Heavy quark diffusion and radiation at intermediate momentum

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Heavy quark diffusion and radiation are discussed in an intermediate-momentum regime where finite mass effects can be significant. Diffusion processes are described in the Fokker-Planck approximation for soft momentum transfer, while radiative ones are taken into account by nearly collinear gluon emission from a single scattering in the Boltzmann equation. There are also radiative corrections to the transverse momentum diffusion coefficient, which are $\mathcal{O}(g^2)$ suppressed more than the leading-order diffusion coefficient but logarithmically enhanced. Numerical results show that the heavy quark distribution function depends on the energy loss mechanism so that the medium modifications by diffusion and radiation are distinguishable. The nuclear modification factor is estimated by employing the heavy quark diffusion coefficient which is constrained by lattice quantum chromodynamics data. The suppression factor exhibits a transition from diffusion at low momentum to radiation at high momentum. The significance of the radiative effects at intermediate momentum depends on the diffusion coefficient and the running coupling constant.

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I. INTRODUCTION

Heavy quarks are important probes for high-temperature quantum chromodynamics (QCD) matter created in relativistic heavy-ion collisions, as they are mostly produced at an early stage and conserved during the evolution. Slowly moving heavy quarks experience a Brownian motion in quark-gluon plasmas, and gluon-bremsstrahlung can affect the high-momentum spectra. Medium modifications of heavy quark production can be described by the collisional and radiative energy loss. Heavy quark transport and the related energy loss have been thoroughly investigated by various models (for recent review, see Refs. [1–3]). Many of the transport models treat medium-induced gluon emission as an additional contribution to heavy quark diffusion or analogously to jet quenching with multiple scatterings. Previously, a recoil force term due to gluon radiation has been introduced in the Langevin equation for Brownian motion [4], and the radiative energy loss has been estimated independently of the collisional energy loss [5,6]. In these studies, it is not easy to distinguish two energy-loss effects and to find out which mechanism is more influential, depending on momentum. This work introduces a heavy-quark transport approach that allows a different treatment of gluon-bremsstrahlung from diffusion while describing two mechanisms consistently with a single transport parameter. Concentrating on an intermediate-momentum regime where heavy mass effects can be significant, the transition between diffusion and radiation from a single scattering is investigated.

The interaction between heavy quarks and dynamic thermal media is characterized by transport coefficients. Especially, the heavy quark diffusion coefficient depending on momentum and temperature is important because it controls the rate of equilibration in high-temperature QCD plasmas. The leading-order momentum diffusion coefficient has been calculated by hard-thermal-loop (HTL) perturbation theory [7–9], and its $\mathcal{O}(g)$ correction has been obtained in the soft sector [10]. For a realistic value of the strong coupling constant, the classical correction is so large that nonperturbative determination is required. Similar to the jet transport parameter $\hat{q}$, there are also quantum corrections which are suppressed by $\mathcal{O}(g^2)$ but double-logarithmically enhanced [11,12]. Recently, a Bayesian analysis and transport model comparison have been performed to determine the heavy quark transport coefficients from phenomenological studies [13–15]. While most models are able to describe experimental data with some adjustment of parameters, the extracted diffusion coefficients vary due to the large differences between models.

The distribution function of heavy quarks can be described by the Boltzmann equation

$$\left(\frac{\partial}{\partial t}+\boldsymbol{v}\cdot\frac{\partial}{\partial \boldsymbol{x}}\right)f(\boldsymbol{p})=C_{\rm col}[f]+C_{\rm rad}[f], \tag{1}$$

where the collision terms correspond to elastic scattering and gluon emission for the collisional and radiative energy loss, respectively. In a leading-log approximation the first term can be formulated as a Fokker-Planck operator, while the second term is radiative corrections to the collision kernel responsible for diffusion. For heavy quarks with intermediate momentum, the transport equation can be formulated only in terms of the momentum diffusion coefficient which can be constrained by lattice QCD computations. With this single transport parameter, two types of energy loss are treated consistently and the relative importance of each mechanism can be studied in the transition region.

The outline of the paper is as follows. Section II presents a brief review of the leading-log heavy quark diffusion with a Fokker-Planck equation. Then, the radiative effects, nearly collinear gluon-emission and radiative corrections to the transverse momentum diffusion coefficient, are discussed in Sec. III. Section IV presents the numerical results for the medium modifications of the heavy quark spectrum. Employing the heavy quark diffusion coefficient constrained by lattice QCD data and the running coupling constant, the nuclear modification factors of heavy quarks are estimated for a Bjorken expansion. Finally, a summary is given in Sec. V. The details on gluon emission are provided in the Appendix.

II. HEAVY QUARK DIFFUSION

This section begins with a brief review on the collisional energy loss of heavy quarks in a relatively low-momentum regime [7–9,16,17]. Traversing quark-gluon plasmas, heavy quarks with $m, p \gg T$ undergo diffusion by elastic scattering. For spacelike soft-gluon exchange, the leading collision term in Eq. (1) can be approximated as a Fokker-Planck operator,

$$C_{\rm col}[f] = \frac{\partial}{\partial p^i}[\eta(\boldsymbol{p})p^i f(\boldsymbol{p})] + \frac{1}{2}\frac{\partial^2}{\partial p^i \partial p^j}[\kappa^{ij}(\boldsymbol{p})f(\boldsymbol{p})], \quad (2)$$

where $\eta(\boldsymbol{p})$ is the drag coefficient and $\kappa^{ij}(\boldsymbol{p}) = \kappa_L(p)\hat{p}^i\hat{p}^j + \kappa_T(p)(\delta^{ij} - \hat{p}^i\hat{p}^j)$ is the momentum diffusion tensor.

