Thermal effect in hot QCD matter in strong magnetic fields

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The quasiparticle model is improved by the free magnetic contribution to investigate the QCD matter in a strong magnetic field. The temperature-dependent bag function is determined by the thermodynamic consistency to represent the difference in energy density between physical vacuum and the lowest state of QCD. It is found that the positive bag function vanishes at high temperature, indicating the deconfinement. The rapid decrease of the bag function in stronger magnetic fields reveals the so-called inverse magnetic catalysis. The interaction measure at high temperature remains so large that the usual Stefan-Boltzmann limit cannot be reached. We suggest a limit $|q_i B_m| T^2/4$ for each Landau level pressure. Finally, it is demonstrated that the positive magnetization modified by the bag function and free magnetic contribution indicates the paramagnetic characteristic of QCD matter.

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I. INTRODUCTION

It has been observed that the strongly interacting matter, quark-gluon plasma (QGP), produced in the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC) behaves more like a near-perfect fluid. The QCD theory in the magnetic background may reveal a better understanding of the QGP that have rich collective effects [1,2]. Many efforts have been made in theoretical work, revealing interesting properties of strongly interacting matter in the strong magnetic field [3,4]. With the development of relativistic heavy ion collisions, study of the medium effect of quark-gluon plasma has become more active. Quark self-energy at high temperature receives the contribution of both the electric scale and magnetic scale, which has a profound impact on the confinement effects on thermal quark collective excitation [5,6].

In hot dense quark matter, one of the most important medium effects is the effective mass generated by the nonperturbative interaction of the particles with the system. In literature, the phenomenological models overcome the difficulty of the QCD theory at finite temperature and/or chemical potentials. A natural mechanism for quark confinement is given by the MIT bag model [7]. The bag model was proposed to explain hadrons and quark confinement, which artificially constrains the quarks inside a finite region in space. However, the bag model is not able to well exhibit the phase transition of the deconfinement. The quark quasiparticle model, as an extended bag model, has been developed in studying the bulk properties of the dense quark matter at finite density and temperature [8–11] and the strangelets in finite size [12]. The quasiparticle description has been assumed to be valid also in the case of sufficiently high temperature [13–15]. The advantage of the quasiparticle model is the introduction of the medium-dependent quark mass scale to reflect the nonperturbative QCD properties [16] and color confinement mechanism [17]. The transport properties of the quark-gluon plasma have been well investigated by the noninteracting and weakly interacting particles with effective masses [18]. The hard thermal loop (HTL) approximation can also be used to calculate the effective quark mass, but these calculations are valid only in the perturbative regime of QCD [15,19,20]. There are also self-consistent quasiparticle models and single-parameter quasiparticle models [21–23]. The medium effects are taken into account by considering quarks and gluons as quasiparticles. Their temperature-dependent masses are proportional to the plasma frequency. More recent developments have shown that quasiparticles with effective fugacity have been successful in describing the lattice QCD results [24,25], which was initially proposed by Chandra and Ravishankar [25,26].

It is well known that quarks are bound inside hadrons through strong interaction, and it has not yet been found that quarks can exist independently. Based on this fact, the nature of quark confinement was derived. According to lattice simulations, the deconfinement phase transition is of first order [27]. In some phenomenological models, an order parameter of the deconfinement transition is the Polyakov loop, which is the trace of a Wilson line along a closed loop in the time direction [28]. For the SU(3) pure gauge theory, the deconfined phase transition at high temperature corresponds to the spontaneous Z(3) symmetry breaking. However, the study of symmetry can be complicated in QCD due to the quark dynamics. In particular, the quark confinement and vacuum energy density can be well described by the density-dependent bag constant [9,16,29]. In the present work, the formula of QGP has been described as a noninteracting gas of quarks at zero chemical potential but finite temperature, taking into account the phenomenological bag constant. As for gluon gas, the equation of state for gluons has been excellently described using the ideal gas approximation at high temperatures in lattice calculations [13,30].

