Phenomenological reexamination of two-photon-exchange corrections to the proton recoil-polarization ratio $\mu_p G_E/G_M$ utilizing electron-proton elastic scattering experimental data

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(Received 23 October 2023; revised 10 February 2024; accepted 18 March 2024; published 2 April 2024)

In this work, we have phenomenologically reexamined the ε dependence of the proton's recoil-polarization ratio $R = \mu_p G_E/G_M$ for possible two-photon-exchange (TPE) corrections beyond the one-photon-exchange (OPE) or Born approximation. High-precision Rosenbluth measurements of $\sigma_R(\varepsilon, Q^2)$ taken at $Q^2 = 2.64$, 3.20, and 4.10 GeV^2 were used to extract the TPE coefficients needed to provide an estimate of the size of the TPE corrections to the ratio R, and construct the two TPE $Y_{(M,E)}(\varepsilon, Q^2)$ amplitudes. Our results suggest that the ratio R shows very small (negligible) enhancement with ε, less than 0.40% in magnitude, with values consistent with the OPE prediction for the Q^2 range studied. We also compare our results to previous phenomenological extractions and several theoretical calculations.

DOI: 10.1103/PhysRevC.109.045202

I. INTRODUCTION

The proton's elastic electromagnetic form factors (FFs), electric $G_E(Q^2)$ and magnetic $G_M(Q^2)$ FFs, are key ingredients needed to characterize the internal structure of the proton, which help enhance and extend our understanding of hadronic physics and quantum chromodynamics (QCD). They are also key inputs to many studies and analyses aimed at understanding composite particles and their nuclear structures [1–5]. However, the experimentally reported proton's FF ratio $R = \mu_p G_E/G_M = \mu_p R_p$ as measured using the Rosenbluth separation method [6] and the high-Q^2 recoil polarization method [7–9] in the one-photon-exchange (OPE) or Born-approximation differs almost by a factor of three at high Q^2 [10–12]. Such discrepancy in the ratio R has suggested a systematic difference between the two techniques. To reconcile the two ratios, it was speculated that missing higher-order radiative corrections, and in particular a two-photon exchange (TPE) correction [13–15] to the elastic electron-proton (ep) scattering cross section $\sigma_R(\varepsilon, Q^2)$, should be applied. Here, ε is the virtual photon longitudinal polarization parameter, and Q^2 is the four-momentum transfer squared.

Several dedicated studies aimed at understanding the effect of TPE on elastic ep scattering observables were performed. On the theoretical side [16–60], the TPE corrections are rather small, on the few percentile level, but exhibit significant angular dependence at high Q^2. Phenomenologically [13,61–87], several studies have attempted to quantify the size of TPE contributions to σ_R by using combined ep elastic-scattering experimental data, and assuming that the recoil-polarization ratio R is rather insensitive to ε, the TPE effect is linear in ε and does not destroy the experimentally observed linearity in σ_R, and it vanishes as $\varepsilon \to 1$ (Regge limit) [13,63–77]. Other studies tried to extract the TPE amplitudes using fewer assumptions and constraints but rather with relatively large uncertainties [66,67,69,76,79]. See Refs. [88–90] for detailed reviews. On the experimental side, some studies focused on examining the ε dependence of σ_R [63,64,68], but no deviation from linearity as predicted in the Born-approximation was observed. Some measurements were performed to examine the ε dependence of the ratio R at $Q^2 = 2.50$ GeV^2 [91], GEp-2γ Collaboration, and the ratio was found to be essentially independent of ε, in agreement with the OPE prediction. Some experiments were dedicated to measure the size of TPE contributions by measuring the positron-proton and electron-proton elastic-scattering cross sections ratio $R_{e^+e^-}(\varepsilon, Q^2)$ [92–95], because any deviation of the ratio $R_{e^+e^-}$ from unity is an evidence for TPE effect. All these measurements were carried out for $Q^2 \leqslant 2.10$ GeV^2, below where the discrepancy on the ratio R is significant, and reported significant TPE contributions at low ε and moderate Q^2, in agreement with hadronic TPE predictions [17]. The CLAS [92,93] and VEPP-3 Collaborations [94] measured the ratio $R_{e^+e^-}$ in the range 0.2 GeV$^2 \leqslant Q^2 \leqslant 1.5$ GeV^2, and provided precise measurements of $R_{e^+e^-}$ at $Q^2 \approx 1.0$ and 1.5 GeV^2. The measured ratio $R_{e^+e^-}$ is larger than unity and exhibits ε dependence at low ε points, which is a clear evidence for a sizable hard-TPE correction at larger Q^2 values consistent with the ratio R discrepancy at Q^2 values of 1.0–1.6 GeV^2. In addition, the ratio $R_{e^+e^-}$ showed clear deviation and change of sign from the exact calculations, high proton mass limit at $Q^2 = 0$ [96], and the finite-Q^2 calculations for a point-proton [88]. The OLYMPUS experiment [95] measured the ratio $R_{e^+e^-}$ in the range 0.165 GeV$^2 \leqslant Q^2 \leqslant 2.038$ GeV^2. The measured ratio is below unity at high ε points, showing a dip below unity for $\varepsilon \geqslant 0.7$, and then it changes sign and starts to increase gradually, above unity, with decreasing ε showing a clear enhancement for $\varepsilon \leqslant 0.6$ reaching about 2% at $\varepsilon = 0.46$. As world data on the ratio $R_{e^+e^-}$ are all accumulated for

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$Q^2 < 2.1$ GeV^2, below the region where the discrepancy on the ratio R is significant, the assumption whether hard-TPE corrections could account for the discrepancy on R is still an open question. Therefore, precise $R_{e^+e^-}$ measurements at high Q^2 are clearly needed. In addition, the $Q^2 = 2.50$ GeV^2 value used in Ref. [91] to measure and examine the ε dependence of the ratio R was not a good choice for the experiment because the TPE correction to R is very small as the elastic and $\Delta(1232)$ resonance contributions very much cancel each other [66]. Therefore, in this work we extend the Q^2 range and reexamine phenomenologically the ε dependence of the ratio R at $Q^2 = 2.64$, 3.20, and 4.10 GeV^2. We provide an estimate of the size of the TPE correction to the ratio R and extract the TPE amplitudes using combined unpolarized and polarized ep elastic scattering experimental data.

