Level densities for ^{69,71}Ga nuclei using a particle-evaporation technique

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Proton spectra and double differential cross sections were measured from ^{7}Li-induced reactions on ^{63,65}Cu with a 15.5 MeV lithium beam. Protons were measured at four angles: 37, 52, 97, and 142 degrees in the laboratory frame. The spectra measured at 97 and 142 degrees were used to test level density models for ^{69,71}Ga nuclei populated by protons. Also, the level density excitation energy functions were extracted and compared directly with model calculations and with previous results on level densities for near mass nuclei ^{74,76}Ge. It was found that the level density for ^{69}Ga is consistent with the prediction of the back-shifted Fermi-gas model, while the level density for ^{71}Ga is consistent with the ones for ^{74,76}Ge, which all are systematically lower than this model predicts. The models based on the Gilbert and Cameron approach and microscopic model are not supported by experimental data.

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I. INTRODUCTION

The nuclear level density is a physical property of nuclei, which is defined as a number of nuclear levels per unit of excitation energy, most commonly per MeV. It is widely used as an input parameter for reaction cross section calculations and embedded in various reaction codes such as TALYS [1], EMPIRE [2], NON-SMOKER [3], etc.

The special interest in the nuclear level density is associated with astrophysics reaction rate calculations. Slow and rapid neutron capture processes as well as reactions with proton and α particles are considered to be main mechanisms responsible for the formation of elements heavier than iron in astrophysical environments. At astrophysical temperatures ($1–2\times10^{9}$ K), energy of colliding nuclei is low (from nuclear physics point of view) such that the compound reaction model implemented into the Hauser-Feshbach (HF) theory of nuclear reactions [4] is a main tool for reaction rate calculations and the nuclear level density is considered to be a critical parameter.

The nuclear data evaluation is another area where reaction modeling uses HF approach, which heavily relies on nuclear level densities modeling. Because level densities for most of the nuclei are not constrained experimentally, adjusting the level density parameters is often used in cross section evaluation procedures.

All level density models embedded in nuclear reactions codes are based on experimental data on *s*-wave neutron resonance spacings [5,6], which are available at excitation energies in vicinity of the neutron binding energy. Because of the limited spin range populated by slow neutrons, the number of observed resonances constitutes only 10–20 % of the total number of levels at this excitation energy. A limited set of data is not sufficient for constraining models. Such important parameters as the spin distribution, the parity ratio, and the excitation energy dependence of the level density remain largely unsupported by experimental data and constitute the main source of uncertainties in calculation of level densities and reaction cross sections. Therefore, experimental data on level densities from an alternative experimental technique is needed to constrain level density models. The important problem is to understand to what extent the level density based on the data from neutron resonance spacings are able to describe cross section of nuclear reactions.

The technique, based on the measurement of particle evaporation spectra from compound nuclear reactions, allows one to obtain information about the level densities of nuclei populated by these particles [7]. In the HF reaction model [4], the differential cross sections $d\sigma(E)/dE$ (they are also referred to as evaporation spectra) of outgoing particles are very sensitive to input level density parametrization. Specifically, energy spectra are determined by particle transmission coefficients and level densities of product nuclei. Because transmission coefficients can be calculated with higher accuracy from optical model potentials, the level density of product nuclei can be studied from analysis of particle evaporation spectra. There is still lack of study about the ability of level density models parameterized with neutron resonance data to reproduce differential spectra from compound nuclear reactions. This is especially true for the systematic studies, over range of nuclei. It would be important to investigate systematically, how well level these density models were able to reproduce particle evaporation spectra from compound nuclear reactions.

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The level density of ^{74,76}Ge isotopes was studied with proton evaporation spectra from ^{68,70}Zn(^{7}Li, p) reactions in our recent work [8]. It was shown that all level density models based on neutron resonance parameters result in the slope, which is too steep compared to experimental level densities obtained from analysis of proton evaporation spectra. It suggests that the total level density values at the excitation energy around the neutron binding energy might be overestimated from current models by a factor of two. It would, therefore, be important to study the level density of nuclei within the same mass range using the same technique to determine if any systematic trends could be identified.