The paper is organized as follows. In Sec. II we present the thermodynamics of the magnetized QGP in the quasiparticle model. Section III discusses the numerical results for the confinement bag function and the thermodynamic quantities in the strong magnetic field at finite temperatures. The last section is a short summary.

II. THERMODYNAMICS OF THE QUASIPARTICLE MODEL IN STRONG MAGNETIC FIELDS

The important feature of the quasiparticle model is the medium dependence of quark masses in describing QCD nonperturbative properties. The quasiparticle quark mass is derived at the zero-momentum limit of the dispersion relations from an effective quark propagator by resuming one-loop self-energy diagrams in the hard dense loop (HDL) approximation. The dynamical information for gluonic degrees of freedom can also be accessed through the effective gluon mass. In this paper the effective quark mass m_q and gluon mass m_g are adopted as [15,31,32]

$$m_g(T) = \sqrt{\frac{1}{6}\left(N_c + \frac{1}{2}N_f\right)g^2T^2}, \tag{1}$$

$$m_q(T) = \frac{1}{2}\left(m_{i0} + \sqrt{m_{i0}^2 + \frac{N_c^2-1}{2N_c}g^2T^2}\right), \tag{2}$$

where m_{i0} denotes the quark current mass of the quark flavor i. The constant g is the strong interaction coupling. In order to reflect the asymptotic freedom of QCD, one can also use a running coupling constant $g(Q/\Lambda)$ in the equations of state of strange matter [33]. The parametrization of the coupling as a function of temperature close to the theory is adopted as [30]

$$g^2(T, T_c) = \frac{48\pi^2}{(11N_c - 2N_f)\ln[\lambda^2(T/T_c - T_s/T_c)^2]}\left(\frac{\Lambda T_c}{T}\right)^\eta. \tag{3}$$

The current mass can be neglected for up and down quarks, while the strange quark is taken to be massive. Because the vanishing current mass is assumed for up and down quarks, Eq. (2) is reduced to the simple form

$$m_{u,d} = \frac{1}{\sqrt{3}}gT. \tag{4}$$

As a typical treatment in the quasiparticle model, one implements confinement by introducing a bag pressure, measuring the level difference between the physical vacuum and the ground state in the colorful world of QCD [34]. To account for the essential nonperturbative features, the corresponding thermodynamics is based on the ideal gas partition function with additional contributions,

$$T\ln Z = T\ln(Z_0 Z_{\text{vac}} Z_{\text{mag}}), \tag{5}$$

where the vacuum partition depends on the bag function $T\ln Z_{\text{vac}} = -B(T)V$ [34], and the term Z_{mag} stands for the free magnetic field contribution. The conventional matter contribution is introduced by Z_0. So the total partition function can produce the thermodynamic quantities from the self-consistent relation. Within the framework of the temperature-dependent mass $m(T)$, the pressure of the system is expressed as

$$\begin{aligned} P(T, B_m) &= \frac{T}{V}\ln Z(T, B_m) \\ &= -\sum_{i=q,g}[\Omega_i(T, B_m) + B_i(T, B_m)] \\ &\quad - B_0 - \mathcal{V}(T_0, B_m). \end{aligned} \tag{6}$$

The first term Ω_i is the conventional matter contribution. The bag constant B_0 stands for the vacuum energy density at zero temperature. The variant term $B_i(T, B_m)$ resembles the interaction term from quasiparticles and can be interpreted as the thermal vacuum energy density. We assume the Maxwell term $\mathcal{V}(T_0, B_m)$ is independent on the quark mass but dependent on the magnetic field, which represents the free pressure of magnetic contribution [35–37],

$$\begin{aligned} \mathcal{V}(T_0, B_m) = -\sum_{i=q}\frac{N_c|e_iB_m|^2}{2\pi^2}\bigg[\zeta'(-1, x_i) - \zeta'(-1, 0) \\ - \frac{1}{2}(x_i^2 - x_i)\ln(x_i) + \frac{x_i^2}{4}\bigg], \end{aligned} \tag{7}$$

