Effects of short-range correlations on neutron star cooling

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(Received 17 January 2024; accepted 9 May 2024; published 3 June 2024)

Cooling curves of a neutron star are investigated within the minimal cooling scenario based on the Akmal-Pandharipande-Ravenhall (APR) equation of state. The Fermi surface depletion (Z factor) of β-stable nuclear matter is calculated by employing a Brueckner-Hartree-Fock approach with the AV18 two-body force supplemented by a microscopic three-body force. The Z factor of β-stable nuclear matter is computed to study its effects on three physical properties (superfluidity, neutrino emissivity, and heat capacity) which determine the neutron star thermal evolution. The calculated cooling curves reproduce well the mostly observed data when the Z factor was taken into account in the gap energies, neutrino emissivity, and heat capacity.

DOI: 10.1103/PhysRevC.109.065801

I. INTRODUCTION

Neutron stars (NSs) have a lot of extreme features such as rapid rotation, extremely strong magnetic field, superstrong gravitation, superprecise spin period, superfluidity, and superconductivity in the crust and core [1]. Up to now, the information of NS interior has not yet been uncovered sufficiently through the current observations due to the complexity of NS system. Thermal luminosities and ages of neutron stars can be inferred from observations [2–7]. The thermal evolution of a neutron star (NS) is quite sensitive to the equation of state (EOS) of NS matter, in particular to its composition and superfluid states [8–14]. Therefore, the NS cooling provides an important way to understand the information in NS interior.

Inspired by these abundant observed data, many approaches have been proposed to compute the cooling curve of neutron star. However, no approach is currently predominant. Two major theoretical scenarios have been developed to explain this observation: one is standard cooling or minimal cooling [15–21], and the other one is enhanced cooling by very fast direct Urca cooling [22–26].

In all models about neutron star cooling processes, superfluidity is a key ingredient in the description of cooling curves. Theoretical understanding of superfluidity is rooted in the microscopic theory of superconductivity proposed by Bardeen, Cooper, and Schrieffer (BCS) [27–29]. Various many-body theories have been developed to provide insight on different aspects of singlet and triplet pairing in pure neutron matter and isospin-asymmetric nuclear matter in β-stable conditions [30–40].

It is well known that most calculations of the superfluidity focus on a perfect degenerate Fermi system. However, in such a nuclear-many body system, the short-range correlations (SRCs) are so strong that the momentum distribution around the Fermi level departs significantly from the typical profile of a degenerate Fermi system. This Fermi surface depletion is characterized by the so-called Z factor [41].

Since the Fermi surface depletion hinders particle transitions around the Fermi surface, the pairing gap is expected to be reduced. Recent calculations show that the effects of the SRC quench the pairing gaps of dense matter in NS core greatly [42,43]. In addition, the SRC is also found to reduce the neutrino emissivity and the heat capacity of the stellar interior [43]. In the present work, the Z factor effects on the pairing gaps and various neutrino processes, and heat capacity are considered to calculate cooling curves. Our aim is to interpret all observational data using the fixed EOS [44] and the gap energies as inputs to calculate the cooling curve.

II. THEORETICAL FRAMEWORK

In the present work, the cooling curves of neutron stars are calculated using the publicly available code NSCool [45], which solves the general-relativistic equations of energy balance and energy transport:

$$\frac{1}{4\pi r^2 e^{2\Phi}}\sqrt{1-\frac{2Gm}{r}}\frac{\partial}{\partial r}(e^{2\Phi}L_r) = -Q_\nu - \frac{C_\upsilon}{e^\Phi}\frac{\partial T}{\partial t}, \tag{1}$$

$$\frac{L_r}{4\pi r^2\kappa} = -\sqrt{1-\frac{2Gm}{r}}e^{-\Phi}\frac{\partial(Te^\Phi)}{\partial r}, \tag{2}$$

where Φ is the metric function. Q_ν is the neutrino emissivity. C_υ and κ are the specific heat capacity and the thermal conductivity, respectively. Local luminosity L_r and temperature T depend on radial coordinate r and time t. The details of numerical calculations can be found in code NSCool [45].

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The minimal cooling scenario [15,16,45] assumed that the cooling processes with no charged meson condensate, no hyperons, no confinement quarks in NSs. Superfluidity of neutrons and/or protons in neutron-star cores affects the heat capacity of nucleons and reduces neutrino reactions involving superfluid nucleons. Moreover, superfluidity initiates an additional neutrino emission mechanism associated with Cooper pairing of nucleons. All these effects of superfluidity are incorporated into cooling code NSCool [45].

