Measurement of $g_{9/2}$ strength in the stretched 8^- state and other negative parity states via the ^{51}V$(d,p)^{52}$V reaction

I. C. S. Hay,[1] P. D. Cottle,[1] L. A. Riley,[2] L. T. Baby,[1] S. Baker,[1] A. L. Conley,[1] J. Esparza,[1] K. Hanselman,[1] M. Heinze,[2] D. Houlihan,[1] B. Kelly,[1] K. W. Kemper,[1] G. W. McCann,[1] R. Renom,[1] A. Sandrik,[1] D. Simms,[2] M. Spieker,[1] and I. Wiedenhöver[1]

[1]*Department of Physics, Florida State University, Tallahassee, Florida 32306, USA*

[2]*Department of Physics and Astronomy, Ursinus College, Collegeville, Pennsylvania 19426, USA*

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We performed a measurement of the ^{51}V$(d,p)^{52}$V reaction at 16 MeV using the Florida State University Super-Enge Split-Pole Spectrograph (SE-SPS) to search for single-neutron transfer strength for the $g_{9/2}$ intruder orbit. Measurements of $\nu g_{9/2}$ strength with (d,p) reactions in the $N = 29$ isotones ^{49}Ca, ^{51}Ti, ^{53}Cr, and ^{55}Fe have concluded that much of the expected $\nu g_{9/2}$ strength is "missing"; that is, the summed strength is much smaller than the sum rule. In odd-odd $N = 29$ isotones, we expect a significant amount of $\nu g_{9/2}$ strength to be located in the "stretched" 8^- states with $\pi f^n_{7/2}\nu g_{9/2}$ structure that were systematically observed in the odd-odd $N = 29$ isotones via the (α,d) reaction. ^{52}V is the only one of these odd-odd isotones in which a stable target is available for single-neutron transfer reactions. We report on a determination of $\nu g_{9/2}$ strength for the stretched 8^- state and ten other negative parity states populated via $L = 4$ transfer in the ^{51}V$(d,p)^{52}$V reaction. This is the first measurement of spectroscopic strength for $L = 4$ states in ^{52}V via (d,p). In total, the $L = 4$ strength observed here sums to only 28.9(11)% of the sum rule for $g_{9/2}$ neutron strength, a result that is consistent with the summed strengths observed in recent (d,p) measurements of the even-Z $N = 29$ isotones ^{51}Ti, ^{53}Cr, and ^{55}Fe. The $(\alpha,{}^3$He) reaction and the use of particle-γ coincidences would provide more sensitivity to search for the missing $g_{9/2}$ strength.

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I. INTRODUCTION

While the nuclear structure of odd-odd nuclei is generally very complex, high-spin "stretched states" of odd-odd isotopes populated selectively by the (α,d) direct reaction have simple structures dominated by the coupling of a proton and neutron, each located in a high-angular momentum orbit, to the maximum total J^π value. These states are particularly pure when they involve either the proton or neutron being in an intruder orbit; that is, they involve a high angular momentum orbit pushed down from its native oscillator shell to the next lower major shell by the spin-orbit force [1]. For example, Okada *et al.* [2] used the (α,d) reaction to identify stretched $J^\pi = 8^-$ states in the odd-odd $N = 29$ isotones ^{52}V, ^{54}Mn, and ^{56}Co that have $\pi f^n_{7/2}\nu g_{9/2}$ structure. These states are particularly pure because the only other configurations that provide 8^- states occur at much higher excitation energies.

In recent (d,p) studies of $N = 29$ isotones [3–6], less than half of the expected $\nu g_{9/2}$ strength has been observed. The stretched 8^- states in the odd-odd $N = 29$ isotones provide a starting point for determining whether $g_{9/2}$ neutron strength is missing in these isotones as well. However, we would expect that $g_{9/2}$ neutron strength would also be located in other negative parity states with $\pi f^n_{7/2}\nu g_{9/2}$ structure; that is, in negative parity states with $J = 1$–7. Performing an inventory of the $g_{9/2}$ neutron strength in all of the negative parity states is necessary to determine whether $g_{9/2}$ neutron strength is missing in the odd-odd $N = 29$ isotones as it is in the even-Z isotones.