For a heavy quark moving in the z direction, the longitudinal and transverse momentum diffusion coefficients are defined as

$$\kappa_L(p) = \int d^3\boldsymbol{q}\frac{d\Gamma(\boldsymbol{q})}{d^3\boldsymbol{q}}\boldsymbol{q}_z^2,$$
$$\kappa_T(p) = \frac{1}{2}\int d^3\boldsymbol{q}\frac{d\Gamma(\boldsymbol{q})}{d^3\boldsymbol{q}}\boldsymbol{q}_T^2, \quad (3)$$

where $\boldsymbol{q}$ is the soft momentum transfer. Because the heavy quark mass is larger than a typical parton momentum of $\mathcal{O}(T)$, the dominant contribution comes from t-channel gluon exchange. In the Coulomb gauge, the collision rate is given by

$$\begin{aligned} C(\boldsymbol{q}) &\equiv (2\pi)^3\frac{d\Gamma(\boldsymbol{q})}{d^3\boldsymbol{q}} \\ &= \frac{\pi}{2}g^2C_F m_D^2\int d\omega\,\delta(\omega - \boldsymbol{q}\cdot\boldsymbol{v})\frac{T}{q} \\ &\quad\times\left[\frac{2}{|\boldsymbol{q}^2 + \Pi_L(Q)|^2} + \frac{(q^2-\omega^2)(q^2v^2-\omega^2)}{q^4|\boldsymbol{q}^2 - \omega^2 + \Pi_T(Q)|^2}\right], \end{aligned} \quad (4)$$

where the interaction rate can be expressed in terms of the imaginary part of the heavy quark self-energy [7]. Taking account of heavy quark interactions with both gluons and light quarks, the Debye screening mass is $m_D^2 = \frac{2N_{c,f}g^2}{T}\int\frac{d^3\boldsymbol{k}}{(2\pi)^3}n(k)[1 \pm n(k)] = (N_c + \frac{N_f}{2})\frac{g^2T^2}{3}$ and HTL resummations are [18,19]

$$\Pi_L(Q) = m_D^2\left[1 - \frac{\omega}{2q}\left(\ln\frac{q+\omega}{q-\omega} - i\pi\right)\right],$$
$$\Pi_T(Q) = m_D^2\left[\frac{\omega^2}{2q^2} + \frac{\omega(q^2-\omega^2)}{4q^3}\left(\ln\frac{q+\omega}{q-\omega} - i\pi\right)\right]. \quad (5)$$

In a leading-log approximation [9]

$$\kappa_L(p) = \kappa_0\frac{3}{2}\left[\frac{E^2}{p^2} - \frac{E(E^2-p^2)}{2p^3}\ln\frac{E+p}{E-p}\right],$$
$$\kappa_T(p) = \kappa_0\frac{3}{2}\left[\frac{3}{2} - \frac{E^2}{2p^2} + \frac{(E^2-p^2)^2}{4Ep^3}\ln\frac{E+p}{E-p}\right], \quad (6)$$

where $\kappa_0 \equiv \kappa_L(p=0) = \kappa_T(p=0) = \frac{g^4C_FT^3}{18\pi}(N_c + \frac{N_f}{2})[\ln\frac{T}{m_D} + \mathcal{O}(1)]$.

As the heavy quark distribution must approach the thermal equilibrium, $f(\boldsymbol{p}) \propto e^{-E_p/T}$, the drag coefficient and the longitudinal diffusion coefficient are related by $\eta(p) = \kappa_L(p)/(2TE)$ to leading order in T/E. At this order, the collisional energy loss of heavy quarks, $-\frac{dE}{dz} = p\eta(p)$, is also proportional to the longitudinal diffusion coefficient.

III. RADIATIVE EFFECTS

The collisional energy loss by diffusion is dominant for low-momentum heavy quarks, whereas the medium-induced gluon emission starts to contribute as the heavy quark momentum increases. Unlike quasiparticle dynamics where both collisional and radiative processes contribute at leading order [20], gluon emission off slow heavy quarks is $\mathcal{O}(g^2)$ suppressed more than elastic scatterings at weak coupling. While there is $\mathcal{O}(1/g^2)$ enhancement for light partons with soft gluon exchange and collinear gluon emission [21,22], radiation from heavy quarks depends on their momentum extent because the heavy quark mass cannot be ignored in an intermediate-momentum regime. At higher orders, diffusion and radiation are not clearly distinguished [23]. It will be observed that a part of radiative effects will contribute to the transverse momentum diffusion coefficient.

The radiative energy loss of ultrarelativistic partons, known as jet quenching, has been extensively studied using different formalisms: the path-integral formulation, a Schrödinger-like equation, opacity and high-twist expansions, and a summation of ladder diagrams [24–28]. Gluon emission from light partons takes some time (called the formation time), $t_f \sim 1/(g^2T)$ which is of the same order as the mean free path. In that case, one needs to sum multiple scatterings which reduce the emission rate due to the coherence (LPM) effect [29,30]. The radiative energy loss of heavy quarks has been evaluated within the frameworks of high-twist and opacity expansions [5,31–34]. This work will follow a diagrammatic approach of Ref. [22] to evaluate gluon emission from heavy quarks with $p \gg m$.

