Stripping and pickup contributions to the optical potentials for ^{3}H and ^{3}He on ^{40}Ca

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(Received 17 October 2023; accepted 16 February 2024; published 12 March 2024)

Background: Well established coupled channel (CC) and coupled reaction channel (CRC) processes make contributions to the elastic scattering optical model potential (OMP) that are not represented in local density folding models.
Purpose: We aim to establish and characterize the contribution to the ^{3}He and ^{3}H OMPs generated by coupling to proton or neutron pickup and stripping channels, in particular for 33 MeV ^{3}He and ^{3}H OMPs on ^{40}Ca; also stripping for 40 MeV ^{3}He.
Methods: CRC calculations provide the elastic channel S matrix, S_{lj}, generated by the included processes. Inversion of S_{lj} then produces a local potential, including a spin-orbit term, that yields, in a single channel calculation, the elastic scattering observables from the CRC calculation. Subtracting the bare potential then gives a local and l-independent representation of the dynamical polarization potential, DPP.
Results: Pickup generates greater OMP absorption for ^{3}He than for ^{3}H. Neutron stripping for ^{3}H generates much greater absorption than proton stripping for ^{3}He. Neutron pickup for ^{3}He has a substantial repulsive effect. All stripping and pickup processes reduce the rms radius of the OMP. None of the coupling effects can be represented by a uniform renormalization of the OMP. A comparison of 33 and 40 MeV ^{3}He stripping yields evidence of energy dependence of DPPs with available data.
Conclusions: The DPPs challenge the notion that local density folding models can provide a satisfactory description of elastic scattering from nuclei. No standard form global OMP could fit precisely ^{3}He or ^{3}H elastic scattering from ^{40}Ca. For both pickup and stripping coupling, the increase in reaction cross section is much *less* than the cross section to the coupled channel states, which is the opposite of what was found in many other cases.

DOI: 10.1103/PhysRevC.109.034605

I. INTRODUCTION

We have previously shown that transfer reactions make a significant contribution to optical potentials in ways that cannot be represented by uniform renormalization of folding model potentials. The main subject of the present study is the contribution generated by reaction channel coupling to the dynamic polarization potential, DPP. In particular, we determine the contribution to the local equivalent DPP: the local and l-independent, S-matrix equivalent of the formal nonlocal and l-dependent DPP; see Refs. [1,2]. The local equivalent DPP derived in this way reveals the limitations of local density folding models as applied in local phenomenology. The volume integrals and other general properties of the local DPP can be related to purely phenomenological potentials, but the radial form of the complete potential, with added DPP, will not resemble a Woods Saxon form, having irregular features that reflect the l dependence and nonlocality of the formal DPP. This phenomenon is not limited to transfer reactions since both inelastic scattering and breakup have been found to generate similar undulatory DPPs; see, e.g., Refs. [3–5] and [6–8], respectively.

In the present work we compare the reaction contributions of stripping and pickup to the optical potentials for both ^{3}He and ^{3}H scattering on ^{40}Ca. That is, we compare the reaction contributions to the OMPs for ^{3}He and ^{3}H on ^{40}Ca for the following couplings: (^{3}He, d), (^{3}H, d), (^{3}He, ^{4}He), and (^{3}H, ^{4}He). This work is part of an ongoing study and reference will be made to the pickup results presented in Refs. [3,4].

The case of proton stripping of ^{3}He on ^{40}Ca is of particular interest since only the ground state of ^{41}Sc is bound with respect to proton emission. However, a full comparison with the neutron stripping of ^{3}H on ^{40}Ca to states of ^{41}Ca must compare the effect of coupling to an appropriate range of formally particle unstable states of ^{41}Sc. How this is achieved for the (^{3}He, d) reaction will be described below.

A. Motivation

The calculation of the interaction between mass-3 nuclei and heavier target nuclei is not only a basic challenge for nuclear theory, but is also motivated by the general need to

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establish suitable interactions, "optical model" potentials, for the analysis of direct nuclear reactions. Folding models have had various degrees of success, but they seldom give precise fits to elastic scattering data that are of high quality and extensive angular range; see, for example, Refs. [9–11]. A uniform renormalization of the folding model potential to improve the fit to elastic scattering data is not satisfactory and does not give good fits to high-quality data. The source of the problem becomes obvious when the local potential that includes the effect of channel coupling is examined: such potentials do not represent a uniform renormalization of a smooth potential. Elastic scattering from ^{40}Ca is of particular interest because of the relative failure of global potentials for this nucleus; see Ref. [12] where a "regional" potential was necessary for this target.

The problem can be viewed from another perspective: model-independent fits to high quality elastic scattering data typically lead to potentials that have an oscillatory structure, in some cases pronounced. For deuteron scattering, it was found that precise model independent fitting, at two energies, of precise and wide angular range elastic scattering data, did lead to potentials with pronounced wavy features; see Refs. [13,14]. Similar effects were found for proton scattering, Refs. [15,16]. In the last case it seems that the attempts to get a better fit were abandoned when waviness (undularity) began to appear; such is the reluctance to accept wavy potentials. Such waviness is now understood as an indicator of underlying l dependence. In fact there are sound theoretical reasons for coupling effects to generate potentials that are l dependent, and it is known that the l-independent potential that has the same S matrix as an l-dependent potential will be "wavy" [17]. Scrutiny of the l-independent potentials that we find by inversion thus has scope to identify underlying l dependence as a property of the DPPs generated by stripping and pickup coupling.