In this work proton evaporation spectra from lithium-induced reactions ^{63,65}Cu(^{6,7}Li, Xp) are studied experimentally. High-energy, first-stage protons from (^{7}Li, p) populate ^{69,71}Ga isotopes. The motivation of this work is to study the level density of gallium isotopes experimentally from proton evaporation spectra. Level density models are tested by directly comparing the measured proton double differential cross section with HF model calculations. Also the level density excitation energy function is extracted from experimental spectra and directly compared with available level density models based on neutron resonance spacing parametrizations, such as those from the RIPL-3 database [5].

II. EXPERIMENT AND RESULTS

We measured proton spectra from 15.5 MeV ^{7}Li-induced reactions on 2 mg/cm^2 thick cooper self-supporting foils enriched to 95% ^{63,65}Cu. The ^{7}Li beam was produced by the Edwards Laboratory tandem electrostatic accelerator. the ΔE-E telescope consisting of a thin (0.25 mm Si) and a thick [5 mm Si(Li)] detectors was consecutively placed at the 37°, 52°, 97°, and 142° angles in respect to the beam direction inside the chamber to register high-energy protons (from 7 up to 25 MeV). Because a cross section for high-energy protons is very small, the telescope was placed at the distance of about 25 cm from the target to increase the solid angle and counting statistics. The energy calibration of Si detectors was performed with the reaction of ^{12}C(^{7}Li, p), in the low energy range (up to 13 MeV), and with ^{27}Al(^{7}Li, p)^{33}P, in the high-energy range, where the structure due to discrete levels population in ^{33}P is seen in proton spectra between 20 and 24 MeV.

The loss of the beam energy in the targets was about 1.2 MeV causing the fusion cross section, according to EMPIRE calculations, to drop from initial 244 mb to 90 mb at the exit of the target. Therefore, in order to convert proton spectra from the laboratory to the center-of-mass system, and to compare them with calculations, the measured spectra were assumed to be produced with the average lithium beam energy of 14.9 MeV. It was checked with calculations that the average proton spectrum is a good approximation (within a 5% of a scaling factor) of the real proton spectrum calculated exactly taking into account a gradual energy loss along the path of the incoming beam in the target.

The proton spectra in the form of double differential cross sections converted to the center-of-mass frame are presented in Fig. 1. Comparing spectra at different angles helps us to understand the reaction mechanism. It is seen from the figure that spectra at 97° and 142° are identical while spectra at forward angles exhibit a flatter slope indicating contribution from the preequilibrium reaction mechanism at higher proton energies. We assume that the contribution of the preequilibrium reaction mechanism is diminished at backward angles so the spectra can be analyzed with the compound reaction HF model. We therefore sum up spectra taken at 97° and 142° for further analysis of nuclear level density models.

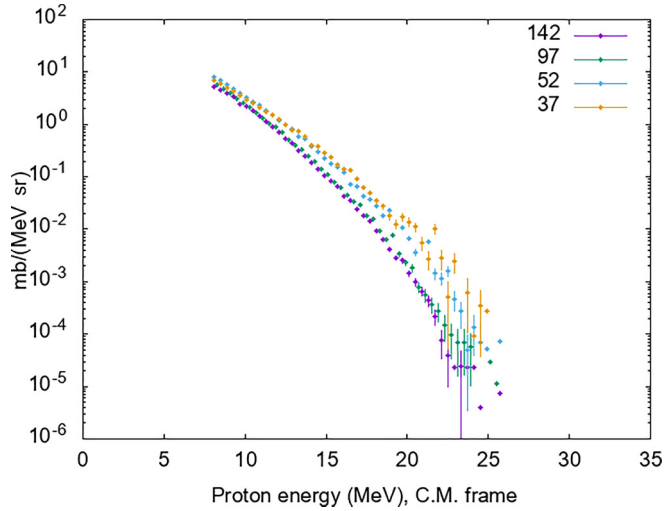

FIG. 1. Proton spectra at different angles.

For the level density studies we used two approaches. The first one is to test proton double differential cross section calculations with compound reaction Hauser-Feshbach (HF) model against experimental data. These cross sections, as it will be shown in the next section, are affected by choice of level density models for all product nuclei populated by neutrons, protons, and α particles. In this case global level density model prescriptions are tested. The second approach consists of extraction of level density data points for ^{69,71}Ga directly from high-energy part of experimental proton spectra according to the method of Ref. [9]. This will allow us to study level density excitation energy dependence specifically for gallium isotopes.