For energetic heavy quarks, soft collisions induce collinear gluon-bremsstrahlung. Figure 1 shows diagrams for the radiative contributions [35]. The radiative energy loss is dominated by hard gluon emission ($k \sim T$), even though the energy of gluon is still much smaller than that of heavy quark ($k \ll E_{\boldsymbol{p}}$).

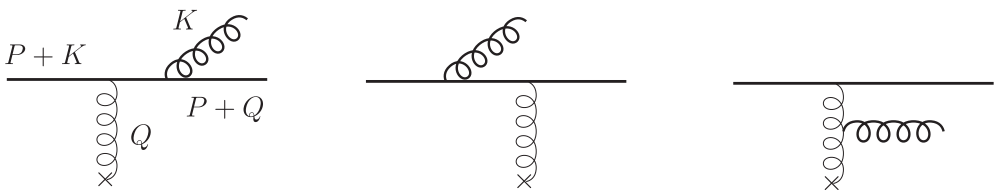

FIG. 1. Gluon radiation off heavy quarks interacting with soft classical fields. Thick solid lines denote heavy quarks, thick and thin wiggly lines are hard ($K \sim T$) and soft ($Q \sim gT$) gluons, respectively, and crosses are for thermal scattering centers.

In the collinear limit, the emitted gluon has transverse momentum, $k_T \sim gT$. The radiative process is then factorized into elastic scattering and the gluon emission factor allowing enhancement so that radiation can be as important as elastic scattering.

The energy change in the radiation process is the inverse formation time

$$\frac{1}{t_f} = \delta E = E_{\boldsymbol{p}} + k^0 - E_{\boldsymbol{p}+\boldsymbol{k}} \simeq \frac{\boldsymbol{k}_T^2 + m^2x^2 + m_g^2}{2k(1-x)}, \tag{7}$$

where $x = k/E_{\boldsymbol{p}+\boldsymbol{k}}$ and $m_g^2 = m_D^2/2$ is the thermal mass of the emitted gluon. The initial transverse momentum of heavy quark has been chosen to be zero, $\boldsymbol{p}_T + \boldsymbol{k}_T = 0$. If the heavy quark momentum is so large that $mx \sim gT$, then one needs to consider multiple soft scatterings as light partons. On the other hand, radiation rarely occurs from heavy quarks with $p \lesssim m$ if $mx \sim T$. To smoothly interpolate between the two limits, only the $gT \ll mx \ll T$ case will be considered. Then the formation time is shorter than the mean free path, allowing for a discussion only on gluon emission from a single scattering.

Gluon emission from quark-gluon plasmas has been computed by summing multiple scatterings during the emission process [22,28,36]. Without the LPM effect, the radiative corrections to the collision kernel and the transport coefficient $\hat{q}$ have been evaluated for ultrarelativistic partons [11,12,37], but not for heavy quarks with finite mass effects. Adopting a similar approach to $\hat{q}$ in a single scattering, the heavy quark case will be considered in this work. In this way, heavy quark diffusion and radiation can be consistently calculated by using the transverse momentum diffusion coefficient in an intermediate-momentum regime.

The gluon emission rate is given by [38]

$$\frac{d\Gamma(E_{\boldsymbol{p}}, k)}{dk} = \frac{g^2C_F}{8\pi k^3}[1+n_B(k)][1-n_F(E_{\boldsymbol{p}-\boldsymbol{k}})]\frac{(1-x)^2+1}{(1-x)^2} \times \int \frac{d^2\boldsymbol{p}_T}{(2\pi)^2}\boldsymbol{p}_T \cdot \mathrm{Re}\,F(\boldsymbol{p}_T), \tag{8}$$

where $\Gamma(E_{\boldsymbol{p}}, k)$ is the rate for a heavy quark with momentum $\boldsymbol{p}$ to emit a gluon with energy k, $n_B(k)$ and $n_F(E_{\boldsymbol{p}-\boldsymbol{k}})$ are the Bose-Einstein and Fermi-Dirac thermal distributions, respectively, and $F(\boldsymbol{p}_T)$ is the solution of a linear integral equation which sums ladder diagrams. For a single scattering (see Appendix),

$$\mathrm{Re}\,F(\boldsymbol{p}_T) = \frac{2}{\delta E(\boldsymbol{p}_T)}\int \frac{d^3\boldsymbol{q}}{(2\pi)^3}C(\boldsymbol{q}) \times \left[\frac{\boldsymbol{p}_T}{\delta E(\boldsymbol{p}_T)} - \frac{\boldsymbol{p}_T+\boldsymbol{q}_T}{\delta E(\boldsymbol{p}_T+\boldsymbol{q}_T)}\right], \tag{9}$$

where $C(\boldsymbol{q})$ is the collision kernel in Eq. (4). Taking the real processes [39] it is assumed that the emitted gluon has a larger transverse momentum than the soft momentum of gluon exchange, $p_T \gg q_T$ [1]:

$$\begin{aligned}\int \frac{d^2\boldsymbol{p}_T}{(2\pi)^2}\boldsymbol{p}_T \cdot \mathrm{Re}\,F(\boldsymbol{p}_T) &= \int \frac{d^2\boldsymbol{p}_T}{(2\pi)^2}\int \frac{d^3\boldsymbol{q}}{(2\pi)^3}C(\boldsymbol{q})\left[\frac{\boldsymbol{p}_T}{\delta E(\boldsymbol{p}_T)} - \frac{\boldsymbol{p}_T+\boldsymbol{q}_T}{\delta E(\boldsymbol{p}_T+\boldsymbol{q}_T)}\right]^2 \\ &\simeq 8\kappa_T k^2(1-x)^2\int \frac{d^2\boldsymbol{p}_T}{(2\pi)^2}\frac{1}{\left(\boldsymbol{p}_T^2+m^2x^2+m_g^2\right)^2},\end{aligned} \tag{10}$$

where the definition of the transverse momentum diffusion coefficient, Eq. (3), has been used. Except for employing the collision kernel responsible for heavy quark diffusion instead of a static Debye-screened potential or the same kernel of light partons in a dynamical medium [41], this corresponds to the incoherent limit of the $N = 1$ opacity expansion [5,32,33].