In this work, for simplicity, we assume the NS models with the cores composed of neutrons, protons, electrons, and muons. The fraction of each component being determined by the Akmal-Pandharipande-Ravenhall (APR) EOS, the symmetry energy is assumed ($E_{\rm sym}(\rho) = B/A(\rho, 0) - B/A(\rho, 0.5)$) for the calculation of β-stable matter.

The deviation of a correlated Fermi system from the ideal degenerate Fermi gas is measured by the quasiparticle strength [41]

$$Z(k) = \left[1 - \frac{\partial \Sigma(k,\omega)}{\partial \omega}\right]^{-1}_{\omega=\epsilon(k)}, \tag{3}$$

where $\Sigma(k, \omega)$ is the self-energy as a function of momentum k and energy ω. The Z factor at the Fermi surface equals the discontinuity of the occupation number at the Fermi surface. The Z factors is calculated in the framework of the Brueckner-Hartree-Fock (BHF) approach [43] by employing the AV18 two-body force with a microscopic three-body force [46].

Next, we first calculate the gap energy of the proton $^1S_0$ channel. By considering the Z factor induced by the SRC, the gap equation is given as

$$\Delta(k) = -\frac{1}{\pi}\int_0^\infty V(k,k')\frac{\Delta(k')Z(k)Z(k')}{\sqrt{(\epsilon(k')-\mu)^2+\Delta^2(k')}}k'^2 {\rm d}k', \tag{4}$$

where $V(k, k')$ is the spin-singlet component of the realistic NN interaction adopted for the BHF calculation of the self-energy, which includes three-body force. The calculated gap energy in β-stable nuclear matter is displayed in Fig. 1(a) without (solid squares) and with (hollow squares) taking into account the Z factor. One can see from Fig. 1(a) that the quasiparticle strength quenches the gap peaks more than one order of magnitude. The superconducting density domain remarkably shrinks and the peak value drops to 0.02 MeV with the inclusion of the Z factor. If this is true, the proton $^1S_0$ superconductivity almost plays no role in NS cooling.

The $^3PF_2$ superfluidity in the β-equilibrium state has been investigated in our previous work [43] with the inclusion of the Z factor. The $^3PF_2$ pairing gaps are determined by the following coupled equations:

$$\begin{pmatrix}\Delta_L(k)\\ \Delta_{L+2}(k)\end{pmatrix} = -\frac{1}{\pi}\int_0^\infty k'^2{\rm d}k'\frac{Z(k)Z(k')}{E_{k'}} \times \begin{pmatrix}V_{L,L}(k,k') & V_{L,L+2}(k,k')\\ V_{L+2,L}(k,k') & V_{L+2,L+2}(k,k')\end{pmatrix} \times \begin{pmatrix}\Delta_L(k')\\ \Delta_{L+2}(k')\end{pmatrix} \tag{5}$$

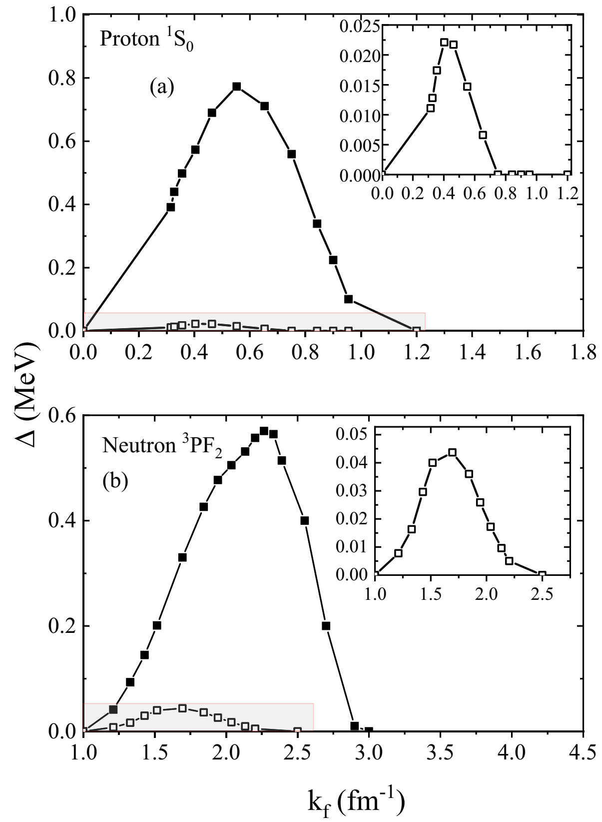

FIG. 1. (a) Proton $^1S_0$ superfluidity gap as a function of Fermi wave number in β-stable nuclear matter, where the AV18 interaction is used for BHF calculations. The calculations without and with the Z factor inclusion of SRC are shown for comparison. (b) Similar as (a) but for neutron $^3PF_2$ gap energy.