where the parameter $x_i = m_i^2/(2|q_iB_m|)$ is defined at the moderate temperature $T_0 = 150$ MeV in the thermal bag. The constant $\zeta'(-1, 0) = -0.165\,421...$ was introduced in Ref. [37], which is helpful to maintain a positive magnetic pressure. The presence of $\mathcal{V}(T_0, B_m)$ would not change the thermodynamically self-consistent relation $\partial P/\partial m_i = 0$ in the quasiparticle model. So we have the following differential equation:

$$\frac{dB_i}{dT}\frac{dT}{dm_i} = -\frac{\partial\Omega_i}{\partial m_i}. \tag{8}$$

So the temperature-dependent term $B_i(T, B_m)$ is

$$B_i(T, B_m) = -\int_0^T \frac{\partial\Omega_i}{\partial m_i}\frac{dm_i}{dT}dT. \tag{9}$$

The derivative $\partial\Omega_i/\partial m_i$ is

$$\frac{\partial\Omega_i}{\partial m_i} = \frac{d_i|e_iB_m|}{\pi^2}\sum_{\nu=0}^{\infty}(2 - \delta_{\nu 0})\int_0^\infty f(\varepsilon_i)\frac{m_i}{\varepsilon_i}dp_z, \tag{10}$$

where the fermion distribution function is $f(\varepsilon_i) = 1/[1 + \exp(\frac{\varepsilon_i}{T})]$ and the single-particle energy is $\varepsilon_i = \sqrt{p_z^2 + m_i^2 + 2\nu e_iB_m}$ due to the quantization of orbital motion of charged particles in the presence of a strong magnetic field along the z axis [38]. The derivative of the mass $m(T)$ with respect to the temperature is

$$\frac{dm_i}{dT} = \frac{N_c^2 - 1}{4N_c} \frac{g^2 T}{\sqrt{m_0^2 + \left(N_c^2 - 1\right) g^2 T^2/(2N_c)}} + \frac{\partial m_i}{\partial g^2}\frac{\partial g^2}{\partial T}, \quad (11)$$

which will be simplified as $\frac{dm_i}{dT} = \frac{g}{\sqrt{3}}$ for zero current mass and the constant coupling $\frac{\partial g}{\partial T} = 0$. If one takes into account the running coupling $g(T)$, the derivative of the mass $m_i(T)$ with respect to the T should be calculated through the full differential relation.

The entropy density, as a measure of phase space, is unaffected by $B_i(T, B_m)$ [9], which is clearly understood from relation (8). Similar to the number density, the entropy density is written based on the fundamental thermodynamic relation,

$$s_i = -\frac{\partial \Omega_i}{\partial T} = \frac{d_i|e_i B_m|}{\pi^2}\sum_{\nu=0}^{\infty}(2-\delta_{\nu 0})\int_0^\infty f(\varepsilon_i)\frac{p^2+\varepsilon_i^2}{T\varepsilon_i}dp_z. \quad (12)$$

The net effect of the bag function is to cancel the entropy density contribution, which would arise from the dependence of the mass $m(T)$ on the temperature. It is well known to us that the energy density and pressure should vanish in vacuum. So the pressure should be normalized by requiring zero pressure at zero temperature as

$$P^{\text{eff}}(T, B_m) = P(T, B_m) - P(0, B_m). \quad (13)$$

The magnetization is an important thermodynamic quantity in understanding the QCD matter [39]. The development of the study on the magnetization in various methods has been summarized in Ref. [40]. At zero temperature, the magnetization is found to be positive and to be responsible for the anisotropic pressures [41,42]. We propose the expression of the magnetization in the quasiparticle model as