In the Boltzmann equation (1), the radiation term is given by [28,38]

$$C_{\mathrm{rad}}[f] \sim \int dk\left[f(\boldsymbol{p}+\boldsymbol{k})\frac{d\Gamma(E_{\boldsymbol{p}+\boldsymbol{k}}, k)}{dk} - f(\boldsymbol{p})\frac{d\Gamma(E_{\boldsymbol{p}}, k)}{dk}\right], \tag{11}$$

where $\boldsymbol{p}+\boldsymbol{k} \simeq (p+k)\hat{\boldsymbol{p}}$ in the eikonal approximation. $k < 0$ corresponds to gluon absorption which is required for detailed balance. Heavy quark radiation is different from light partons in that gluon emission is suppressed at smaller angles than m/E [42]. This dead-cone effect can be observed if m^2x^2 is larger than the other terms in the denominator of Eq. (10). In the region $gT \ll mx \ll T$, the radiation term can be larger than $\mathcal{O}(g^6)$ but smaller than $\mathcal{O}(g^4)$ of the ultrarelativistic limit. If the energy carried by an emitted gluon is soft ($k \sim gT$), the first term in Eq. (11) can be expanded, contributing to the longitudinal diffusion at next-to-leading order $\mathcal{O}(g^5)$ [23].

The collision kernel $C(k_T)$ is the rate for heavy quark to acquire a transverse momentum k_T. After gluon emission in Fig. 1, radiative corrections arise:

$$\delta C(k_T) = \frac{g^2C_F\kappa_T}{\pi}\int \frac{dk}{k}[(1-x)^2+1]\frac{1}{\left(\boldsymbol{k}_T^2+m^2x^2+m_g^2\right)^2}, \tag{12}$$

[1] The approximations and power-counting used in this section are similar to those for semicollinear emission [23,39,40] or soft-collinear effective theory [34].

which has been obtained in the same approximation as Eq. (10). Then the radiative correction to the transverse momentum diffusion coefficient is

$$\delta\kappa_T(p) = \frac{1}{2}\int \frac{d^2\boldsymbol{k}_T}{(2\pi)^2} k_T^2 \delta C(k_T)[1+n_B(k)]. \tag{13}$$

Using the kinematic boundaries, $k_{T,\max} \sim k$, $k_{\max} \sim p$, and $k_{\min} \sim T$,

$$\delta\kappa_T(p) \sim g^2\kappa_T \ln\frac{E}{m}\ln\frac{p}{T}. \tag{14}$$

In comparison to the leading-order coefficient κ_T, $\delta\kappa_T$ is $\mathcal{O}(g^2)$ suppressed but logarithmically enhanced in the high-momentum limit. This is analogous to quantum corrections to the transverse momentum broadening coefficient [11,12], except for the different phase space boundaries and the heavy quark mass regulating the collinear singularity. The importance of the factor $[1+n_B(k)]$ in Eq. (13) has been discussed in Ref. [37]: it is needed to account for Bose enhancement for $k \lesssim T$, connecting to $\mathcal{O}(g)$ classical corrections for soft-gluon emission. A numerical estimate for this potentially large correction is given in Fig. 3(a) in the next section. The correction increases with the heavy quark momentum and becomes comparable to the leading-order coefficient at high momentum.

The final form of the radiation term is given by

$$C_{\rm rad}[f] = \int dk\left[f((p+k)\hat{\boldsymbol{p}})\frac{d\Gamma(E_{(p+k)\hat{\boldsymbol{p}}},k)}{dk} - f(\boldsymbol{p})\frac{d\Gamma(E_{\boldsymbol{p}},k)}{dk}\right] + \frac{1}{2}\nabla^2_{\boldsymbol{p}_T}[\delta\kappa_T(p)f(\boldsymbol{p})]. \tag{15}$$

Because the emission rate in Eq. (11) can be as small as $\mathcal{O}(g^6)$ at low momentum, the radiative correction ($\delta\kappa_T$ term) to the eikonal approximation has been included.

IV. NUMERICAL ANALYSIS

The heavy quark Boltzmann equation has been formulated with diffusion and radiation in Eqs. (2) and (15), respectively. Using the leading-log momentum dependence, Eq. (6), the two collision terms involve only one parameter, the static momentum diffusion coefficient, $\kappa_{L,T}(p=0) \equiv \kappa_0 = 2T^2/D_s$ (D_s is the spatial diffusion coefficient at $p=0$). Since the perturbative expansion poorly converges at a realistic value of the strong coupling constant [10], κ_0 from lattice QCD data will be used so that nonperturbative effects can be absorbed in this transport coefficient. Employing κ_0 in this way amounts to effectively changing the coupling constant and the thermal masses of light partons in the collision kernel, Eq. (4).