The search for $g_{9/2}$ neutron strength in the odd-odd $N = 29$ isotones via single-neutron stripping reactions is constrained by the fact that only one odd-Z $N = 28$ isotone is stable: ^{51}V. Hence, in the present work we present a new measurement of $1g_{9/2}$ neutron strength in the $N = 29$ isotone ^{52}V via the ^{51}V$(d,p)^{52}$V reaction at a beam energy of 16 MeV. We have extracted angular distributions for eleven states populated via the transfer of four units of angular momentum ($L = 4$) and located between excitation energies of 3.5 and 5.7 MeV. In total, the $g_{9/2}$ strength observed in these states exhausts only 28.9(11)% of the sum rule. This result is qualitatively consistent with the sum rule percentages observed for $g_{9/2}$ neutron states in the even-Z $N = 29$ isotones; that is, a considerable amount of $g_{9/2}$ neutron strength is missing in ^{52}V as it is in the even-Z $N = 29$ isotones.

We compare the present results to those from a study of the ^{51}V$(t,d)^{52}$V reaction at a beam energy of 33 MeV by Karban *et al.* [7]. The (d,p) reaction has an advantage over the (t,d) in that the (d,p) angular distributions differ more strongly between different L values than the (t,d) angular distributions do, making it easier to identify $L = 4$ states using the (d,p) reaction. The overarching conclusion here—that a significant amount of $g_{9/2}$ neutron strength is missing—is the same as that reached by Karban *et al.* It is also worth noting that previous ^{51}V$(d,p)^{52}$V studies, all performed at beam energies much lower than the 16 MeV we used, did not report any $L = 4$ spectroscopic strengths [8–14].

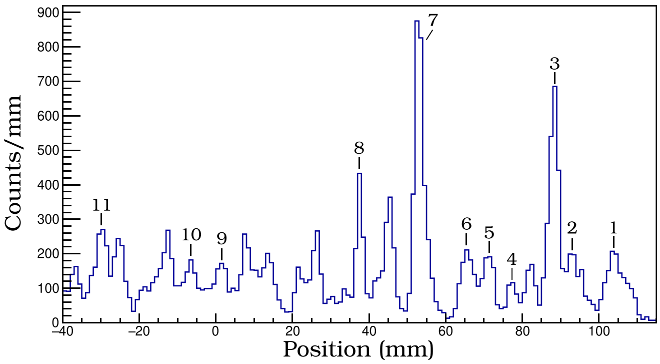

FIG. 1. Proton-momentum spectum at a laboratory angle of 40° for the 7400 G magnetic field setting. The $L = 4$ peaks from ^{52}V are labeled with the numbers listed in Table I. The remainder of the peaks in this spectrum correspond to ^{52}V states that are not populated via $L = 4$ transfer. The spectrum is shown as a function of position in the focal plane.

II. EXPERIMENTAL DETAILS AND RESULTS

A deuteron beam, produced by a SNICS (source of negative ions by cesium sputtering) source with a deuterated titanium cone, was accelerated to an energy of 16 MeV by the 9 MV Super FN Tandem Van de Graaff Accelerator at the John D. Fox Superconducting Accelerator Laboratory of Florida State University (FSU). The beam was delivered to a natural self-supporting vanadium target of thickness 610 μg/cm^2 that was mounted in the target chamber of the Super-Enge Split-Pole Spectrograph (SE-SPS). The natural abundance of ^{51}V is 99.75%.

The spectrograph, which accepted a solid angle of 2.3 msr, was rotated from scattering angles of 15° to 50° at increments of 5° to measure angular distributions of protons from the ^{51}V$(d,p)^{52}$V reaction. Further details of the experimental setup are described in Refs. [4,15].

The proton momentum spectrum collected at a scattering angle of 40° and with a magnetic field setting of 7400 G is shown in Fig. 1. We have chosen to display a 40° spectrum here because the $L = 4$ states are most prominent at this angle. The spectrum taken with the 7400 G setting is shown because it contains all of the $L = 4$ states observed in this experiment.

The magnetic rigidity spectrum measured at each scattering angle was fit using a linear combination of Gaussian functions with a cubic background. The energy resolution was 40 keV (full width at half maximum). Because of the density of states populated in this experiment, some of the structures formed by multiple peaks in a small range of magnetic rigidity were quite complex. For example, the peak corresponding to the 3538 keV state (which is labeled "1" in the spectrum figure) was in a structure consisting of four peaks.