$$\mathcal{M} = \frac{\partial P^{\text{eff}}}{\partial B_m} = -\sum_{i=u,d,s}\left(\frac{\partial \Omega_i}{\partial B_m} + \frac{\partial B_i}{\partial B_m} + \frac{\partial \mathcal{V}_i}{\partial B_m}\right), \quad (14)$$

where the first term is the conventional contribution from the pure quasiparticle [43]. The second term demonstrates that the effective bag function would have additional contribution to the magnetization, which reveals the medium effect on the hot quark matter. It can be written as

$$\frac{\partial B_i}{\partial B_m} = \frac{N_c|e_i|m_i}{\pi^2}\sum_{\nu=0}^{\infty}(2-\delta_{\nu 0})\int_0^\infty f(\varepsilon_i) \times \left(\frac{\nu B_m}{\varepsilon^2} + f(\varepsilon_i)\frac{\nu|e_i|B_m}{\varepsilon_i T} - 1\right)\frac{dp_z}{\varepsilon_i}. \quad (15)$$

III. NUMERICAL RESULT AND CONCLUSION

In the framework of the preceding quasiparticle model, we have done the numerical calculations with the quark current mass values $m_u = m_d = 0$, and $m_s = 120$ MeV. The constant term B_0 is $(145\ \text{MeV.})^4$ The effective bag constant acts as an energy penalty for the deconfined phase. In Fig. 1 the effective bag constant $B(T, B_m)$ is shown as a function of the temperature at different magnetic fields $B_m = 0.2, 0.4, 0.6$ GeV^2. It falls to zero at high temperature, which means that the deconfined state has larger pressure and is energetically preferred. Compared with the fixed coupling constant $g = 3$, the running coupling constant $g(T)$ leads to a decrease of $B(T, B_m)$ at higher temperature. The increase of the magnetic field will not change the vacuum energy $B(T, B_m)$ at $T = 0$. But the decrease of the bag constant would be more rapidly in stronger magnetic fields, which indicates a lower value of the critical temperature for the deconfinement transition.

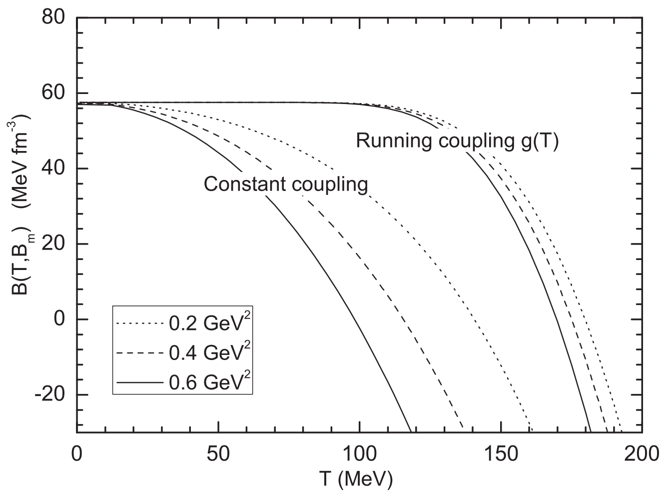

FIG. 1. The effective bag function $B(T, B_m)$ is shown as a function of temperature denoting the deconfinement transition. The $B(T, B_m)$ decreases more rapidly with increasing temperatures at stronger magnetic fields for both constant coupling and running coupling constants.

In the quasiparticle model, the decrease of the effective bag constant $B(T, B_m)$ denotes the deconfinement transition. The critical temperature can be determined by the position of half-height of the bag constant. In Fig. 2 the pseudocritical temperature is plotted by the red solid curve. For the convenience of comparison, the result from PNJL is marked by the black dashed curve [44]. It can be clearly seen that the trend of the decrease of the T_{pc} with the magnetic field is close to the lattice QCD (LQCD) result marked by the shadow band in the panel [45]. Moreover, the pseudocritical temperature decreases by about 10 percent of its original value at zero magnetic field. Our result is in agreement with the so-called inverse magnetic catalysis effect revealed by the LQCD.

