Polarization effects in elastic deuteron-electron scattering

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The differential cross section and polarization observables for the elastic reaction induced by deuteron scattering off electrons at rest are calculated in the one-photon-exchange (Born) approximation. Specific attention is given to the kinematical conditions, that is, to the specific range of incident energy and transferred momentum. The peculiar interest of this reaction is to access very small transferred momenta. Numerical estimates are given for polarization observables that describe the of single- and double-spin effects, provided that the polarization components (both vector and tensor) of each particle in the reaction are determined in the rest frame of the electron target.

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I. INTRODUCTION

In a previous work [1] we calculated the cross section in the Born approximation for the elastic scattering of deuteron on atomic electrons as well as the first order radiative corrections for lepton emission. In the present paper we complete and extend this study to a number of polarization observables. Expressions for some of the observables have already been published [2].

The main feature of the reaction $d + e \to d + e$ where electrons are at rest is the inverse kinematics: the projectile is much heavier than the target, transferring then an extremely small squared momentum, Q^2, compared to the incident energy. Reactions induced by proton and deuteron beams on atomic electrons, give the possibility to measure the electromagnetic hadron form factors at very small transferred four-momentum, that is unachievable by direct kinematics.

Inverse kinematics was used in a number of the experiments to measure the pion or kaon radius in the elastic scattering of negative pions (kaons) from atomic electrons in a liquid-hydrogen target [3–8]. Recently, low-Q^2 data were used to determine the hadron charge radius, r_c. In the cases of protons and deuterons, renewed interest in the charge radius is due to the discrepancy between several experiments based on different methods. The most recent CODATA evaluation gives an updated value of the proton root mean squared (rms) radius of $\langle r^2\rangle_p = 0.8414(19)$ fm for the proton and $\langle r^2\rangle_d = 2.12799(74)$ fm for the deuteron, see [9] and references therein. Any revision of the static (and dynamic) properties of the proton affects directly the description of light nuclei, in particular of the deuteron. At relatively large internal distances (small Q^2 values) the deuteron is considered to be a bound system of a proton and a neutron and any small correction would introduce additional effects beyond this simple picture.

Large interest in inverse kinematics is related to polarization phenomena. Polarization observables are essential to disentangle the hadron structure and the reaction mechanism so to be able to test the validity and the predictions of hadron models having in addition interesting applications.

In the case of the $p + e$ elastic scattering the possibility to build beam polarimeters for high energy polarized proton beams in the BNL relativistic heavy ion collider energy range was put forward in Ref. [10]. The calculation of the spin correlation parameters, for the case of polarized proton beam and electron target, are sizable and a polarimeter based on this reaction can measure the polarization of the proton beam [10]. Numerical estimations of other polarization observables were done in Ref. [11]. They showed that polarization effects may be sizable in the GeV range, and that the polarization transfer coefficients for $\vec{p} + e \to \vec{p} + e$ could be used to measure the polarization of high energy proton beams.

In this work we consider the scattering of a polarized deuteron beam on a polarized electron target, assuming that the electron target is at rest and the deuteron beam interacts through the exchange of one photon with four-momentum

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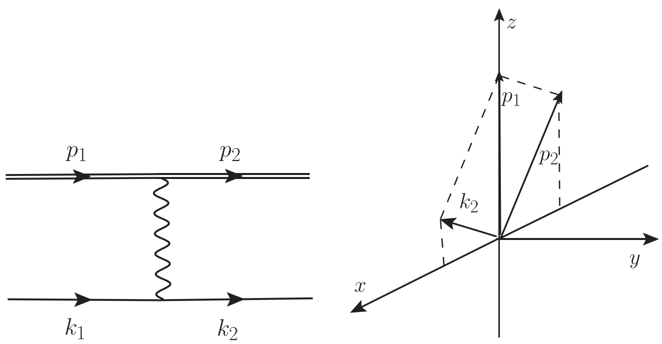

FIG. 1. One photon exchange diagram of the process (1) and chosen coordinate axes in the rest frame of the initial electron.

squared $-k^2 = Q^2 > 0$. We follow the formalism from Ref. [11], which was developed in the one-photon-exchange approximation for the process of the elastic proton-electron scattering $p + e^- \to p + e^-$ and extended to unpolarized deuteron in Ref. [1]. Numerical estimations are given for various polarization observables. The possibility to build a polarimeter based on elastic deuteron-electron scattering is also discussed.

The paper is organized as follows. In Sec. II we give the details of the order of magnitude and the range which is accessible to the kinematic variables, as they are very specific for this reaction (Sec. II A). The spin structure of the matrix element and the unpolarized cross section are derived and calculated in Sec. II B in terms of deuteron form factors, which parametrization is discussed in the Appendix. Section III is devoted to the calculation of the polarization observables for the reaction $d + e \to d + e$. Analyzing powers in the $\vec{d}^{\,T} + e \to d + e$ reaction (when deuteron beam is tensor polarized) are calculated in Sec. III A. The tensor polarization coefficients in the $d + e \to \vec{d}^{\,T} + e$ reaction (when the scattered deuteron is tensor polarized) are given in Sec. III B. The polarization transfer coefficients from a polarized target to the polarized recoil electron are calculated in Sec. III C. In Secs. III D–III G we give the expressions for double polarization observables and derive various combinations the coefficients of the polarization correlation and polarization transfer between deuteron and electron, provided they have vector polarization. In Sec. III H the vector polarization transfer from the initial deuteron to the scattered one is calculated. Section IV is devoted to discussion and conclusion.

II. GENERAL FORMALISM

Let us consider the reaction (Fig. 1)

$$d(p_1) + e^-(k_1) \to d(p_2) + e^-(k_2), \tag{1}$$

where the particle momenta are indicated in parentheses, and $k = k_1 - k_2 = p_2 - p_1$ is the four-momentum of the virtual photon. The reference system is the laboratory (Lab) system, where the electron target is at rest. A general characteristic of all reactions of elastic and inelastic hadron scattering by atomic electrons (which can be considered at rest) is the small value of the four-momentum squared even for relatively large energies of colliding hadrons. The derivation of the spin structure of the matrix element and of the unpolarized and polarized observables is given below, following the description of the specific kinematics and the illustration of the accessible kinematical range and of the order of magnitudes involved.

A. Kinematics

In the Lab system the four-momentum transfer squared is a linear function of the scattered electron energy ϵ_2,

$$-k^2 \equiv Q^2 = -(k_1 - k_2)^2 = 2m(\epsilon_2 - m), \tag{2}$$

where m is the electron mass. The conservation of the four-momentum in the reaction (1) leads to the following relation between the energy ϵ_2 and the scattering angle θ_e of the final electron:

$$\cos\theta_e = \frac{(E+m)(\epsilon_2 - m)}{p\sqrt{\epsilon_2^2 - m^2}}, \tag{3}$$

where E is the deuteron beam energy, $p = \sqrt{E^2 - M^2}$ is the modulus of the three-momentum (M is the deuteron mass). From Eq. (3) one can see that $\cos\theta_e \geqslant 0$ (because $\epsilon_2 > m$) and the electron can never scatter backward. The following relations hold:

$$\epsilon_2 = m\frac{(E+m)^2 + (E^2 - M^2)\cos^2\theta_e}{(E+m)^2 - (E^2 - M^2)\cos^2\theta_e},$$
$$Q^2 = \frac{4m^2p^2\cos^2\theta_e}{(E+m)^2 - p^2\cos^2\theta_e}, \tag{4}$$

which show that final electron has maximal energy when it is emitted forward ($\cos\theta_e = 1$),

$$\epsilon_{2\max} = m\frac{2E(E+m) + m^2 - M^2}{M^2 + 2mE + m^2}, \tag{5}$$

giving the maximum value of the momentum transfer squared $Q^2_{\max}$,

$$Q^2_{\max} = \frac{4m^2\,p^2}{M^2 + 2mE + m^2}. \tag{6}$$

From Eqs. (5), (6) it appears that in the inverse kinematics the available kinematical regions are reduced to small values of ϵ_2 and Q^2 (compared with E and E^2) which are proportional to m and m^2, respectively. For example, at $E = 10$ GeV one has $\epsilon_2 \leqslant 25$ MeV and $Q^2 \leqslant 25$ MeV^2. The upper limits of these quantities increase approximately as E^2.

As in the proton case to one deuteron angle may correspond two values of the deuteron energy with two corresponding values for the recoil-electron energy and angle and for the transferred momentum squared. This is a typical situation when the velocity of the center of mass, where all angles are allowed for the recoil electron, is larger than the velocity of the projectile. The momentum conservation gives the following relation between the energy and the angle of the scattered deuteron E_2 and θ_d:

$$E_2^{\pm} = \frac{(E+m)(M^2 + mE) \pm p^2\cos\theta_d\sqrt{m^2 - M^2\sin^2\theta_d}}{(E+m)^2 - p^2\cos^2\theta_d}. \tag{7}$$

The two solutions coincide when the angle between the initial and final hadron takes its maximum value, which is determined by the ratio of the electron and scattered hadron masses, $\sin\theta_{d,max} = m/M$. Nevertheless, at fixed values of ϵ_2 or Q^2 the energy of the scattered deuteron is unambiguous

$$E_2 = E + m - \epsilon_2 = E - \frac{Q^2}{2m}. \quad (8)$$

Summarizing, hadrons are scattered from atomic electrons at very small angles: the larger is the hadron mass, the smaller is the available angular range for the scattered hadron.

B. Unpolarized cross section

In the one-photon-exchange approximation the matrix element $\mathcal{M}$ of the reaction (1) can be written as

$$\mathcal{M} = -\frac{e^2}{k^2} j^\mu J_\mu, \quad (9)$$

where $j_\mu(J_\mu)$ is the lepton (hadron) electromagnetic current and e is the electron charge. The sign "minus" is not relevant for the present derivation in frame of one photon exchange, but it becomes important for radiative correction calculations, as for example, for the two photon exchange term. The lepton current is

$$j_\mu = \bar{u}(k_2)\gamma_\mu u(k_1), \quad (10)$$

where $u(k_1)$ $(u(k_2))$ is the bispinor of the incoming (outgoing) electron. Following the requirements of Lorentz invariance, current conservation, parity, and time-reversal invariance of the hadron electromagnetic interaction, the general form of the electromagnetic current for the deuteron (being a spin-one particle) is fully described by three form factors. The hadron electromagnetic current can be written as [12]

$$J_\mu = (p_1 + p_2)_\mu \left[-G_1(Q^2) U_1 \cdot U_2^* + \frac{1}{M^2} G_3(Q^2) \left(U_1 \cdot k U_2^* \cdot k + \frac{Q^2}{2} U_1 \cdot U_2^* \right) \right] + G_2(Q^2)(U_{1\mu} U_2^* \cdot k - U_{2\mu}^* U_1 \cdot k), \quad (11)$$

where $U_{1\mu}$ and $U_{2\mu}$ are the polarization four vectors for the initial and final deuteron states. The functions $G_i(k^2)$, $i = 1, 2, 3$, are the deuteron electromagnetic form factors, depending only on the virtual photon four-momentum squared. Due to the hermiticity of the current these form factors are real functions in the region of space-like momentum transfer.

These form factors are related to the standard deuteron form factors G_C (charge monopole), G_M (magnetic dipole), and G_Q (charge quadrupole) by the following relations:

$$G_M(Q^2) = -G_2(Q^2), \quad G_Q(Q^2) = G_1(Q^2) + G_2(Q^2) + 2G_3(Q^2),$$
$$G_C(Q^2) = \frac{2}{3}\tau[G_2(Q^2) - G_3(Q^2)] + \left(1 + \frac{2}{3}\tau\right) G_1(Q^2), \quad \tau = \frac{Q^2}{4M^2} \quad (12)$$

with the normalizations

$$G_C(0) = 1, \quad G_M(0) = \frac{M}{m_N}\mu_d, \quad G_Q(0) = M^2 \mathcal{Q}_d, \quad (13)$$

where m_N is the nucleon mass, $\mu_d = 0.857$, [13] ($\mathcal{Q}_d = 0.2857$ fm^2 [14]) is the deuteron magnetic (quadrupole) moment. Note also that

$$G_1(Q^2) = \frac{1}{1+\tau}\left[\tau G_M(Q^2) + G_C(Q^2) + \frac{\tau}{3} G_Q(Q^2) \right],$$
$$G_3(Q^2) = \frac{1}{2(1+\tau)}\left[G_M(Q^2) - G_C(Q^2) + \left(1 + \frac{2\tau}{3}\right) G_Q(Q^2) \right]. \quad (14)$$

The matrix element squared is

$$|\mathcal{M}|^2 = 16\pi^2 \frac{\alpha^2}{Q^4} L^{\mu\nu} H_{\mu\nu}, \quad (15)$$

where $\alpha = e^2/4\pi = 1/137$ is the electromagnetic fine structure constant. The lepton $L_{\mu\nu}$ and hadron $H_{\mu\nu}$ tensors are defined as

$$L^{\mu\nu} = j^\mu j^{\nu *}, \quad H_{\mu\nu} = J_\mu J_\nu^*. \quad (16)$$

The lepton tensor, $L_{\mu\nu}^{(0)}$, for unpolarized initial and final electrons (averaging over the initial electron spin) has the form

$$L_{\mu\nu}^{(0)} = -Q^2 g_{\mu\nu} + 2(k_{1\mu}k_{2\nu} + k_{1\nu}k_{2\mu}). \quad (17)$$

The contribution to the lepton tensor from a polarized electron target is

$$L_{\mu\nu}^{(p)}(s_1) = 2im\epsilon_{\mu\nu\rho\sigma} k^\rho s_1^\sigma, \quad \epsilon_{0123} = -1, \quad (18)$$

where $s_{1\sigma}$ is the initial electron polarization four-vector satisfying the conditions $k_1 \cdot s_1 = 0$, $s_1^2 = -1$.

The hadron tensor $H_{\mu\nu}$ is calculated in terms of the deuteron electromagnetic form factors using the explicit form of the electromagnetic current (11). The spin density matrices of the initial and final deuterons have the following expressions:

$$\rho^{(i)}_{\alpha\beta} = -\frac{1}{3}\left(g_{\alpha\beta} - \frac{1}{M^2}p_{1\alpha}p_{1\beta}\right) + \frac{i}{2M}\varepsilon_{\alpha\beta\lambda\rho}\eta_1^\lambda p_1^\rho + \mathcal{Q}^{(i)}_{\alpha\beta},$$
$$\rho^{(f)}_{\alpha\beta} = -\left(g_{\alpha\beta} - \frac{1}{M^2}p_{2\alpha}p_{2\beta}\right) + \frac{i}{2M}\varepsilon_{\alpha\beta\lambda\rho}\eta_2^\lambda p_2^\rho + \mathcal{Q}^{(f)}_{\alpha\beta}. \tag{19}$$

Here, $\eta_{1\alpha}\,(\eta_{2\alpha})$ and $\mathcal{Q}^{(i)}_{\alpha\beta}\,(\mathcal{Q}^{(f)}_{\alpha\beta})$ are the four vectors and tensors describing the vector and tensor (quadrupole) polarization of the initial (final) deuteron, respectively. The four-vector of the vector polarization of the initial (final) deuteron satisfies the following conditions: $\eta_1^2 = -1, \eta_1 \cdot p_1 = 0\,(\eta_2^2 = -1, \eta_2 \cdot p_2 = 0)$. The tensor $\mathcal{Q}^{(i)}_{\alpha\beta}$ satisfies the conditions $\mathcal{Q}^{(i)}_{\alpha\beta}\, g^{\alpha\beta} = 0,\ \mathcal{Q}^{(i)}_{\alpha\beta} = \mathcal{Q}^{(i)}_{\beta\alpha},\ \mathcal{Q}^{(i)}_{\alpha\beta}\, p_1^\alpha = 0$. The tensor $\mathcal{Q}^{(f)}_{\alpha\beta}$ satisfies the same conditions, after substituting: $i \to f$ and $p_1^\alpha \to p_2^\alpha$.

The hadron tensor $H_{\mu\nu}(0)$, for the case of unpolarized initial and final deuterons can be written in the standard form in terms of two spin-independent structure functions:

$$H_{\mu\nu}(0) = H_1(Q^2)\tilde{g}_{\mu\nu} + \frac{1}{M^2}H_2(Q^2)P_\mu P_\nu, \tag{20}$$

where $\tilde{g}_{\mu\nu} = g_{\mu\nu} - (k_\mu k_\nu)/k^2,\ P_\mu = (p_1 + p_2)_\mu$. Averaging over the spin of the initial deuteron, the structure functions $H_i(Q^2), i = 1, 2$, can be expressed in terms of the electromagnetic form factors as

$$H_1(Q^2) = -\tfrac{2}{3}Q^2(1+\tau)G_M^2(Q^2),$$
$$H_2(Q^2) = M^2\left[G_C^2(Q^2) + \tfrac{2}{3}\tau G_M^2(Q^2) + \tfrac{8}{9}\tau^2 G_Q(Q^2)\right]. \tag{21}$$

The differential cross section is related to the matrix element squared (15) by

$$d\sigma = \frac{(2\pi)^4\overline{|\mathcal{M}|^2}}{4\sqrt{(k_1\cdot p_1)^2 - m^2M^2}}\frac{d^3\vec{k}_2}{(2\pi)^3 2\epsilon_2} \times \frac{d^3\vec{p}_2}{(2\pi)^3 2E_2}\delta^4(k_1 + p_1 - k_2 - p_2) \tag{22}$$

with

$$\overline{|\mathcal{M}|^2} = 16\pi^2\frac{\alpha^2}{Q^4}L^{\mu\nu}(0)\,H_{\mu\nu}(0),$$

where $\vec{p}_2\,(E_2)$ is the three-momentum (energy) of the scattered deuteron.