B. Method

To calculate the local and l-independent representation of the DPP generated by specific channel coupling, we apply an S-matrix-to-potential inversion procedure recently applied to ^{3}H scattering cases [4,18]. For more general discussion of the inversion method, in the context of deuteron scattering, see Ref. [7], which refers to relevant review articles. In brief outline, such inversion of the S-matrix generated by a calculation including a specific coupling yields a local potential that includes the coupling contribution. Subtracting the local potential employed in the coupling calculation, the "bare" potential, yields a local and l-independent representation of the coupling contribution, the DPP. The DPP will not be smooth and will not represent a uniform renormalization of the bare potential.

II. TRANSFER CONTRIBUTIONS TO MASS-3 INTERACTIONS WITH ^{40}Ca

Coupled reaction channel (CRC) calculations were carried out with the code FRESCO [19] for the ^{3}H + ^{40}Ca and ^{3}He + ^{40}Ca systems. The calculations included coupling to just the (^{3}H, ^{2}H) and (^{3}He, ^{2}H) stripping reactions, respectively. Entrance channel bare OMPs were taken from Refs. [4] and [3], allowing a comparison of the stripping DPPs with the DPPs obtained for the corresponding (^{3}H, ^{4}He) and (^{3}He, ^{4}He) proton or neutron pickup reactions. Exit channel deuteron OMPs were calculated using the global parameter set of Ref. [20]. The $\langle ^3\mathrm{H} \mid ^2\mathrm{H} + n\rangle$ and $\langle ^3\mathrm{He} \mid ^2\mathrm{H} + p\rangle$ overlaps were both taken from Ref. [21]. The $\langle ^{41}\mathrm{Ca} \mid ^{40}\mathrm{Ca} + n\rangle$ overlaps for the ^{40}Ca(^{3}H, ^{2}H)^{41}Ca reaction were taken from Ref. [22], but, since data are available for stripping to the ground state of ^{41}Ca [23] at a ^{3}H incident energy of 33 MeV, the spectroscopic factor for this overlap was adjusted to give the best fit to these data, resulting in a value 23% smaller than that of Ref. [22]. All other spectroscopic factors for the target overlaps were left unchanged.

For the ^{40}Ca(^{3}He, ^{2}H)^{41}Sc reaction, since only the ground state of ^{41}Sc is bound with respect to proton emission, the other low-lying levels being quasibound narrow resonances, a different approach was adopted for determining the $\langle ^{41}\mathrm{Sc} \mid ^{40}\mathrm{Ca} + p\rangle$ overlaps, as now outlined. Data exist for the ^{40}Ca(^{3}He, ^{2}H)^{41}Sc stripping reaction at an incident ^{3}He energy of 40 MeV [24] as well as for the corresponding ^{40}Ca + ^{3}He elastic scattering at 41 MeV [25]. The elastic scattering data were fitted along with the stripping data by a CRC calculation. The $p + {}^{40}$Ca binding potential geometry parameters were the same as those used in Ref. [24], and, while the radial wave function for the ^{41}Sc ground state was calculated using the standard well depth prescription, the wave functions for the resonant levels were calculated within continuum bins, the depth of the central part of the binding potential being adjusted to give a resonance at the experimental energy, defined as where the phase shift angle $\delta = 90^\circ$. The spectroscopic factors were adjusted to fit the stripping data of Ref. [24]. These form factors were then used as input to the ^{3}He + ^{40}Ca stripping calculations at 33 MeV. There are data for stripping to the ground state of ^{41}Sc at this incident energy [26] and these are well described by the corresponding CRC calculation, the magnitude of the cross section being overpredicted by about 20%, within the usual uncertainties of absolute spectroscopic factors. Further details of the calculations are given in the Supplemental Material [27].

We note that some of the results for the ^{40}Ca(^{3}He, ^{2}H)^{41}Sc stripping reaction at 33 MeV departed somewhat from expectations, giving interest to the 40 MeV results. Since the only differences between the 33- and 40-MeV calculations were in the entrance and exit channel optical potentials it should be meaningful to compare the DPPs and other results with those found at 33 MeV, although see our later discussion of this point.

In Figs. 1 and 2 the ^{3}H + ^{40}Ca and ^{3}He + ^{40}Ca elastic scattering angular distributions resulting from the no-coupling ("Bare"; dashed lines), pickup only ("PU only"; solid lines), and stripping only ("Strip only"; dotted lines) calculations at 33 MeV are compared with the data of Refs. [28,29]. In Fig. 3 the elastic scattering angular distributions for 40-MeV ^{3}He + ^{40}Ca elastic scattering resulting from calculations including coupling to stripping only ("Strip only"; dotted line) and no coupling ("Bare"; dashed line) are compared with the data of Ref. [25].

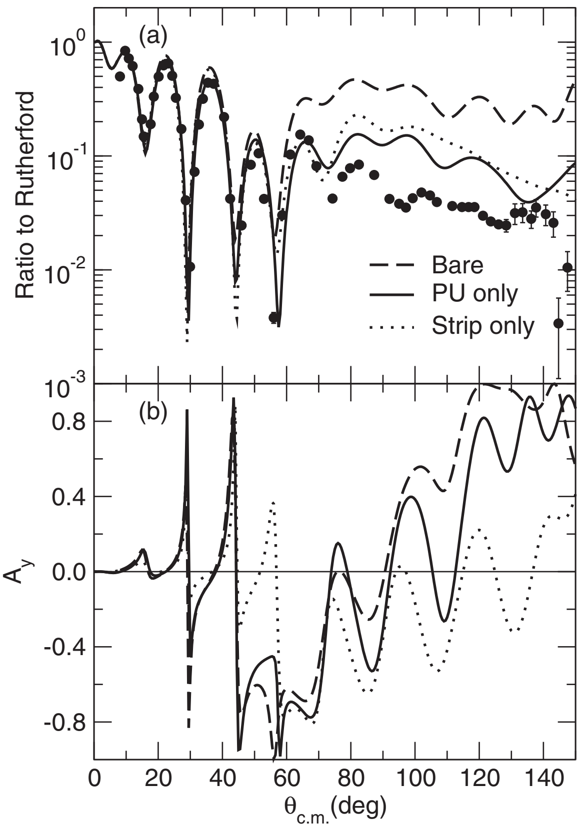

FIG. 1. Elastic scattering angular distributions for 33 MeV $^3H + {}^{40}Ca$ for the no-coupling ("Bare"), (^3H, ^4He) pickup only ("PU only") and (^3H, ^2H) stripping only ("Strip only") CRC calculations. (a) Differential cross section. (b) Analyzing power. The data are taken from Ref. [28].