with $E_k = \sqrt{[\epsilon(k)-\mu]^2 + \Delta_k^2}$ and $\Delta = \sqrt{\Delta_L^2 + \Delta_{L+2}^2}$. $V_{L,L'}(k, k')$ is the matrix element of the realistic NN interaction including three-body forces. Here, the angle-average approximation is adopted, and the relation between the gap at $T = 0$ and the critical temperature T_c is given by $k_B T_c = 0.57\Delta(T = 0)$ [38].

The calculated neutron $^3PF_2$ gaps for the β-stable matter are depicted in Fig. 1(b). We noticed that the Z factor effect quenches the peak value about one order of magnitude, and it is extremely sizable at higher densities. The superfluidity domain shrinks to 1–2.5 fm^{-1} and the peak value drops to 0.04 MeV. Therefore, the role of the $^3PF_2$ superfluidity in NS cooling is limited.

The gap energy is significantly dependent on the nucleon-nucleon interaction model and the many-body theory employed. Up to now, the significant differences of gap energy for the neutron $^3PF_2$ channel are obtained from different theoretical methods. It is well known that the gap energy of $^3PF_2$ partial-wave channel is harder to calculate, because the various nucleon-nucleon interactions are not constrained at high density by the phase-shift analysis needed for this channel. In addition, many-body effects such as three-body forces, short-range correlations, and screening play a substantial role in this channel. Therefore, there is currently much uncertainty in estimates of the spin-triplet pairing gap.

Recently, the influence of short-range correlations on the $^3PF_2$ pairing gap in pure neutron matter at high density has been calculated, they found that the $^3PF_2$ gap is strongly suppressed [47]. In addition, the medium-induced spin-orbit interaction leads to a reduction of the $^3P_2$ ($^3PF_2$) pairing gap by considering the full polarization contributions for noncentral interactions [48].

The neutrino emissivity has been depressed by the SRC since the depletion of Fermi surface hinders particle-hole excitations around the Fermi level. The various neutrino processes and the corresponding emissivity have been reviewed [8,11,12]. With the inclusion of the Z factor caused by the SRC, the corresponding neutrino emissivity $Q^{(DU)}$ under the β-equilibrium condition is given as [43]

$$Q^{(DU)} = Z_{F,n} Z_{F,p} Q_0^{(DU)} \tag{6}$$

where $Q_0^{(DU)}$ is the neutrino emissivity of the DU process without the SRC effect. In the present work, the expression for the neutrino emissivity $Q_0^{(DU)}$ in the direct Urca process has been generalized to the case of superfluid matter by introducing the so-called reduction factors R [8]. The reduction factors R arise due to taking into account the energy gap near the Fermi surface in the Fermi distribution itself. The parameters in the simple fit expressions for reduced factors R take the default values in the program [45]. A similar treatment can be applied to other neutrino emissivity processes, such as modified URCA (MU), nucleon-nucleon bremsstrahlung (NNB), Cooper pair breaking and formation (PBF) processes. The MU, NNB, and PBF neutrino emissivity processes with the inclusion of the Z factor are addressed in detail in Ref. [43].

Many publications have given the effect of various neutrino emissivity processes on the cooling curves [8–16], but the work about the effects of short-range correlation on the neutrino emissivity seems to be rare [49,50]. The modified URCA process and neutrino bremsstrahlung process were computed using a simplified one-pion-exchange interaction in the Born approximation, in which effects of short-range correlations on the tensor part are approximated through insertion of a cutoff in the one-pion-exchange potential. However, the results are still controversial and inconclusive [49,50].