From this point on, the formalism differs from the standard elastic electron-deuteron scattering because we introduce a reference system where the electron is at rest. In this system the differential cross section is written as

$$\frac{d\sigma}{d\epsilon_2} = \frac{1}{32\pi}\frac{\overline{|\mathcal{M}|^2}}{m\,p^2}. \tag{23}$$

The average over the spins of the initial particles has been included in the lepton and hadron tensors. Using Eq. (2) one can write

$$\frac{d\,\sigma}{d\,Q^2} = \frac{1}{64\pi}\frac{\overline{|\mathcal{M}|^2}}{m^2\,p^2}. \tag{24}$$

The differential cross section over the electron solid angle can be written as

$$\frac{d\sigma}{d\Omega_e} = \frac{1}{32\pi^2}\,\frac{1}{m\,p}\,\frac{|\vec{k}_2|^3}{Q^2}\,\frac{\overline{|\mathcal{M}|^2}}{E+m}, \tag{25}$$

where $d\,\Omega_e = 2\pi\, d\cos\theta$ (due to the azimuthal symmetry) and we used the relation

$$d\epsilon_2 = \frac{p}{E+m}\frac{|\vec{k}_2|^3}{m(\epsilon_2 - m)}\frac{d\Omega_e}{2\pi}. \tag{26}$$

The differential cross section over Q^2 for unpolarized deuteron-electron scattering (24) (in the coordinate system where the electron is at rest) can be written as

$$\frac{d\sigma}{dQ^2} = \frac{\pi\,\alpha^2}{2\,m^2\,p^2}\frac{\mathcal{D}}{Q^4}, \quad \mathcal{D} = \frac{1}{2}L^{\mu\nu}(0)\,H_{\mu\nu}(0) \tag{27}$$

with

$$\mathcal{D} = (-Q^2 + 2m^2)H_1(Q^2) + 2[-Q^2M^2 + 2mE(2mE - Q^2)]\frac{H_2(Q^2)}{M^2}. \tag{28}$$

The term $\mathcal{D}$ has the following form in terms of the deuteron form factors:

$$\mathcal{D} = \tfrac{4}{3}\tau[4m^2(E^2 - M^2) - Q^2(m^2 - M^2 + 2mE - 2M^2\tau] \times G_M^2(Q^2) + 2[-M^2\,Q^2 + 2mE(2mE - Q^2)] \times \left[G_C^2(Q^2) + \tfrac{8}{9}\tau^2 G_Q^2(Q^2)\right]. \tag{29}$$

To perform the numerical estimations one needs to know the behavior all three form factors ($G_M,\ G_C,\ G_Q$) in the region of small momentum transfer squared. We choose the parametrization from Ref. [15], that is reported in the Appendix. At $Q^2 = 0$ form factors are normalized to the static values of charge, magnetic and quadrupole moments but they are in principle functions of Q^2 and their derivative becomes important for some applications, typically for extracting the hadron radius. We restrict ourselves to the maximum deuteron beam energy $E = 200$ GeV, i.e., $Q^2_{\rm max}$ does not exceed 0.012 GeV^2.

III. POLARIZATION OBSERVABLES

Several polarization observables can be measured and calculated for elastic deuteron-electron scattering. Besides the electron polarization, the initial and final deuterons may have vector and tensor polarizations. Let us focus here on single and double polarization observables. Among single-spin observables, we consider effects which arise within the one-photon exchange approximation when the amplitude of the process (1) is real. In such approximation, single-spin effects arise due to the tensor polarization of the initial or final deuteron only. In this respect we note that in presence of the two photon exchange contribution the scattering amplitude contains an imaginary part: additional single-spin effects arise due to the target electron polarization or due to vector polarization of the deuteron beam which lead to an azimuthal asymmetry of the cross section similar to the one in elastic electron-proton scattering [16,17].

We calculate the following single- and double-spin effects due to the tensor polarization of the initial or final deuteron:

(1) The analyzing powers (asymmetries) due to the tensor polarization of the deuteron beam, $\vec{d}^{\,T} + e \to d + e$.
(2) The tensor polarization of the scattered deuteron when the other particles are unpolarized, $d + e \to \vec{d}^{\,T} + e$.
(3) The polarization transfer coefficients which describe the polarization transfer from the polarized electron target to the recoil electron in the $d + \vec{e} \to d + \vec{e}$ reaction.
(4) The spin correlation coefficients when the deuteron beam is vectorially polarized and the initial electron has arbitrary polarization, $\vec{d}^{\,V} + \vec{e} \to d + e$.
(5) The polarization transfer coefficients which describe the vector polarization transfer from a polarized electron target to the scattered deuteron, $d + \vec{e} \to \vec{d}^{\,V} + e$.
(6) The spin correlation coefficients when the scattered deuteron has vector polarization and the final electron has arbitrary polarization, $d + e \to \vec{d}^{\,V} + \vec{e}$.
(7) The polarization transfer coefficients which describe the polarization transfer from the vector-polarized deuteron beam to the recoil electron in the $\vec{d}^{\,V} + e \to d + \vec{e}$ reaction.
(8) The depolarization coefficients which define the dependence of the scattered deuteron vector polarization on the vector polarization of the deuteron beam, $\vec{d}^{\,V} + e \to \vec{d}^{\,V} + e$.

The following orthogonal system is chosen: the z axis is directed along the direction of the deuteron beam momentum $\vec{p}$, the momentum of the recoil electron $\vec{k}_2$ lies in the xz plane (θ_e is the angle between the deuteron beam and the recoil electron momenta), and the y axis is directed along the vector $\vec{p} \times \vec{k}_2$ (see Fig. 1). So, the components of the deuteron beam and recoil electron momenta are

$$p_x = p_y = 0, \quad p_z = p, \quad k_{2x} = k_2 \sin\theta_e,$$
$$k_{2y} = 0, \quad k_{2z} = k_2 \cos\theta_e,$$

where $p\,(k_2)$ is the magnitude of the deuteron beam (recoil electron) momentum.

To calculate the polarization observables, the polarization three-vectors of all particles as well the components of both deuteron tensor polarizations have to be defined in their rest frames. All observables are calculated in the Lab coordinate system shown in Fig. 1.

The corresponding polarization observables are analytically calculated as functions of Q^2 at fixed deuteron beam energy and their dependence on the kinematical variables is plotted similarly to the unpolarized cross section discussed in the Appendix.

A. Analyzing powers or asymmetries, A_{ij}, unpolarized electrons, tensor polarized deuteron beam, $\vec{d}^{\,T} + e \to d + e$

We consider here the scattering of a tensor polarized deuteron beam on an unpolarized electron target. The hadron tensor can be written in the following general form:

$$H_{\mu\nu}(\mathcal{Q}^{(i)}) = H_3(Q^2)\bar{Q}^{(i)}\tilde{g}_{\mu\nu} + H_4(Q^2)\frac{\bar{Q}^{(i)}}{4M^2}P_\mu P_\nu + H_5(Q^2)(P_\mu\tilde{Q}^{(i)}_\nu + P_\nu\tilde{Q}^{(i)}_\mu) + H_6(Q^2)\tilde{\mathcal{Q}}^{(i)}_{\mu\nu}, \tag{30}$$

where

$$\tilde{Q}^{(i)}_\mu = \mathcal{Q}^{(i)}_{\mu\nu}k^\nu + \frac{k_\mu}{Q^2}\bar{Q}^{(i)}, \quad \tilde{Q}^{(i)}_\mu k^\mu = 0,$$
$$\tilde{\mathcal{Q}}^{(i)}_{\mu\nu} = \mathcal{Q}^{(i)}_{\mu\nu} + \frac{k_\mu k_\nu}{Q^4}\bar{Q}^{(i)} + \frac{k_\nu k^\alpha}{Q^2}\mathcal{Q}^{(i)}_{\mu\alpha} + \frac{k_\mu k^\alpha}{Q^2}\mathcal{Q}^{(i)}_{\nu\alpha}, \tag{31}$$
$$\tilde{\mathcal{Q}}^{(i)}_{\mu\nu}k^\nu = 0, \quad \bar{Q}^{(i)} = \mathcal{Q}^{(i)}_{\alpha\beta}k^\alpha k^\beta.$$

The structure functions $H_i(k^2)$ are related to the deuteron electromagnetic form factors by

$$H_3(Q^2) = -G_M^2,$$
$$H_4(Q^2) = G_M^2 + \frac{4}{1+\tau}G\,G_Q,$$
$$H_5(Q^2) = -\tau(G_M + 2G_Q)G_M, \tag{32}$$
$$H_6(Q^2) = Q^2(1+\tau)G_M^2,$$
$$G = \tau\,G_M + G_C + \frac{\tau}{3}G_Q.$$

In an arbitrary reference frame the contraction of the spin independent lepton $L^{\mu\nu}(0)$ and spin dependent hadron tensors $H_{\mu\nu}(\mathcal{Q}^{(i)})$ gives

$$C(\mathcal{Q}^{(i)}) = L^{\mu\nu}(0)\,H_{\mu\nu}(\mathcal{Q}^{(i)}) = a\,k_1^\mu\,k_1^\nu\,\mathcal{Q}^{(i)}_{\mu\nu} + b\,k_1^\mu\,k^\nu\,\mathcal{Q}^{(i)}_{\mu\nu} + c\,k^\mu\,k^\nu\,\mathcal{Q}^{(i)}_{\mu\nu}, \tag{33}$$

where the functions a, b, and c are expressed in terms of the deuteron electromagnetic form factors (in the rest frame of the electron target) as

$$a = 4(1+\tau)Q^2\,G_M^2,$$
$$b = -16\tau G_M[(M^2 + m\,E)G_M + 2(m\,E - \tau M^2)\,G_Q],$$
$$c = \left[Q^2 - 4m^2 + 4\frac{m^2}{M^2}E^2\right]G_M^2 + 16\tau m\,E\,G_M\,G_Q + \frac{4}{M^2}\frac{G_Q\,G}{1+\tau}[-Q^2(M^2 + 2m\,E) + 4m^2\,E^2]. \tag{34}$$

From the condition $p_1^\mu\,\mathcal{Q}^{(i)}_{\mu\nu} = 0$ one can write the time components of the quadrupole polarization tensor in terms of the space components of this tensor. These relations are

$$\mathcal{Q}^{(i)}_{00} = \frac{p^2}{E^2}\mathcal{Q}^{(i)}_{zz}, \quad \mathcal{Q}^{(i)}_{0x} = \frac{p}{E}\mathcal{Q}^{(i)}_{xz}, \quad \mathcal{Q}^{(i)}_{0y} = \frac{p}{E}\mathcal{Q}^{(i)}_{yz}, \quad \mathcal{Q}^{(i)}_{0z} = \frac{p}{E}\mathcal{Q}^{(i)}_{zz}. \tag{35}$$

The components of the quadrupole polarization tensor $\mathcal{Q}^{(i)}_{ij}$ defined in the Lab system can be related to the ones defined in the rest system of the deuteron beam (denoted as R_{ij}) by the following relations:

$$\mathcal{Q}^{(i)}_{xx} = R_{xx}, \quad \mathcal{Q}^{(i)}_{yy} = R_{yy}, \quad \mathcal{Q}^{(i)}_{xz} = \frac{E}{M}R_{xz}, \quad \mathcal{Q}^{(i)}_{zz} = \frac{E^2}{M^2}R_{zz}.$$

The Q^2 dependence of the differential cross section of the reaction (1) on the polarization characteristics of the (tensor polarized) deuteron beam is

$$\frac{d\sigma}{dQ^2}(\mathcal{Q}^{(i)}) = \left(\frac{d\sigma}{dQ^2}\right)_{un}[1 + A_{xx}(\mathcal{Q}_{xx} - \mathcal{Q}_{yy}) + A_{xz}\mathcal{Q}_{xz} + A_{zz}\mathcal{Q}_{zz}], \tag{36}$$

where $A_{ij},\ i, j = x, y, z$ are the analyzing powers (asymmetries) which characterize the $\vec{d}^{\,T} - e$ scattering when the deuteron beam is tensor polarized.

The expressions of these analyzing powers in terms of the deuteron electromagnetic form factors are

$$\begin{aligned}
\mathcal{D}A_{xx} &= \frac{x^2}{M^2}\left[(m^2p^2 + \tau M^4)G_M^2 + mE\,Q^2\,G_M\,G_Q + \frac{(4m^2E^2 - M^2Q^2 - 2mE\,Q^2)}{1+\tau}G_Q\,G\right],\\
\mathcal{D}A_{xz} &= 2\frac{x\tau(M^2 + mE)}{mpE}\left\{M^2Q^2\,G_M^2 - 2\left[\frac{m^2p^2(4mE - Q^2)}{M^2 + mE} - 2mE\,Q^2\right]G_M\,G_Q + \frac{4(4m^2E^2 - M^2Q^2 - 2mE\,Q^2)}{1+\tau}G_Q\,G\right\},\\
\mathcal{D}A_{zz} &= \frac{Q^2}{E^2}\left\{\left[m^2p^2\left(1 + \frac{Q^2}{Q^2_{\max}}\right) + \frac{Q^4}{8} - \frac{3}{4}M^2x^2\right]G_M^2 + \left[\frac{Q^4}{2} + mE[4\tau(Q^2 - 2mE) - 3x^2]\right]G_M\,G_Q\right.\\
&\quad\left. + \frac{Q^2(M^2 + 2mE) - 4m^2E^2}{1+\tau}\left(1 - 2\tau - 3\frac{Q^2}{Q^2_{\max}}\right)G_Q\,G,\right\},\\
x &= k_{2x} = -p_{2x} = \left[Q^2\left(1 - \frac{Q^2}{Q^2_{\max}}\right)\right]^{1/2}.
\end{aligned} \tag{37}$$

The asymmetries due the tensor polarization of the deuteron beam are plotted in Fig. 2 for the chosen form factors.

B. Tensor polarization coefficients, P_{ij}, unpolarized electrons, tensor polarized scattered deuteron, $d + e \to \vec{d}^{\,T} + e$

We consider here the scattering of an unpolarized deuteron beam on an unpolarized electron target, when the polarization of the recoil electron is not measured and the scattered deuterons becomes tensor polarized. The hadron tensor is written in the following general form:

$$H_{\mu\nu}(\mathcal{Q}^{(f)}) = \bar{H}_3\bar{Q}^{(f)}\tilde{g}_{\mu\nu} + \bar{H}_4\frac{\bar{Q}^{(f)}}{4M^2}P_\mu P_\nu - \bar{H}_5(P_\mu\tilde{Q}^{(f)}_\nu + P_\nu\tilde{Q}^{(f)}_\mu) + \bar{H}_6\tilde{Q}^{(f)}_{\mu\nu}, \tag{38}$$

where the structure functions $\bar{H}_i$, averaged over the spin of the initial deuteron, are written in terms of the deuteron electromagnetic form factors: $\bar{H}_i = H_i/3,\ i = 3, 4, 5, 6$. Note that the tensor structures in this case can be obtained from Eq. (30) by the substitution $(p_1 \leftrightarrows -p_2)$, wherein the structure accompanying $\bar{H}_5$ changes sign.