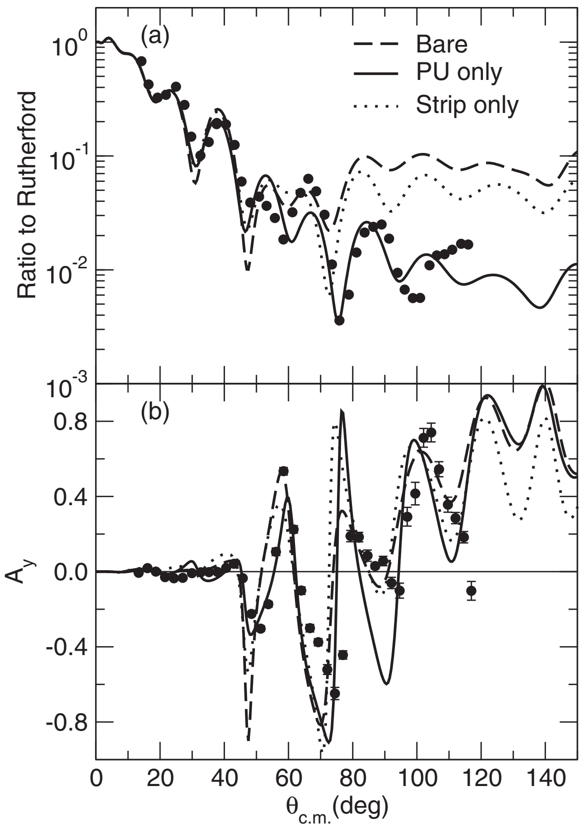

FIG. 2. Elastic scattering angular distributions for 33 MeV $^3He + {}^{40}Ca$ for the no-coupling ("Bare"), (^3He, ^4He) pickup only ("PU only") and (^3He, ^2H) stripping only ("Strip only") CRC calculations. (a) Differential cross section. (b) Analyzing power. The data are taken from Ref. [29].

The angular distributions for coupling to pickup only in Figs. 1 and 2 are taken from Refs. [4,3] for the $^3H + {}^{40}Ca$ and $^3He + {}^{40}Ca$ systems, respectively. In Fig. 1 we see that for $^3H + {}^{40}Ca$ stripping and pickup have similar, substantial effects on the differential cross section, whereas the stripping clearly has a more important influence on the predicted analyzing power (unfortunately there are no analyzing power data available for this system). For the ^3He case, Fig. 2 shows that for $^3He + {}^{40}Ca$ the neutron pickup has a very important effect on the differential cross section, much larger than that of the proton stripping. The influence on the analyzing power is similar for pickup and stripping.

Note that for both systems neither the pickup-only nor the stripping-only calculations closely fit the elastic scattering angular distributions. This is because the bare potentials used in the present calculations were obtained from CRC fits to the respective elastic scattering data including both ^{40}Ca inelastic and pickup coupling [3,4].

The stripping coupling has a more pronounced effect on the elastic scattering of the $^3He + {}^{40}Ca$ system at 40 MeV, as can be seen in the difference between the dotted and dashed curves in Fig. 3. The effect is similar in magnitude to that of the stripping in the $^3H + {}^{40}Ca$ system at 33 MeV; see Fig. 1. Note that the dotted curve in Fig. 3 provides a better description of the data than either of the 33 MeV cases, since for this energy the bare potential was adjusted so that the CRC calculation including the stripping gave a good fit to the data.

Underlying the present work is the assumption that the extracted DPPs are, to a significant degree, independent of the bare potential; see Ref. [30]. A fixed bare potential is particularly important when studying the additivity of DPPs since meaningful results then require added DPPs to be derived from a fixed bare potential. However, as discussed in Sec. III A, the 33 MeV ^3He stripping only DPP exhibits apparently anomalous behavior which was found to be linked to the characteristics of the particular bare potential employed.

The elastic channel S matrices, S_{lj}, resulting from the CRC calculations were subjected to S_{lj}-to-potential inversion to provide the DPPs (by subtraction of the appropriate bare OMP from the inverted potential). Significant characteristics of these DPPs, in particular the volume integrals and the consequent change in rms radius of the real central term, are presented in Table I. The same quantities for the corresponding pickup reactions are also given, taken from Refs. [3,4]. In many previous works, volume integrals have been found usefully to characterize the DPPs, but in the present cases the oscillatory nature of some DPPs suggests caution in this respect.

TABLE I. In lines 1 and 2, for ^{3}H scattering from ^{40}Ca at 33 MeV, volume integrals ΔJ (in MeV fm^3) of the four components of the DPP induced by (^{3}H, ^{2}H) stripping ("Strip") and (^{3}H, ^{4}He) pickup ("PU") coupling, respectively. The "t" column indicates proton or neutron transfer. The $\Delta R_{\rm rms}$ column gives the change in rms radius of the real central component (in fm). Line 1(gs) is for stripping to the ground state of ^{41}Ca only. Lines 1a and 2a present corresponding values of the same quantities for the case of ^{3}He on ^{40}Ca at the same energy. Line 1a(gs) is for proton stripping to the (bound) ground state of ^{41}Sc only. Line 1c is for stripping for 40 MeV ^{3}He. The ground state to ground state Q values, $Q_{\rm gg}$, in column 3, are in MeV. The final four columns present the changes induced by the respective couplings: in the total reaction cross section, the calculated cross section to the specific coupled channels, the ratio R defined as $R = \Delta({\rm CS})/\Delta J_{\rm IM}$, and the ratio $R_{\rm CS} = \Delta({\rm CS})/{\rm State\ CS}$. The quantities Δ(CS) and State CS are given in mb.