A. Model calculations

Calculations of proton cross sections of lithium-induced reactions have been performed with the computer code EMPIRE [2] since it allows calculating reactions with projectiles heavier than α particle. We assume that for proton spectra measured at backward angles, the compound reaction mechanism is dominant. Generally, some contribution from other reaction mechanisms is possible. The most important one is the preequilibrium mechanism of nuclear reactions when particles are emitted before a compound equilibration takes place. Cross sections for high-energy outgoing particles tend to be higher compared to what would have been expected from compound mechanism. This can potentially lead to distortion of purely compound particle emission spectra and make level density analysis more difficult and uncertain.

This mechanism can optionally be taken into account in EMPIRE for ion projectiles using, as a basis, the Griffin exciton model [10]. However, because of lack of capabilities of calculating the angular distribution, it is difficult to estimate the preequilibrium contribution for backward angles at which we measured our proton spectra. For the moment we assume that this contribution is negligible. This assumption is supported by our earlier experimental observations [11], which show that proton spectra measured at backward angles from two different reactions ^{55}Mn(^{6}Li, p)^{60}Co and ^{57}Fe(α, p)^{60}Co can be reproduced with compound reaction calculations using the same level density model parametrization. Therefore, experimental spectra in this work are compared to calculations based purely on the compound nuclear reaction model using the HF theory [4].

The general HF expression is the following:

$$\frac{d\sigma}{d\varepsilon_b}(\varepsilon_a, \varepsilon_b) = \sum_{J\pi} \sigma^{\mathrm{CN}}(\varepsilon_a) \frac{\sum_{I\pi} \Gamma_b(U, J, \pi, E, I, \pi)\rho_b(E, I, \pi)}{\Gamma(U, J, \pi)} \tag{1}$$

with

$$\Gamma(U, J, \pi) = \sum_{b'} \left(\sum_{k} \Gamma_{b'}(U, J, \pi, E_k, I_k, \pi_k) + \sum_{I'\pi'} \int_{E_c}^{U-B_{b'}} dE' \, \Gamma_{b'}(U, J, \pi, E', I', \pi') \, \rho_{b'}(E', I', \pi') \right). \tag{2}$$

Here $\sigma^{\mathrm{CN}}(\varepsilon_a)$ is the fusion cross section, ε_a and ε_b are energies of relative motion for incoming and outgoing channels ($\varepsilon_b = U - E_k - B_b$, where B_b is the separation energy of particle b from the compound nucleus), the Γ_b are transmission coefficients of an outgoing particle, and the quantities (U, J, π) and (E, I, π) are the energy, the angular momentum, and the parity of the compound and residual nuclei, respectively. The energy E_c is the discrete levels cutoff, above which the number of levels is calculated using a level density model. For energies below E_c, the known excitation energies, spins, and parities of discrete levels are used. In practice, the E_c is already established in RIPL files with discrete levels, which are used as input files in EMPIRE and TALYS. When comparing calculations with experimental data points, it is important to see how well experimental data are reproduced in both continuum and discrete level regions.

One can see that probability of the compound nucleus formation is determined by the fusion cross section σ^{CN}. Generally, to calculate the fusion reaction cross section, one can use the optical reaction model that is common approach in reaction codes for light projectiles such as neutrons, protons, and α particles. For heavier projectiles, the EMPIRE code is set up to use the barrier distribution and coupled channel models [2]. In our calculations we used a coupled channel model to calculate fusion cross section for ^{7}Li projectiles.

The differential cross section in respect to the energy of the ejectile, such as neutron, proton, or α particle, is determined by particle transmission coefficients and level density of the nucleus populated by this particle. Assuming that transmission coefficients $T(E_p)$ are calculated with a better precision, at least regarding its energy dependence (or functional shape) than the nuclear level density, the energy dependence of differential cross sections are expected to be sensitive to input level density models used in HF calculations. It is also important to mention that a measured proton spectrum includes the first-stage protons from (^{7}Li, p) as well as second- and subsequent-stage protons from (^{7}Li, Xp), where X might, in general, be any number of neutrons, protons, or α particles allowed energetically. The Xp protons as well as X particles populate different nuclei and affected by corresponding level densities of those nuclei. The shape of total proton spectrum in high-energy region, where emitting of a second particle is not possible energetically, is determined by just level density of final nuclei, which are ^{69,71}Ga in our case. A proton spectrum in the low-energy region includes all stage protons and affected by level density of more nuclei populated by corresponding particles. Therefore, the proton spectra measured in the broad range of energies is sensitive to level densities of more than one nuclei allowing testing global systematic parametrization of level density models.