Figure 2 shows how the b quark distribution with an initial δ function evolves in a static medium, under the influence of two different types of energy loss. It is noteworthy that how the distributions are spread out with time depends on the energy loss mechanism. The diffusion process is characterized by Gaussian fluctuations, whereas the radiative one develops non-Gaussian distributions. It has been discussed that there are significant differences between Langevin and Boltzmann approach for heavy quark diffusion unless the ratio m/T is large: the Langevin (Fokker-Planck) approach is a good approximation for bottom quark diffusion [43]. In this work, the radiation term of Eq. (11) is not expanded for soft gluon emission, so it is not a diffusion operator. This difference between diffusion and radiation might allow the medium modifications by two mechanisms to be qualitatively distinguishable from each other.

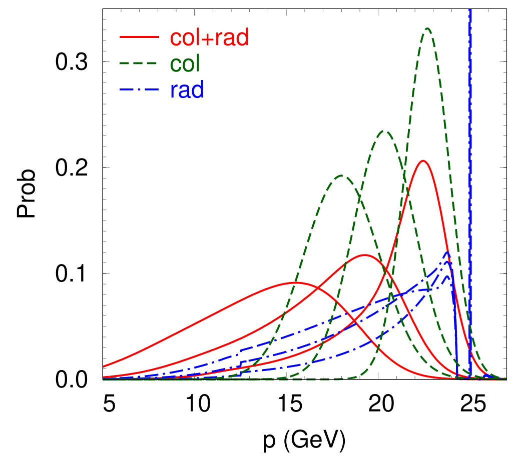

FIG. 2. The probability distribution of b quarks with initial momentum $p_0 = 25$ GeV in a static medium at $T = 300$ MeV, using $m = 4.5$ GeV, $(2\pi T)D_s = 6$, and $\alpha_s = 0.3$ for gluon emission. From right to left, $t = 5,\ 10,$ and 15 fm.

The transport coefficients and their dependence on momentum and temperature are crucial to analyze experimental data. Figure 3 shows the momentum and temperature dependence of the transport coefficients employed in this work. As the momentum of heavy quark increases, the momentum diffusion coefficient and energy loss increase. At the leading-log order, the momentum dependence of the longitudinal and transverse diffusion coefficients is modest, shown as the solid and dashed lines, respectively. As mentioned in the previous section, $\delta\kappa_T(p)$ (the radiative correction to κ_T) also grows with momentum and becomes considerable at high momentum, especially for a strong coupling constant $\alpha_s \sim 0.3$.

The temperature dependence of $(2\pi T)D_s$ comes from running of the coupling constant.[2] An infrared-finite effective running coupling has been developed and employed for the spacelike momentum transfer [47,48]. Replacing the coupling constant in the t-channel amplitude by the running coupling and using the one-loop result,

$$\alpha_s(Q^2) = \frac{12\pi}{(11N_c - 2N_f)\ln\left(Q^2/\Lambda^2_{\rm QCD}\right)}, \tag{16}$$

at the scale $Q^2 \sim t$ ($\Lambda_{\rm QCD} \approx 200$ MeV), resummations and nonperturbative effects can be implemented [47–49]. This

[2]The running coupling constant is related to nonperturbative effects in heavy quark diffusion. These effects have also been considered in the T-matrix approach [44] and using a rather strong coupling with large quasiparticle masses near T_c [45,46].

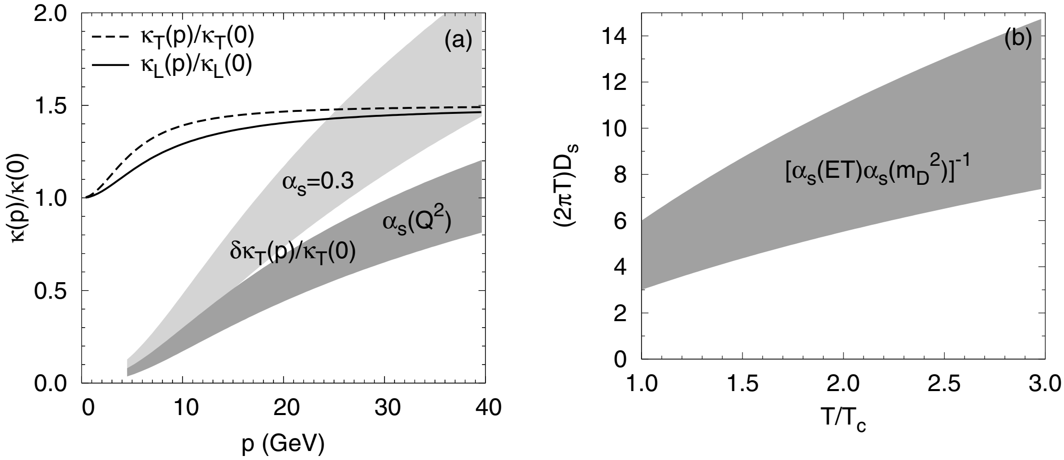

FIG. 3. (a) The momentum dependence of the heavy quark transport coefficients. The light and dark shaded regions represent the momentum-dependent $\delta\kappa_T(p)$ using fixed and running coupling constants, respectively. The upper (lower) lines of the shaded regions correspond to $T = 157(475)$ MeV. (b) The temperature dependence of $(2\pi T)D_s$.