The proton yields corresponding to each state in ^{52}V were used to produce the measured proton angular distributions shown in Figs. 2 and 3. The absolute cross sections were determined to be accurate within an uncertainty of 15%, with contributions from uncertainties in charge integration, target thickness and solid angle.

We used the compilation of Ref. [14] to identify peaks in our spectrum. The energies (and uncertainties on those energies) in Table I, which lists the states observed in the present experiment that are populated via $L = 4$ transfer, were taken from Ref. [14]. During the analysis, we extracted an energy calibration that reproduced the adopted energies from Ref. [14] to within 14 keV or better.

To extract spectroscopic factors from the present angular distributions, calculations that use the adiabatic approach for generating the entrance channel deuteron optical potentials (as developed by Johnson and Soper [17]) were used. The potential was produced using the formulation of Wales and Johnson [18]. Its use takes into account the possibility of deuteron breakup and has been shown to provide a more consistent analysis as a function of bombarding energy [19] as well as across a large number of (d,p) and (p,d) transfer reactions on $Z = 3$–24 target nuclei [20]. The proton-neutron and neutron-nucleus global optical potential parameters of Koning and Delaroche [21] were used to produce the deuteron potential as well as the proton-nucleus optical potential parameters needed for the exit channel of the (d,p) transfer calculations, in keeping with the nomenclature of Ref. [19]. The angular momentum transfer and spectroscopic factors found in Table I were determined by scaling these calculations, made with the FRESCO computer program [16], to the proton angular distributions. Optical potential parameters are listed in Table II. The overlaps between ^{52}V and ^{51}V $+n$ were calculated using binding potentials of Woods-Saxon form whose depth was varied to reproduce the given state's binding energy with geometry parameters of $r_0 = 1.25$ fm and $a_0 = 0.59$ fm and a Thomas spin-orbit term of strength $V_{so} = 5.65$ MeV that was not varied. We did not perform any normalization of these calculations.

Previous experiments did not determine J^π values for any of the $L = 4$ states observed here (although the 4327 keV state is tentatively assigned $J^\pi = 8^-$ because it was populated strongly in the ^{50}Ti$(\alpha,d)^{52}$V reaction [2]). Fortunately, the quantity

$$S_{lj}(\alpha_f I_f)\frac{2I_f+1}{2I_i+1}, \tag{1}$$

can be determined for a state using FRESCO without knowing I_f. In this expression, S_{lj} is the spectroscopic factor for the neutron orbit with orbital angular momentum l and total angular momentum j, and I_i is the total angular momentum of the ground state of the target nucleus.

We determined these values for eleven states that were populated via $L = 4$ transfers, indicating that they included $g_{9/2}$ neutron components. For nine of these states, including the states at 3538, 3687, 4034, 4120, 4327, 4533, 5080, 5187, and 5548 keV, we were able to isolate these peaks from neighboring peaks using a peak-fitting program. In these cases, we were able to deduce these values in the usual way: by scaling the calculated $L = 4$ angular distribution generated using FRESCO to best fit the data. For the 3777 keV state, the resolution of the spectrograph was not sufficient to allow us to separate this state from the neighboring 3733 keV state. We were able to extract a result for the 3777 keV state by fitting the experimentally measured angular distribution of the doublet (extracted via the peak-fitting program) with the sum of $L = 1$ (for the 3733 keV state) and $L = 4$ (for the