II. TWO-PHOTON-EXCHANGE AMPLITUDES

Depending on the Q^2 values, the TPE amplitudes (generalized form factors) are usually calculated based on hadronic- and pQCD-based calculations. At small to moderate Q^2 values, the hadronic approach is mainly valid, and the TPE processes are typically mediated by the production of virtual hadrons and/or hadronic resonances in the intermediate state. Based on the hadronic intermediate state involved, the TPE amplitudes can be described in terms of elastic pure nucleon, and inelastic multiparticles processes such as $p\pi$, $p\pi\pi$ with emphasis on the $\Delta(1232)$ resonance, Roper resonance, and πN (pion + nucleon) contributions. The πN contribution can also split further into contributions coming from different particle waves or channels such as the P_{33} channel and higher total angular momentum $J = 1/2$ and 3/2 contributions. These contributions are only the first terms of an infinite expansion of the total πN contribution, and it is unclear when such a series will converge eventually. At high Q^2 values, the quark approach is applicable, and the nucleon is treated as a system of interacting partons in the intermediate state, where their interactions are described by perturbative quantum chromodynamics (pQCD).

In the hadronic-type approach, the most important contribution is the elastic contribution, which impacts mainly G_M. Kondratyuk *et al.* [18] investigated the effect of adding the $\Delta(1232)$ resonance [18], assuming zero width for the $\Delta(1232)$ resonances, and other several light resonances [19] on the cross section. The effect is smaller than the elastic contribution, with the largest contribution coming from the $\Delta(1232)$ resonance, and other resonances contributions ended up partially canceling each other. Ahmed, Blunden, and Melnitchouk [54] calculated the contributions of excited intermediate-state resonances to the TPE corrections to σ_R in the range of 0.50 GeV$^2 \leqslant Q^2 \leqslant 5.00$ GeV^2. They accounted for all four- and three-star spin-1/2 and spin-3/2 resonances with mass below 1.80 GeV, including the six isospin-1/2 states $N(1440)1/2^+$, $N(1520)3/2^-$, $N(1535)1/2^-$, $N(1650)1/2^-$, $N(1710)1/2^+$, and $N(1720)3/2^+$ and the three isospin-3/2 states $\Delta(1232)3/2^+$, $\Delta(1620)1/2^-$, and $\Delta(1700)3/2^-$. In the low-Q^2 region up to ≈ 1 GeV^2 and aside from the $\Delta(1232)3/2^+$, both the $N(1520)3/2^-$ and $N(1535)1/2^-$ resonances gave the most significant contributions and with almost a complete cancellation of the $N(1520)3/2^-$ contribution by that coming from the sum of other higher-mass resonances. That results in a net correction that is well approximated by that coming from the $\Delta(1232)3/2^+$ resonance. For $Q^2 \geqslant 2.0$ GeV^2, the $\Delta(1232)3/2^+$ contribution to the TPE correction is overtaken by that of the $N(1520)3/2^-$, which has opposite sign, and therefore the suppression of the TPE cross section relative to the nucleon elastic contribution by the $\Delta(1232)3/2^+$ is neutralized by the $N(1520)3/2^-$ contribution. Therefore, for $Q^2 \geqslant 3.0$ GeV^2, higher mass resonances increase the magnitude of the TPE correction to σ_R and this is due mainly to the growth of the negative $N(1520)3/2^-$ and $N(1535)1/2^-$ resonances contributions, where they tend to overcompensate the positive $\Delta(1232)3/2^+$ contributions. Blunden and Melnitchouk [44] examined the TPE corrections to σ_R within the dispersive approach, where they included both the nucleon and intermediate states involving the spin-3/2 and isospin-3/2 Δ baryons, and also reported a suppression of the $\Delta(1232)$ contributions to the TPE cross section. Kondratyuk and Blunden [19] performed calculation of resonance TPE contributions and have also identified the $N(1520)3/2^-$ and $\Delta(1232)3/2^+$ resonance contributions as the most significant. Their nucleon and $\Delta(1232)$ contributions at $Q^2 = 4.0$ GeV^2 and small ε are in excellent agreement with those obtained in Ref. [54], however, their $N(1520)$ is smaller in magnitude due to the different parametrization used of the resonance electrocoupling at the hadronic vertices. In a similar study, Borisyuk and Kobushkin [34] also investigated the impact of adding the $\Delta(1232)$ resonance contribution with zero width on both the cross section and the TPE amplitudes. The $\Delta(1232)$ resonance has affected mainly the $\delta G_E/G_M$ amplitude, while the elastic contribution impacted G_M. The size of the $\Delta(1232)$ resonance contribution is found to grow in magnitude with increasing Q^2, exceeding that of the elastic contribution at large Q^2, resulting in a relatively large correction to the recoil-polarization ratio R. For $Q^2 > 3.0$ GeV^2, the total correction to R coming from both elastic and $\Delta(1232)$ contributions $\delta R = (\delta^{el} + \delta^{\Delta})$ was way larger than the experimentally quoted systematic uncertainty. However, when the δR correction was applied to the experimental R data, R became negative at $Q^2 = 8.5$ GeV^2. The effect of πN hadronic intermediate state with emphasis on the P_{33} channel including a realistic $\Delta(1232)$ resonance width, shape, and corresponding background [35] is also investigated. The $\Delta(1232)$ resonance contribution is found to be very negligible compared with the elastic one at low Q^2. However, the impact on the TPE $\delta G_E/G_M$ amplitude is large and grows in magnitude for $Q^2 \geqslant 2.50$ exceeding that of the elastic intermediate state in agreement with their previous results [34]. However, the size of the correction to the ratio R at high Q^2 is $\approx$30% smaller than their previous results. Calculations of the TPE amplitudes accounting for hadronic intermediate state, which included the πN system with higher total angular momentum, $J = 1/2$ and 3/2, for eight πN different channels were also performed [36]. In these calculations, a finite resonance width, realistic resonance shape and form factors, and nonresonant background were considered. The largest contributions came from channels with quantum numbers of the lightest resonances dominated mainly by the contribution of the P_{33} channel. However, the correction to the ratio R at high Q^2 is smaller but sizable and grows roughly linearly with increasing Q^2