Line	Reaction	t	$Q_{\rm gg}$	$\Delta J_{\rm R}$	$\Delta J_{\rm IM}$	$\Delta J_{\rm RSO}$	$\Delta J_{\rm IMSO}$	$\Delta R_{\rm rms}$	Δ(CS)	State CS	R	$R_{\rm CS}$
1	Strip ^{3}H	n	2.11	1.29	21.95	0.559	0.990	−0.103	25.5	62.0	1.16	0.41
1(gs)	Strip ^{3}H	n	2.11	4.18	10.77	0.995	−0.349	−0.076	3.6	19.8	0.33	0.18
2	PU ^{3}H [4]	p	11.49	−8.34	15.74	−0.197	0.075	−0.053	7.2	13.9	0.46	0.52
1a	Strip ^{3}He	p	−4.41	1.15	5.17	−0.073	−0.439	−0.061	21.8	51.3	4.21	0.43
1a(gs)	Strip ^{3}He	p	−4.41	2.88	5.05	0.812	0.189	−0.062	8.6	27.8	1.70	0.31
2a	PU ^{3}He [3]	n	4.94	−15.81	33.47	1.022	0.837	−0.042	10.4	14.7	0.31	0.71
1c	Str 40 ^{3}He	p	−4.41	4.49	20.63	−1.094	−0.640	−0.036	22.3	44.2	1.08	0.50

The Q values for the various cases strongly influence the reactions and are as follows: ^{40}Ca(^{3}H, ^{4}He)^{39}K, $Q = +11.49$ MeV; ^{40}Ca(^{3}H, ^{2}H)^{41}Ca, $Q = +2.11$ MeV; ^{40}Ca(^{3}He, ^{4}He)^{39}Ca, $Q = +4.94$ MeV; ^{40}Ca(^{3}He, ^{2}H)^{41}Sc, $Q = -4.41$ MeV.

The difference between the Q values for stripping and pickup is about the same for both projectiles, although the stripping value is negative for ^{3}He while the pickup value is large and positive for ^{3}H. The implications are unclear. The reactions are well into the Fraunhofer scattering regime here, so matching conditions may not be much of a guide since they are based on semiclassical concepts that will be much less realistic here than for heavy ions at near-barrier energies. The charge on the transferred proton would appear to have more influence for ^{3}H, suppressing the pickup effect somewhat compared to ^{3}He; any influence of the charge seems to be less marked for the stripping. Figures 1 and 2 together suggest that neutron pickup to ^{4}He has a greater effect than proton pickup to ^{4}He. Comparing Figs. 1 and 2, the conspicuously small effect of stripping for helions, at least at backward angles, is remarkable and may relate to certain surprising results described later. The larger effect of stripping at 40 MeV, Fig. 3, also relates to these results.

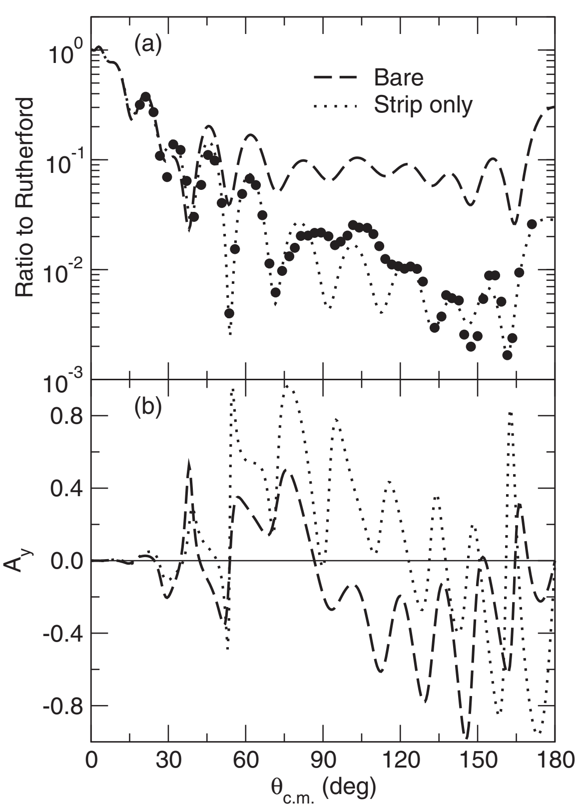

FIG. 3. Elastic scattering angular distributions for 40 MeV ^{3}He + ^{40}Ca for the no-coupling ("Bare"), and the (^{3}He, ^{2}H) stripping only ("Strip only") CRC calculations. (a) Differential cross section. (b) Analyzing power. The data are taken from Ref. [25].