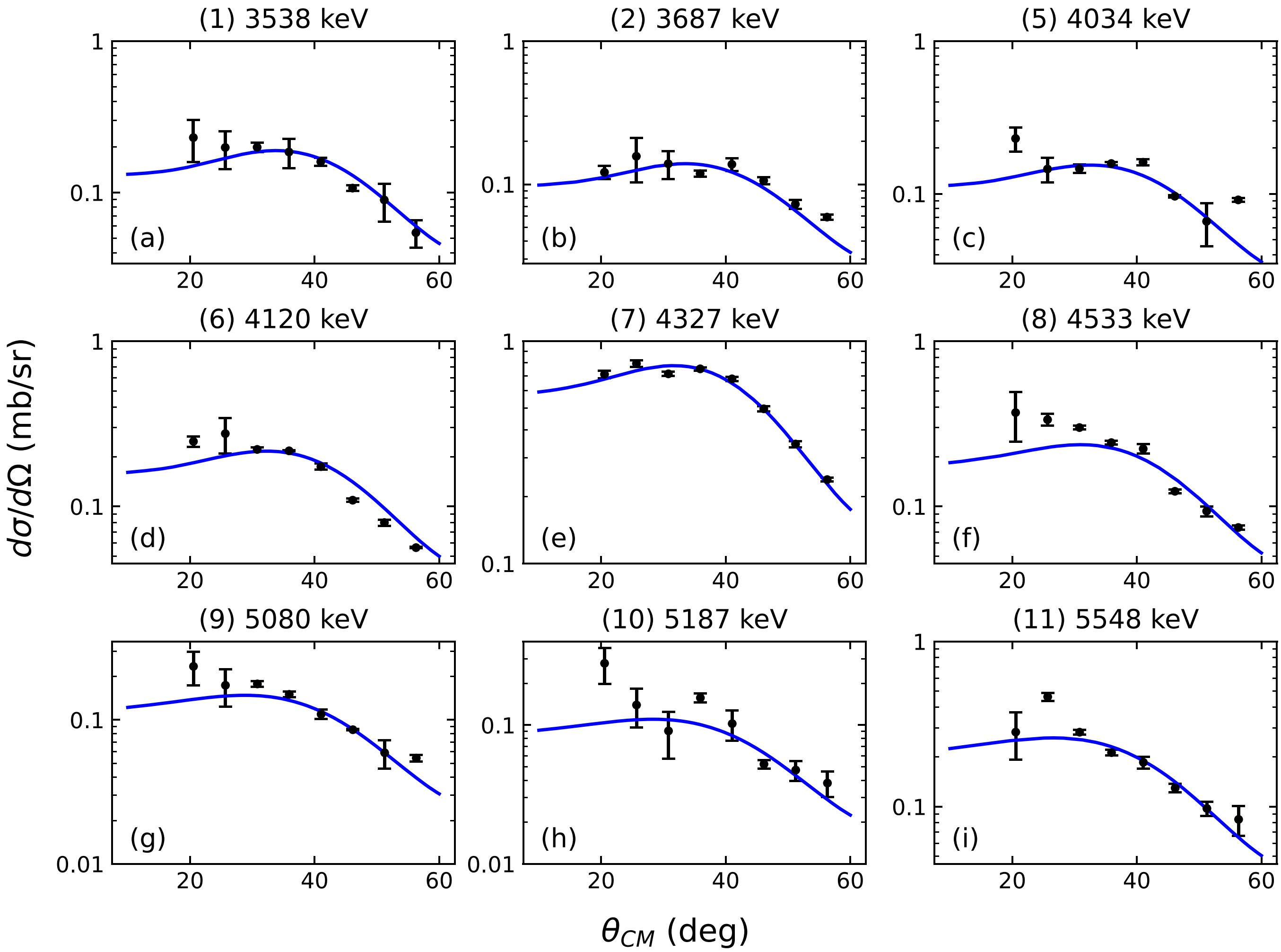

FIG. 2. Measured proton angular distributions for states populated in the $^{51}V(d,p)^{52}V$ reaction via $L = 4$ transfer that could be fitted as single peaks with distributions calculated as described in the text. Each state energy is listed with the peak number from Table I.

3777 keV state) angular distributions. The $L = 1$ and $L = 4$ strengths were determined using a chi-square fit. Likewise, we were unable to separate the 3940 ($L = 4$) and 3960 ($L = 1$) keV states sufficiently to analyze their angular distributions separately. So we used the same procedure as we used for the 3733 and 3777 keV states. These fits are shown in Fig. 3.

We have confidence in the analyses shown in Fig. 3 because of the success of the $L = 4$ fits in Fig. 2 and because fits to several low-lying states populated via $L = 1$ transfer are also successful. The previous study of the $^{51}V(d,p)^{52}V$ reaction with a beam energy of 10.1 MeV by Catala *et al.* [13] identified six states below an excitation energy of 850 keV that were

TABLE I. States populated via $L = 4$ transfer in the present experiment. Excitation energies are from Ref. [14], J^π assignments are from Ref. [14], and spectroscopic strengths are from the present work and Ref. [7].