work follows Ref. [50] to consider the dependence on a wide range of t scales, from $\mathcal{O}(m_D^2)$ up to $\mathcal{O}(ET)$. Then $\kappa_0 \propto \alpha_s(ET)\alpha_s(m_D^2)T^3$, where m_D is self-consistently determined by [51]

$$\ln\left(\frac{m_D^2}{\Lambda_{\rm QCD}^2}\right) = \frac{N_c(1+N_f/6)}{11N_c - 2N_f}\left(\frac{4\pi T}{m_D}\right)^2. \tag{17}$$

As the temperature decreases, the running coupling becomes stronger near T_c where nonperturbative effects enter. For temperature and momentum considered in this work, $\alpha_s \sim 0.23$–0.68 which is of the same order as the effective coupling from Ref. [48]. While the coupling constant decreases with increasing temperature, $(2\pi T)D_s = 4\pi T^3/\kappa_0 \propto [\alpha_s(ET)\alpha_s(m_D^2)]^{-1}$ increases by a factor of ∼2.5 in Fig. 3(b), aligning closely with the lattice QCD data from Refs. [52–54]. Although the degree of increase might vary with a different choice of effective coupling, the temperature dependence is expected to be qualitatively consistent with the current study. For the radiation process, the running coupling constant is determined at the scale $Q^2 = (\boldsymbol{k}_T^2 + m^2x^2 + m_g^2)/x$ [55].

The nuclear modification factor of heavy mesons is an important observable to measure the thermal medium effects in heavy-ion collisions. It is affected by the initial production of heavy quarks, medium evolution, and hadronization as well as heavy quark interactions in quark-gluon plasmas. This work focuses on the energy loss effects in quark-gluon plasmas, especially the qualitative difference between two energy loss mechanisms. To isolate significant uncertainties related to medium expansion and hadronization, a simple model is employed. The initial spectrum of b quarks is given by the differential cross section of B meson production measured in pp collisions [56], fit to the following form:

$$\frac{dN}{p_T dp_T} \propto \frac{1}{\left(\boldsymbol{p}_T^2 + \Lambda^2\right)^{\alpha}}, \tag{18}$$

where $\Lambda = 6.07$ GeV and $\alpha = 2.85$. Then the plasma evolution is described by a Bjorken expansion, $T(t) = T_0(t_0/t)^{1/3}$ [57] with $t_0 = 0.6$ fm and $T_0 = 475$ MeV [58] until $T_c = 157$ MeV [59]. These initial conditions depend on centrality and collision energy, but the variations of the values have little impact on the qualitative analysis of the momentum spectrum. After solving for the heavy quark distribution, the ratio of the final spectrum to the initial one is used to estimate the suppression factor

$$R_{AA}(p_T) = \frac{\left.\frac{dN}{dp_T}\right|_{t=t_f}}{\left.\frac{dN}{dp_T}\right|_{t=t_0}}. \tag{19}$$

Figure 4 shows the nuclear modification factor for b quarks. The solid lines are the results using the momentum-dependent diffusion coefficients and the running coupling constant, while the dashed lines are the results with constant diffusion coefficient and coupling constant. At $p = 0$ and $T = T_c$, the diffusion coefficient is fixed, $(2\pi T_c)D_s(T_c) = 3$–6, closely aligning with the lattice QCD data from Refs. [52–54]. The value of α_s directly affects the suppression by the radiative energy loss: the stronger the coupling, the smaller the R_{AA} factor. As expected from Fig. 2, the collisional and radiative effects exhibit distinct momentum behaviors. The R_{AA} by the radiative energy loss consistently decreases with momentum, while the R_{AA} by the collisional energy loss decreases at low momentum but increases at intermediate momentum. Thus, as the heavy quark momentum rises, the dominant energy loss shifts from collisional to radiative. It is noteworthy that the momentum at which this transition occurs depends on the transport coefficients and their dependence on momentum and temperature. In the current numerical analysis, the transition takes place (and radiation becomes effective) at higher momentum when $\kappa_{L,T}$ increases with momentum and α_s decreases with energy and temperature, compared to when they are constant.

While the momentum-dependence of Eq. (6) is valid to leading logarithm in T/m_D, higher-order terms can influence the flatness of the suppression factor. To estimate this effect, if a 30% increase in the diffusion coefficients' growth rate

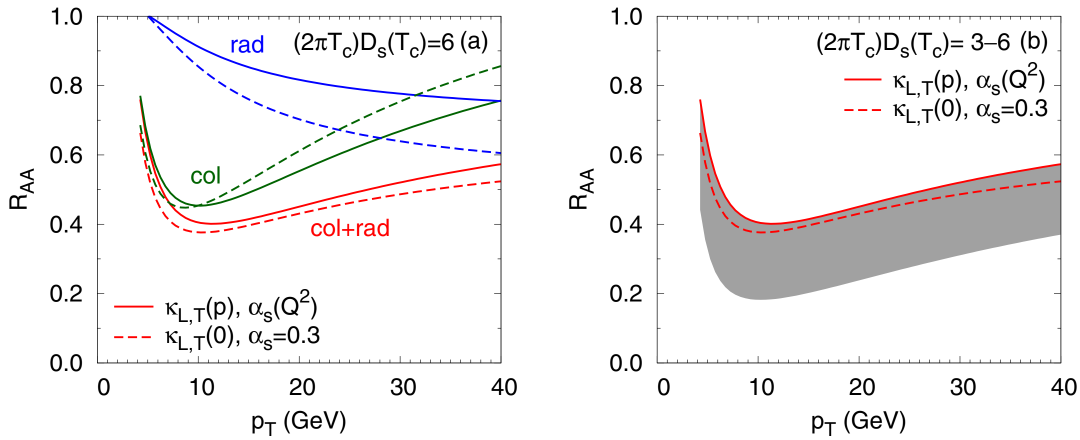

FIG. 4. The nuclear modification factor R_{AA} for b quarks. (a) The solid lines show the results using the momentum-dependent $\kappa_{L,T}$ and running α_s, with $(2\pi T_c)D_s(T_c) = 6$ fixed at $p = 0$ and $T = T_c$. The dashed lines show the results with constant $\kappa_{L,T}$ and α_s. (b) The upper and lower bounds of the shaded region correspond to $(2\pi T_c)D_s(T_c) = 6$ and 3, respectively.