III. RESULTS AT 33 AND 40 MeV

A. Quantifying coupling effects

Table I presents the key properties of the DPPs as calculated for the various reactions. The volume integral of the real central term of the DPP is $\Delta J_{\rm R}$, the difference between the real central volume integral of the inverted potential and that of the bare potential; similarly for the imaginary central term, $\Delta J_{\rm IM}$, and the spin-orbit terms, $\Delta J_{\rm RSO}$ and $\Delta J_{\rm IMSO}$, with $\Delta R_{\rm rms}$ being the change in the rms radius of the real central potential due to the coupling. The quantity Δ(CS) is the change in the total reaction cross section due to the coupling, and is positive in each case. An unambiguous difference between stripping and pickup effects is evident in the column labeled "State CS." This quantity refers to the cross section(s) to the state, or states, fed by the pickup or stripping. The stripping cross sections for both systems (State CS in lines 1 and 1a) are significantly greater than the cross sections for the corresponding pickup processes (State CS in lines 2 and 2a). The difference is more marked for ^{3}H. However, while this difference is reflected in the $\Delta J_{\rm IM}$ values for ^{3}H, for ^{3}He the reverse is the case (lines 1a and 2a), and it is the pickup which leads to by far the greater absorption, as measured by the $\Delta J_{\rm IM}$ values, than the corresponding stripping, despite its significantly smaller State CS value. Remarkably, for stripping, although $\Delta J_{\rm IM}$ is 4.25 times larger for ^{40}Ca(^{3}H,^{2}H) compared to ^{40}Ca(^{3}He,^{2}H), the ratio of the change in the reaction cross section to the stripping cross section, $R_{\rm CS}$, is essentially identical for the two systems.

It has been found in almost all previous studies that reaction channel coupling and inelastic coupling lead to overall repulsion in the real central potential, as signified by negative $\Delta J_{\rm R}$. This applies here to the couplings in lines 2 and 2a which do lead to an overall repulsive effect. However, the stripping cases on lines 1, 1(gs), 1a, 1a(gs), and 1c are unusual, and the coupling leads to a small degree of overall attraction. For pickup coupling, the repulsion is substantial for both ^{3}H and ^{3}He.

It is notable that *all* the couplings lead to a reduction in the rms radius of the real part of the potential, of concern regarding analyses of elastic scattering for nuclear size determination.

The ^{3}He stripping result for 33 MeV is remarkable for its anomalously low value of $\Delta J_{\rm IM}$, underlined by the large value of R, the ratio of Δ(CS) to $\Delta J_{\rm IM}$. Conversely, it seems that $\Delta J_{\rm IM}$ for ^{3}He pickup, line 2a, is anomalously *large* in magnitude in view of the modest State CS. We appear to have a system where the DPPs (or at least their volume integrals) do not seem to be correlated with the physical observables in any obvious fashion.

Concerning lines 1(gs) and 1a(gs), line 1a(gs) refers to stripping to the ground state, the only stable state, of ^{41}Sc; we discuss below the stripping to excited states. The characteristics revealed in lines 1a and 1a(gs) of Table I are surprising and need to be understood. In order to get some understanding we carried out the comparison case of neutron stripping to just the ground state of ^{41}Ca. This led to the results reported in line 1(gs), which do not appear anomalous.

The low value of $\Delta J_{\rm IM}$ for (^{3}He, ^{2}H) stripping at 33 MeV (line 1a) does not occur at 40 MeV (line 1c): the anomalously low value of $\Delta J_{\rm IM}$ in line 1a is replaced by a more expected value in line 1c. This is puzzling, and might suggest a threshold effect to account for the apparent significant variation in the ratio R for a relatively small increase in the incident ^{3}He energy. However, note that while the bare potential at 33 MeV is taken from Ref. [3] to facilitate comparison with the DPP for pickup-only coupling established in that work, the bare potential at 40 MeV was derived from a CRC fit to the corresponding elastic scattering data with stripping couplings only. Two markedly different imaginary central terms are involved: the 33 MeV bare potential has predominantly volume absorption whereas the 40 MeV bare potential has surface absorption only. A test calculation at 33 MeV using the bare potential employed at 40 MeV produced a significantly different DPP: while $\Delta J_{\rm R}$ and $\Delta J_{\rm IM}$ remain positive, they are both considerably larger than the values given in Table I, $\Delta J_{\rm IM}$ in particular being now similar in magnitude to that for the ^{40}Ca(^{3}H, ^{2}H)^{41}Ca stripping of line 1. However, both Δ(CS) and State CS remain relatively unchanged so that in that case R is similar to the values given in lines 1, 1a(gs), and 1c.

We thus find that the apparently anomalous behavior of the 33 MeV ^{3}He stripping-only DPP is intimately linked with the

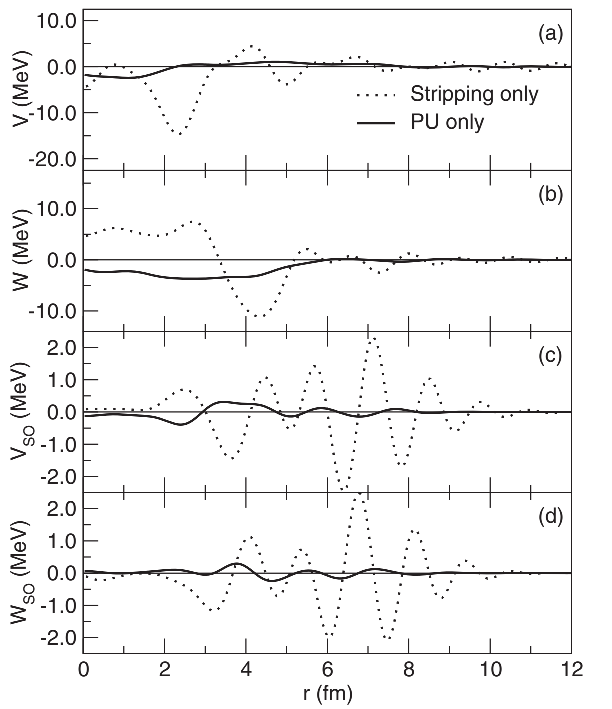

FIG. 4. For 33 MeV ^{3}H + ^{40}Ca, DPPs for the (^{3}H, ^{4}He) pickup only ("PU only") and (^{3}H, ^{2}H) stripping only ("Stripping only") CRC calculations.

choice of bare potential, so that, in contrast to previous cases [30], in the present system the DPP *does* depend significantly on the bare potential. Why this should be so in this particular case and how common it may be (we have not encountered other examples) requires a dedicated study beyond the scope of this work. Consequently, it should be borne in mind that detailed comparisons of the 33 MeV ^{3}He stripping-only DPP with the corresponding ^{3}H stripping-only DPP must carry a caveat.