The contraction of the spin independent lepton $L^{(0)}_{\mu\nu}$ and spin dependent hadron tensors $H_{\mu\nu}(Q^{(f)})$ (due to the tensor polarization of the scattered deuteron) in an arbitrary reference frame, gives

$$C(\mathcal{Q}^{(f)}) = L^{\mu\nu}(0)H_{\mu\nu}(\mathcal{Q}^{(f)}) = \bar{a}\,k_1^\mu k_1^\nu\,\mathcal{Q}^{(f)}_{\mu\nu} + \bar{b}\,k_1^\mu p_1^\nu\,\mathcal{Q}^{(f)}_{\mu\nu} + \bar{c}\,k^\mu k^\nu\,\mathcal{Q}^{(f)}_{\mu\nu}, \tag{39}$$

where the coefficients $\bar{a}$, $\bar{b}$, and $\bar{c}$ are written in terms of the deuteron electromagnetic form factors as

$$\begin{aligned}
\bar{a} &= \frac{2}{3}(1+\tau)Q^2G_M^2,\\
\bar{b} &= \frac{8}{3}\tau G_M[(M^2 + 2\tau M^2 - k_1\cdot p_1)G_M + 2(\tau M^2 - k_1\cdot p_1)G_Q],\\
\bar{c} &= \frac{2}{3M^2}\{[(k_1\cdot p_1)^2 - Q^2k_1\cdot p_1 - m^2M^2 + \tau(1+4\tau)M^4]G_M^2 - k^2(2\tau M^2 - k_1\cdot p_1)G_MG_Q\\
&\quad + (1+\tau)^{-1}[4(k_1\cdot p_1)^2 - Q^2(M^2 + 2k_1\cdot p_1)]G_Q\,G\}.
\end{aligned} \tag{40}$$

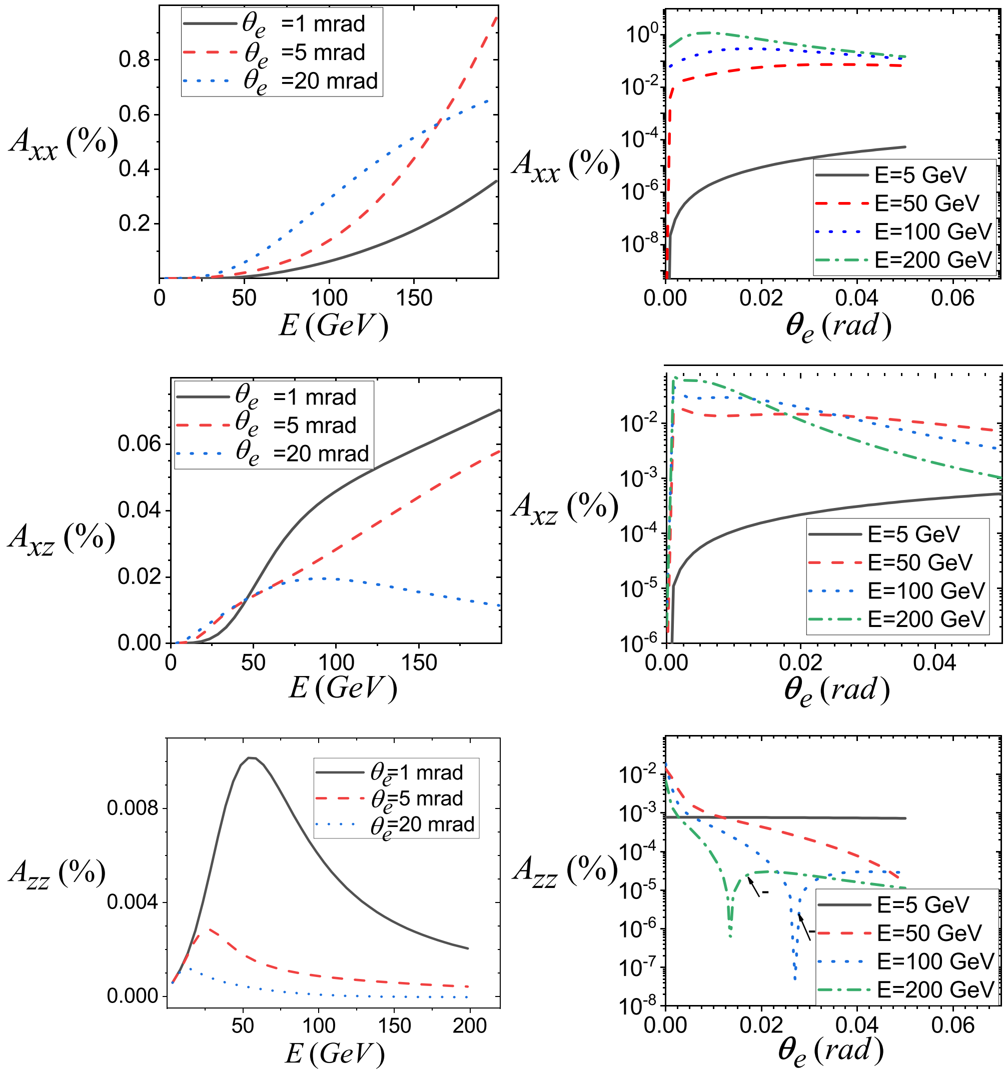

FIG. 2. Tensor asymmetries with unpolarized electrons, tensor polarized deuteron beam, Eqs. (36), (37), as functions of the deuteron beam energy E for different θ_e values (left column): $\theta_e = 1$ mrad (solid black line), $\theta_e = 5$ mrad (red dashed line), $\theta_e = 20$ mrad (dotted blue line) and as function of the electron scattering angle θ_e for different values of the energy (right column): $E = 5$ GeV (solid black line), $E = 50$ GeV (dashed red line), $E = 100$ GeV (dotted blue line), and $E = 200$ GeV (dash-dotted green line). The arrow with "minus" sign indicates that the corresponding asymmetry may become negative and the absolute value is plotted.

From the condition $p_2^\mu\, Q^{(f)}_{\mu\nu} = 0$ one finds the time components of the scattered deuteron quadrupole polarization tensor in terms of the space components of this tensor:

$$Q^{(f)}_{00} = \frac{x^2 Q^{(f)}_{xx} - 2xz Q^{(f)}_{xz} + z^2 Q^{(f)}_{zz}}{E_d^2}, \quad Q^{(f)}_{0z} = \frac{z Q^{(f)}_{zz} - x Q^{(f)}_{xz}}{E_d},$$

$$Q^{(f)}_{0x} = \frac{z Q^{(f)}_{xz} - x Q^{(f)}_{xx}}{E_d}, \quad Q^{(f)}_{0y} = \frac{z Q^{(f)}_{yz} - x Q^{(f)}_{xy}}{E_d}, \quad E_d = E - \frac{Q^2}{2m}, \tag{41}$$

where E_d is the energy of the scattered deuteron and

$$z = p_{2z} = p\left[1 - \frac{(E+m)\,Q^2}{2m\,p^2}\right].$$

The components of the quadrupole polarization tensor $Q^{(f)}_{ij}$ in the Lab system are related to the corresponding ones in the rest system of the scattered deuteron, denoted as V_{ij}, by the relations

$$\begin{aligned}
\mathcal{Q}_{xx}^{(f)} &= (1+x^2y)^2V_{xx} - 2xyz(1+x^2y)V_{xz} + (xyz)^2V_{zz}, \quad \mathcal{Q}_{yy}^{(f)} = V_{yy},\\
\mathcal{Q}_{zz}^{(f)} &= (xyz)^2V_{xx} - 2xyz(1+z^2y)V_{xz} + (1+yz^2)^2V_{zz},\\
\mathcal{Q}_{xz}^{(f)} &= -xyz(1+z^2y)V_{xx} + [1+y(x^2+z^2)+2(xyz)^2]V_{xz} - xyz(1+yz^2)V_{zz},
\end{aligned} \tag{42}$$

where $y=[M(E_d+M)]^{-1}$.

In the Lab system, the Q^2 dependence of the differential cross section on the polarization characteristics of the scattered deuteron with unpolarized initial particles is

$$\frac{d\sigma}{dQ^2}(\mathcal{Q}^{(f)}) = \left(\frac{d\sigma}{dQ^2}\right)_{un}\big[1 + P_{xx}(\mathcal{Q}_{xx}^{(f)} - \mathcal{Q}_{yy}^{(f)}) + P_{xz}\mathcal{Q}_{xz}^{(f)} + P_{zz}\mathcal{Q}_{zz}^{(f)}\big], \tag{43}$$

where P_{ij}, $i, j = x, y, z$ are the components of the tensor polarization of the scattered deuteron and are written in terms of the deuteron electromagnetic form factors as

$$\begin{aligned}
\mathcal{D}P_{zz} &= \frac{2}{3M^2d\,E_d}\big[a_1\,G_M^2 + a_2\,G_M\,G_Q + a_3\,G_Q\,G\big], \quad d = 2E_d^2 - x^2,\\
a_1 &= (2z^2-x^2)\,E_d\{E^2(m^2p^2+\tau M^4) - Q^2(E+m)[mp^2 - \tau M^2(E+m)]\}\\
&\quad + p\,d[-mzQ^2(M^2+2\tau M^2 - mE) + (pE_d - 2Ez)(m^2p^2 - mEQ^2 + \tau M^4 + \tau M^2Q^2)],\\
a_2 &= Q^2\big\{-p^2d\,m\,E_d^2 - 2p\,d\,z[\tau M^2(2E+m) - mE(E+m)] + (2z^2-x^2)E\,E_d[2\tau M^2(E+m) - mE(E+2m)]\big\},\\
a_3 &= \frac{4m^2E^2 - Q^2(M^2+2mE)}{1+\tau}[(2z^2-x^2)E^2\,E_d + p\,d(p\,E_d - 2E\,z)],\\
\mathcal{D}P_{xx} &= \frac{2\,x^2}{3d\,M^2}[b_1\,G_M^2 + b_2\,G_M\,G_Q + b_3\,G_Q\,G],\\
b_1 &= mp^2[mE^2 - Q^2(E+m)] + \tau M^2[E^2M^2 + Q^2(E+m)^2],\\
b_2 &= -EQ^2[2(mE-\tau M^2)(E+m) - mE^2],\\
b_3 &= \frac{E^2}{1+\tau}[4m^2E^2 - Q^2(M^2+2mE)],\\
\mathcal{D}P_{xz} &= \frac{2x}{3dM^2E_d}\big[c_1G_M^2 + c_2G_MG_Q + c_3G_Q\,G\big],\\
c_1 &= 4pQ^2(E+m)E_d[2mp^2 - Q^2(E+m)] - 4zE_d[Q^2\tau M^2(E+m)^2 + E^2(m^2p^2+\tau M^4)\\
&\quad + p^2Q^2m(E+m)] + pd[2E(m^2p^2+\tau M^4+\tau Q^2M^2) - mQ^2(mE+E^2+p^2-2\tau M^2)],\\
c_2 &= -4zEQ^2E_d[mE^2 + 2(E+m)(\tau M^2 - mE)] - pdQ^2[2mE(E+m) - 2\tau M^2(2E+m)],\\
c_3 &= \frac{2E}{1+\tau}[(4m^2E^2 - Q^2(M^2+2mE)](pd - 2zEE_d).
\end{aligned} \tag{44}$$

The results for the tensor polarizations of the scattered deuteron are shown in Fig. 3.

C. Polarization transfer coefficients, t_{ij}, polarized target, polarized recoil electron, $d+\vec{e}\to d+\vec{e}$

We consider the scattering of an unpolarized deuteron beam on a polarized electron target when the polarization of the recoil electron is measured and the polarization of the scattered deuteron is not measured.

The contribution to the lepton tensor due to a polarized target and a polarized recoil electron is

$$\begin{aligned}
L_{\mu\nu}(s_1,s_2) &= -\left(k_1\cdot s_2k_2\cdot s_1 - \frac{Q^2}{2}s_1\cdot s_2\right)g_{\mu\nu} - \frac{Q^2}{2}(s_{1\mu}s_{2\nu} + s_{1\nu}s_{2\mu}) - s_1\cdot s_2(k_{1\mu}k_{2\nu} + k_{1\nu}k_{2\mu})\\
&\quad + k_1\cdot s_2(s_{1\mu}k_{2\nu} + s_{1\nu}k_{2\mu}) + k_2\cdot s_1(s_{2\mu}k_{1\nu} + s_{2\nu}k_{1\mu}),
\end{aligned} \tag{45}$$

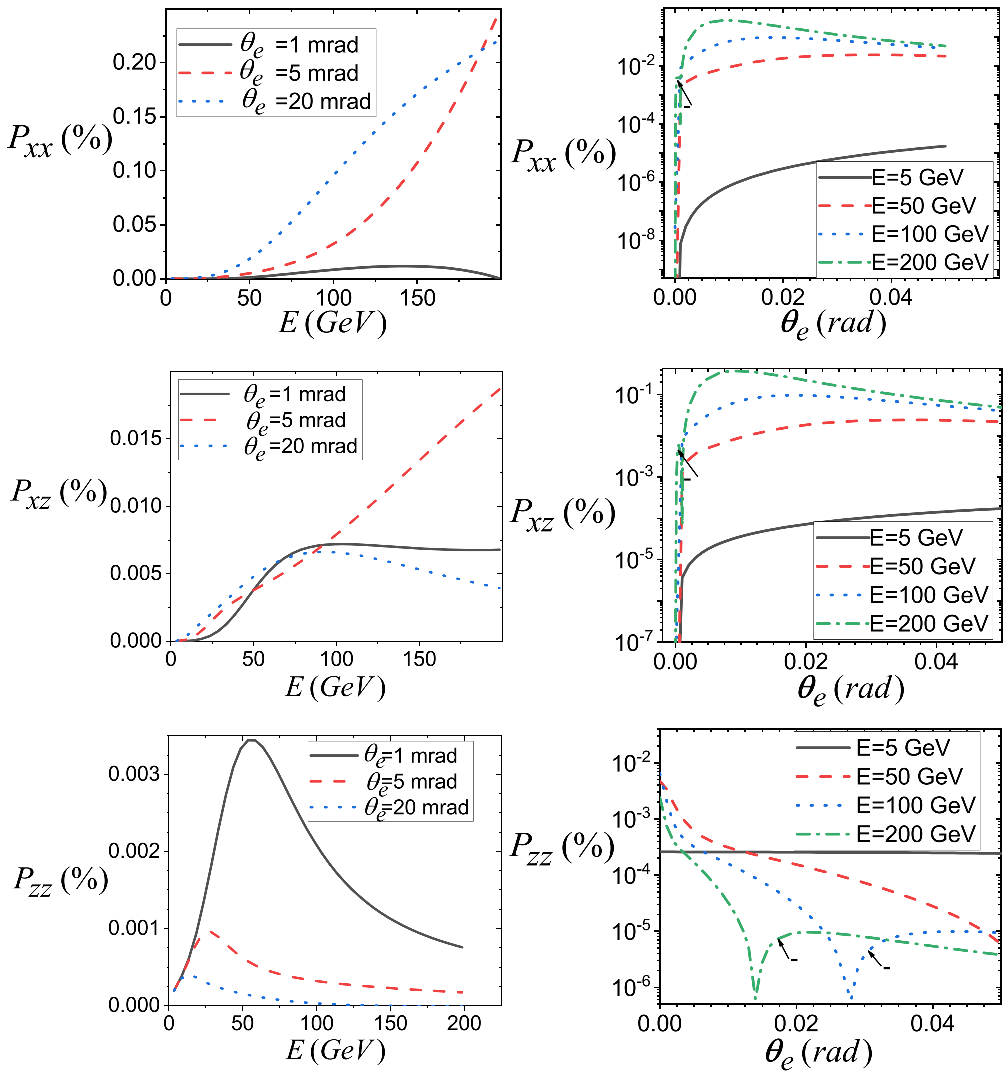

FIG. 3. Same as Fig. 2 but for the tensor polarization coefficients P_{ij}, with unpolarized electrons, tensor polarized scattered deuteron, Eqs. (43), (44).

where $s_{2\mu}$ is the polarization four-vector of the recoil electron, satisfying the following conditions: $k_2 \cdot s_2 = 0$, $s_2^2 = -1$. In the Lab system, where the target electron is at rest, the polarization four-vector of the recoil electron is

$$s_2 = \left(\frac{\vec{k}_2 \cdot \vec{\xi}_2}{m}, \vec{\xi}_2 + \frac{\vec{k}_2(\vec{k}_2 \cdot \vec{\xi}_2)}{m(m+\epsilon_2)} \right), \tag{46}$$

where $\vec{\xi}_2$ is the unit vector describing the polarization of the recoil electron in its rest system and ϵ_2 is the recoil electron energy.

The contraction of the spin dependent lepton tensor $L_{\mu\nu}(s_1, s_2)$ and the spin independent hadron tensor $H_{\mu\nu}(0)$, is written in an arbitrary reference frame as

$$\begin{aligned} C(s_1, s_2) &= L^{\mu\nu}(s_1, s_2) H_{\mu\nu}(0) \\ &= -2m^2 s_1 \cdot s_2 H_1(Q^2) + \frac{H_2(Q^2)}{M^2} \Bigg\{ -2 s_1 \cdot s_2 \left[(k_1 \cdot P)^2 - \frac{P^2 Q^2}{4} \right] \\ &\quad - 4M^2(1+\tau) k_1 \cdot s_2 k_2 \cdot s_1 - Q^2 P \cdot s_1 P \cdot s_2 + 2P \cdot k_1 (k_1 \cdot s_2 P \cdot s_1 + k_2 \cdot s_1 P \cdot s_2) \Bigg\}, \end{aligned} \tag{47}$$

where the structure functions $H_{1,2}(Q^2)$ are given by Eq. (21).