Figure 3 illustrates how the pressure and the energy density (in unit of T^4) depend on the temperature of the medium. In the region around and just above the critical temperature, the energy density rises much more rapidly than the pressure, which gives rise to a observed rapid increase of the interaction measure. At stronger magnetic fields, the larger pressure and energy density are obtained at the high temperature. However, the temperature is not the only scale with the dimension of the energy. The pressure as well as the energy density cannot asymptotically converge to their Stefan-Boltzmann value $P^{\text{SB}}/T^4 = \text{constant}$ at $T \to \infty$. The deviations from the usual

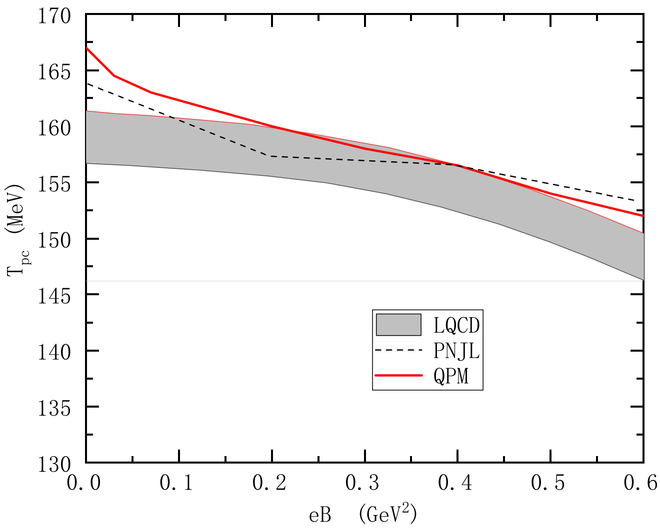

FIG. 2. The pseudocritical temperature for the deconfinement transition in a quasiparticle model (red curve) compared with the results of the PNJL model (dashed curve) [44] and of LQCD (shadow band) [45].

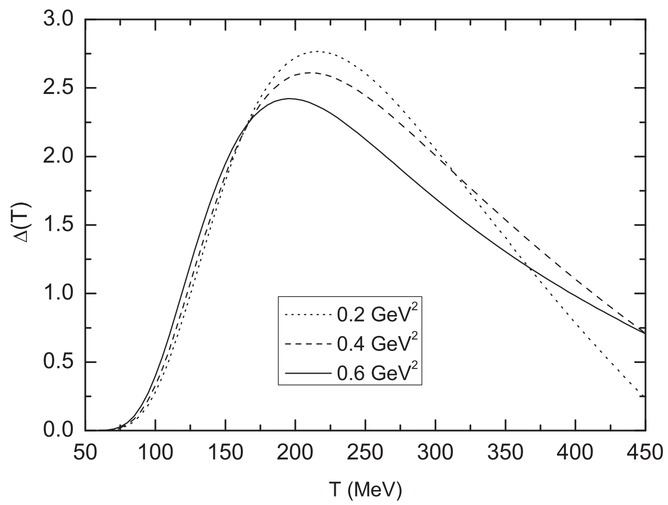

FIG. 4. The interaction measure is shown as a function of temperature in different magnetic fields. It remains so much larger at high temperatures that the interaction must be still present at $eB_m = 0.6$ GeV^2, marked by the solid curve.

Stefan-Boltzmann values are due to the quarks being constrained by the Landau level in strong magnetic fields.

Interaction measure is the trace of energy-momentum tensor. For noninteracting massless constituents the "conformal" limit is zero, so that the temperature is the only scale. In a strong magnetic field, the interaction measure is defined as $\Delta(T) \equiv (\varepsilon - 3P)/T^4$ for quark gluon plasma. In Fig. 4 the trace anomaly of the energy-momentum tensor is plotted as a function of temperature. The so-called interaction measure normalized by T^4 gives the deviation from the free gas relation between the energy density and the pressure and is also a measure of the breakdown of conformal symmetry. Even though the temperature-dependent coupling constant is employed to realize the asymptotic freedom at high temperature, the nonzero value indicates that some interactions must still be present due to the Landau levels in magnetic fields. The $\Delta(T)$ at high temperature will remain larger at stronger magnetic fields, which is in agreement with LQCD that the interaction measure remains large even at very high temperatures where the Stefan-Boltzmann (S-B) limit is not yet reached.