B. Visualizing the DPPs

We compare the DPPs on a point-by-point basis for stripping and pickup for ^{3}H in Fig. 4 and for ^{3}He in Fig. 5. Figures 6 and 7 compare the DPPs for the cases with stripping to all states with the DPPs for stripping to the ground states only of ^{41}Ca and ^{41}Sc, respectively. We note that the observables (angular distributions of cross section and analyzing power) are sensitive to the DPPs in to radii of 2 fm for the ^{3}He + ^{40}Ca system and 1.5 fm for the ^{3}H + ^{40}C system.

When compared as in Fig. 4 to Fig. 7, various aspects of the DPPs appear surprising. Considering first the stripping, it is clear from Fig. 4 that the central terms of the ^{3}H stripping DPP are considerably greater in magnitude than those due to pickup. In addition, they are qualitatively very different and display features that are not easy to interpret. In particular there is a significant emissive region in the stripping imaginary DPP for $r < 3$ fm. This is not so obvious in Fig. 5 for ^{3}He for which the stripping imaginary DPP is much smaller in magnitude, with the imaginary central term being largely absorptive with just a small emissive region near 3 fm. Con-

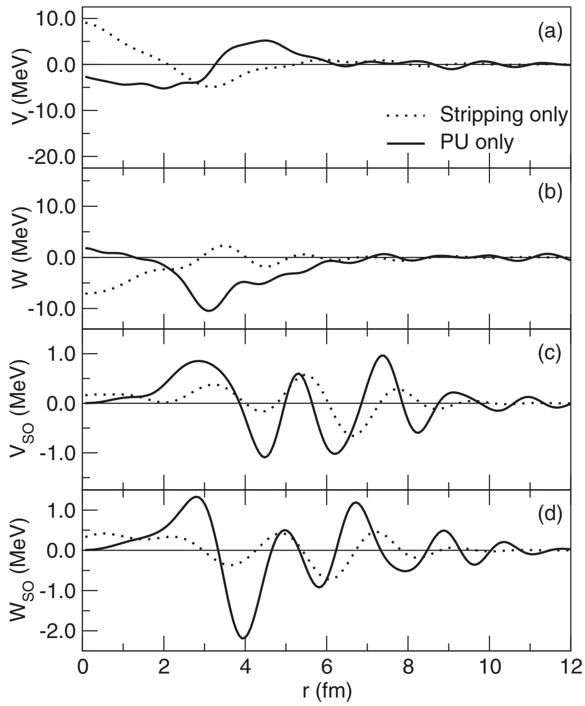

FIG. 5. For 33 MeV ^{3}He + ^{40}Ca, DPPs for the (^{3}He, ^{4}He) pickup only ("PU only") and (^{3}He, ^{2}H) stripping only ("Stripping only") CRC calculations.

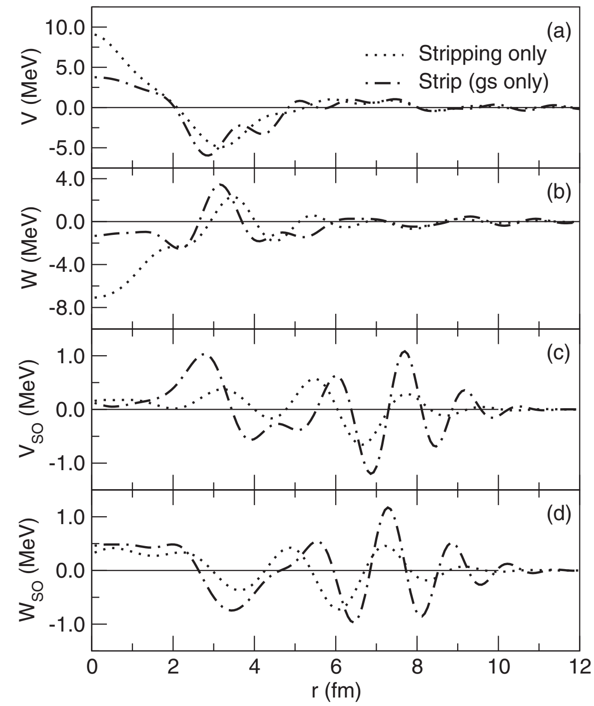

FIG. 7. For 33 MeV ^{3}He + ^{40}Ca, DPPs for (^{3}He, ^{2}H) stripping only ("Stripping only") and (^{3}He, ^{2}H) stripping to ^{41}Sc ground state only ("gs only") CRC calculations.

cerning the real central parts of the stripping DPPs, which have ΔJ_R small and positive (attractive) for both ^{3}H and ^{3}He, in each case there is cancellation between effects over different radial ranges, with repulsion near $r = 0$ dominating for the ^{3}He case in Fig. 5.