III. COMPARISON WITH MODEL CALCULATIONS

(^{7}Li, Xp) differential cross sections have been calculated with the Hauser-Feshbach model, which is part of the EMPIRE reaction code [2]. Proton optical potentials were parameterized according to the Koning and Delaroche compilation [1]. The coupled channel model was used to calculate the fusion cross section of ^{7}Li + ^{63,65}Cu reactions. This model was tested in our recent work [8] for ^{7}Li-induced reactions on near mass ^{68,70}Zn isotopes and has proven to give accurate results.

The following most popular level density models used in both EMPIRE and TALYS codes were selected to be tested in calculations:

(i) Back-shifted Fermi-gas (BSFG) model with the energy-dependent level density parameter a as suggested in Ref. [12].

$$\rho(U) = \frac{\exp(2\sqrt{aU})}{12\sqrt{2}\sigma a^{1/4} U^{5/4}}, \tag{3}$$

where the effective excitation energy U is determined through the actual excitation energy E and the back-shift parameter δ as $U = E - \delta$. The level density parameter a is determined to be energy dependent according to the Ignatyuk expression of Ref. [12]

$$a(U) = \tilde{a}\left[1 + \frac{\delta W}{U} \cdot (1 - \exp[-\gamma \cdot U])\right], \tag{4}$$

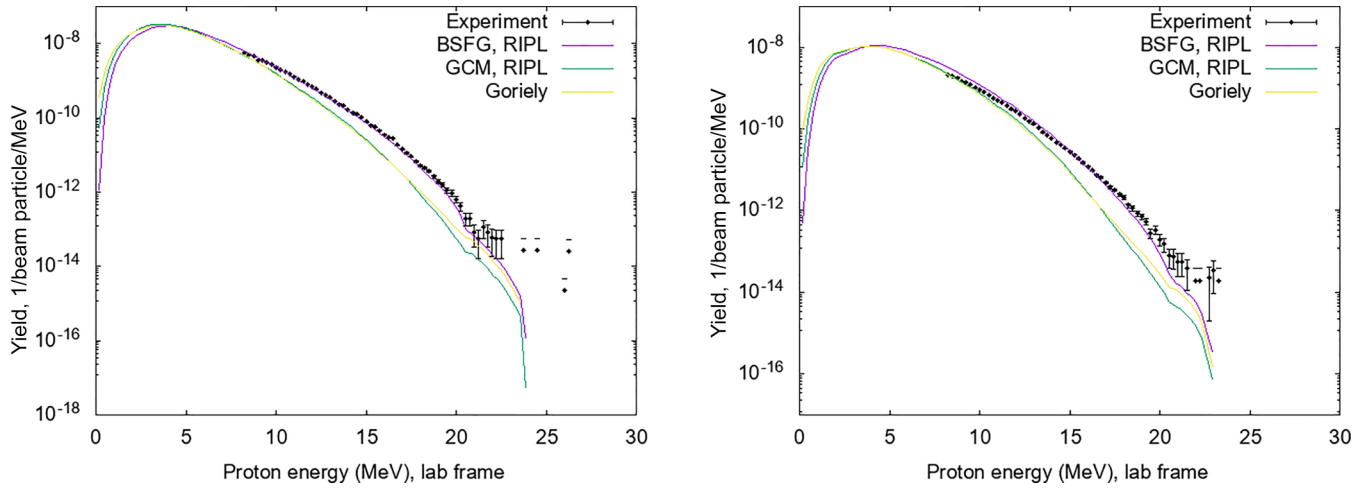

FIG. 2. Proton spectra from ^{63}Cu(^{7}Li, Xp) (left panel) and ^{65}Cu(^{7}Li, Xp) (right panel) compared to model calculations. RIPL and Goriely refer to Refs. [5,14], respectively.

where $\tilde{a}$ is the asymptotic level density parameter, δW is the shell correction, and γ is the damping parameter. Parameters are calculated according to the RIPL global systematics of Ref. [5]. The spin cutoff parameter, $\sigma(U)$, is determined according to the following expression:

$$\sigma^2(U) = 0.0138 \cdot A^{5/3}\sqrt{U \cdot a(U)}/\tilde{a}. \qquad (5)$$

Parameters were used from the reference input parameter library (RIPL) [5]. Since data on neutron resonance spacings are not available for ^{69,71}Ga isotopes, the global parameter systematic formulas were used.