as a result of cancellation of the different channels contributions. Tomalak, Pasquini, and Vanderhaeghen [46] evaluated the TPE correction to σ_R within the dispersive framework in the extended range 0.064 GeV$^2 \leqslant Q^2 \leqslant$ 1.0 GeV^2. In addition to the elastic contributions, they have accounted for all πN intermediate-state contributions. A contour integration in the complex plane was performed in order to evaluate the imaginary part of the TPE amplitudes with nucleon intermediate state. To utilize the dispersion relation, the contour was allowed to be deformed and a dipole shaped nucleon form factors were used so as the integral can be analytically continuous into some unphysical region. They also used input pion electroproduction amplitudes taken from the MAID 2007 parametrizations [97]. Their results are in good agreement with the empirical extraction of the TPE cross section of Ref. [61] at high ε, but disagree at low ε.

At high Q^2 values, the TPE corrections are calculated mainly within the framework of GPDs [26,27] and pQCD [20–22,33,43]. Afanasev *et al.* [27] calculated TPE corrections to σ_R assuming different formalisms for GPDs. Calculations of TPE corrections to σ_R within the framework of pQCD for a proton target incorporating wave functions based on QCD sum rules were performed by Borisyuk and Kobushkin [33]. The TPE $\delta G_M/G_M$ amplitude shows linearity in ε, and grows logarithmically with increasing Q^2 reaching about 3.5% of the Born amplitude at $Q^2 = 30.0$ GeV^2. They also suggested the possibility of smooth connection with hadronic calculations, assuming an elastic intermediate state, at lower Q^2 values. At high Q^2, on the other hand, both the GPDs- and pQCD-type calculations yielded different results, indicating the inadequacy of the hadronic approach for $Q^2 \geqslant 3.0$ GeV^2.

Calculations of TPE corrections to σ_R based on QCD factorization approach within the framework of the soft-collinear effective theory (SCET) arising from both the soft- and hard-spectator scattering contributions at moderately large Q^2 values of 2.64, 3.20, and 4.10 GeV^2 were also performed [22]. The cross section σ_R showed some deviation from linearity at small ε at $Q^2 = 2.64$ GeV^2. The nonlinearity with ε increased with increasing Q^2 in agreement with hadronic-type calculations at moderate Q^2 values. The two TPE Y_M and Y_3 amplitudes showed weak Q^2 dependence and behaved oppositely with ε with nonvanishing values as $\varepsilon \to 1$.

III. PHENOMENOLOGICAL TWO-PHOTON EXCHANGE APPROACH

In this section, we lay down the procedure together with the constraints and assumptions used to estimate the size of the TPE correction to the recoil-polarization ratio R and extract the TPE amplitudes. Based on the theoretical framework of Borisyuk and Kobushkin [66], the reduced cross section σ_R and the recoil-polarization ratio R after correction for TPE contributions are expressed as

$$\sigma_R(\varepsilon, Q^2) = G_M^2\left[1 + \frac{\varepsilon R_p^2}{\tau} + 2Y_M(\varepsilon, Q^2) + \frac{2\varepsilon R_p^2}{\tau}Y_E(\varepsilon, Q^2)\right], \tag{1}$$

$$R(\varepsilon, Q^2) = R_p\left[1 + Y_E(\varepsilon, Q^2) - Y_M(\varepsilon, Q^2) - \frac{\varepsilon(1-\varepsilon)}{1+\varepsilon}Y_3(\varepsilon, Q^2)\right], \tag{2}$$

where G_M is the true magnetic FF of the proton, R_p is the Born value of the recoil-polarization ratio R, and $Y_{(E,M,3)}$ are the real parts of the TPE amplitudes and functions of ε and Q^2.

(1) As the TPE amplitudes are on the few percentile levels, the last term $\varepsilon(1-\varepsilon)/(1+\varepsilon)Y_3$ in Eq. (2) is too small compared with the remaining terms and can be safely neglected. Therefore, in this work, we only extract the two TPE amplitudes $Y_{(M,E)}(\varepsilon, Q^2)$.

(2) Based on Eq. (2), the ratio R is a function of both ε and Q^2. Therefore, we Taylor expand $R(\varepsilon, Q^2)$ in terms of ε and Q^2 as $R(\varepsilon, Q^2) = a_0(Q^2) + a_1(Q^2)\varepsilon + a_2(Q^2)\varepsilon^2$, where $a_0(Q^2) = R_p$ is the Born (OPE) value of R and is a function of Q^2 only. Because the TPE correction to R vanishes as $\varepsilon \to 1$ (Regge limit), that suggests that $a_1(Q^2) = -a_2(Q^2)$, and the ratio R can finally be expressed as

$$R(\varepsilon, Q^2) = R_p + a_1(Q^2)\varepsilon(1-\varepsilon), \tag{3}$$

where the second term represents the TPE correction to the ratio R.

(3) Equating the ratio R from Eqs. (2) and (3), we can express the two TPE amplitudes Y_E and Y_M as

$$Y_E(\varepsilon, Q^2) = Y_M(\varepsilon, Q^2) + \frac{a_1\varepsilon(1-\varepsilon)}{R_p}. \tag{4}$$

(4) Because of the experimentally observed linearity of σ_R, where σ_R exhibits no (or very weak) nonlinearity in ε, and to reserve as possible the linearity of σ_R, we parametrize $Y_M(\varepsilon, Q^2)$ linearly in ε as

$$Y_M(\varepsilon, Q^2) = b_1(Q^2)(\varepsilon - 1), \tag{5}$$

which clearly vanishes in the Regge limit.