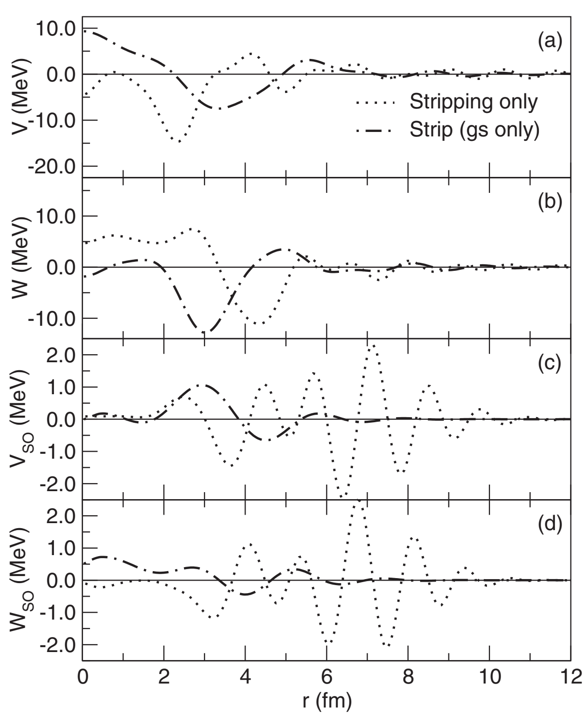

FIG. 6. For 33 MeV ^{3}H + ^{40}Ca, DPPs for (^{3}H, ^{2}H) stripping only ("Stripping only") and (^{3}H, ^{2}H) stripping to ^{41}Ca ground state only ("gs only") CRC calculations.

For stripping-only coupling, the imaginary parts are not even qualitatively similar, the large emissive region for $r < 3$ fm seen in Fig. 4 for ^{3}H having no counterpart for ^{3}He. The imaginary central DPP for ^{3}He is generally absorptive except for a small region near 3 fm. The small magnitude at larger radii is consistent with the anomalously low value of ΔJ_{IM} for ^{3}He.

The stripping spin-orbit DPPs are much closer for ^{3}H and ^{3}He, both qualitatively and quantitatively, than the corresponding components of the pickup DPPs, although there remain important differences of detail.

Comparing the DPPs for the PU-only calculations, the central real terms are qualitatively similar for ^{3}H (Fig. 4) and ^{3}He (Fig. 5), with the ^{3}He potential being of much larger magnitude. This is not very apparent in the volume integrals (lines 2 and 2a in the table) in which the r^2 factor plays an important role. In both cases there is a clear switch from attraction at small radii to repulsion further out. This is a clear example of why coupling effects can not be related to a uniform renormalization of the potential.

Concerning the spin-orbit components, for ^{3}H it is clear that the stripping DPP is much greater in magnitude than the pickup DPP. However, for ^{3}He we find that the stripping and pickup spin-orbit DPPs (both real and imaginary) have similar magnitudes and shapes.

C. Coupling to the ground state only

For ^{3}He stripping to states other than the ground state, special measures are required. Stripping to the ground state

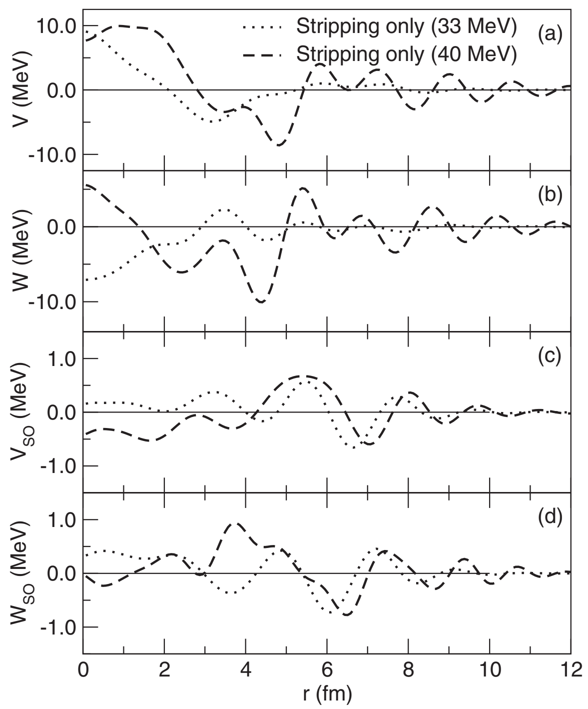

FIG. 8. The DPPs for 33 and 40 MeV (^{3}He, ^{2}H) stripping compared.

is straightforward, leading to the results in line 1a(gs). This suggested comparing the stripping to the ground states for ^{3}H and ^{3}He, lines 1(gs) and 1a(gs) in the table, but also in Figs. 6 and 7. In Fig. 6 it is clear that for ^{3}H stripping the additional states make a large difference. Among other changes, the strong emissive region between the center and 3 fm is seen to be associated with the addition of stripping to the excited states of ^{41}Ca. However, in Fig. 7, it is clear that including stripping to the excited states of ^{41}Sc leads to an almost identical real central DPP, while the imaginary central term becomes strongly absorptive at less than 2 fm but is otherwise little different. There is little change to the spin-orbit terms.

D. Energy dependence of ^{3}He stripping

There are insufficient data for a full account of energy dependence apart from the case of stripping at 40 MeV. In principle, comparison of lines 1a and 1c of the table gives some measure of the overall energy dependence of the DPPs but does not convey the energy dependence of their radial shape. In Fig. 8 the stripping only DPPs at 33 and 40 MeV are compared, and a considerable change in the radial shape is evident. $\Delta J_{\rm IM}$ for 40 MeV is more similar to the value in line 1 for ^{3}H than that for 33 MeV ^{3}He in line 1a.

E. The reaction cross sections

Table I presents values of the State CS, the cross section of the pickup or stripping state(s) fed by the reaction, as well as Δ(CS), the change in the total reaction cross section induced by the particular coupling. The value of the State CS gives a measure of the strength of the reaction. It shows, for example, that the small value of $\Delta J_{\rm IM}$ in lines 1a and 1a(gs) cannot be attributed to the weakness of the stripping reaction.