The heat capacity of superfluid Fermi systems is well known [8,15]. There is a sharp increase due to the effect of pairing interactions on the specific heat. These effects can be taken into account by using reduction factors R_V. A general expression for the reduction factors R_V is obtained by introducing the dispersion relation [8]. In the present work, the parameters in the simple fit expressions for reduced factors R_V take the default values in the program [45]. The heat capacity of the stellar interior is also suppressed by the SRC. When the effect of the Fermi surface depletion is included, the specific heat of the nucleon i is given by [43]

$$c_{v,i} = Z_{F,i}\left(\frac{m_i^* p_{F,i}}{3\hbar^3}\right) k_B^2 T \times R_V, \quad i = n, p. \tag{7}$$

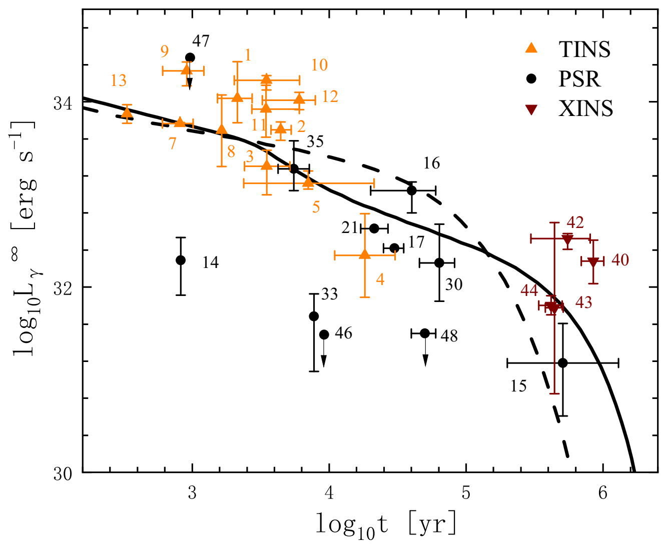

FIG. 2. Comparison of predictions of the cooling curves with the observed data [6], the cooling of $1.4M_\odot$ stars built using the EOS of APR, without (dashed line) and with (solid line) considering the effect of the Z factor on the nucleon pairing gap. Details of the number near the observed data mark similar to Fig. 1 in Ref. [6], where only a subset of neutron stars with known "true ages" (named t* in Table 2 in Ref. [6]) has been used. The TINS, PSR, and XINS represent different types of neutron stars, see Ref. [6] for details.

Here, the $Z_{F,n}$ and $Z_{F,p}$ are denoted as the calculated Z factor at the Fermi surface for nuclear matter in a β-stable state, respectively.

III. NUMERICAL RESULTS AND DISCUSSIONS

In the present work, for simplicity, the cooling curves were calculated for the model of nonmagnetized heat-blanketing envelope made of iron. The calculated cooling curve for a superfluid neutron star of mass $1.4M_\odot$ with the APR EOS is shown in Fig. 2. The observed thermal luminosities and ages and of neutron stars are represented by black squares with error bars, the data were taken from the review article [6]. The cooling curve (dashed line) is calculated using superfluid gap energies obtained from BHF method with the two-body force and three-body force. We found that the observed data are reasonably compatible with the minimal cooling provided by using APR EOS. However, most old and warm stars are still not covered by the cooling curve (dashed line).

The calculated cooling curve (solid line) was shown in Fig. 2 when the Z factor was taken into account in the gap energies. As is shown in Fig. 1, the Z factor significantly suppresses the neutron $^3PF_2$ and proton $^1S_0$ superfluidity. Therefore, the cooling curve (solid line) of the neutron star for the first 1×10^3 years is gently retarded compared to the case without taking into account the Z factor (dashed line). The cooling curve from 1×10^3 to 1×10^5 is significantly accelerated when the Z factor is taken into account. The critical temperature T_c for neutron $^3PF_2$ superfluidity is quite low as a result of the inclusion of Z factor. The PBF cooling

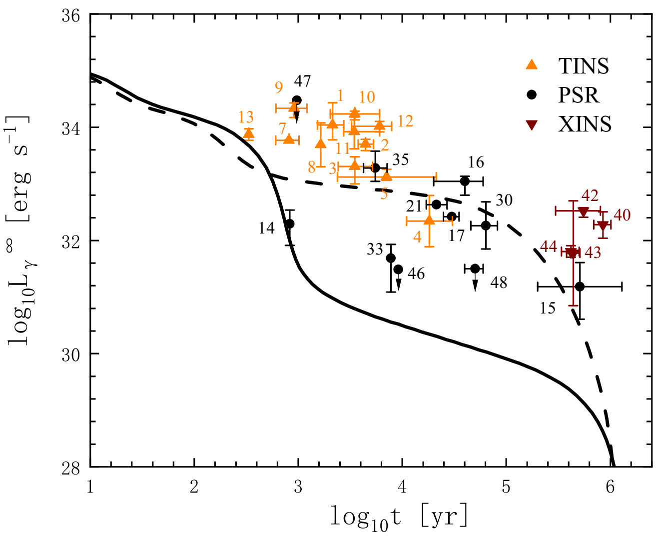

FIG. 3. Comparison of predictions of the cooling curves with the observed data [6], the cooling of $2.0M_\odot$ stars built using the EOS of APR, without (dashed line) and with (solid line) considering the effect of the Z factor on the nucleon pairing gap. The description of the observed data are illustrated in Fig. 2.