Label	E_x (keV) Ref. [14]	J^π Ref. [14]	$S(2I_f+1)/(2I_i+1)$ Present work	$S(2I_f+1)/(2I_i+1)$ Ref. [7]
1	3538.52(5)		0.24(1)	0.42
2	3687(8)		0.17(2)	0.29
3	3777.09(3)		0.51(4)	0.90
4	3940(10)		0.072(22)	0.20
5	4034(10)		0.17(2)	0.32
6	4120(10)		0.24(3)	0.34
7	4327(15)	$(8)^-$	0.82(4)	1.18
8	4533(10)		0.24(7)	0.33
9	5080(8)		0.13(1)	0.20
10	5187(10)		0.093(55)	0.38
11	5548(8)		0.20(1)	0.26

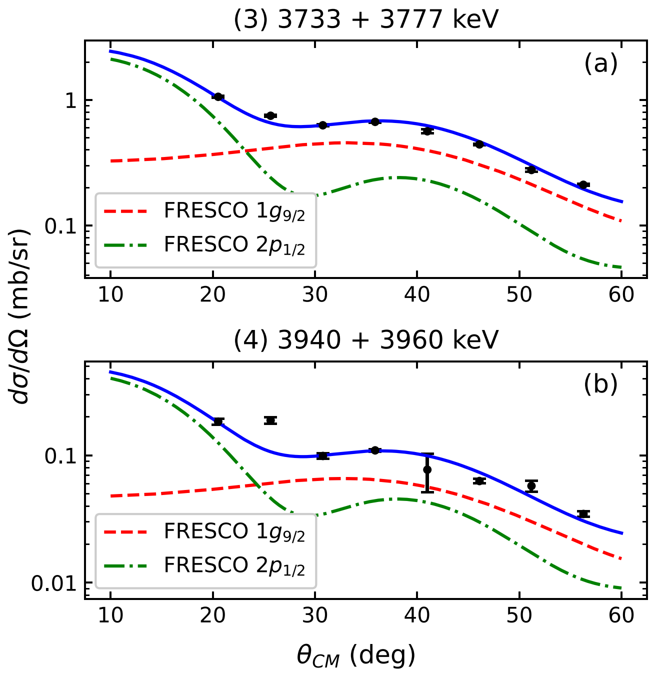

FIG. 3. Measured proton angular distributions for two doublet peak structures. Panel (a) shows the 3733 and 3777 keV states. The 3777 keV state is populated via $L = 4$ transfer and the 3733 keV state is populated via $L = 1$ transfer. The second doublet peak structure, shown in panel (b), consists of the 3940 and 3960 keV states. The 3940 keV state is populated via $L = 4$ transfer and the 3960 keV state is populated via $L = 1$ transfer. The spectroscopic factors were determined with a chi-square fitting procedure.

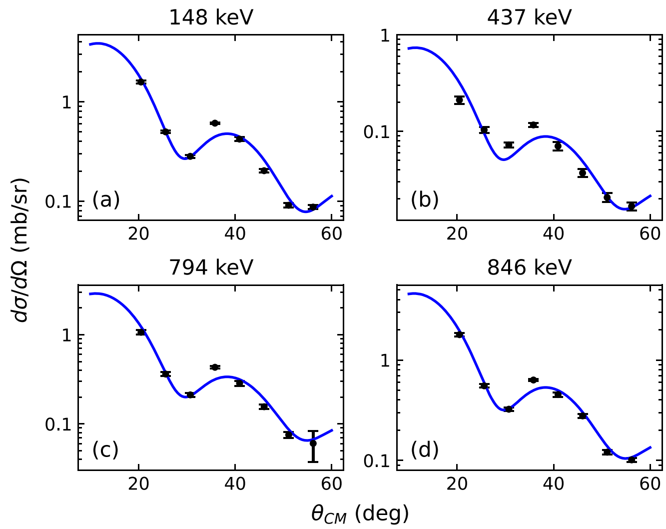

FIG. 4. Measured proton angular distributions for states populated in the ^{51}V$(d, p)^{52}$V reaction via $L = 1$ transfer at excitations energies of 148, 437, 794, and 846 keV that could be fitted as single peaks with distributions calculated as described in the text.