In fact, the S-B limit for the nth Landau level can be defined as

$$\frac{P_{\rm SB}^{(n)}}{|q_i B_m| T^2} = \frac{1}{4}. \tag{16}$$

In Fig. 5 the pressures and entropy of i-flavor quarks in the nth Landau level are shown as a function of temperature. At sufficiently high temperatures, the scaled pressure $P^{(n)}/(|q_i B_m| T^2)$ on the left panel can approach the limit marked by the

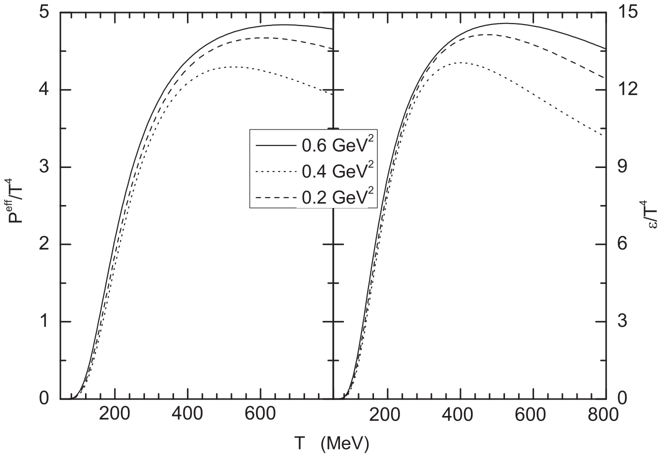

FIG. 3. The scaled pressure $P^{\rm eff}/T^4$ on the left panel and energy density ϵ/T^4 on the right panel are always increasing functions of temperature. It is apparent that the Stefan-Boltzmann (S-B) limit is absent at high temperatures in strong magnetic fields.

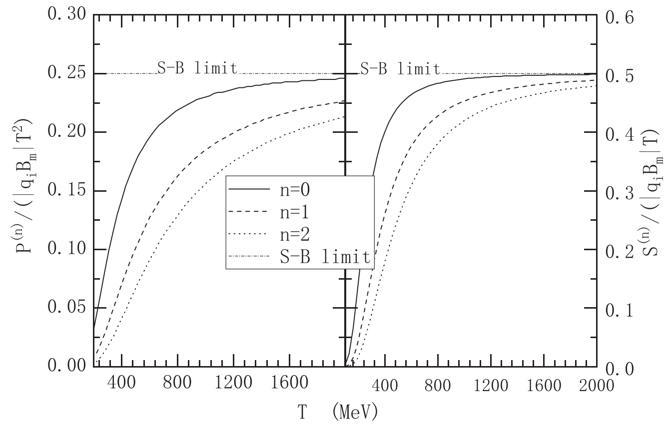

FIG. 5. The scaled pressure and entropy of i-flavor quarks lying in Landau levels $n = 0$, 1, and 2. At high enough temperatures, the three lines approach the Stefan-Boltzmann limit marked by the dash-dotted horizontal line. It is characteristic that the limit is early approached in the lowest Landau level.