(5) Using Eqs. (4) and (5), σ_R as given by Eq. (1) is now expressed as

$$\sigma_R(\varepsilon, Q^2) = G_M^2\left[1 + \frac{\varepsilon R_p^2}{\tau} + 2\left(1 + \frac{\varepsilon R_p^2}{\tau}\right)b_1(Q^2)(\varepsilon - 1) + \frac{2a_1(Q^2)R_p}{\tau}\varepsilon^2(1-\varepsilon)\right]. \tag{6}$$

(6) Precision Rosenbluth measurements of σ_R from Ref. [10] taken at $Q^2 = 2.64$, 3.20, and 4.10 GeV^2 are used in this analysis. At fixed Q^2 value, we constrain both the ratio R_p and G_M^2 in Eq. (6). For R_p, we constrain its value along with its associated uncertainty to $\mu_p R_p = [1/1 + 0.1430Q^2 - 0.0086Q^4 + 0.0072Q^6]$, with an absolute uncertainty

$\delta^2_{R_p}(Q^2)=\mu_p^{-2}\{(0.006)^2 + [0.015\ln(1+Q^2)]^2\}$ based on the parametrization of Ref. [75]. For $G_M^2(Q^2)$, and because of the experimentally observed linearity of σ_R with ε, we first fit σ_R linearly to the form $\sigma_R = [c_1(Q^2) + c_2(Q^2)\varepsilon]$, where $c_{(1,2)}(Q^2)$ are the fit parameters, and then we extract $G_M^2(Q^2)$ by equating the two expressions for $\sigma_R(\varepsilon = 1, Q^2)$ yielding $G_M^2(Q^2) = [c_1(Q^2) + c_2(Q^2)]/[1 + (R_p^2/\tau)]$ [69,76]. Finally, σ_R data at $Q^2 = 2.64$, 3.20, and 4.10 GeV^2 are fitted to Eq. (6), and the parameters of the fit $a_1(Q^2)$ and $b_1(Q^2)$ at each Q^2 value are extracted. The TPE corrections to R and the TPE amplitudes at each Q^2 value are then constructed using Eqs. (3), (4), and (5). This will be referred to as the "σ_R Fit" throughout the text.

Relevant to this work, Guttmann *et al.* [69] (based on the theoretical framework of Guichon and Vanderhaeghen [13]) determined the ε dependence of the three TPE amplitudes $Y_{(M,E,3)}$ around $Q^2 = 2.50$ GeV^2 using high-precision data on the ratios R and $P_l/P_l^{\rm Born}$ determined by the GEp-2γ Collaboration [91], and σ_R at $Q^2 = 2.64$ GeV^2 from the super-Rosenbluth experiment [10]. The ratio R was fitted to the functional form $R = R_p + B\varepsilon^c(1-\varepsilon)^d$, where R_p is the Born (OPE) value, and B, c, and d are constants. The constant B was effectively zero for a range of different values of c and d, and all values of R were equal within their error bars. They concluded that the precision of the experimental data of Ref. [69] did not allow for the ε-dependent part in addition to the constant value of R to be extracted. They fitted R to its Born value, which yielded: $R = 0.693 \pm 0.006_{\rm stat} \pm 0.010_{\rm sys}$. The ratio $P_l/P_l^{\rm Born}$ shows a decrease for $\varepsilon \to 0$. Although in qualitative agreement with perturbative QCD (pQCD) [20,33], the ratio $P_l/P_l^{\rm Born}$ at $Q^2 = 2.50$ GeV^2 falls off faster than the pQCD prediction. Therefore, the ratio $P_l/P_l^{\rm Born}$ was fit to two different functional forms: $P_l/P_l^{\rm Born} = 1 + a_l\varepsilon^4(1-\varepsilon)^{1/2}$ (Fit I), and $P_l/P_l^{\rm Born} = 1 + a_l\varepsilon\ln(1-\varepsilon)(1-\varepsilon)^{1/2}$ (Fit II), giving a value of $a_l = 0.11 \pm 0.03_{\rm stat} \pm 0.06_{\rm sys}$ for Fit I, and $a_l = -0.032 \pm 0.008_{\rm stat} \pm 0.020_{\rm sys}$ for Fit II. Throughout the text, the TPE amplitudes extracted using Fit I (II) will be referred to as the "Guttmann Fit I" ("Guttmann Fit II"). Note that our functional form for R, Eq. (3), is similar to that used in Ref. [69] with $B = c = d = 1$, although no justification was given to why such a functional form was used in Ref. [69]. It is worth mentioning here that the three TPE amplitudes $Y_{(M,E,3)}(\varepsilon, Q^2)$ defined in Ref. [13] and extracted in Refs. [69,76] are the real parts of the TPE amplitudes relative to the magnetic form factor G_M (generalized form factors): $Y_M = \delta G_M/G_M$, $Y_E = \delta G_E/G_M$, and $Y_3 = (\nu/M^2)\delta G_3/G_M$. On the other hand, the TPE amplitudes $Y_{(M,E,3)}(\varepsilon, Q^2)$ based on the framework of Borisyuk and Kobushkin [66], Eqs. (1) and (2), are defined as $Y_M = \delta\mathcal{G}_M/G_M$ and $Y_E = \delta\mathcal{G}_E/G_E$, and $Y_3 = \delta\mathcal{G}_3/G_M = (\nu/4M^2)\delta G_3/G_M$. While the amplitude Y_3 is the same in both Refs. [13,66] with the definition of ν differs by a factor of four, the two amplitudes $Y_{(M,E)}$ are different. For better comparison between the different theoretical extractions, we put the $Y_{(M,E)}$ amplitudes from Refs. [13,66] on equal footing and relate them using $Y_M = \delta\mathcal{G}_M/G_M = (\delta G_M/G_M + \varepsilon Y_3)$ and $Y_E = \delta\mathcal{G}_E/G_E = (\delta G_E/G_M + Y_3)/R_p$.