populated via $L = 1$ transfer. Two of those states, the ground state and a state to which Catala *et al.* assigned an excitation energy of 20(9) keV, could not be resolved in the present experiment (the compilation of Ref. [14] lists two states at 17.2 and 22.8 keV and does not identify which of these two states corresponds to the state listed at 20(9) keV by Catala *et al.*). However, Catala *et al.* also identified $L = 1$ states at 145(9), 431(11), 787(12), and 838(12) keV that we were able to resolve. The compilation of Ref. [14] assigns energies of 147.845(3), 436.634(9), 793.544(12), and 845.945(12) keV to these states, respectively. The calculations reproduce the shapes of the angular distributions well, as can be seen in Fig. 4. The $S_{lj}(2I_f + 1)/(2I_i + 1)$ values [as shown in Eq. (1)] determined for these states were 0.27(5), 0.049(13), 0.18(3), and 0.29(4), respectively. These strength results for the first two states compare to those extracted by Karban *et al.* of 0.42 and 0.10, respectively. Karban *et al.* were unable to resolve the 794 and 846 keV states, and they extracted a summed value of $S_{lj}(2I_f + 1)/(2I_i + 1) = 0.64$ for these two states. For comparison, our individual values for these two states sum to 0.47(5).

The values of $S_{lj}(2I_f + 1)/(2I_i + 1)$ determined here for states populated both via $L = 4$ and $L = 1$ transfer were, on average, 30–40% smaller than those determined for corresponding states in the (t, d) reaction by Karban *et al.* [7]. However, Karban *et al.* used a normalization constant of 4.43 in their distorted-wave Born approximation (DWBA) analysis to describe their (t, d) results. No normalization constant was applied in the present work. This alone could account for the 30–40% differences between the our results and those of Karban *et al.*

III. DISCUSSION

Before we address whether there is $\nu g_{9/2}$ strength missing in the present result as there is in ^{49}Ca, ^{51}Ti, ^{53}Cr, and ^{55}Fe, we must be clear about the sum rule for single nucleon strength in a stripping reaction like (d, p). According to Ref. [22], the spectroscopic factor sum rule in a stripping reaction is given by

$$G_{lj} = \sum_{\alpha_f I_f} S_{lj}(\alpha_f I_f)\frac{2I_f + 1}{2I_i + 1} = 2j + 1. \qquad (2)$$

TABLE II. Optical model potential parameters used in FRESCO [16] calculations in the present work determined using Refs. [17,18] as described in the text.

	V_V (MeV)	r_V (fm)	a_V (fm)	W_V (MeV)	r_W (fm)	a_W (fm)	W_D (MeV)	r_D (fm)	a_D (fm)	V_{so} (MeV)	W_{so} (MeV)	r_{so} (fm)	a_{so} (fm)	r_C (fm)
$d + {}^{51}$V	104.3	1.195	0.702	1.23	1.197	0.702	14.98	1.283	0.583	11.31	−0.13	1.013	0.621	1.25
$p + {}^{52}$V	56.9	1.195	0.670	0.596	1.195	0.670	8.12	1.284	0.545	5.71	−0.030	1.011	0.590	1.25

This equation assumes that the orbital being populated in the stripping reaction is completely unoccupied prior to the reaction. Therefore, we should compare the sum of the $S_{lj}(2I_f + 1)/(2I_i + 1)$ values for the present work in Table I to the sum rule value of 10 (since $j = 9/2$). The sum for the present work is 2.89(11). We would conclude from this result that about 70% of the $g_{9/2}$ strength is missing. Even the 4327 keV 8^- state has a smaller spectroscopic strength [$S = 0.39(2)$] than we might have expected from the results of the study of the ^{50}Ti$(\alpha, d)^{52}$V reaction by Okada *et al.* [2]. This result indicates that the 4327 keV state does not have the pure stretched configuration for which Okada *et al.* argued. Instead, this 8^- state must mix with others of the same J^π value.

In (d, p) studies of the even-Z $N = 29$ isotones ^{49}Ca [3], ^{51}Ti [4], ^{53}Cr [6], and ^{55}Fe [5], the sums of the $S_{lj}(2I_f + 1)/(2I_i + 1)$ values observed for the $g_{9/2}$ neutron orbit were also much smaller than 10 (of course, for all of these reactions, $I_f = 9/2$ and $I_i = 0$). The sums were 5.3 in ^{49}Ca [3], 2.0(3) in ^{51}Ti [4], 2.6(1) in ^{53}Cr [6], and 3.2(4) in ^{55}Fe [5]. The observation of 28.9(11)% of the sum rule strength in ^{52}V is qualitatively consistent with the results for ^{51}Ti, ^{53}Cr, and ^{55}Fe. The ^{49}Ca result is significantly higher.