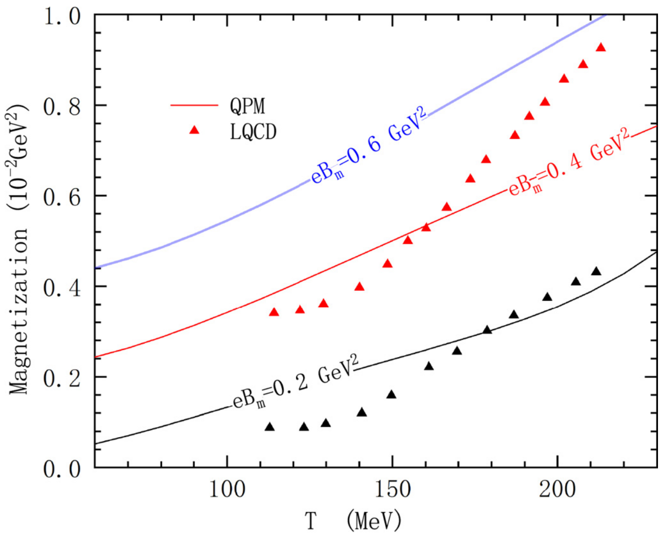

FIG. 6. Magnetization of the strange quark matter is shown as a function of temperature at $eB_m = 0.2$, 0.4, 0.6 GeV^2. The LQCD result at $eB_m = 0.2$ and 0.4 GeV^2 is marked by scattering triangles for comparison [47].

dash-dotted horizontal line. In particular, the pressure from the lowest Landau level ($n = 0$) marked by the solid line is close to the S-B limit line. Moreover, a much higher temperature is required for any excited levels ($n = 1, 2$) to reach the limit. It can be accounted for by the fact that the higher level leads to the larger effective mass and therefore results in a larger deviation from the S-B limit. Correspondingly, the entropy $S^{(n)}/(|q_iB_m|T)$ in the n level has the S-B limit shown on the right panel in the strong magnetic field.

In Fig. 6 the magnetization of strange quark matter is shown as a function of temperature at fixed magnetic fields $eB_m = 0.2$, 0.4, 0.6 GeV^2 marked by the black, red, and blue solid curves, respectively. The positive value produces the paramagnetic characteristic for the whole temperature range [46]. The magnetization increases with increasing temperature at fixed magnetic fields. It can be understood that the more Landau levels at high temperature the stronger the magnetization. The effective bag function marked by the red dotted line enhances the magnetization at finite temperature. The discrepancy of the magnetization at low temperature is sizable due to the free magnetic contribution. By comparison with the lattice result marked by the scattering triangles at $eB_m = 0.2$ and 0.4 GeV^2 [47], it can be concluded that the ascending trend and the magnetic effect are consistent. In particular, the increase of the magnetic field enhances the magnetization of the quark matter.

IV. SUMMARY

In this paper we have investigated the hot QCD matter exposed to sufficiently high magnetic fields, which could be generated in RHIC experiments. The quasiparticle model is extended by including the free magnetic contribution and the effective thermomagnetic bag constant, which is self-consistently derived to represent the confinement. The running coupling constant has been employed to reflect the asymptotic freedom of QCD. It has been found that the decrease of the effective bag constant at high temperature indicates the occurrence of the deconfinement transition. Moreover, the stronger magnetic field results in a more rapid decrease of the effective bag constant, which provides a novel method to account for the so-called inverse magnetic catalysis effect. Moreover, the paramagnetic characteristic of QCD is obtained in the quasiparticle model. The effective bag constant would have an additional contribution in the new definition of the magnetization due to the medium effect. It is concluded that the magnetization modified by the function and free magnetic contributions can only account for the trend revealed by the lattice result. It would be of some interest to improve the quasiparticle model to quantitatively interpret the lattice results in the future.

For the quark-gluon plasma in strong magnetic field, the interaction measure remains larger even at very high temperature and indicates some interactions are present. Therefore, the usual S-B limit is not applicable. The deviation from the S-B limit becomes remarkable with the massive effective mass led by the stronger magnetic field. Not only the temperature but also the magnetic field are the scale of energy. We suggested that for single Landau level, the S-B limit of the quark pressure can be defined as $|q_iB_m|T^2/4$. It has been shown that the lowest Landau level is close to the S-B limit, while higher temperature is required for the excited levels to approach the limit.

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