IV. RESULTS AND DISCUSSION

Following the procedure outlined in Sec. III, we fit precision σ_R measurements from Ref. [10] taken at $Q^2 = 2.64$, 3.20, and 4.10 GeV^2 to Eq. (6), and extract the TPE coefficients $a_1(Q^2)$ and $b_1(Q^2)$ at each Q^2 value. Figure 1 shows the results of the fit at the Q^2 value listed in the figure. Excellent fits were achieved, with the reduced χ^2 values of the fit χ^2_ν are generally reasonable with values $\chi^2_\nu = 0.37$, 0.51, and 0.13, for $Q^2 = 2.64$, 3.20, and 4.10 GeV^2, respectively. The results of the fits are listed in Table I. Unlike the TPE coefficient $b_1(Q^2)$, the coefficient $a_1(Q^2)$ is clearly less constrained with uncertainty way larger than its central value. In addition, we perform a simultaneous global fit combining the data for the ε dependence of R at $Q^2 = 2.50$ GeV^2 from Ref. [91], 3ε points, and σ_R measurements at a similar Q^2 value of 2.64 GeV^2 from Ref. [10] with 5ε points. When performing the fit, Eq. (3) for R and Eq. (6) for σ_R were used, and the best fit parameters, the TPE coefficient $a_1(Q^2)$ and $b_1(Q^2)$, were obtained by minimizing the χ^2 function defined as

$$\chi^2 = \sum_{i=1}^{N_\sigma}\left[\frac{\sigma^{(i)}_{R,\rm data} - \sigma^{(i)}_{R,\rm theo.}}{\Delta\sigma^{(i)}_{R,\rm data}}\right]^2 + \sum_{i=1}^{N_{\rm pol.}}\left[\frac{R^{(i)}_{p,\rm data} - R^{(i)}_{p,\rm theo.}}{\Delta R^{(i)}_{p,\rm data}}\right]^2. \tag{7}$$

As the measurements were taken at slightly different Q^2 value, we set the Born R_p value in Eq. (2) to $R_p = 0.6930 \pm 0.006$, as given by Ref. [69], and $R_p = 0.6896 \pm 0.020$ in Eq. (6), as determined based on the parametrization of Ref. [75], to account for the difference. Note, however, that the one-parameter fit of Ref. [91] yielded $R = R_p = 0.6923 \pm 0.0058$. This will be referred to as the "Glob. Fit" throughout the text. A reasonable fit was achieved with $\chi^2_\nu = \chi^2/\nu = 1.32$ for $\nu = (N_{\rm point} - N_{\rm param.}) = 6$ degrees of freedom. The global fit at $Q^2 = 2.64$ GeV^2 is in excellent agreement with the σ_R Fit and is also shown in Fig. 1 for comparison. While both fits yield a consistent value for $b_1(Q^2)$ at $Q^2 = 2.64$ GeV^2, the $a_1(Q^2)$ value obtained using the global fit has an opposite sign with much improved uncertainty compared with that obtained using the σ_R Fit.

Having obtained the TPE coefficients $a_1(Q^2)$ and $b_1(Q^2)$, the ε dependence of the ratio R and the two TPE amplitudes $Y_{(E,M)}$ can now be constructed at each Q^2 value. Figure 2 shows the ε dependence of the ratio R as extracted from this work. For $Q^2 = 2.64$ GeV^2, we show the results of both the σ_R and global fits along with their uncertainty bands as computed using the covariance matrix of the fits. Note that the one-point experimental R value from Ref. [98], shown as a solid red triangle in Fig. 2, was not included in the Global Fit and was only shown for comparison. We also compare the results to several previous theoretical predictions: hadronic calculations which account for all the proton intermediate states [17] "Hadronic," partonic calculations which account for hard scattering of the electron by embedded quarks inside the nucleon through generalized parton distributions (GPDs) [27] "GPD," perturbative QCD (pQCD) calculations [20], which used two different light-front-proton distribution amplitude parametrizations from Ref. [60] "COZ," and Ref. [43] "BLW," and electron structure function calculations which account for

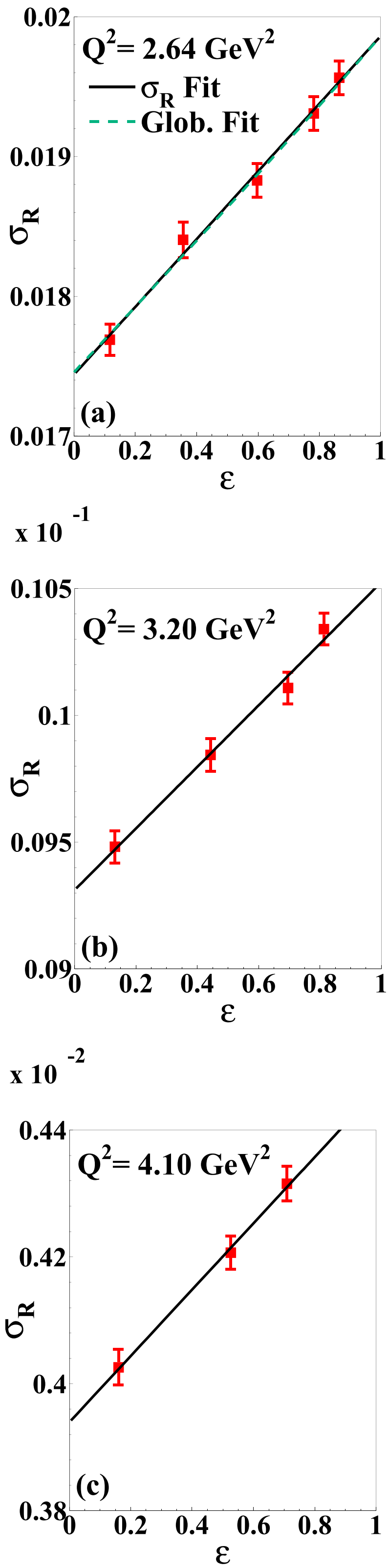

FIG. 1. The reduced cross section σ_R as a function of ε (solid red squares) from Ref. [10] at the Q^2 value listed in the figure. Also shown are our σ_R Fit based on Eq. (6) (solid black line), and the global fit based on Eq. (3) (long dashed dark-green line) for comparison. The global fit is only performed at $Q^2 = 2.64$ GeV^2.