Kay, Schiffer, and Freeman [23] argued that spectroscopic strengths in single nucleon transfer reactions are quenched by short-range correlations between nucleons, just as strengths in $(e, e'p)$ are [24]. They analyzed data from a large number of single nucleon transfer reactions involving targets from $A = 16$ to 208 and concluded that spectroscopic strengths are quenched by a constant factor of 0.55(10) from those expected from mean-field theory. If we adopt this factor, then we would conclude that we are missing approximately 50% of the expected total $g_{9/2}$ neutron strength in ^{51}Ti, ^{52}V, ^{53}Cr, and ^{55}Fe. Furthermore, we would conclude that there is no missing $g_{9/2}$ neutron strength in ^{48}Ca.

There are two possible explanations for the large amount of missing $\nu g_{9/2}$ strength in ^{51}Ti, ^{52}V, ^{53}Cr, and ^{55}Fe. One is that the majority of this strength is located in the continuum, above the particle threshold. In ^{52}V, the neutron separation energy is 7311 keV while the proton separation energy is higher, 9001 keV [14]. The other possible explanation is that this strength is so fragmented into many states that are weakly populated in our (d, p) reaction that we do not have sufficient sensitivity in the present experiment to detect it.

The possibility that the bulk of the $g_{9/2}$ neutron strength is in the continuum is given credibility by the results of a calculation performed in the framework of covariant density functional theory that is shown in Ref. [6]. This calculation predicts that the $g_{9/2}$ neutron orbit is unbound in ^{48}Ca, ^{50}Ti, and ^{52}Cr, and bound by only 1.4 MeV in ^{54}Fe.

In either case, finding the "missing" $g_{9/2}$ neutron strength would require a more sensitive experimental probe than the (d, p) reaction with 16 MeV deuterons used in the present work and in Refs. [4–6]. As noted by (for example) Szwec *et al.* [25], single nucleon transfer reactions vary in their sensitivities to populating orbits of different L values. In the reaction studied in the present work, the difference in the angular momenta of the incoming deuteron and outgoing proton is $1.1\hbar$ (calculated using a semiclassical approximation). Therefore, this reaction is most sensitive to the $p_{3/2}$ and $p_{1/2}$ orbits. In contrast, the $(\alpha, ^3\text{He})$ reaction is more sensitive to orbits with higher angular momenta. For example, the difference between the angular momenta of the incoming α particle and outgoing ^{3}He nucleus in the ^{51}V$(\alpha, ^3\text{He})^{52}$V reaction at 32 MeV (an energy that is accessible at the Fox Laboratory) is $6.8\hbar$ (once again calculated using a semiclassical approximation). Consequently, this reaction would be more sensitive to neutron orbits having larger orbital angular momenta such as $g_{9/2}$.

Detecting γ rays in coincidence with particle detection in the SE-SPS could provide additional selectivity that would be especially helpful in reactions like the one studied here in which the spectrum of excited states is crowded. $CeBr_3$ scintillators can provide resolution of 4% or better at energies above 500 keV while providing resilience in the presence of large neutron fluxes like those present during (d, p) experiments [26]. Five $CeBr_3$ detectors are already available for particle-γ coincidence experiments at the SE-SPS.

IV. CONCLUSIONS

We performed a measurement of the ^{51}V$(d, p)^{52}$V reaction at 16 MeV using the FSU SE-SPS, focusing on states populated via $L = 4$ transfer. We measured angular distributions for eleven of these states. Spectroscopic factors had not been determined for any of these states via the (d, p) reaction previously.

The summed $g_{9/2}$ neutron strength observed here in the eleven $L = 4$ states is only 28.9(11)% of the sum rule. This is similar to the situation in the even-Z $N = 29$ isotones ^{49}Ca, ^{51}Ti, ^{53}Cr, and ^{55}Fe. The remaining $g_{9/2}$ strength may be located in the continuum, or it may be fragmented among many bound states. The $(\alpha, ^3\text{He})$ reaction may provide a more sensitive probe for the missing $g_{9/2}$ neutron strength. In addition, particle-γ coincidence experiments with $CeBr_3$ detectors may provide additional sensitivity for identifying these missing fragments.

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