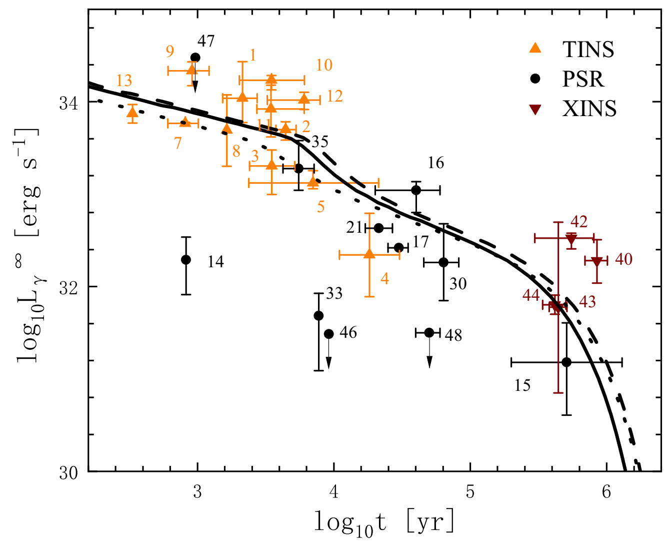

FIG. 4. Comparison of predictions of the cooling curves of cooling of $1.4M_\odot$ stars with the observed data [6], the stellar structure is built with the APR EOS. The calculated cooling curves with Z factors only in superfluidity (dotted line), with Z factors both in superfluidity and neutrino emission (dashed line) and with Z factors in the three inputs (solid line) are shown for comparison. The description of the observed data are illustrated in Fig. 2.

channel opens as soon as the stellar temperature falls below the T_c, which leads to a relatively fast cooling because it is more efficient than MU processes. The theoretical results (solid line) show that most of the observed data [6] can be explained when the Z factor was taken into account in the gap energies.

To study the effect of the Z factor on the cooling curves of a massive neutron star, the calculated cooling curve for a superfluid neutron star of mass $2.0M_\odot$ with the APR EOS is also shown in Fig. 3. As seen from Fig. 3, the dramatic effect of the Z factor on the thermal evolution of a massive neutron star is demonstrated by comparing dashed and solid lines. The thermal evolution of massive neutron star (dashed line) drops to a value after 10^2 yr and decreases slowly for the next 10^5 yr when the Z factors in the gap energies were not considered. It is well known that the strong superfluidity mixes the cooling types. The qualitative effect of superfluidity is to halt the rapid cooling of a massive neutron star, the enhanced cooling may look like standard cooling while the standard cooling may look like enhanced. However, the Z factor quenches the peak value of proton ${}^1S_0$ and neutron ${}^3PF_2$ about one order of magnitude. Therefore, a massive neutron star shows fast cooling via the direct Urca process.

In order to analyze the effect of the Z factor induced reduction for the neutrino emissivity and heat capacity on the cooling curves, the calculated cooling curve for neutron stars with $1.4M_\odot$ within the Z factor in gap energies (dot line), Z factor in gap energies and neutrino emissivity (dashed line), and Z factor in three inputs (solid line) are shown in Fig. 4. The Z factor substantially reduces the neutrino emission of modified URCA (MU), nucleon-nucleon bremsstrahlung (NNB), and PBF processes [43]. Therefore, the Z factor retards the thermal luminosity in the neutrino cooling era by comparing dotted and dashed lines. The heat capacity of the neutron star is also reduced by the Z factor, but this effect is not so sharp in the neutrino cooling era as shown in Fig. 4. In the photon cooling era, however, heat capacity reduced by the Z factor significantly accelerates the cooling process.