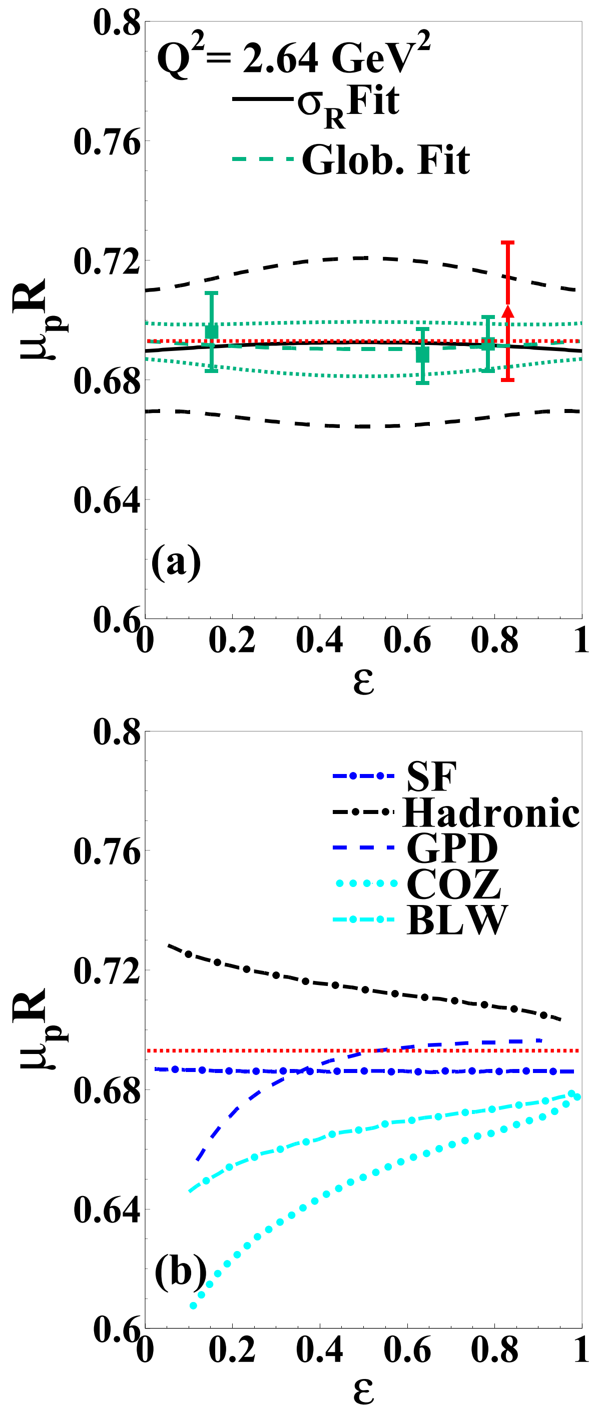

FIG. 2. (a) The ratio R as a function of ε at $Q^2 = 2.50$ GeV^2 from Ref. [91] (solid dark-green squares) and Ref. [98] (solid red triangle). Also shown are our σ_R Fit (solid dark line), "Glob. Fit" (long dashed dark-green line) at $Q^2 = 2.64$ GeV^2 along with their uncertainty bands (long dashed black line) and (small dashed dark-green line), respectively, and the Born set value (small dashed-red line) as given by Ref. [69]. (b) The theoretical predictions: "Hadronic" [17] (long dashed-dotted black line), "GPD" [27] (long dashed blue line), pQCD [20] ("COZ" [60] (large dotted cyan line), and "BLW" [43] (long dashed-dotted cyan line), and "SF" [28] (long dashed-dotted blue line).

TABLE I. The values of the fit parameters for the TPE coefficients $a_1(Q^2)$ and $b_1(Q^2)$. The χ^2_ν value of the fit is also listed.

Fit type	Q^2 (GeV^2)	$(b_1 \pm \Delta_{b_1}) \times 10^{-2}$	$(a_1 \pm \Delta_{a_1}) \times 10^{-2}$	χ^2_ν
σ_R Fit	2.64	$+2.49 \pm 0.32$	$+0.42 \pm 2.80$	0.37
Global Fit	2.64	$+2.48 \pm 0.39$	-0.39 ± 0.98	1.32
σ_R Fit	3.20	$+3.27 \pm 0.36$	-0.54 ± 3.62	0.51
σ_R Fit	4.10	$+4.53 \pm 0.41$	$+0.41 \pm 6.10$	0.13

all higher-order radiative corrections in the leading-logarithm approximation from Ref. [28] "SF." Note that our results based on the two fits have slightly different R value at $\varepsilon = 0$ (Born R_p value) due to the slightly different constrained R_p value in each fit. All theoretical calculations have widely predictions and sizable ε dependence for R, except for the "SF" calculations which do not predict any measurable ε dependence. Note that the "SF" prediction in Fig. 2 has been offset by -0.0067 for clarity with respect to the one-parameter fit result of $R = 0.6923 \pm 0.0058$ obtained by Ref. [91]. Both fits suggest that the ratio R exhibits no (very negligible) ε dependence, with R reaching maximum deviation from its Born R_p value of $\delta_R = R - R_p = +0.29\%(-0.27\%)$ at $\varepsilon = 0.50$ based on the σ_R (Global) fit result. Our results are in excellent agreement with the experimental data within their uncertainties, and in a good agreement with the "SF" calculations, but disagree strongly with the other shown theoretical predictions.

For $Q^2 = 3.20$ GeV^2, the ratio R decreases below its Born R_p value with increasing ε reaching maximum deviation of $\delta_R = -0.38\%$ at $\varepsilon = 0.50$. The ratio then starts to increase again reaching R_p at $\varepsilon = 1$. For $Q^2 = 4.10$ GeV^2, the ratio R behaves rather differently as it increases above its Born R_p value with increasing ε reaching maximum deviation of $\delta_R = +0.29\%$ at $\varepsilon = 0.50$ and then starts to decrease again reaching R_p at $\varepsilon = 1$. Therefore, and for the three Q^2 points considered in this study, the ratio R shows very small (negligible) enhancement with ε, with a value consistent with the Born-approximation (OPE) prediction.