with respect to momentum is assumed, the R_{AA} factor with $(2\pi T_c)D_s(T_c) = 6$ would be reduced by at most 20% at high momentum, flattening R_{AA}. Despite the stronger momentum-dependence, it would still be within the shaded region in Fig. 4(b) due to the large uncertainties of D_s. The qualitative behavior discussed in the previous paragraph remains consistent because the momentum-dependence enters both diffusion and radiation simultaneously. This phenomenological study estimates the suppression factor with the leading momentum-dependence of the diffusion coefficients, allowing for the implicit inclusion of higher-order effects through the nonperturbative lattice QCD data and the running coupling constant.

The qualitative distinction between diffusion and radiation in the momentum spectra might be useful to identify the relevant energy loss process.[3] The radiative effect makes the nuclear modification factor flatter than the suppression entirely by the collisional one, as seen in Fig. 4(a). Although it is premature to compare the numerical results with experimental data, the suppression factor calculated with $(2\pi T_c)D_s(T_c) =$ 3–6 is comparable with the R_{AA} factor of B mesons [56,62]. A Bjorken expansion has been employed in this work, while $(3+1)$-dimensional expansion provides the time evolution of the spatial distribution of temperature and collective flow velocity. The energy loss of the heavy quark will be influenced by a modified profile of quark-gluon plasmas, determined by different temperature, lifetime, and expansion rate of $(3+1)$-dimensional evolution. However, similar medium modifications, averaged over position, are expected through the adjustment of D_s. In future work, I plan to perform a more quantitative analysis with realistic hydrodynamic evolution and hadronic effects.

It should be mentioned that the valid momentum range, where gluon emission from a single scattering is applicable, is not clear. In a high-momentum regime, the emission rate must be computed in multiple soft scatterings. Although gluon emission is more involved than photon emission (because gluons carry color) [28], the LPM effect on the photon emission rate for $k \gtrsim 2T$ is less than 30% [36]. If this suppression is included in the radiation term, the R_{AA} factor is expected to increase slightly with momentum, approximately ∼10% at most. However, the momentum dependence of the heavy quark spectrum does not change significantly. The radiative contribution is still expected to be distinguishable from the diffusion effects in an intermediate-momentum regime.

Compared to bottom quarks, charm quarks have 3 times smaller mass, thus the energy loss is expected to be larger. Although the heavy quark conditions and approximations assumed in this work may be only marginally satisfied for charm quarks, this formulation has been applied to demonstrate the impact of the heavy quark mass (see Fig. 5). Charm quarks are more suppressed by elastic scattering and gluon-bremsstrahlung than bottom quarks, while the R_{AA} factor depends similarly on momentum and temperature through the transport coefficients. The transition between diffusion and radiation occurs at relatively lower momentum, and thus the radiative effects become more significant to determine the intermediate-momentum spectrum.

V. SUMMARY

In this work, the heavy-quark Boltzmann equation has been formulated with diffusion and radiation from a single scattering in an intermediate-momentum regime. A part of the radiative effects has been shown to contribute to quantum corrections to the transverse momentum diffusion coefficient, which are $\mathcal{O}(g^2)$ suppressed than the leading-order diffusion coefficient but logarithmically enhanced in the high-energy limit. Employing the same collision kernel consistently for both processes, this formulation has only a single transport parameter, the static diffusion coefficient which can be constrained by nonperturbative determination. Although this approach is based on perturbation, the running coupling

[3]To discriminate between the collisional and radiative energy loss mechanisms, angular correlations of heavy quark pairs have also been studied [60,61].

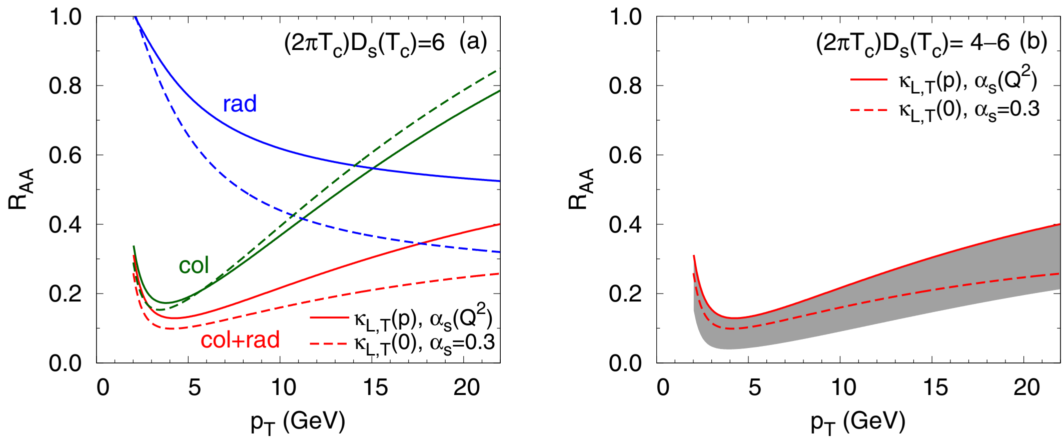

FIG. 5. The estimated R_{AA} factor for c quarks with $m = 1.5$ GeV and the initial spectrum given by the differential cross section of D meson [63].

constant and the diffusion coefficient given by lattice QCD data allow for nonperturbative effects at low momentum and temperature.