In Fig. 5, we plot a family of cooling curves calculated using APR EOS when the Z factor was taken into account

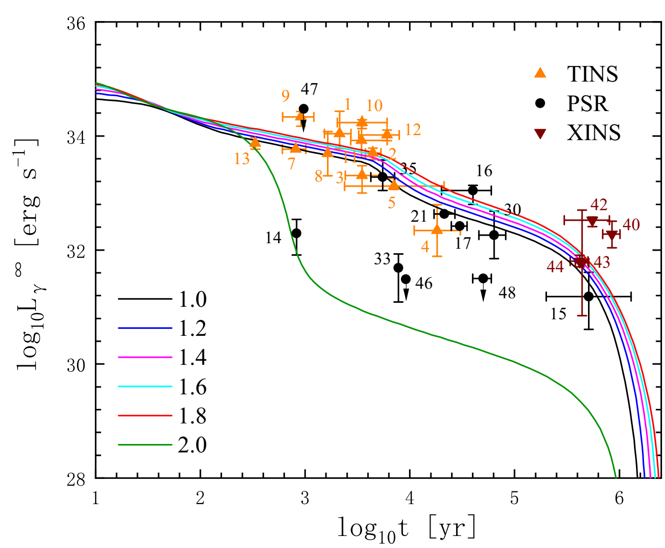

FIG. 5. Thermal luminosities and ages of observed data for NSs compared with cooling curves with masses from 1.0 to 2.0 $M_\odot$. The theoretical cooling curves were computed when the Z factor was taken into account in the gap energies, neutrino emissivity, and heat capacity. The description of the observed data are illustrated in Fig. 2.

in the gap energies, neutrino emissivity, and heat capacity, the curves refer to neutron stars with gravitational masses $M_\odot$ from 1.0 to 2.0$M_\odot$. It can be seen from Fig. 5 that the thermal luminosities increase when increasing M from 1.0 to 1.8$M_\odot$. At first glance, we see that there is almost no mass effect both during the neutrino and the photon cooling processes. Careful comparison of the observed data [6] with the calculated results based on different masses reveal that the mostly observed data can be covered when the mass dependence of the cooling curve was taken into account.

We have investigated some aspects of the effects of short-range correlation on neutron star cooling. One finds that: (i) The short-range correlation (so-called Z factor) significantly suppresses the proton $^1S_0$ and neutron $^3PF_2$ superfluidity. (ii) The present results support that no direct Urca process occurs during the cooling of low mass and medium mass neutron stars, the massive neutron stars show fast cooling via the direct Urca process. The conclusion is consistent with the theoretical results in Refs. [6,15–18] obtained from a different method. (iii) When the direct Urca processes can be activated in the most massive neutron stars, the calculated cooling curves with a very small mass shift $M + \Delta M$ can abruptly switch to a rapid cooling mode and go beyond the observed points. The thermal conductivity is sensitive to the presence of the accretion of light elements and magnetic fields in the blanketing envelope [10], however, the cooling curves of our work were calculated for the model of a nonmagnetized heat-blanketing envelope made of iron for simplicity. Further discussing the mechanisms of a possible smoothing of the transition to the rapid cooling regime by considering the accretion of light elements and magnetic fields in the blanketing envelope is needed.

IV. SUMMARY

In summary, the Z factor is calculated in the framework of the BHF approach using the two-body AV18 force plus a microscopic three-body force. We found that the superfluidity, neutrino emissivity, and heat capacity are reduced by the Z factor. Therefore, the effect of the Fermi surface depletion (Z factor) is needed to be included in a theoretically rigorous exploration of the NS thermal evolution. Based on the above results, we calculated the cooling curve using the APR EOS. The theoretical results show that most of the observed data [6] can be explained when the Z factor is taken into account in the gap energies, neutrino emissivity, and heat capacity. When the direct Urca processes can be activated in the most massive neutron stars, the calculated cooling curves with a very small mass shift $M + \Delta M$ can abruptly switch to a rapid cooling mode and go beyond the observed points. The thermal conductivity is sensitive to the presence of the accretion of light elements and magnetic fields in the blanketing envelope [10], however, the cooling curves of our work were calculated for the model of a nonmagnetized heat-blanketing envelope made of iron for simplicity. The mechanisms of a possible smoothing transition to the rapid cooling regime by considering the accretion of light elements and magnetic fields in the blanketing envelope need to be further discussed.

ACKNOWLEDGMENTS

This work ws supported by the National Natural Science Foundation of China (Grants No. 12222511, No. 12275080, No. 12175064, No. U2167203), Hunan Outstanding Youth Science Foundation (2022JJ10031), and the stable support program for basic research youth teams of Chinese Academy of Sciences YSBR-088.

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