(ii) The Gilbert and Cameron empirical model (GCM) [13] with parameters from Ref. [5]. This model is based on the composite formula consisting of the constant temperature model $\rho(E) = 1/T \exp[(E - E_0)/T]$ with parameters T and E_0, at excitation energies below or at around the neutron separation energy, and the Fermi-gas model (3) at higher excitation energies.

(iii) The microscopic calculations based on Hartree-Fock-Bogolubov (HFB) combinatorial model allowing calculations of parity- and spin-dependent level densities [14]. Results of model calculations are available in table form for all nuclei across the nuclear chart.

EMPIRE code was modified to be able to use RIPL parameter systematics for both BSFG and GCM models. Experimental and calculated proton spectra are presented in Fig. 2. These spectra were calculated in the laboratory frame taking into account the gradual energy loss energy when a beam passes through the target foil. One can see that both the shape and the absolute proton yield for the reaction on ^{63}Cu is well described by calculations using the BSFG level density model, while both GCM and HFB models give a steeper slope, which is not consistent with experimental data points. For the reaction on ^{65}Cu, the BSFG model still gives the best description, however, the slope of the model level density function becomes steeper that causes a small discrepancy with the experimental data points. Both GCM and HFB model calculations result in ^{65}Cu level density slope, which is too steep compared with experimental data points.

The experimental level density excitation energy dependence $\rho(E^*)_{\rm exp}$ is obtained from proton spectra $dY(\varepsilon_p)/d\varepsilon_p$ using the following formula:

$$\rho(E^*)_{\rm exp} = \rho(E^*)_{\rm calc}\frac{(dY(\varepsilon_p)/d\varepsilon_p)_{\rm exp}}{(dY(\varepsilon_p)/d\varepsilon_p)_{\rm calc}} \cdot K_{\rm norm}, \qquad (6)$$

where $K_{\rm norm}$ is a normalization coefficient calculated as a ratio of spectra integrals in the high-energy region between E_1 and E_2, where primary protons populate discrete levels of product nuclei, ^{69,71}Ga in this case, i.e., where the density of levels is known:

$$K_{\rm norm} = \frac{\left(\int_{E_1}^{E_2} dY(\varepsilon_p)/d\varepsilon_p \cdot d\varepsilon_p\right)_{\rm calc}}{\left(\int_{E_1}^{E_2} dY(\varepsilon_p)/d\varepsilon_p \cdot d\varepsilon_p\right)_{\rm exp}}. \qquad (7)$$

Maximum excitation energies up to which energies (but not necessary spins) of all levels are considered to be known are presented in the RIPL-3 data table. For the ^{69,71}Ga nuclei these energies are 3 and 2.1 MeV with number of levels 46 and 23 respectively. In the laboratory frame corresponding energy intervals (E_1, E_2) populated by protons are 20.4–23.3 and 20.5–22.5 MeV. In Fig. 2 there are data points in these energy intervals, which were used to calculate the normalization coefficient (7).

The level densities extracted with Eqs. (6)–(7) are shown in Fig. 3 along with BSFG model calculations. Experimental level densities appear to be almost identical for ^{69,71}Ga while the BSFG model predict them to be different being higher and with a steeper slope for ^{71}Ga. The difference in the model is equally due to both increase of the mass number and to increase of the shell correction factor, which, according to the BSFG with the energy-dependent a parameter [12], results in its increase. These factors cause other models, such as GCM, behave a similar way.