The momentum dependence of the heavy quark spectrum and the suppression factor, determined by the two types of heavy quark energy loss, has been investigated. For nearly collinear gluon emission from a single scattering, the medium modifications by radiation are found to be distinguishable from those by diffusion so that the relevant energy loss mechanism can be identified. The numerical results indicate that, at low and high momentum, the R_{AA} factor is primarily influenced by the collisional and radiative energy loss, respectively. Meanwhile, the importance of the radiative effects at intermediate momentum is determined by the momentum-dependent diffusion coefficient and the running coupling constant.

This work has concentrated on the qualitative features of the heavy quark momentum spectra in quark-gluon plasmas. Eventually to describe the experimental data of heavy mesons, it is necessary to consider other effects such as hadronization, finite-size medium, viscous corrections in the hydrodynamic expansion [64–66], and possible pre-equilibrium dynamics [67,68]. In the same framework, it is also essential to describe the elliptic flow induced by the spatial anisotropy of thermal media. Although various transport models for heavy quarks have been developed, incorporating both elastic and inelastic scatterings [69–72], there still exist large uncertainties in an intermediate-momentum regime. I hope that this approach provides a way to understand the transition between diffusion and radiation and to distinguish the radiative effects in the heavy quark momentum spectra.

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APPENDIX: GLUON EMISSION

In the high-momentum limit, gluon emission from heavy quarks is akin to that from light partons involving multiple scatterings. References [22,28] provide the rigorous derivation of an integral equation which sums multiple scatterings. In this Appendix, the same approach is used to evaluate a single gluon exchange diagram (Fig. 6) which is relevant to the radiative energy loss of heavy quarks. Although the emitted gluon can also interact with soft background fields, the emission rate can be simplified by assuming the real processes with $k_T \gg q_T$ as in Sec. III.

A heavy quark loop in ladder diagrams involves the following frequency integral:

$$\int \frac{dp^0}{2\pi}\frac{1}{p^0 - E_{\boldsymbol{p}} + i\Gamma/2}\,\frac{1}{p^0 + k^0 - E_{\boldsymbol{p}+\boldsymbol{k}} - i\Gamma/2} \simeq \frac{1}{i\delta E + \Gamma}, \tag{A1}$$

where $\Gamma/2$ is the heavy quark damping rate [73]. In the ultrarelativistic limit ($\delta E \sim g^2 T$), this allows $\mathcal{O}(1/g^2)$ enhancement so that gluon-bremsstrahlung contributes at leading order.

In the kinematic regime with $t_f \ll 1/(g^2 T)$, soft gluon exchange is perturbation. Based on a Bethe-Salpeter equation for the gluon vertex from either side of the diagram, Fig. 6 is roughly expressed as the sum of the loop diagrams without

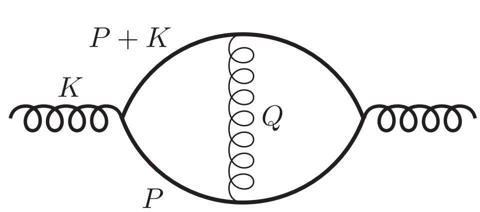

FIG. 6. Gluon emission from a single scattering. P and K are nearly collinear ($k_T \sim gT$) and the gluon exchange is soft ($Q \sim gT$).

and with a single gluon exchange,

$$F(\boldsymbol{p}_T) = \frac{2\boldsymbol{p}_T}{i\delta E + \Gamma} + \frac{1}{i\delta E + \Gamma}\int \frac{d^3\boldsymbol{q}}{(2\pi)^3} C(\boldsymbol{q}) F(\boldsymbol{p}_T + \boldsymbol{q}_T), \tag{A2}$$

where $\boldsymbol{p}_T$ is the transverse projection with respect to $\boldsymbol{k}$, and $C(\boldsymbol{q})$ is the collision kernel of Eq. (4). Then, multiplying both sides by $i\delta E + \Gamma$ and using $\Gamma = \int \frac{d^3\boldsymbol{q}}{(2\pi)^3} C(\boldsymbol{q})$, the integral equation is obtained as follows:

$$2\boldsymbol{p}_T = i\delta E\, F(\boldsymbol{p}_T) + \int \frac{d^3\boldsymbol{q}}{(2\pi)^3} C(\boldsymbol{q})[F(\boldsymbol{p}_T) - F(\boldsymbol{p}_T + \boldsymbol{q}_T)]. \tag{A3}$$

Since δE is larger than $\int \frac{d^3\boldsymbol{q}}{(2\pi)^3} C(\boldsymbol{q}) \sim g^2 T$, it can be solved perturbatively. The leading-order solution is pure imaginary, $F^0(\boldsymbol{p}_T) = 2\boldsymbol{p}_T/(i\delta E)$. Substituting this into the equation, the next-order solution is obtained and its real part determines the emission rate in Eqs. (8) and (9).

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