The differential cross section for polarized initial and recoil electrons has the following form:

$$\frac{d\sigma}{dQ^2}(\vec{\xi}_1, \vec{\xi}_2) = \frac{1}{2} \left(\frac{d\sigma}{dQ^2} \right)_{un} [1 + t_{xx}\xi_{1x}\xi_{2x} + t_{yy}\xi_{1y}\xi_{2y} + t_{zz}\xi_{1z}\xi_{2z} + t_{xz}\xi_{1x}\xi_{2z} + t_{zx}\xi_{1z}\xi_{2x}], \tag{48}$$

where t_{ij}, $i, j = x, y, z$ are the coefficients of the polarization transfer from the target to the recoil electron. In the Lab system, the polarization transfer coefficients are written in terms of the deuteron form factors as

$$
\begin{aligned}
Dt_{xx} &= (2m^2 + Q^2\sin^2\theta_e)\left\{H_1(Q^2) + 4\frac{H_2(Q^2)}{M^2}[E^2 - \lambda(M^2 + 2mE)]\right\}\\
&\quad + \frac{4H_2(Q^2)|\vec{k}_2|\ \sin^2\theta_e}{M^2}[4\lambda mpE\cos\theta_e + |\vec{k}_2|(M^2 - 2E^2)],\\
Dt_{yy} &= 2m^2\left\{H_1(Q^2) + 4\frac{H_2(Q^2)}{M^2}[E^2 - \lambda(M^2 + 2mE)]\right\},\\
Dt_{zz} &= 4|\vec{k}_2|\cos\theta_e\frac{H_2(Q^2)}{M^2}[(M^2 - 2E^2)|\vec{k}_2|\cos\theta_e + 4\lambda mp(m + E + E\cos^2\theta_e)]\\
&\quad + 2m^2(1 + 2\lambda\cos^2\theta_e)\left\{H_1(Q^2) + 4\frac{H_2(Q^2)}{M^2}[E^2 - \lambda(2E^2 - M^2 + 2mE)]\right\},\\
Dt_{xz} &= 4|\vec{k}_2|\sin\theta_e\frac{H_2(Q^2)}{M^2}[(M^2 - 2E^2)|\vec{k}_2|\cos\theta_e + 2mpE]\\
&\quad + 4\lambda\cos\theta_e\sin\theta_e\left\{m^2H_1(Q^2) + \frac{H_2(Q^2)}{M^2}[4mE(mE + p|\vec{k}_2|\cos\theta_e) - Q^2(M^2 + 2mE)]\right\},\\
Dt_{zx} &= 4|\vec{k}_2|\sin\theta_e\frac{H_2(Q^2)}{M^2}[(M^2 - 2E^2)|\vec{k}_2|\cos\theta_e + 2mp(2m\lambda + 2\lambda E - E)]\\
&\quad + 4\lambda\cos\theta_e\sin\theta_e\left\{m^2H_1(Q^2) + \frac{H_2(Q^2)}{M^2}[4mE(mE + p|\vec{k}_2|\cos\theta_e) - Q^2(E^2 + p^2 + 2mE)]\right\},
\end{aligned}
\tag{49}
$$

where $\lambda = Q^2/(4m^2)$. We recall that $\sin\theta_e$ and $\cos\theta_e$ are functions of Q^2 and of the deuteron beam energy E, namely,

$$
|\vec{k}_2|\cos\theta_e = \frac{Q^2}{2m}\sqrt{1 + \frac{4m^2}{Q^2_{\max}}},\quad |\vec{k}_2|\sin\theta_e = \sqrt{Q^2\left(1 - \frac{Q^2}{Q^2_{\max}}\right)},\quad |\vec{k}_2| = \sqrt{Q^2(1 + \lambda)}
$$

with $Q^2_{\max}$ given in Eq. (6).

The electron polarization transfer coefficients t_{ij} are plotted in Figs. 4 and 5.

D. Spin correlation coefficients, C_{ij}, polarized electron target, vector polarized deuteron beam, $\vec{d}^{\,V} + \vec{e} \to d + e$

Let us consider the scattering of a vector polarized deuteron beam where the polarizations of the final particles are not measured. In the one photon exchange approximation, nonzero polarization effects arise only when the electron target is also polarized. The part of the hadron tensor $H_{\mu\nu}(\eta_1)$ related to the vector polarized deuteron beam and unpolarized scattered deuteron can be written as

$$
\begin{aligned}
H_{\mu\nu}(\eta_1) &= 2iM\,G_M\left[(1 + \tau)\widetilde{G}\varepsilon_{\alpha\beta\lambda\rho}\eta_1^\lambda k^\rho + \frac{\eta_1\cdot k}{4M^2}(G_M - 2\widetilde{G})\varepsilon_{\alpha\beta\lambda\rho}p_1^\lambda k^\rho\right],\\
\widetilde{G} &= G_C + \frac{\tau}{3}G_Q.
\end{aligned}
\tag{50}
$$

One can see that all correlation coefficients in $\vec{d}^{\,V} + \vec{e} \to d + e$, and polarization transfer coefficients in the reaction $d + \vec{e} \to \vec{d}^{\,V} + e$ when the deuteron is vector polarized are proportional to the deuteron magnetic form factor. This is also true for the corresponding polarization observables in *ed* elastic scattering.

The contraction of the spin-dependent lepton $L^{(p)}_{\mu\nu}(s_1)$ and hadron $H_{\mu\nu}(\eta_1)$ tensors, in an arbitrary reference frame, gives

$$
\begin{aligned}
C(s_1, \eta_1) &= L^{(p)}_{\mu\nu}(s_1)H^{\mu\nu}(\eta_1)\\
&= 4mMG_M\{\tau k\cdot\eta_1(k\cdot s_1 + 2p_1\cdot s_1)G_M + 2\,\widetilde{G}[Q^2(1 + \tau)s_1\cdot\eta_1 + k\cdot\eta_1(k\cdot s_1 - 2\tau p_1\cdot s_1)]\}.
\end{aligned}
\tag{51}
$$

In the frame where the target electron is at rest, the polarization four-vectors of the electron target, s_1, and of the deuteron beam, η_1, are

$$
s_1 = (0, \vec{\xi}_1),\quad \eta_1 = \left(\frac{\vec{p}\cdot\vec{S}_1}{M}, \vec{S}_1 + \frac{\vec{p}(\vec{p}\cdot\vec{S}_1)}{M(E + M)}\right),
\tag{52}
$$

where $\vec{S}_1$ is the unit vector describing the vector polarization of the deuteron beam in its rest system.

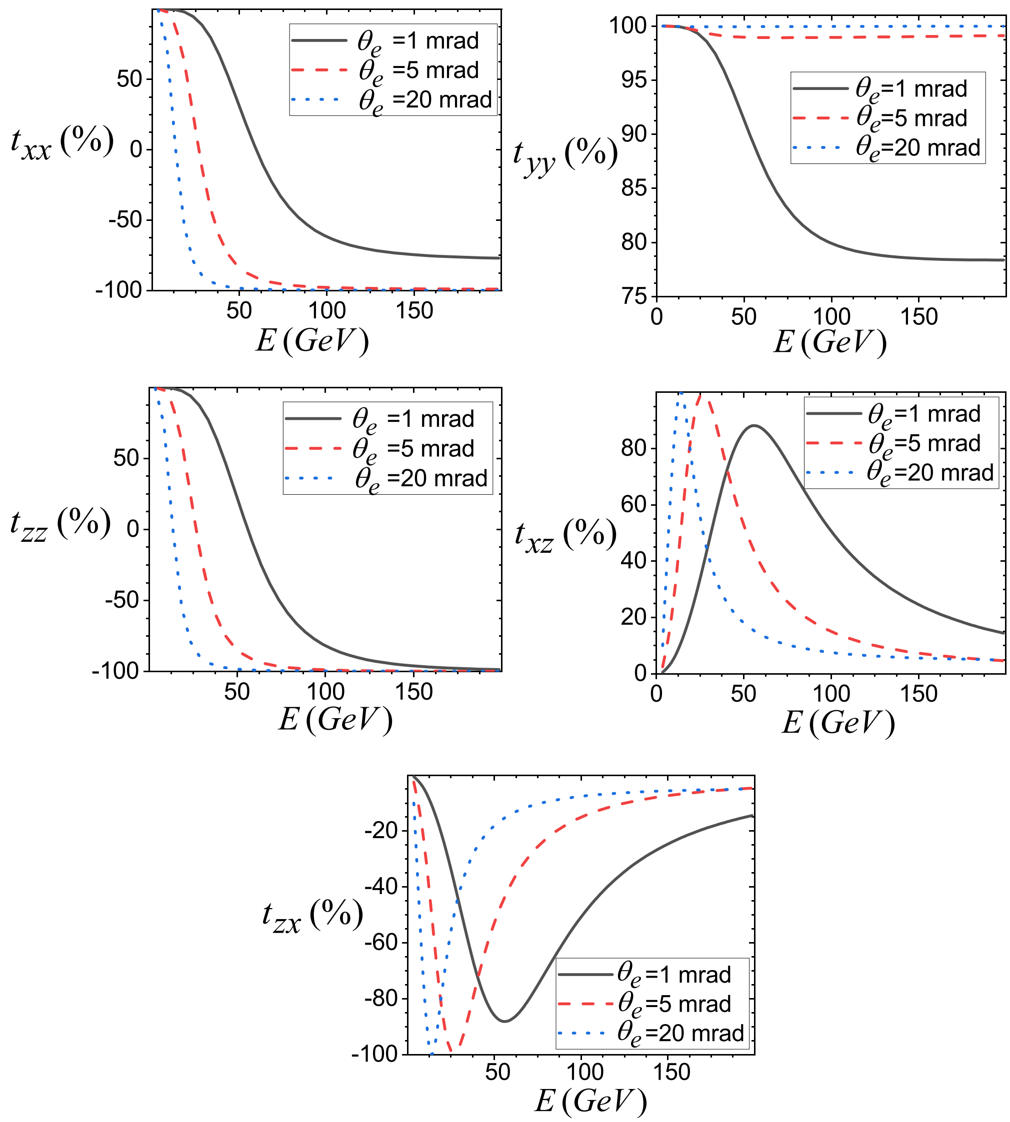

FIG. 4. Coefficients of polarization transfer from polarized target and recoil electrons, Eqs. (48), (49), as functions of energy for different θ_e values: $\theta_e = 1$ mrad (solid black line), $\theta_e = 5$ mrad (red dashed line), $\theta_e = 20$ mrad (dotted blue line).

Applying the P invariance of the hadron electromagnetic interaction, the Q^2 dependence of the differential cross section on the polarization of the initial particles is

$$\frac{d\sigma}{dQ^2}(\vec{\xi}_1, \vec{S}_1) = \left(\frac{d\sigma}{dQ^2}\right)_{un}[1 + C_{xx}\xi_{1x}S_{1x} + C_{yy}\xi_{1y}S_{1y} + C_{zz}\xi_{1z}S_{1z} + C_{xz}\xi_{1x}S_{1z} + C_{zx}\xi_{1z}S_{1x}], \tag{53}$$

where C_{ij}, $i, j = x, y, z$, are the spin correlation coefficients which determine the $\vec{d}^{\,V} - \vec{e}$ scattering, when the deuteron beam is vector polarized and the electron target is arbitrarily polarized.

The explicit expressions of the spin correlation coefficients, as functions of the deuteron form factors is

$$\mathcal{D}C_{yy} = -4mMQ^2(1+\tau)G_M\,\widetilde{G},$$

$$\mathcal{D}C_{xx} = 2\tau mMG_M\left[x^2\,G_M - 2Q^2\left(1 + \frac{4M^2}{Q^2_{\max}}\right)\widetilde{G}\right],$$

$$\mathcal{D}C_{xz} = \frac{Q^2}{p}(mE + M^2)\,x\,G_M\,(\tau G_M + 2\widetilde{G}),$$

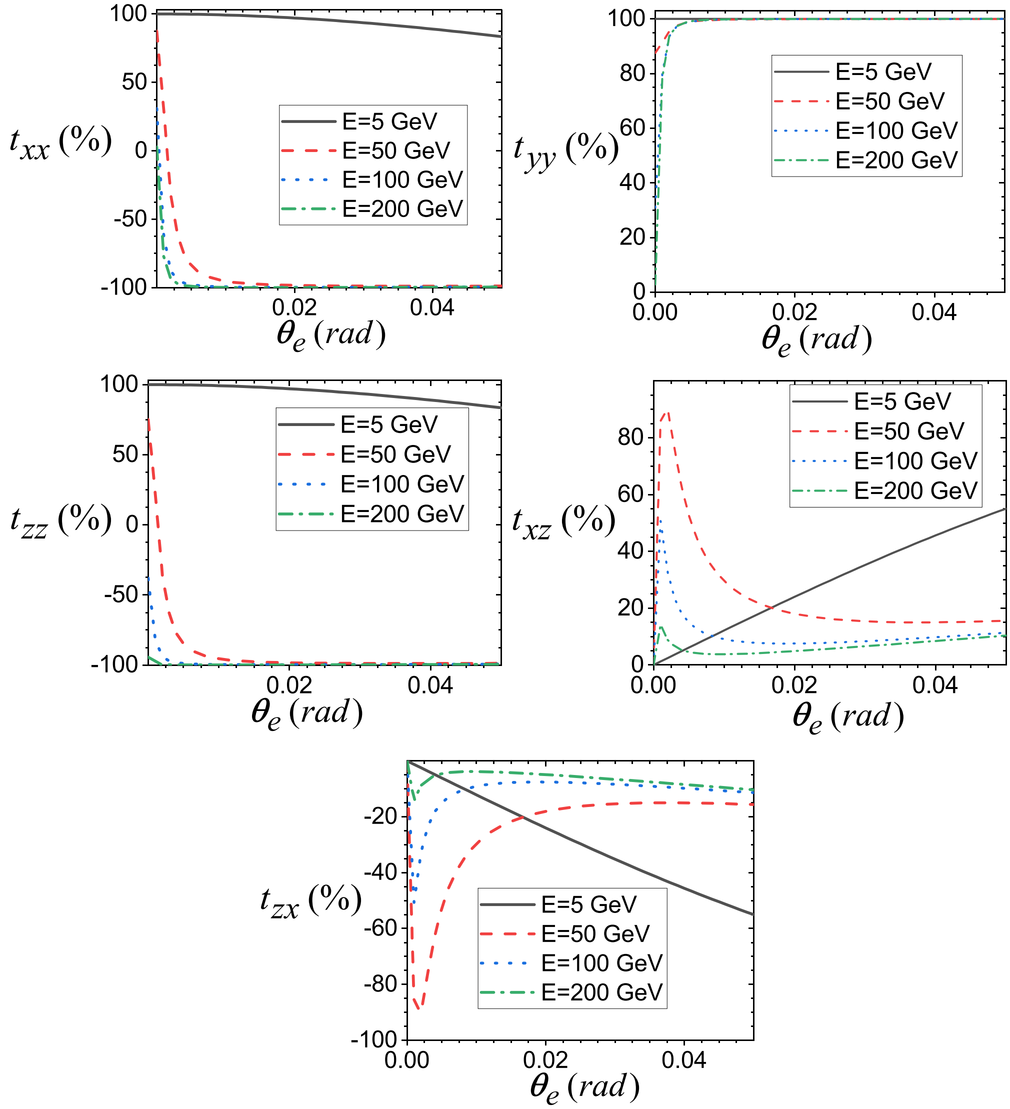

FIG. 5. Same as Fig. 4, but as functions of θ_e, for different values of the energy: $E = 5$ GeV (black solid line), $E = 50$ GeV (red dashed line), $E = 100$ GeV (blue dotted line), and $E = 200$ GeV (green dash-dotted line).

$$\mathcal{D}C_{zx} = -4m\,M\,p\,x\,G_M\left[\tau(G_M - 2\widetilde{G}) - \frac{Q^2(E+m)}{4mp^2}(\tau G_M + 2\widetilde{G})\right],$$

$$\mathcal{D}C_{zz} = -2Q^2 G_M\left[2(mE - \tau M^2)\,\widetilde{G} + \tau(M^2 + mE)G_M - \frac{\tau(E+m)M^2}{mp^2}(M^2 + mE)\,(\tau G_M + 2\,\widetilde{G})\right]. \tag{54}$$

The spin correlation coefficients C_{ij} due to the vector polarizations of the target electron and the deuteron beam are shown in Figs. 6 and 7.

