 4.7° is in accord with the measured value $S_{\mathrm{exp}} = (4.0 \pm 2.0) \times 10^{-7}$, while the result at 1063 MeV overestimates experiment by a factor of 2.5 (but still agrees within theoretical and experimental error bars). It is, however, still an open question how the excitations beyond 135 MeV, leading within the hadronic model to a large negative spin asymmetry, are suppressed by experiment.

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