Figure 3 shows the ε dependence of the two TPE amplitudes $Y_{(M,E)}$ at $Q^2 = 2.64$ GeV^2 as extracted from this work based on both fits. We also compare our results to previous phenomenological extractions from Ref. [76], labeled "IQ," and Ref. [69], labeled Guttmann Fit I and Guttmann Fit II." We also compare our results to several previous hadronic TPE calculations assuming different intermediate states: elastic "BK: elastic" [32], elastic + $\Delta(1232)$ resonance "BK: elastic + $\Delta(1232)$" [34], elastic + πN (P_{33} channel) "BK: elastic + P_{33}" [35], elastic + πN (spin-1/2 and -3/2 channels) "BK: elastic + πN" [36], and to calculations based on QCD factorization within the SCET approach from Ref. [22] "KV." For the KV calculations, we only show the Y_M amplitude but not Y_E as the latter requires knowledge of the quark transverse momenta distribution and cannot be calculated in a leading twist QCD-type calculation.

We start by examining Y_M. The amplitude is on the few-percentage-point level, and behaves linearly in ε. Note that both fits yield identical results for Y_M, and so Y_M based on the σ_R Fit result is only shown. Our results in general are in reasonably good qualitative agreement with previous IQ, Guttmann Fit I, and Guttmann Fit II phenomenological extractions, and hadronic and KV theoretical calculations showing the overall falloff of Y_M with decreasing ε, but they show clear deviation from the IQ extraction at low ε. For Y_E, the

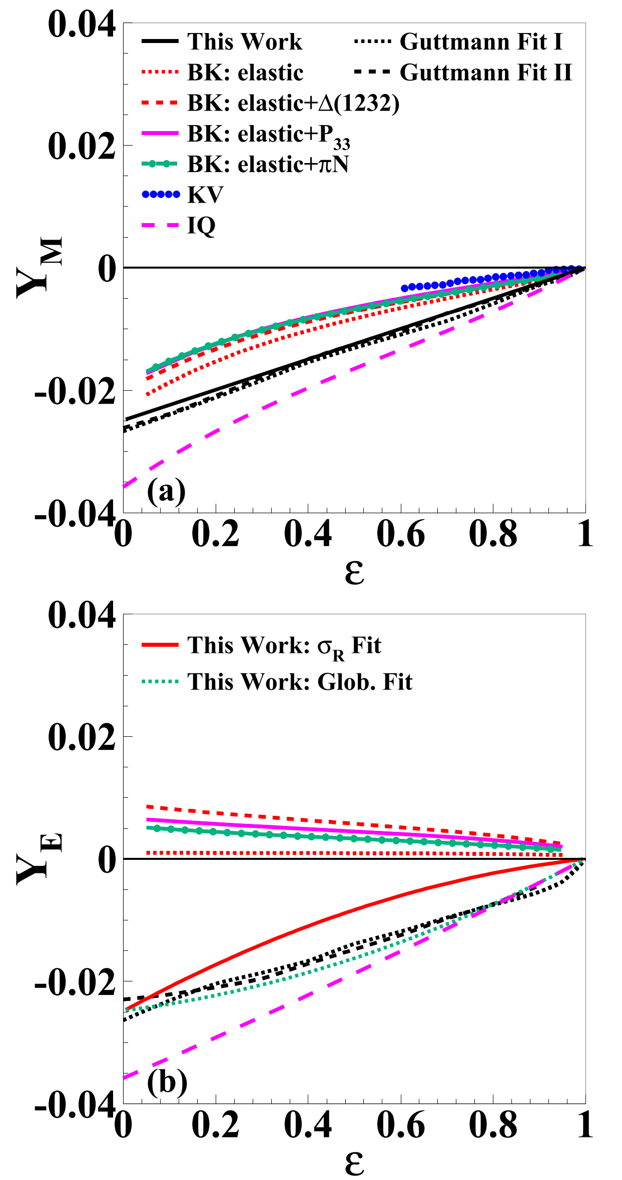

FIG. 3. The ε dependence of the TPE amplitudes at $Q^2 = 2.64$ GeV^2 from this work: Y_M (top) (solid black line), and Y_E (bottom) (σ_R Fit: solid red line, and Glob. Fit: small dashed dark-green line). In addition, we compare our results to previous phenomenological extractions from Ref. [76] IQ (dashed magenta line), and Ref. [69] Guttmann Fit I (dashed black line) and Guttmann Fit II (long-dashed black line). Also shown are several previous hadronic TPE calculations: BK: elastic [32] (dashed red line), BK: elastic + $\Delta(1232)$ [34] (long-dashed red line), BK: elastic + P_{33} [35] (solid magenta line), BK: elastic + πN [36] (dashed-dotted dark-green line), and calculations based on QCD factorization within the SCET approach KV [22] (dotted blue line).