In all the present cases the changes in the reaction cross section induced by the coupling, Δ(CS), are increases, although they are much smaller than the corresponding State CS. Elsewhere, there exist reactions for which Δ(CS) is greater than the State CS, for examples see Refs. [3,4,31,32] and references therein; indeed Δ(CS) can be considerably greater than the State CS (as if that coupling is serving as a doorway to other processes), e.g., Refs. [33,34]. Conversely, cases exist for which Δ(CS) is actually negative (perhaps an "antidoorway") [18] so that the coupling counterintuitively reduces the total reaction cross section.

IV. EXTENSIONS

Each of the cases presented in the table involves a single reaction for either an incident ^{3}H or ^{3}He. However, a ^{3}He or a ^{3}H interacting with a nucleus can also excite vibrational states of the nucleus and undergo a variety of other reactions. The various couplings for a specific projectile could be incorporated in a single calculation, making possible a determination of the DPP when multiple couplings are operative. This opens up the possibility of studying the additivity of DPPs for diverse combinations of couplings. This in turn enables the study of the dynamical nonlocality of the DPPs generated by the various couplings [35]. In this light, the present work is the first step in a comprehensive study of the complex array of reactions contributing to the interaction potential between mass-3 nuclei and ^{40}Ca.

V. CONSEQUENCES

The results presented above have consequences for our understanding of nuclear structure and the interactions between nuclei. One consistent finding of the present work is that, in all ^{3}He and ^{3}H cases studied, reaction channel coupling results in a reduction in the mean square radius of the projectile-nucleus potential. In Table I, $\Delta R_{\rm rms}$ is negative in every case: the coupling leads to a reduced rms radius. A similar consistent effect was found for pickup in the ^{3}H and ^{3}He $+ ^{208}$Pb systems [18]. The effect of channel coupling on rms radius appears generally to have the opposite sign for nucleon-nucleus interactions—see, e.g., Refs. [32,34]—but channel coupling effects must be assumed to modify the radius of the potential. A recent study of nuclear matter radius and neutron-skin thickness [36,37] is based on modern Bruckner-Hartree-Fock folding models. It is unclear whether these models do, in effect, include a representation of particle transfer processes of the kind reported here. The absence of undularity suggests they do not. A signature effect of pickup and stripping processes is some degree of waviness in the consequent interaction. It is therefore reasonable to conclude that, inconveniently, phenomenological potentials exhibiting no undularity lack a representation of certain real physical effects.

Unfortunately there exist few model independent phenomenological fits to high quality elastic scattering data. As we have noted elsewhere, there are cases where waviness has appeared, as for nucleon [15] or (very strongly) for deuteron scattering [13,14]. Unfortunately, when signs of undularity do appear the search may be terminated, as in Ref. [16].

VI. SUMMARY AND CONCLUSIONS

We studied the contributions of proton and neutron stripping and pickup to the interactions between 33 MeV ^{3}He and ^{3}H projectiles and ^{40}Ca. The reaction contributions were quantified in terms of the volume integrals and other properties of the resulting DPPs. The associated reaction cross sections to the consequent states of ^{39}Ca, ^{39}K, ^{41}Ca, and ^{41}Sc were recorded and are presented in Table I. Data for the proton stripping of 40 MeV ^{3}He were also exploited to determine the properties of the DPP for that reaction. The properties of the proton stripping DPP for 40 MeV ^{3}He were closer to expectations than the somewhat unexpected results for proton stripping of 33 MeV ^{3}He. The pickup DPPs for both ^{3}H and ^{3}He were much as expected, as were the results for ^{3}H stripping. One departure from expectations applying to all the stripping cases was that $\Delta J_{\rm R}$ was positive (although not strongly), indicating a small net attractive effect whereas usually the effect of channel coupling is overall repulsion.

For ^{3}H the difference between the DPP for ground state stripping and the coupling to all stripping states is substantial, as shown in Fig. 6, unlike the much lesser difference for ^{3}He as in Fig. 7. There is a substantial difference between the 33 and 40 MeV proton stripping cases, as Fig. 8 clearly shows.

The general conclusion is, once more, that for both ^{3}He and ^{3}H the reaction channel coupling contributes substantially to the OMP in ways that are very far from being describable by a uniform renormalization. Together with our previous results, the conclusion is that interactions between nuclei and incident light ions, such as nucleons and the mass-3 projectiles, are not radially smooth; this strongly suggests an underlying l dependence. Of course, simpler, more systematic l-independent OMPs have a continuing role to play in analyses of experimental data, but it is important to acknowledge the limitations of l-independent local density models when it comes to understanding the interaction between nuclei and incident nucleons or other nuclei. Establishing a local and l-independent nuclear potential requires the description of very precise and wide angular range elastic scattering data, including analyzing powers and spin-rotation measurements, over a range of energies. The present work implies that such a potential would exhibit features arising from underlying l dependence.

There are contributions to the OMPs for both mass-3 projectiles arising from excitation of vibrational states of ^{40}Ca that have not been studied here. Studying these contributions is important and constitutes substantial work for the future which will bring to light questions concerning the nonlocality and nonadditivity of contributions to the DPP [35].

An important aspect of phenomenological OMPs is their global energy dependencies. Studies such as this one have an important role to play in understanding, and eventually predicting, the departures of empirical OMPs from global trends. It is expected that nuclei such as ^{40}Ca have OMPs, for various projectiles, that depart from global predictions. The present work might eventually contribute to an understanding of how mass-3 OMPs depart from global predictions for target nuclei such as ^{40}Ca, which is generally regarded as a (possibly imperfectly) closed shell nucleus.

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