E. Polarization transfer coefficients, T_{ij}, polarized electron target, vector polarized scattered deuteron, $d + \vec{e} \to \vec{d}^{\,V} + e$

The initial electron is arbitrary polarized and the scattered deuteron is vectorially polarized, therefore the contribution to the hadron tensor $H_{\mu\nu}(\eta_2)$ is obtained from Eq. (50) with the substitutions: $\eta_1 \to \eta_2$ and $p_1 \leftrightarrows -p_2$ and the multiplication by a factor $1/3$. The same procedure applies to the calculation of the convolution of the spin dependent parts of lepton and hadron tensors:

$$C(s_1, \eta_2) = L^{(p)}_{\mu\nu}(s_1)H^{\mu\nu}(\eta_2)$$
$$= \tfrac{4}{3}mMG_M\{\tau k\cdot\eta_2(k\cdot s_1 - 2p_2\cdot s_1)G_M + 2\,\widetilde{G}[Q^2(1+\tau)s_1\cdot\eta_2 + k\cdot\eta_2(k\cdot s_1 + 2\tau p_2\cdot s_1)]\}. \tag{55}$$

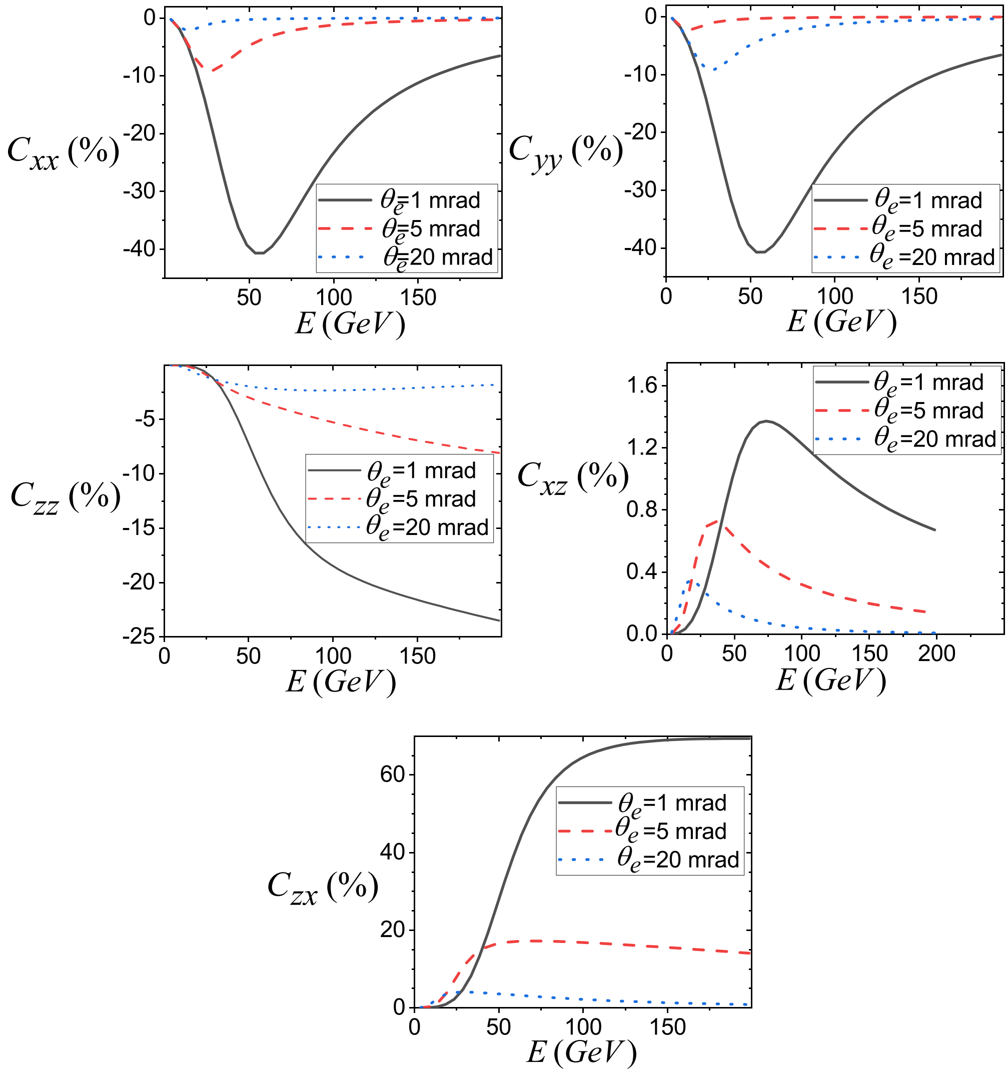

FIG. 6. Spin correlation coefficients, with vector polarized electron target and deuteron beam, Eqs. (53), (54), as functions of energy for different θ_e values. Notations as in Fig. 4.

In the Lab system, the four-vector η_2 is

$$\eta_2 = \left(\frac{\vec{p}_2 \cdot \vec{S}_2}{M}, \vec{S}_2 + \frac{\vec{p}_2(\vec{p}_2 \cdot \vec{S}_2)}{M(E_d + M)} \right), \tag{56}$$

where $\vec{S}_2$ is the three-vector of the scattered deuteron polarization in its rest frame.

The differential cross section can be written as

$$\frac{d\sigma}{dQ^2}(\vec{\xi}_1, \vec{S}_2) = \left(\frac{d\sigma}{dQ^2}\right)_{un} [1 + T_{xx}\xi_{1x}S_{2x} + T_{yy}\xi_{1y}S_{2y} + T_{zz}\xi_{1z}S_{2z} + T_{xz}\xi_{1x}S_{2z} + T_{zx}\xi_{1z}S_{2x}], \tag{57}$$

where T_{ij}, $i, j = x, y, z$ are the polarization transfer coefficients which describe the transfer of polarization from the initial electron to the scattered deuteron, and their explicit expressions in terms of the deuteron form factors read

$$\mathcal{D}T_{yy} = -\frac{4}{3}Q^2 mM(1+\tau)G_M\,\widetilde{G},$$

$$\mathcal{D}T_{xx} = \frac{2}{3}mM\,G_M\{x^2yM[2(M+E+2\tau E)\widetilde{G} - \tau(M+E+2\tau M)G_M] - 2Q^2(1+\tau)\widetilde{G}\},$$

$$\mathcal{D}T_{zx} = \frac{xyM^2\,G_M}{3p}\{(E+M+2\tau M)[m(4p^2-Q^2) - Q^2E]\tau\,G_M$$
$$-2E[4m\tau(p^2 - EM) - Q^2(M+E+2\tau E) - Q^2m(1+2\tau)]\},$$

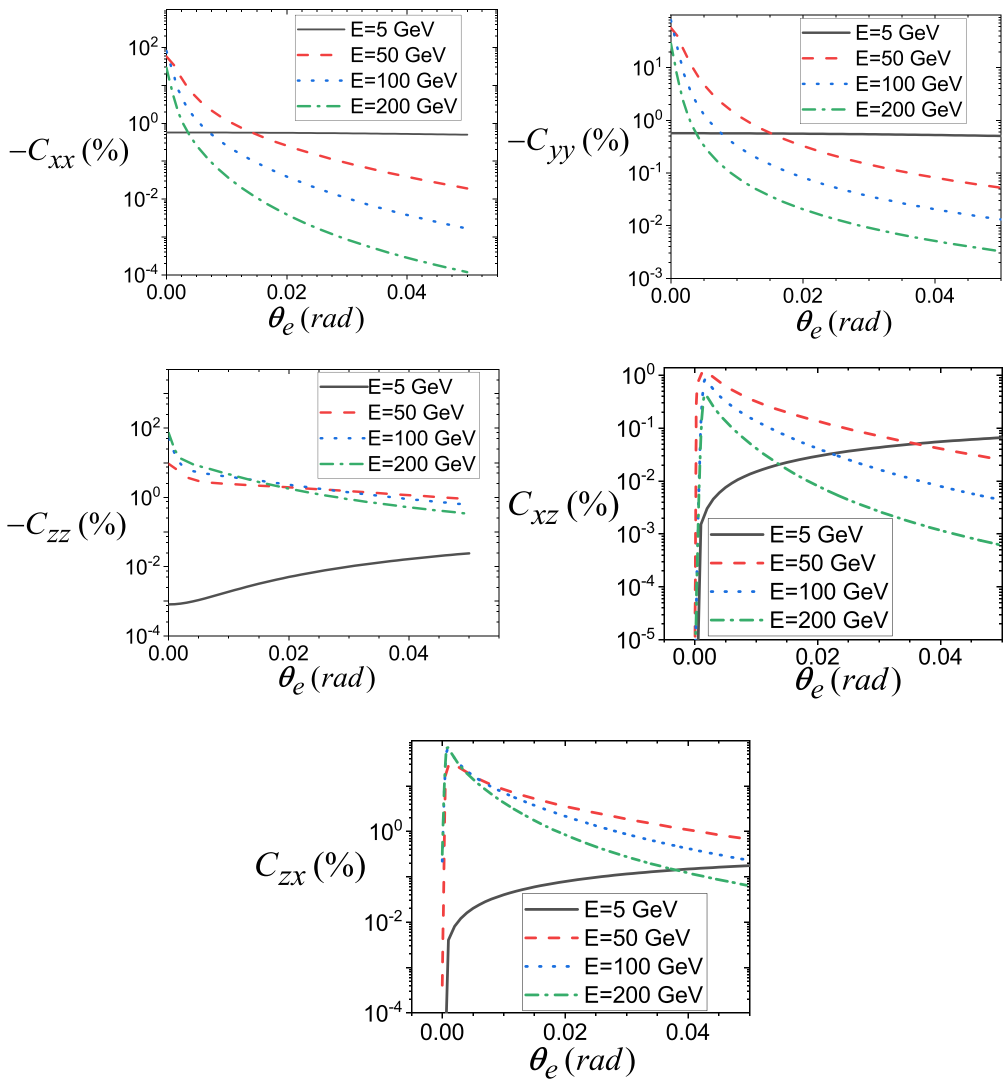

FIG. 7. Same as Fig. 6, but as functions of θ_e, for different values of the energy. Notations as in Fig. 5

$$\mathcal{D}T_{xz} = \frac{xm}{3}\left\{\frac{\tau Q^2\, G_M}{mp}[m(E+m) - yM(E+M)(m+M)^2]G_M + 4M\left[(1+2\tau)[p - yzM(M+E)] + \frac{yzQ^2}{2}\right]\widetilde{G}\right\},$$

$$\mathcal{D}T_{zz} = -\frac{Q^2\, G_M}{3}\left\{\frac{\tau(z+p)}{p}[m(E+m) - yM(E+M)(m+M)^2]G_M\right.$$

$$\left. + \frac{E}{m}\left[Q^2\left[y(m+M)^2 + yz(m+M)\frac{E+m}{p} - \frac{(E+m)^2}{p^2} - \frac{m}{M}\right] + 4m^2\right]\widetilde{G}\right\}, \quad (58)$$

and are plotted in Figs. 8 and 9.

F. Correlation polarization coefficients, c_{ij}, vector polarized scattered deuteron, polarized recoil electron, $d + e \to \vec{d}^{\,V} + \vec{e}$

The scattering of an unpolarized deuteron beam by unpolarized electrons is considered here. A correlation arises between the vector polarization of the scattered deuteron and the polarization of the recoil electron. The part of the hadron tensor $H_{\mu\nu}(\eta_2)$ due to the vector polarized scattered deuteron with unpolarized deuteron beam can be obtained from Eq. (50) with the substitutions $\eta_1 \to \eta_2$ and $p_1 \rightleftarrows -p_2$, so that

$$H_{\mu\nu}(\eta_2) = 2iM\, G_M\left[(1+\tau)\widetilde{G}\varepsilon_{\alpha\beta\lambda\rho}\eta_2^\lambda k^\rho - \frac{\eta_2\cdot k}{4M^2}(G_M - 2\widetilde{G})\varepsilon_{\alpha\beta\lambda\rho}p_2^\lambda k^\rho\right]. \quad (59)$$

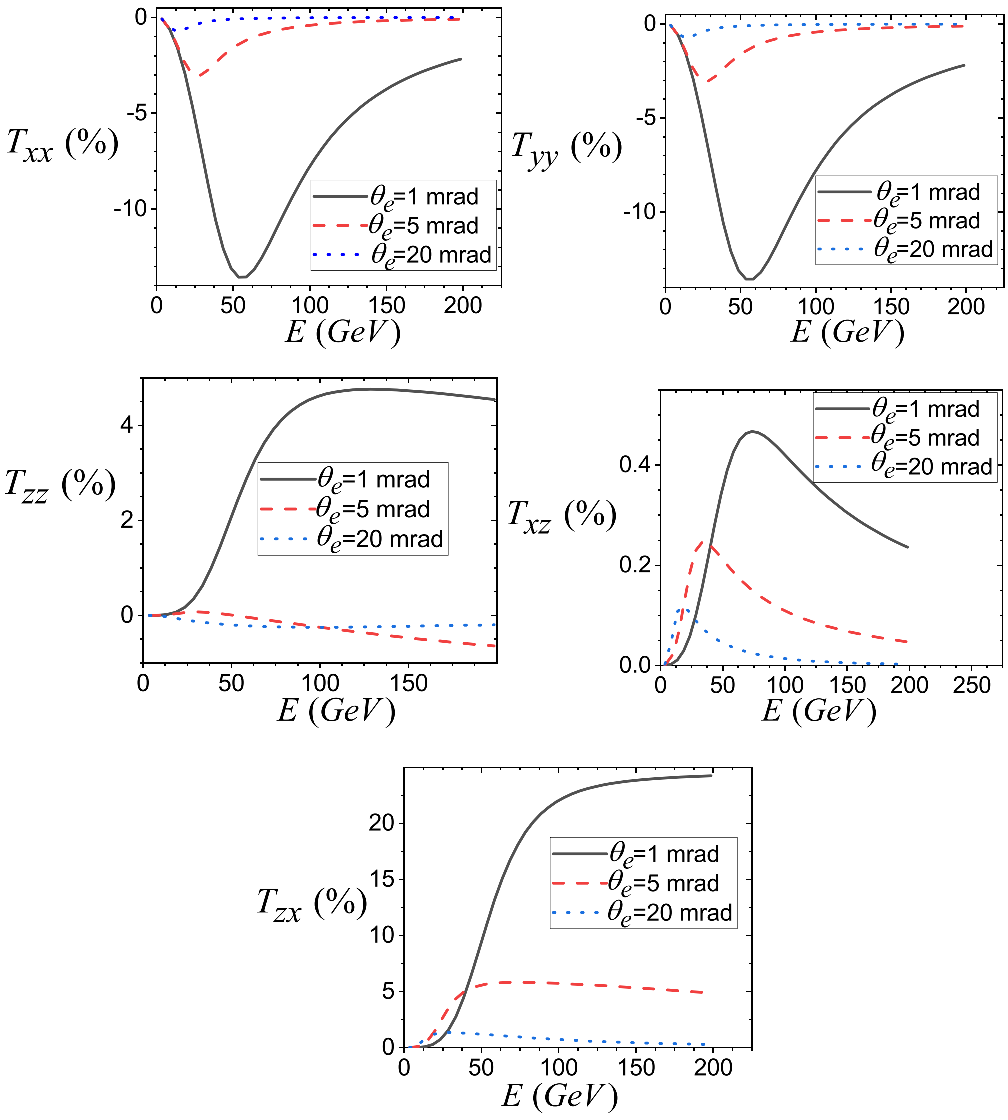

FIG. 8. Polarization transfer coefficients from polarized electron target to vector polarized scattered deuterons, Eqs. (57), (58), as functions of energy, for different values of θ_e. Notations as in Fig. 4.

The lepton tensor, $L_{\mu\nu}$, which corresponds to an unpolarized initial electron target and polarized recoil electron, has the form

$$L_{\mu\nu} = \frac{1}{2}\left[L^{(0)}_{\mu\nu} + L^{(p)}_{\mu\nu}(s_2)\right], \quad L^{(p)}_{\mu\nu}(s_2) = 2im\epsilon_{\mu\nu\rho\sigma}k^\rho s_2^\sigma. \tag{60}$$

The contraction of the spin-dependent lepton $L^{(p)}_{\mu\nu}(s_2)$ and hadron $H_{\mu\nu}(\eta_2)$ tensors, in an arbitrary reference frame, gives

$$C(s_2, \eta_2) = L^{(p)}_{\mu\nu}(s_2)H_{\mu\nu}(\eta_2) = -\frac{4}{3}mMG_M\Big\{\tau k\cdot\eta_2(2p_2\cdot s_2 - k\cdot s_2)G_M + 2\Big(G_C + \frac{\tau}{3}G_Q\Big)[k^2(1+\tau)s_2\cdot\eta_2 - k\cdot\eta_2(k\cdot s_2 + 2\tau p_2\cdot s_2)]\Big\}. \tag{61}$$

All the spin correlation coefficients are proportional to the deuteron magnetic form factor. This is also true for the $\vec{d}^V + e \to d + \vec{e}$ reaction as well for the ed scattering for the corresponding polarization observables.