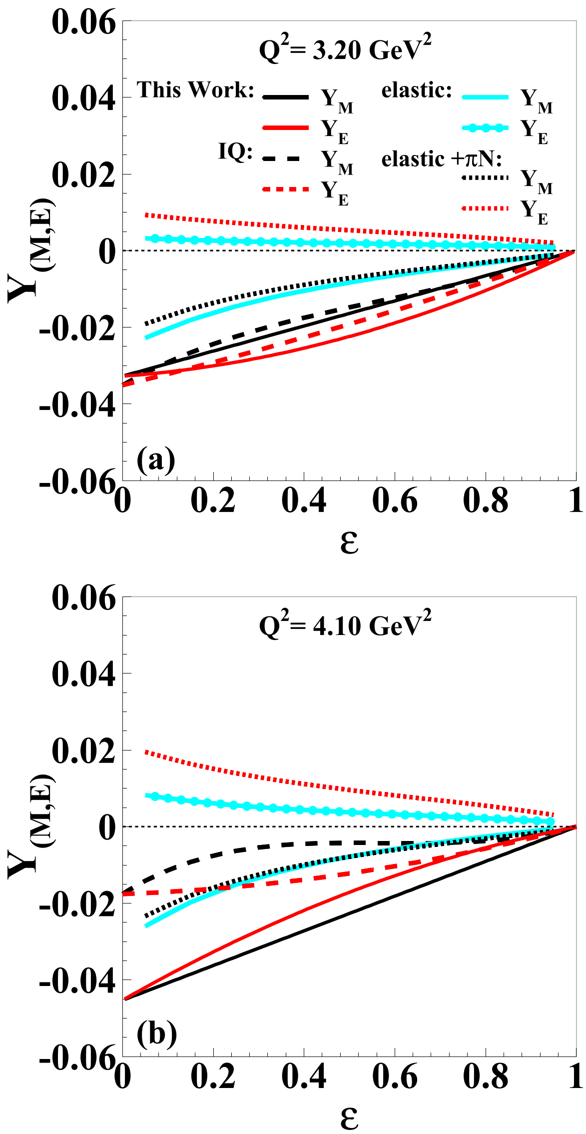

FIG. 4. The ε dependence of the TPE amplitudes $Y_{(M,E)}$ as extracted from this work at the Q^2 listed in the figure: This Work: Y_M, Y_E solid (black, red), respectively. Also shown are previous phenomenological extractions [76]: IQ: Y_M, Y_E long-dashed (black, red) line, respectively, and hadronic calculations assuming different intermediate states: elastic: Y_M, Y_E (solid, dashed-dotted) cyan line, respectively, from Ref. [32], and elastic + πN with spin-1/2 and -3/2 channels labeled as elastic + πN: Y_M, Y_E dotted (black, red) line, respectively, from Ref. [36].

amplitude is also on the few-percentage-point level, and behaves linearly in ε. The two fits yield opposite behavior in ε as the TPE coefficient a_1 has different sign, Table I. Our results are also in reasonably good qualitative agreement with all shown phenomenological extractions with the IQ extraction yielding clear deviation at low ε, but disagree with hadronic calculations because they all predict positive amplitude for the entire ε range.

Figure 4 shows the ε dependence of the $Y_{(M,E)}$ amplitudes as extracted from this work at $Q^2 = 3.20$ and 4.10 GeV^2, labeled as "This Work: Y_M, Y_E". In addition, we compare our results to previous phenomenological extractions [76], labeled as "IQ: Y_M, Y_E", and hadronic calculations labeled as "elastic: Y_M, Y_E" from Ref. [32], and "elastic + πN: Y_M, Y_E" from Ref. [36], which account for both elastic and intermediate-state contributions containing the πN system with higher angular-momentum contributions from $J = 1/2$ and $J = 3/2$ for eight different πN channels. As Q^2 increases, both amplitudes grow in magnitude. The Y_M amplitude continues to behave linearly in ε, and the Y_E amplitude starts to exhibit more linearity in ε with increasing Q^2 value. Our Y_M is in a reasonable qualitative agreement with previous phenomenological extractions and hadronic TPE calculations showing the overall falloff of Y_M with decreasing ε. Our Y_E is also in reasonable qualitative agreement with previous phenomenological extractions but deviates substantially from hadronic TPE calculations as they predict opposite behavior with positive Y_E.

V. CONCLUSIONS

In this work, we have reexamined the ε dependence of the recoil-polarization ratio $R = \mu_p G_E/G_M$ for possible TPE corrections beyond the Born-approximation (OPE). High-precision Rosenbluth measurements of σ_R taken from Ref. [10] at $Q^2 = 2.64$, 3.20, and 4.10 were fitted to Eq. (6), and the two TPE coefficients $a_1(Q^2)$ and $b_1(Q^2)$ were extracted, and then used to construct the TPE correction to R, Eq. (3), and the two TPE amplitudes $Y_{(M,E)}(\varepsilon, Q^2)$, Eqs. (4) and (5). For the three Q^2 points considered in this work, the ratio R shows negligible enhancement with ε with maximum deviation of R from its Born (OPE) R_p value reaching $\delta_R = +0.29\%$, -0.38%, and $+0.29\%$ at $\varepsilon = 0.50$ for $Q^2 = 2.64$, 3.20, and 4.10 GeV^2, respectively, suggesting that the value of R is consistent with the Born-approximation (OPE) prediction. The extracted amplitudes $Y_{(M,E)}(\varepsilon, Q^2)$ are on the few-percentage-points level and increase in magnitude with increasing Q^2. For the entire Q^2 range considered in this study, both $Y_{(M,E)}$ amplitudes behave consistently linearly with ε as Q^2 increases. Our Y_M generally is in a reasonable qualitative agreement with previous phenomenological extractions and theoretical TPE calculations showing the overall falloff of Y_M with decreasing ε. Our Y_E is also is in a reasonable qualitative agreement with previous phenomenological extractions but disagrees strongly with hadronic TPE calculations of Refs. [32,36].

We have also performed a global fit using Eqs. (3) and (6), where we combined data for the ε dependence of R at $Q^2 = 2.50$ GeV^2 from Ref. [91], and σ_R measurements at $Q^2 = 2.64$ GeV^2 from Ref. [10]. The extracted TPE $b_1(Q^2)$ coefficient is consistent with that obtained using the σ_R Fit. On the other hand, while the magnitude of the TPE $a_1(Q^2)$ coefficient is still consistent with that obtained using the σ_R Fit, it has an opposite sign. Again, R shows no significant enhancement with ε with $\delta_R = -0.27\%$ at $\varepsilon = 0.50$ consistent with the results of the σ_R Fit, and in good agreement with the "SF" predictions [28]. Our Y_M is identical to that obtained using the σ_R Fit and is in a reasonable qualitative agreement with previous phenomenological extractions and theoretical TPE calculations. For Y_E, both fits yield opposite behavior in ε, and they are in reasonable qualitative agreement with previous phenomenological extractions but strongly disagree with theoretical TPE calculations.

ACKNOWLEDGMENT

This work was supported by Khalifa University of Science and Technology.

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