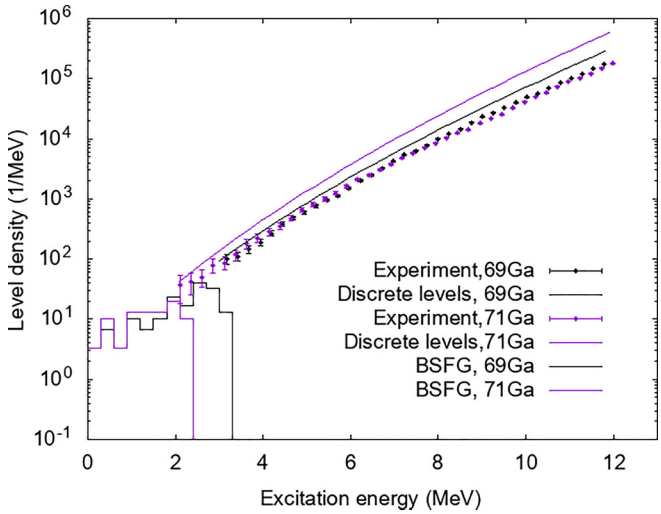

FIG. 3. Level densities of ^{69,71}Ga nuclei compared to model calculations.

IV. COMPARISON WITH PREVIOUS DATA ON LEVEL DENSITIES OF ^{74,76}Ge

In our previous work of Ref. [8] we found that the level densities for ^{74,76}Ge have a flatter excitation energy dependence compared to the prediction of the BSFG model parameterized with data on the neutron resonance spacing. The flatter energy dependence results in a reduced total level density values below the neutron separation energy compared to model predictions. The question of whether this feature indicates a random deviation for particular nuclei, or if it reflects a systematic trend based on some unknown physical features, is yet to be studied. In order to see how level densities of ^{69,71}Ga obtained in this work compare with those for ^{74,76}Ge, all level densities have been fitted with the BSFG model function (3). Parameters of $\tilde{a}$ and δ were obtained and are shown in Table I along with parameters calculated with systematic formulas for the BSFG model from Ref. [5]. The experimental parameters appear to be consistent with the BSFG model predictions for ^{69}Ga, as it is also seen from comparison of the level densities and proton spectra in Figs. 1 and 3. However, the level density parameter $\tilde{a}$ for ^{71}Ga is about the same as for ^{74,76}Ge and is systematically lower than BSFG predictions. The experimental parameter $\tilde{a}$ is about the same within errors for ^{69,71}Ga and ^{74,76}Ge isotopes.

V. CONCLUSION

Comparison of most commonly used in practical calculations level density models with experimental data reveals inconsistency of both GCM and HFB models in comparison with experimental data. The BSFG model based on the RIPL parametrization is consistent with experimental data for ^{69}Ga but produces systematically lower values of the level density parameter $\tilde{a}$ and level density values for ^{71}Ga, which supports our earlier finding for ^{74,76}Ge [8]. If assume that neutron resonance spacing data are accurate in this mass range, the possible explanation might be due to uncertainties in the parity distribution and/or the spin cutoff parameter, which are used to calculate the total level density. It appears that level density model parametrizations, which are solely based on neutron resonance spacing data, are not always capable of reproducing differential reaction cross sections. Such parametrizations still make assumptions about spin and parity distributions, which are not constrained by experimental data. These assumptions might result in incorrect level density values for spin and parities other than ones for neutron resonances. Compiled values for level densities based on resonances counting usually assumed that the spin cutoff parameter is given by the rigid body values. This value is semiclassical and does not require the detailed spin and energy distribution of single-particle states. Two experimental studies give the results that the energy variation of the spin cutoff parameter at the binding energy is not as smooth as a rigid body value estimate implies. Some microscopic calculations also show variations about the rigid body value with A [15,16] at the binding energy, even though they converge to the rigid body value at 20 MeV [17]. Further studies on this potential issue, using different techniques including one based on particle evaporation spectra, would be important for refining the level density models used in applications.

TABLE I. BSFG level density parameters obtained from experiment and from RIPL systematic formulas of Ref. [5] along with shell correction values in Eq. (4).

	^{69}Ga	^{71}Ga	^{74}Ge	^{76}Ge
$\tilde{a}^{\rm exp}$	7.7(2)	7.0(2)	7.5(3)	7.2(2)
$\delta^{\rm exp}$	0.29(28)	−0.13(24)	1.07(38)	0.87(17)
$\tilde{a}^{\rm sys}$	8.3	8.5	8.8	9.0
$\delta^{\rm sys}$	0.5	0.2	1.0	1.4
δW	2.18	3.00	3.22	2.78

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