The differential cross section is

$$\frac{d\sigma}{dQ^2}(\vec{S}_2, \vec{\xi}_2) = \frac{1}{2}\left(\frac{d\sigma}{dQ^2}\right)_{un}[1 + c_{xx}S_{2x}\xi_{2x} + c_{yy}S_{2y}\xi_{2y} + c_{zz}S_{2z}\xi_{2z} + c_{xz}S_{2x}\xi_{2z} + c_{zx}S_{2z}\xi_{2x}], \tag{62}$$

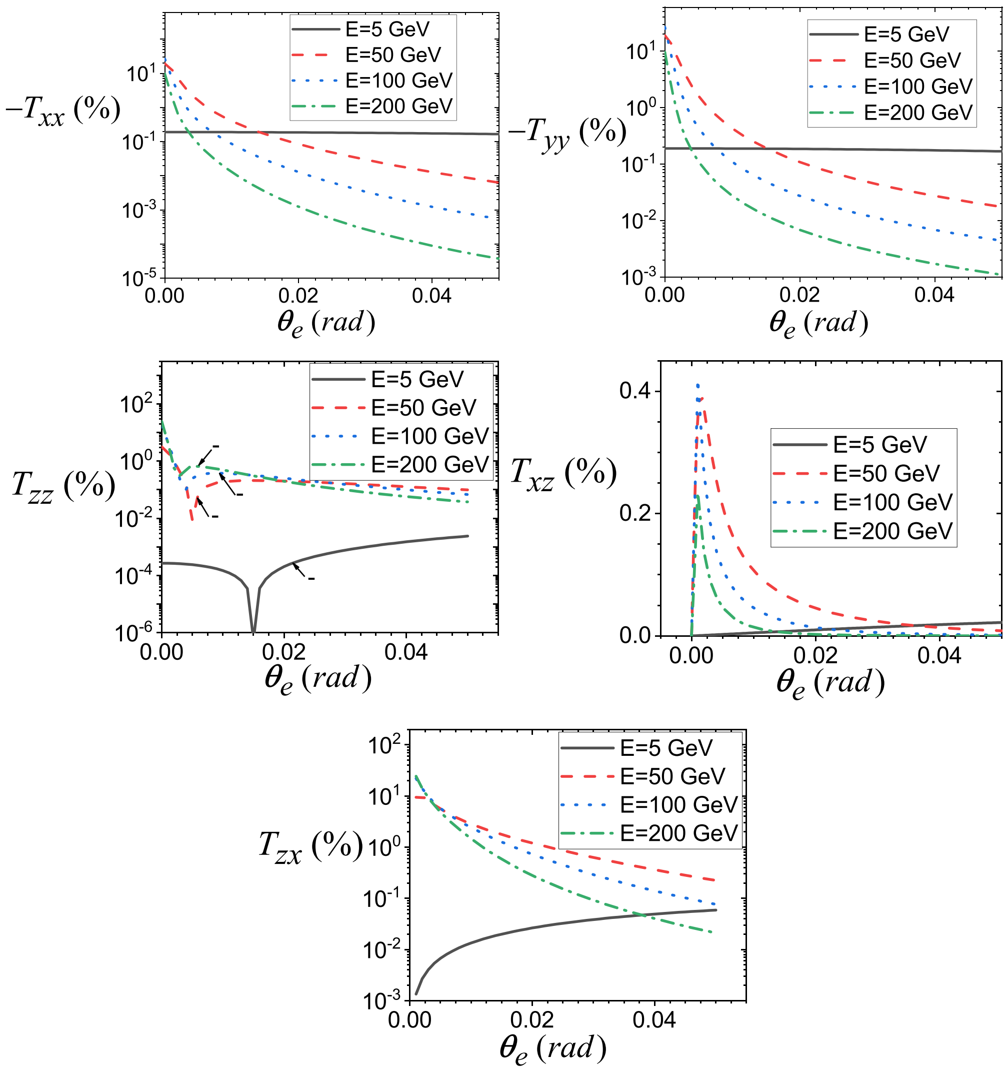

FIG. 9. Same as Fig. 8, but as functions of θ_e, for different values of the energy. Notations as in Fig. 5.

where the explicit expressions of the correlation coefficients c_{ij} are

$$\mathcal{D}c_{yy} = -\frac{4}{3}Q^2 mM(1+\tau)\,G_M\,\widetilde{G},$$

$$\mathcal{D}c_{xx} = \frac{M}{6m(1+\lambda)}\big(B_{xx}\,G_M^2 + 4\tilde{B}_{xx}\,G_M\widetilde{G}\big),$$

$$B_{xx} = yx^2M\tau[Q^2 - 4m(2E+m)][E+(1+2\tau)M],$$

$$\tilde{B}_{xx} = 2yx^2M[(1+2\tau)E+M](m^2-\tau M^2) + 2y\tau m[E(E-M) - 2(1+\tau)M^2] - 2(1+\lambda)(1+\tau)Q^2m^2,$$

$$\mathcal{D}c_{xz} = \frac{4xy\tau M^2}{3m^2p(1+\lambda)}\big(B_{xz}\,G_M^2 - 2\tilde{B}_{xz}\,G_M\widetilde{G}\big),$$

$$B_{xz} = [E+(1+2\tau)M]\{m[m^2p^2 - \tau M^2(mE+M^2)] - \tau M^2(E+m)[m(E+m) - \tau M^2]\},$$

$$\tilde{B}_{xz} = \tau EM^4[E(1+2\tau)+M] + m^3[E^2(E-M) - 2E(1+\tau)M^2]$$
$$+ m^2M^2[4\tau(1+\tau)M^2 - (1-2\tau)EM - E^2(1+4\tau)] + \tau mM^2[2M^3 + 2EM^2(2+3\tau) - E^2(E-M)],$$

$$\mathcal{D}c_{zx} = \frac{4xy\tau M^3}{3m^2p(1+\lambda)}\big(B_{zx}\tau G_M^2 - 2\tilde{B}_{zx}\,\widetilde{G}\big),$$

$$B_{zx} = [\tau M^2 - m(2E+m)]\{M^2[E(1+2\tau) + 2m(1+\tau) + M] - mE(E-m)\},$$

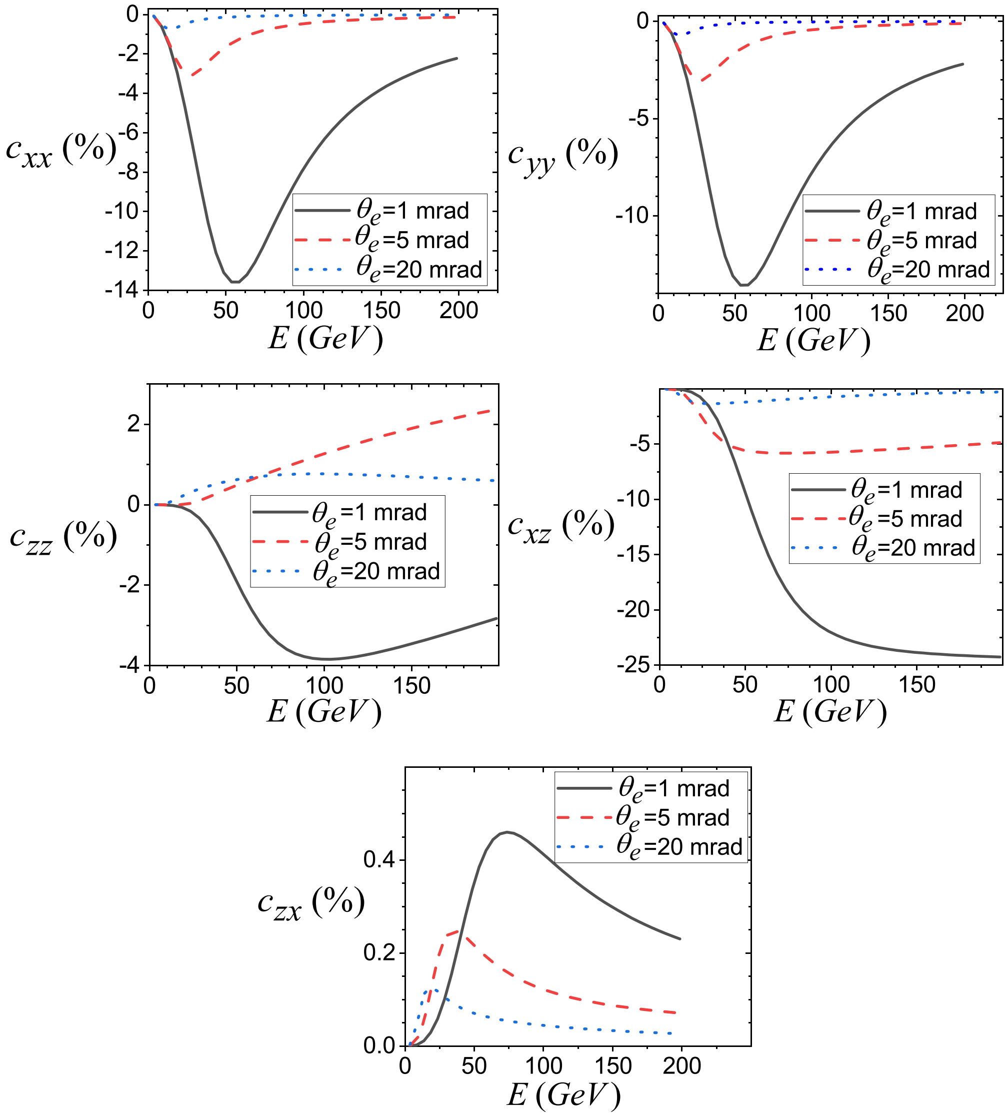

FIG. 10. Correlation coefficients from vector polarized scattered deuterons and recoil electrons, Eqs. (62), (63), as functions of energy for different values of θ_e. Notations as in Fig. 4.

$$\tilde{B}_{zx} = [E + (1+2\tau)M][\tau M^4 - mE(m^2 - 3\tau M^2)] + m^2\{(1+2\tau)M[(1+2\tau)M^2 - E(2E - M)] - 2E^2\},$$

$$\mathcal{D}c_{zz} = \frac{8y\tau M^3}{3m^3p^2(1+\lambda)}\left(B_{zz}\tau\, G_M^2 - 2[E + (1+2\tau)M]G_M\,\widetilde{G}\right),$$

$$B_{zz} = \{M^2[E(1+2\tau) + 2m(1+\tau) + M] - mE(E - M)\}\{m^3p^2 - \tau M^2[(E+m)(mE + m^2 - \tau M^2) + m(mE + M^2)]\},$$

$$\tilde{B}_{zz} = m^4E(p^2 - \tau M^2) + \tau m^2M^2[M^2(5\tau E + 2(1+\tau)m) - E^2(E + 4m)] + \tau M^4[EM^2 + 2m(E^2 + M^2)]. \quad (63)$$

The correlation coefficients c_{ij} between the vector polarizations of the scattered deuteron and the recoil electron are shown in Figs. 10 and 11.

G. Polarization transfer coefficients $\widetilde{T}_{ij}$, vector polarized deuteron beam, polarized recoil electron, $e + \vec{d}^{\,V} \to \vec{e} + d$ reaction

The polarization transfer from the initial vector polarized deuteron to the recoil electron is calculated. The polarized part of the hadron tensor is defined by Eq. (50) and the corresponding part of the lepton one by Eq. (60). The convolution of these polarization dependent terms can be obtained from $C(s_1, \eta_1)$ [see Eq. (51)] with the substitution $s_1 \to s_2$.

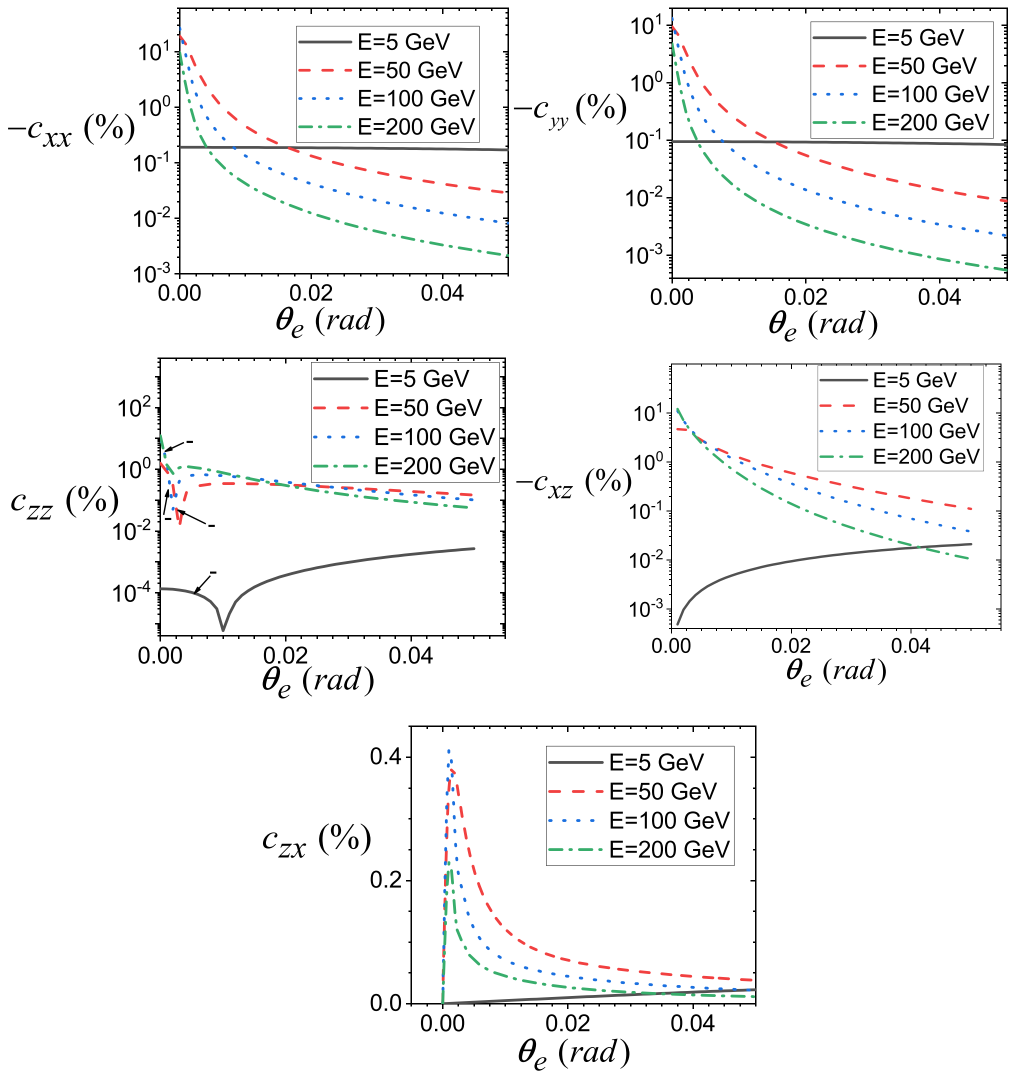

FIG. 11. Same as Fig. 10, but as functions of θ_e, for different values of the energy. Notations as in Fig. 5.

The corresponding differential cross section is written in terms of the polarization transfer coefficients $\widetilde{T}_{ij}$ as

$$\frac{d\sigma}{dQ^2} = \frac{1}{2}\left(\frac{d\sigma}{dQ^2}\right)_{un}[1 + S_{1x}\xi_{2x}\widetilde{T}_{xx} + S_{1y}\xi_{2y}\widetilde{T}_{yy} + S_{1z}\xi_{2z}\widetilde{T}_{zz} + S_{1x}\xi_{2z}\widetilde{T}_{xz} + S_{1z}\xi_{2x}\widetilde{T}_{zx}]. \tag{64}$$

The explicit expressions of the $\widetilde{T}_{ij}$ in terms of the deuteron form factors are

$$\mathcal{D}\widetilde{T}_{yy} = -4mMQ^2(1+\tau)\,G_M\,\widetilde{G},$$

$$\mathcal{D}\widetilde{T}_{xx} = 4mM\left\{x^2\tau\left(\frac{E+m}{m(1+\lambda)} - \frac{1}{2}\right)G_M^2 - \left[x^2\left(1 + \frac{2(\tau E - m)}{m(1+\lambda)}\right) + Q^2(1+\tau)\right]G_M\,\widetilde{G}\right\},$$

$$\mathcal{D}\widetilde{T}_{zz} = \frac{8\tau M^2}{m^2}\left\{\left[mE(\tau M^2 - m^2) - m^2M^2(1-\tau) + \frac{M^2(E+m)^2}{p^2}\left(2M^2 - mE - \frac{2\tau(\tau M^4 - m^3E)}{m^2(1+\lambda)}\right)\right]\tau G_M^2\right.$$
$$\left. + \left[2\tau m^2M^2 + mE(\tau M^2 - m^2) + \frac{\tau M^2(E+m)^2}{p^2}\left(\frac{2(m^3E + \tau M^4)}{m^2(1+\lambda)} - mE - 2\tau M^2\right)\right]2\,G_M\,\widetilde{G}\right\},$$

$$\mathcal{D}\widetilde{T}_{xz} = 4\tau xM\left\{\left[\frac{\tau M^2(E+m)}{p}\left(\frac{2(E+m)}{m(1+\lambda)} - 1\right) - mp\right]G_M^2 + 2\left[mp - \frac{M^2(E+m)}{p}\left(\frac{2m(\tau E - m)}{m^2(1+\lambda)} + 1\right)\right]G_M\,\widetilde{G}\right\},$$

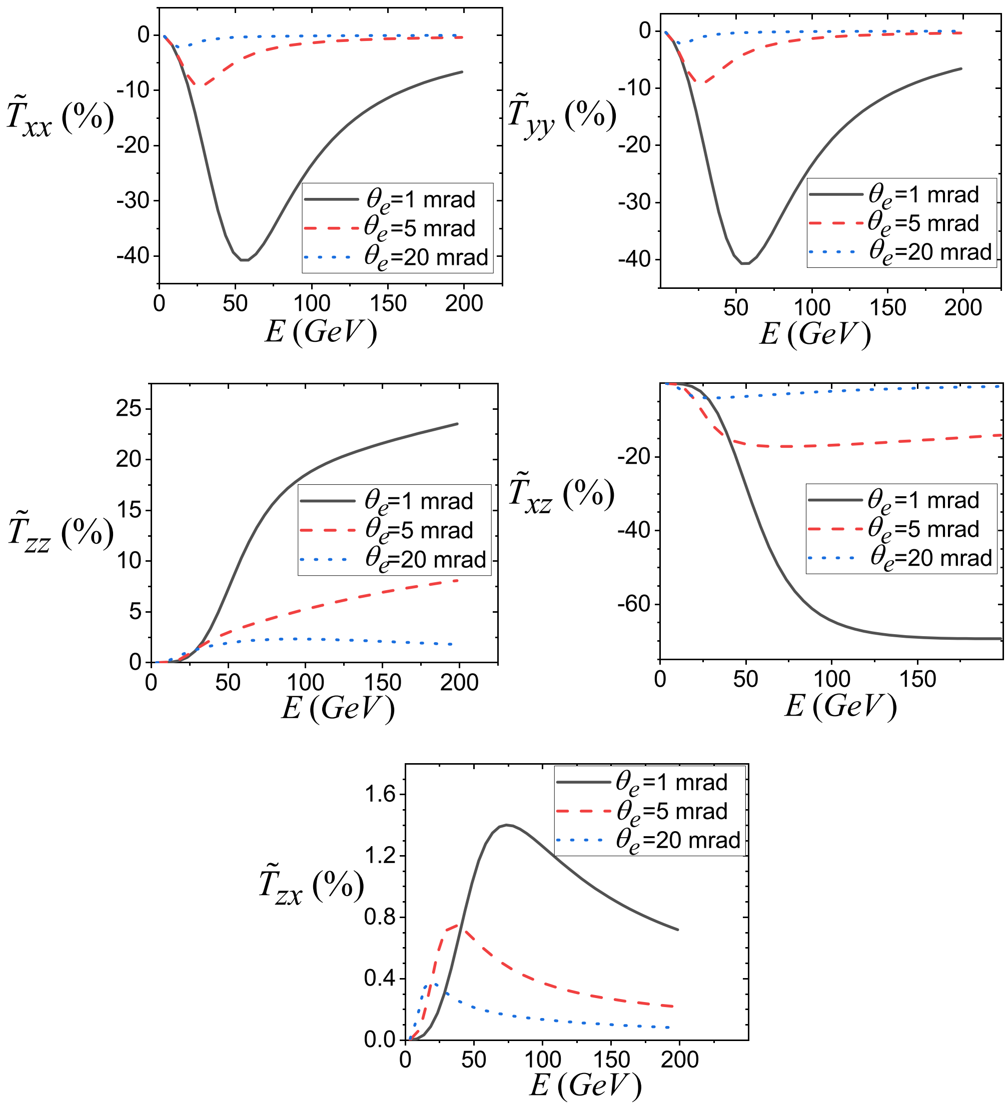

FIG. 12. Polarization transfer coefficients from vector polarized deuteron beam and polarized recoil electrons, Eqs. (64), (65), as functions of energy for different values of θ_e. Notations as in Fig. 4.

$$\mathcal{D}\widetilde{T}_{zx} = \frac{4\tau x M^2}{p}\left\{\left[M^2 - Em + \frac{2M^2E}{m} + \frac{2(E+m)(m^3E - \tau M^4)}{m^3(1+\lambda)}\right]\tau G_M^2 + 2\left[M^2(1-2\tau) - mE - \frac{2\tau M^2 E}{m} + \frac{2(E+m)(m^3E + \tau^2 M^4)}{m^3(1+\lambda)}\right]G_M\,\widetilde{G}\right\}. \quad (65)$$

The polarization transfer coefficients $\widetilde{T}_{ij}$ are plotted in Figs. 12 and 13.

H. Polarization transfer coefficients V_{ij}, vector polarized initial and scattered deuterons, $\vec{d}^{\,V} + e \to \vec{d}^{\,V} + e$

Scattered deuterons from the collision of a vector polarized deuteron beam on an unpolarized electron target can be vector polarized. The corresponding hadron tensor is

$$H_{\mu\nu}(\eta_1,\eta_2) = V_1\tilde{g}_{\mu\nu} + V_2 P_\mu P_\nu + V_3(\tilde{\eta}_{1\mu}\tilde{\eta}_{2\nu} + \tilde{\eta}_{1\nu}\tilde{\eta}_{2\mu}) + V_4(P_\mu\tilde{\eta}_{1\nu} + P_\nu\tilde{\eta}_{1\mu}) + V_5(P_\mu\tilde{\eta}_{2\nu} + P_\nu\tilde{\eta}_{2\mu}),$$
$$\tilde{\eta}_{i\mu} = \eta_{i\mu} - \frac{k\cdot\eta_i}{k^2}k_\mu, \quad (66)$$

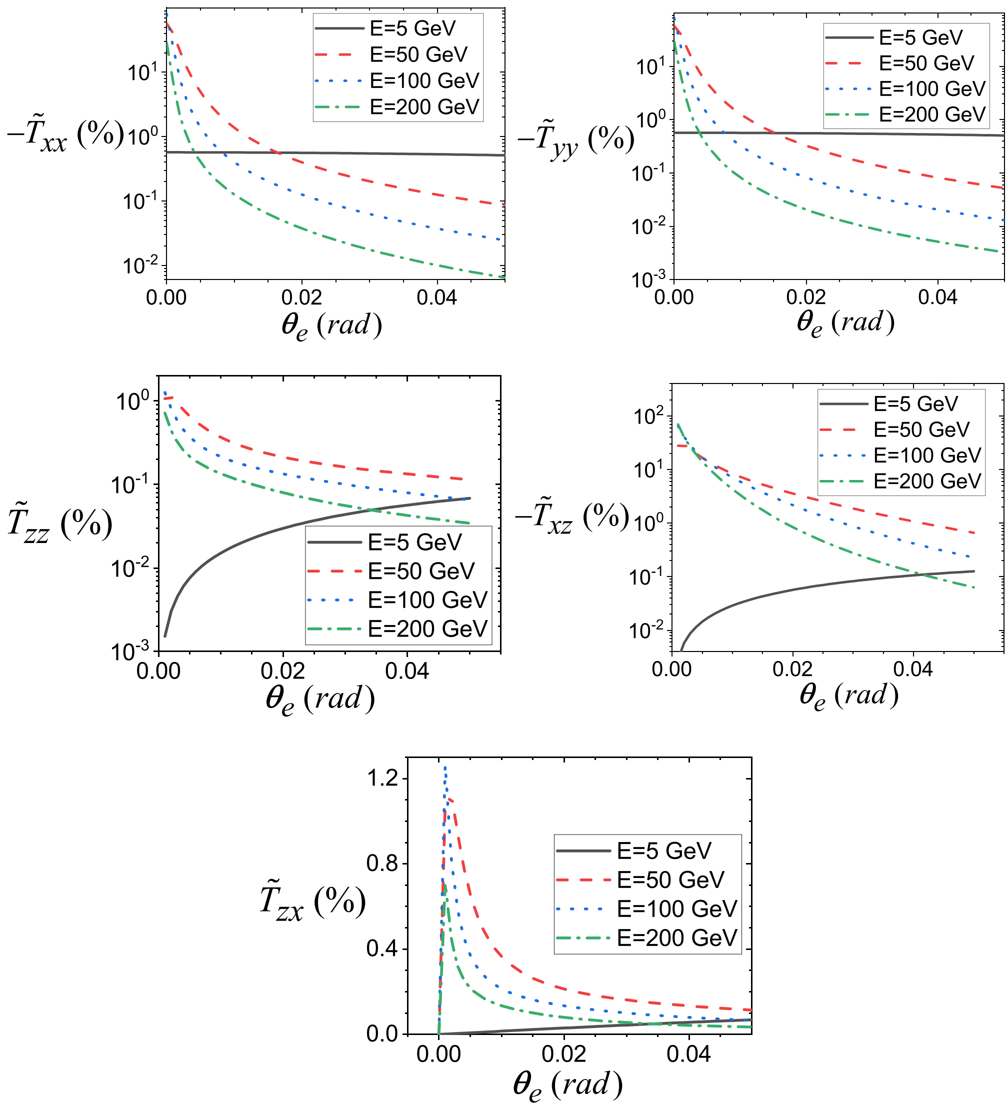

FIG. 13. Same as Fig. 12, but as functions of θ_e, for different values of the energy. Notations as in Fig. 5.

where the structure functions V_i have the following expressions in terms of the deuteron electromagnetic FFs:

$$\begin{aligned}
V_1 &= \frac{1}{2}G_M^2[(1+2\tau)k\cdot\eta_1 k\cdot\eta_2 + 4\tau(1+\tau)M^2\eta_1\cdot\eta_2],\\
V_2 &= -\frac{1}{8M^2}(1+\tau)^{-1}\bigg\{\left(2G_C-\frac{4}{3}\tau G_Q\right)\bigg[\left(G_C+\frac{4}{3}\tau G_Q\right)(k\cdot\eta_1 k\cdot\eta_2+2(1+\tau)M^2\eta_1\cdot\eta_2)+(G_Q-G_M)k\cdot\eta_1 k\cdot\eta_2\bigg]\\
&\quad +(1+\tau)G_M^2(k\cdot\eta_1 k\cdot\eta_2+4\tau M^2\eta_1\cdot\eta_2)\bigg\},\\
V_3 &= -\tau(1+\tau)M^2G_M^2,\\
V_4 &= -\frac{1}{4}k\cdot\eta_2 G_M\left(G_C+\tau G_M-\frac{2}{3}\tau G_Q\right),\\
V_5 &= \frac{1}{4}k\cdot\eta_1 G_M\left(G_C+\tau G_M-\frac{2}{3}\tau G_Q\right). \qquad (67)
\end{aligned}$$

For this configuration of the hadron polarizations it is sufficient to have an unpolarized electron target since the hadron tensor in this case is symmetrical over the μ, ν indices. Thus, the contraction of the (spin independent) lepton $L^{(0)}_{\mu\nu}$ and the (spin dependent) hadron $H_{\mu\nu}(\eta_1,\eta_2)$ tensors, gives the following expressions that are valid in an arbitrary reference frame

$$\begin{aligned}C(\eta_1,\eta_2)=&\frac{4}{M^2}G_M^2\{(k\cdot\eta_1 k\cdot\eta_2-k^2\eta_1\cdot\eta_2)[p_1\cdot k_1(2\tau M^2-p_1\cdot k_1)+M^2((1+\tau)m^2-\tau^2M^2)]\\&+\tau M^2[m^2-(1+\tau)M^2]k\cdot\eta_1 k\cdot\eta s_2+2\tau M^2[(M^2+p_1\cdot k_1)k\cdot\eta_1 k\cdot\eta_2\\&+((1+2\tau)M^2-p_1\cdot k_1)k_1\cdot\eta_1 k\cdot\eta_2-2(1+\tau)M^2k_1\cdot\eta_1 k_1\cdot\eta_2]\}\\&+\frac{8}{(1+\tau)M^2}\left(G_C-\frac{2}{3}\tau G_Q\right)\left\{G_M(\tau M^2-p_1\cdot k_1)[k\cdot\eta_1 k\cdot\eta_2(\tau M^2-p_1\cdot k_1)\right.\\&-(1+\tau)M^2(k\cdot\eta_1 k_1\cdot\eta_2-k_1\cdot\eta_1 k\cdot\eta_2)]+[\tau M^4+2\tau M^2p_1\cdot k_1-(p_1\cdot k_1)^2]\\&\left.\times\left[2(1+\tau)M^2(G_C+\frac{4}{3}\tau G_Q)\eta_1\cdot\eta_2+\left(G_C+\left(1+\frac{4}{3}\tau\right)G_Q\right)k\cdot\eta_1 k\cdot\eta_2\right]\right\}.\end{aligned}\tag{68}$$

The corresponding differential cross section is

$$\frac{d\sigma}{dQ^2}=\left(\frac{d\sigma}{dQ^2}\right)_{un}[1+V_{xx}S_{1x}S_{2x}+V_{yy}S_{1y}S_{2y}+V_{zz}S_{1z}S_{2z}+V_{xz}S_{1x}S_{2z}+V_{zx}S_{1z}S_{2x}]\tag{69}$$

with V_{ij},

$$\begin{aligned}\mathcal{D}V_{xx}=&2h\left\{\frac{x^2}{(1+\tau)M^3}[E-yM(p(p-z)+2(1+\tau)M^2)]-2\right\}G^{(-)}G^{(+)}\\&+\frac{2x^2(\tau M^2-mE)}{(1+\tau)M^3}[yMp(z-p)(\tau M^2-mE)+\tau M^2(E+m)-mp^2]G_M G^{(-)}\\&+\{Q^2[\tau M^2(\tau M^2-2mE-m^2)+m^2p^2]-x^2yM[\tau M^4(M+(1+2\tau)E)+mp^2(m(E+M)-2\tau M^2)]\}\frac{G_M^2}{M^2},\\ \mathcal{D}V_{yy}=&-4hG^{(-)}G^{(+)}+[\tau M^2(\tau M^2-2mE-m^2)+m^2p^2]\tau G_M^2,\\ \mathcal{D}V_{zz}=&\frac{yM}{m^2p^2}\left\{\frac{4(E-M)hh_2}{1+\tau}G^{(-)}G^{(+)}-4\tau M^4[E+(1+2\tau)M][h+(1+\tau)m^2M^2]G_M^2\right.\\&\left.-2Q^2(E-M)(\tau M^2-mE)[h+(1+\tau)m^2M^2]\frac{h_1 G_M G^{(-)}}{(1+\tau)M^2}\right\},\\ \mathcal{D}V_{xz}=&\frac{xyM}{mp}\left\{\frac{2(E-M)}{(1+\tau)M}[-2hh_1G^{(-)}G^{(+)}+h_2(\tau M^2-mE)G_M G^{(-)}]\right.\\&\left.-2\tau M^3(M^2+mE)[E+(1+2\tau)M]G_M^2\right\},\\ \mathcal{D}V_{zx}=&\frac{xyM}{mp}\left\{\frac{2(E-M)}{(1+\tau)M}[2hh_1G^{(-)}G^{(+)}-h_2(\tau M^2-mE)G_M G^{(-)}]\right.\\&\left.-2\tau^2M^3[M^2(M+(1+2\tau)E+2(1+\tau)m)-mE(E-M)]G_M^2\right\},\end{aligned}\tag{70}$$

where we introduced the short notation

$$\begin{aligned}&G^{(-)}=G_C-\frac{2\tau G_Q}{3},\quad G^{(+)}=G_C+\frac{4\tau G_Q}{3},\\&h=\tau M^2(M^2+2mE)-m^2E^2,\quad h_1=\tau M^2-mE-(1+\tau)mM,\\&h_2=-2\tau M^2(h_1+\tau mE)-m^2\{(1+\tau)M[(1+2\tau)M+2E]+(1-\tau)E^2\}.\end{aligned}\tag{71}$$

The polarization coefficients V_{ij} describing the vector polarization transfer from the deuteron beam to the scattered deuteron, also denoted as 'deuteron depolarizations', are shown in Figs. 14 and 15.

IV. DISCUSSION AND CONCLUSION

In this work we calculated the differential cross section and some polarization observables for the elastic reaction induced by deuteron scattering off electrons at rest assuming the

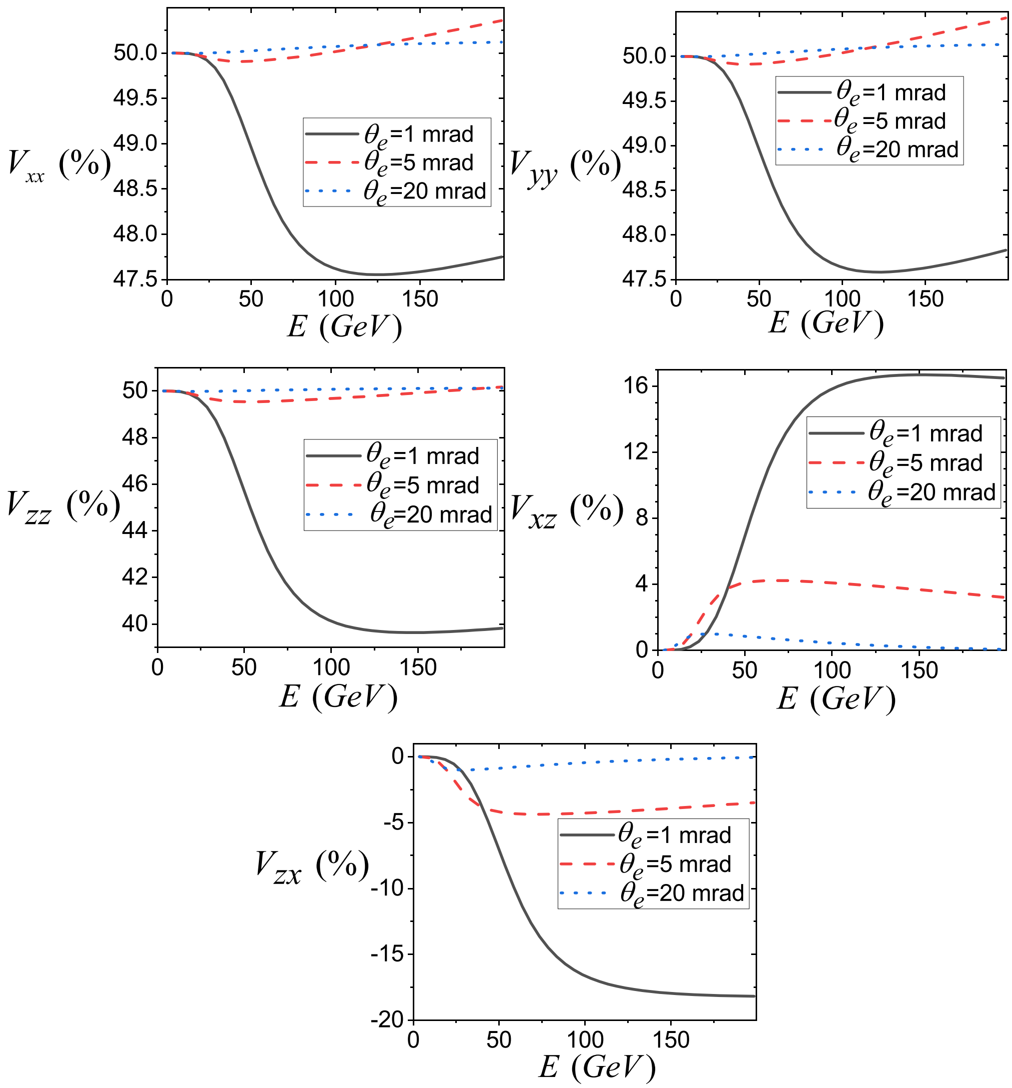

FIG. 14. Polarization transfer coefficients from vector polarized initial and scattered deuterons, Eqs. (69), (70), as functions of energy for different values of θ_e. Notations as in Fig. 4.

one-photon-exchange approximation. We limited the study to the estimation of one-spin effects when one deuteron (initial or scattered) is tensor polarized, and to double-spin effects when two particles are vector polarized. In the last case, all possible polarization states are considered. Our analytical and numerical results are obtained under the condition that all components of the three-vector polarization for every particle involved are defined in the Lab system, as shown in Fig. 1. The same is required for the components of the deuteron tensor polarization. In this respect, it should be noted that the combination of the form factors included in the definition of t_{20} as measured in $e-d$ scattering [18] corresponds to the z-axis along the unit three-vector of the momentum transfer [19,20] in the rest frame of the initial deuteron. Our choice corresponds to the direction opposite to the unit three-vector of the initial electron three-momentum (in the Lab system, the z axis is just along deuteron three-momentum). Therefore A_{zz} gets the contribution of a term $G_M G_Q$ which is absent in t_{20}. Along the numerical calculation we used the parametrization of the deuteron electromagnetic form factors suggested in Ref. [15] and extrapolate it to the small Q^2 region (Fig. 16). Other form factor parametrizations exhibit very similar behavior in this region, being all normalized to the static values for $Q^2 = 0$.

Our result can be applied to measure the polarization of or to polarize the participating particles. Note, that the unpolarized cross section is very large (see Fig. 17) indicating that the number of events in the different polarization conditions can be sufficient to perform fairly accurate measurements despite of the fact that the corresponding effects are at the percent level.

Our formalism is based on the symmetries of the strong and electromagnetic interactions and it is very general. The lepton and hadron tensors are obtained in terms of the deuteron electromagnetic current Eq. (11) and the density matrices of the initial and scattered deuteron, Eq. (19). All the coefficients which describe the single- and double-spin effects are the ratio between the corresponding spin-dependent parts of

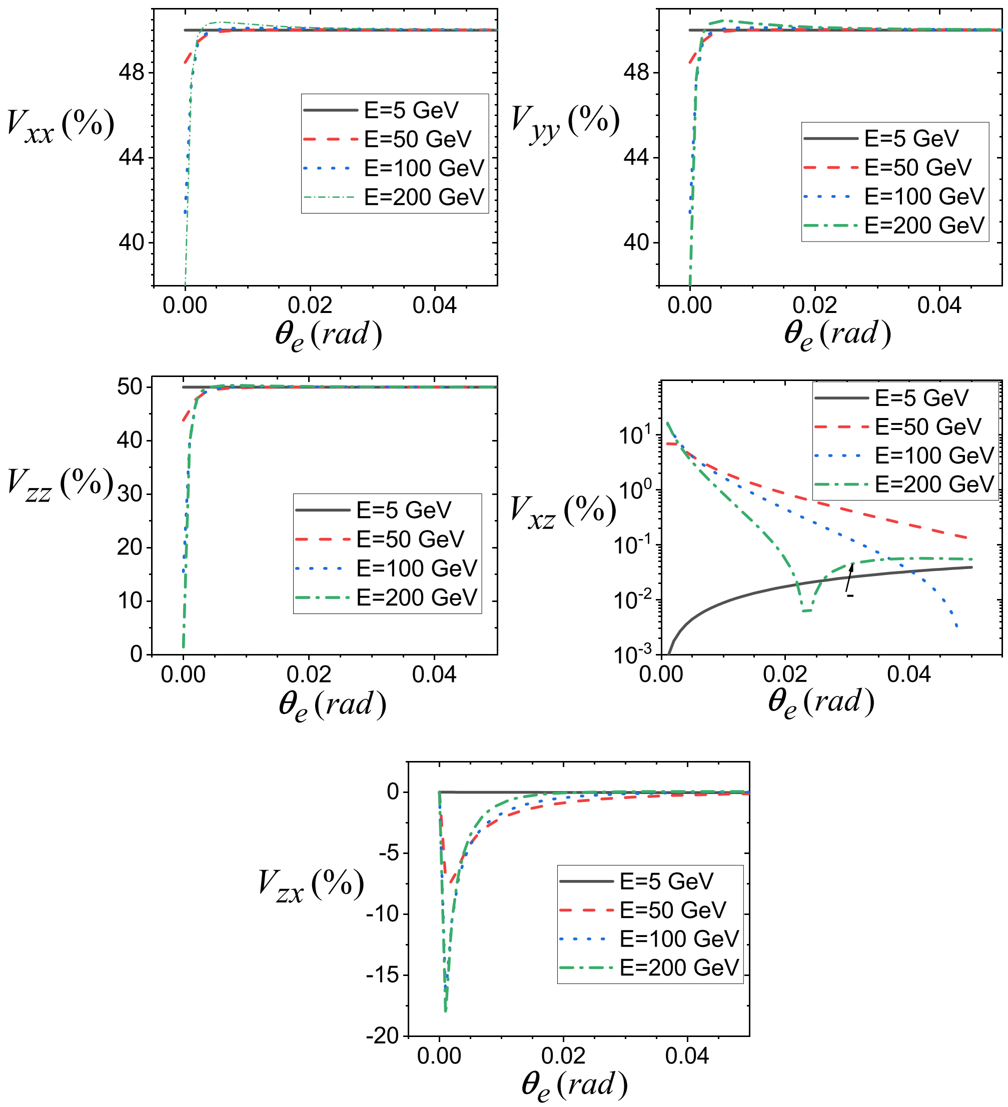

FIG. 15. Same as Fig. 15, but as functions of θ_e for different values of the energy. Notations as in Fig. 5.

the matrix element squared and the spin-independent parts, i.e., $H_{\mu\nu}L^{\mu\nu}/(2\mathcal{D})$, according to the normalization of the unpolarized cross section [Eq. (27)]. The additional factor 1/2 in the differential cross sections when the recoil electron polarization is measured [see Eqs. (48), (62), (64)], arise from the density matrix of the recoil electron. Our main results are illustrated in Figs. 2–15 where we plot different coefficients as functions of the electron scattering angle θ_e at fixed values of the deuteron beam energy E and vice versa.

In the Lab system the tensor asymmetries A_{ij} (Fig. 2) and the tensor polarization coefficients P_{ij} (Fig. 3) are small, not exceeding the order of percent. Nevertheless this situation leaves room for measurements due to the large cross section. The coefficients of the polarization transfer t_{ij} (Figs. 4, 5) from the target to the recoil electron vary in the range -1 to $+1$, making it possible to change the polarization of electrons. The coefficients C_{ij} except C_{xz} (Figs. 6, 7) are on the level a few tens of percent, thus the correlation between the vector polarizations of the deuteron beam and the target electrons is large and measurable.

The possibility to create vector polarized deuterons from polarized target electrons is illustrated by the polarization transfer coefficients T_{ij} (Figs. 8, 9). They are of the order of 10%, except T_{xz}, showing a realistic possibility of applications. The correlation between the vector polarization of the final deuterons and electrons is noticeable although not as large as for the initial ones (Figs. 10, 11). It is quite unexpected that the vector polarization transfer coefficients $\widetilde{T}_{ij}$ (Figs. 12, 13) from the initial deuterons to the recoil electrons are several times larger than T_{ij}. The large values of the coefficients V_{ij}, describing the vector polarization transfer from the initial to the scattered deuterons (Figs. 14, 15) should also be noted.

Our formalism, being very general, gives the essential formulas for deuteron polarization and polarimetry studies. Formulas have been derived analytically and checked within the Mathematica framework [21] that was also used for plotting (the code is available upon request). The specific ingredients of the deuteron structure are contained in the form factors. The sensitivity to different models is

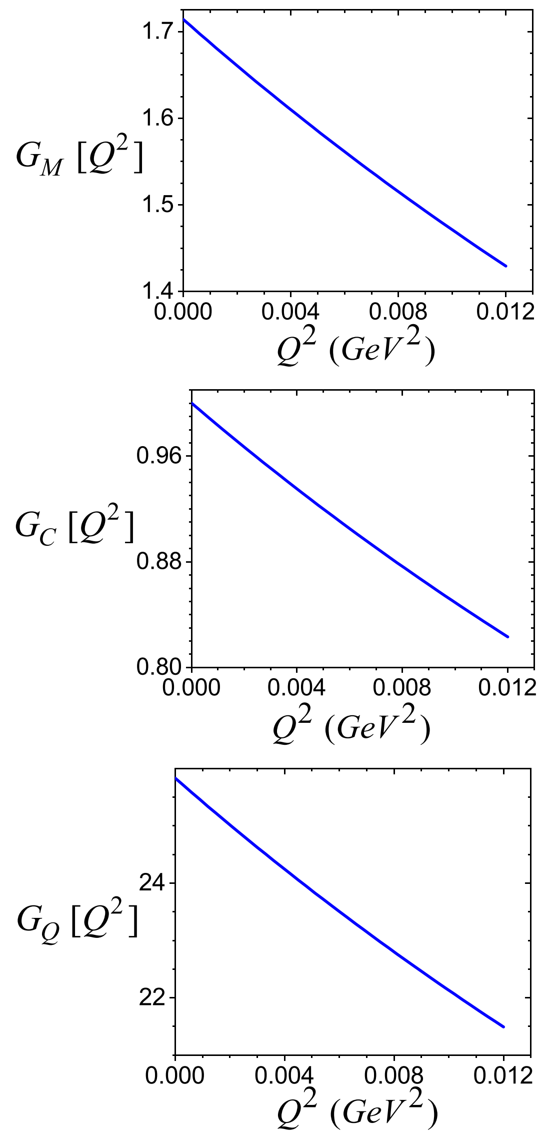

FIG. 16. Deuteron electromagnetic form factors, Eq. (A1), with parameters as in the text.

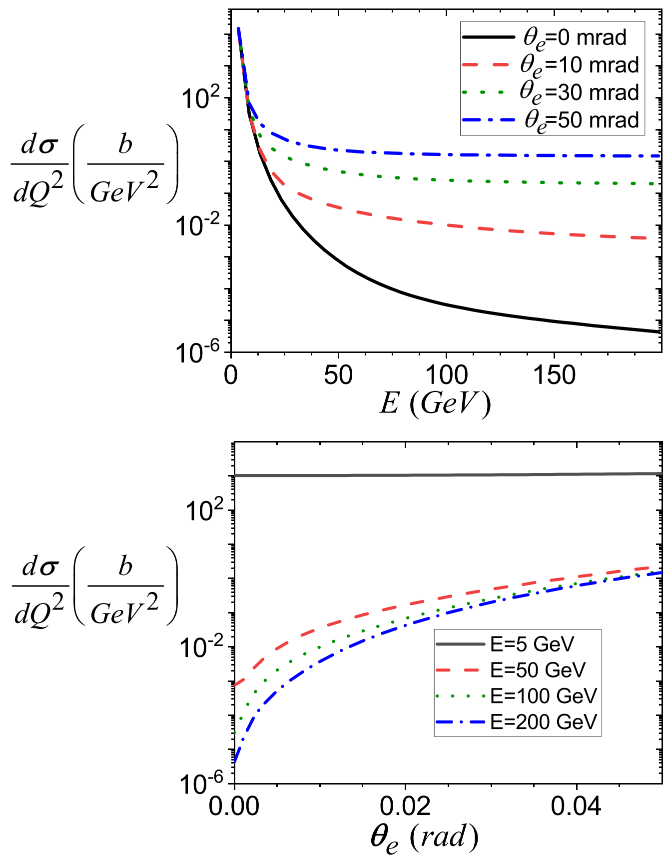

FIG. 17. Unpolarized differential cross section, Eqs. (27) and (29), as a function of incident deuteron beam energy E for different electron scattering angles (top panel) and as a function of the electron scattering angle at different values of the incident energy E (bottom panel).

expected not to be large, because of the low-Q^2 involved and the normalization constrains to the static deuteron properties at $Q^2 = 0$.

ACKNOWLEDGMENTS

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APPENDIX: DEUTERON ELECTROMAGNETIC FORM FACTORS

The deuteron form factors are measured through the differential cross section of electron-deuteron scattering. While the magnetic form factor is uniquely determined by the cross section of unpolarized particles at backward angles, the separation of charge and quadrupole form factors requires polarization measurements, either the tensor analyzing powers T_{20}, T_{22}, T_{21} or the recoil deuteron tensor polarization t_{20}, t_{22}, t_{21} (the electron beam is unpolarized in both cases) [22,23]. This has prompted the development of both polarized deuterium targets and polarimeters for measuring the polarization of recoil hadrons [24].

At storage rings, polarized internal deuteron gas targets from an atomic beam source can be used [25–30]. The high intensity of the circulating electron beam allows achieving acceptable luminosities despite the very low thickness of the gas targets. At facilities with external beams, polarimeters are used to measure the polarization of recoil deuterons [18,31,32]. High beam intensities are a prerequisite because the polarization measurement in this case requires a second scattering, what leads to a loss of a few orders of magnitude in the counting rate.

The angular distribution at a given momentum transfer squared in unpolarized scattering allows to separate G_M^2 and a combination (structure function) of the three form factor squared $A(Q^2) = (2/3)\tau\, G_M^2 + G_C^2 + (8/9)\tau^2\, G_Q^2$. The measurement of T_{20} and T_{21}, or t_{20} and t_{21}, allows to separate also some combination of $G_M\, G_C$, G_Q^2, and G_M^2 and the product $G_M\, G_Q$, respectively [33]. The three electromagnetic deuteron form factors have been experimentally determined up to $Q^2 \simeq 1.7$ GeV^2 [18]. The structure function $A(Q^2)$ has been measured up to $Q^2 = 6$ GeV^2 [34] and $G_M^2(Q^2)$ up to 2.8 GeV^2 [35]. The measurements of the deuteron elastic scattering differential cross section [34] and t_{20} [18] allow to extract G_C and G_Q. This has been done in Ref. [36] where the world data were collected and three different analytical parametrizations were suggested with a number of parameters varying from 12 [37] to 33.

The description of these form factors is a challenge for the deuteron models. The best representation, i.e., very good χ^2 with very small number of parameters, is based on a generalization of the nucleon two-component picture from Refs. [38,39] to the deuteron case [15]. The basic idea of the model is the presence of two components in the deuteron (proton) structure: an intrinsic structure, very compact, characterized by a dipole or monopole Q^2 dependence and a meson cloud, which contains the light vector meson ρ, ϕ, and ω contributions (not the ρ for the deuteron case, due to its isoscalar nature). A very good description of the world data on deuteron electromagnetic form factors has been obtained with as few as six free parameters and few evident physical constraints. The form factors are parametrized as (considering only the contribution of the isoscalar vector mesons, ω and ϕ)

$$G_i(Q^2) = N_i g_i(Q^2) F_i(Q^2), \quad i = C,\ Q,\ M \tag{A1}$$

with

$$F_i(Q^2) = 1 - \alpha_i - \beta_i + \alpha_i \frac{m_\omega^2}{m_\omega^2 + Q^2} + \beta_i \frac{m_\phi^2}{m_\phi^2 + Q^2},$$

where m_ω (m_ϕ) is the mass of the ω (ϕ) meson.

The terms $g_i(Q^2)$ are written as functions of two parameters, also real, γ_i and δ_i, generally different for each form factor:

$$g_i(Q^2) = 1/[1 + \gamma_i Q^2]^{\delta_i}, \tag{A2}$$

and N_i is the normalization of the ith form factor at $Q^2 = 0$, $N_C = G_C(0) = 1$, $N_Q = G_Q(0) = M^2\mathcal{Q}_d = 25.83$, $N_M = G_M(0) = \frac{M}{m_N}\mu_d = 1.714$, where $\mathcal{Q}_d$, and μ_d are the quadrupole and the magnetic moments of the deuteron, m_N is the nucleon mass.

The expression (A1) contains four parameters, α_i, β_i, γ_i, δ_i, generally different for different form factors. We took here the most simple version where $\delta = 1.04$ and $\gamma = 12.1$ are common parameters for the three form factors, $\alpha(G_C,\ G_Q,\ G_M) = 5.75,\ 4.21\ 3.77$ and $\beta(G_C,\ G_Q,\ G_M) = -5.11,\ -3.41,\ -2.86$. With the chosen parametrization, the extrapolation to small values of Q^2 gives the electromagnetic deuteron form factors shown in Fig. 16. In the region of small Q^2 all three form factors are positive and decrease almost linearly with increasing Q^2. The energy dependence of the differential cross section for different angles and the angular dependence for different energies is illustrated in Fig. 17. We restrict ourselves to $E \leqslant 200$ GeV and $\theta_e \leqslant 50$ mrad.

The unpolarized differential cross section is divergent at small values of the energy, as expected from the one-photon exchange mechanism. It is monotonically decreasing not presenting minima when the deuteron energy increases and increases when the electron scattering angle increases.

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