Multichannel R-matrix analysis of ^{12}C compound nucleus reactions

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We have calculated the cross sections (CS), differential cross sections (DCS), and astrophysical S factors for the reactions ^{11}B$(p,\alpha_1)^8$Be*, ^{11}B$(p,\alpha_0)^8$Be, ^{11}B$(p,p_0)^{11}$B, ^{11}B$(p,p_1)^{11}$B*, ^{11}B$(p,\gamma_0)^{12}$C, ^{11}B$(p,\gamma_1)^{12}$C*, and ^{11}B(p, n$_0$)^{11}C through a multichannel-multilevel R-matrix analysis. We have performed calculations for the energy range 100 keV–3.5 MeV and compared the calculated values with the experimental values. As a result, we have obtained the energy value, spin, and parity of some ^{12}C excited levels within this energy interval. Additionally, we have determined the channel spins and angular momentum of the $\alpha_1-{}^8$Be*, $\alpha_0-{}^8$Be, $p_0-{}^{11}$B, $p_1-{}^{11}$B*, and $n_0-{}^{11}$C pairs, as well as the electric and magnetic multipoles of the ^{11}B$(p,\gamma_0)^{12}$C and ^{11}B$(p,\gamma_1)^{12}$C* channels. Although the analysis results and experimental data mostly agree, we have found some inconsistencies in the S-factor values for the ^{11}B$(p,p_1)^{11}$B* channel at the 19.2–19.5 MeV energy interval. For the ^{11}B$(p,\alpha_1)^8$Be* decay channel of the 16.57 MeV (2^-) resonance, the literature discusses which orbital angular momentum quantum value ($l=1$ or $l=3$) is true. We used both values in our analysis, and although no significant difference in the results were observed in the region of the 16.57 MeV (2^-) resonance, we obtained a better fit with $l=3$ values in the high energy region affected by the tail of this resonance.

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I. INTRODUCTION

Carbon is an essential component of all living organisms and plays a crucial role in the development and sustenance of life. It is formed through the triple-alpha process in stellar nucleosynthesis, in which two alpha particles combine to create the ^{8}Be nucleus, which subsequently binds with a third alpha particle to form the ^{12}C nucleus [1]. In addition to the triple alpha process, the ^{11}B$(p,\gamma)^{12}$C reaction is another possible mechanism for ^{12}C production. However, research into the nucleosynthesis of ^{12}C suggests that the ^{11}B$(n,\beta)^{12}$C reaction is more likely than ^{11}B$(p,\gamma)^{12}$C, particularly for proton energies below 100 keV. This is primarily due to the formidable Coulomb barrier of the $p-{}^{11}$B system, which presents a significant challenge for proton capture. Nevertheless, the possibility of proton capture by ^{11}B cannot be completely discounted [2].

Another reaction of astrophysical importance is the ^{11}B$(p,\alpha)^8$Be reaction, which causes a reduction in the abundance of ^{11}B in the inner layers of stars. The significance of this reaction lies in the fact that, when analyzed together with Li and Be abundances, the abundance of ^{11}B detected in the stellar atmosphere can be utilized to determine the depth of stellar convection, providing valuable insights into the internal dynamics of stars [3].

The kinematics and the cross section values of the ^{11}B$(p,\alpha)^8$Be reaction have been the subject of numerous studies, particularly at low energies (~ 163 and ~ 660 keV) corresponding to the 16.10 (2^+) and 16.57 (2^-) resonance levels of the C^{12} nucleus, as this reaction is a key consideration in aneutronic fusion reactor research [4–7]. Investigations into the spin, parity, and partial width (Γ_{p0}, Γ_{p1}, $\Gamma_{\alpha0}$, $\Gamma_{\alpha1}$, $\Gamma_{\gamma0}$, $\Gamma_{\gamma1}$, Γ_{n0}) values of these resonances have been conducted [8–10], as studies into the angular momenta (l) and channel spin (s) of the $p-{}^{11}$B and α-^{8}Be pairs formed during the decay of these resonances [3,11,12].

The 16.11 (2^+) level has been extensively studied, with calculated $\Gamma_{\alpha0}=0.38$ keV reported in Ref. [13]. In Ref. [14], the resonance was observed at an energy of 16.11 MeV, with a width of $\Gamma=6.7$ keV and partial widths of $\Gamma_{p0}=69$ eV, $\Gamma_{\alpha0}=290$ eV, $\Gamma_{\alpha1}=6.3$ keV, $\Gamma_{\gamma0}=0.22$ eV ($E2$ transition), and $\Gamma_{\gamma1}=6.8$ eV. A branching ratio of $\Gamma_{\alpha0}/\Gamma_{\alpha1}=3/97$ was found in Ref. [15]. The study in [16] measured a branching ratio of 5.1(5)% for α_0 breakup, and determined that the decay to the first excited state in ^{8}Be is primarily dominated by d-wave emission. In a more recent study, Ref. [17] updated the partial widths for this state, finding $\Gamma_{p0}=37(7)$ eV, $\Gamma_{\alpha0}=270(30)$ eV, $\Gamma_{\alpha1}=5(2)$ keV, $\Gamma_{\gamma0}=0.35(4)$ eV, and $\Gamma_{\gamma1}=10.5(1.6)$ eV.

The determination of the angular momentum of the α-^{8}Be system formed by the decay of the 16.57 (2^-) level has been the subject of several investigations. Reference [18], using the ^{11}B$(p,\alpha_1)^8$Be* reaction, reported a mixture ratio of $l=1$ to $l=3$ of 10 ± 3. Conversely, Ref. [19] calculated the value to be $l=1$, while Refs. [3,12] found $l=3$. In [8], the resonance

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was observed at an energy of 16.56 MeV (2^-) with a width of $\Gamma = 0.30$ MeV and partial widths of $\Gamma_{p0} = 0.15$ MeV, $\Gamma_{\alpha 0} <$ 0.27 keV, $\Gamma_{\alpha 1} = 0.15$ MeV, $\Gamma_{\gamma 0} < 0.4$ eV, and $\Gamma_{\gamma 1} = 8$ eV.

The 17.23 (1^-) resonance has attracted attention because it is adjacent to these two resonances and because of its potential to influence their shape and amplitude. In Ref. [18], it was suggested that the decay of this state to the α_1 channel is a mixture of $l = 1$ and $l = 3$ with a mixture ratio of 1.1 ± 0.4. On the other hand, Ref. [20] proposed that for $\theta = 60^\circ$ the widths of the $l = 1$ and $l = 3$ states are 0.8 and 0.7 MeV, respectively. In Ref. [8], the resonance was observed at an energy of 17.26 MeV with a width of $\Gamma = 1.15$ MeV, and two possible sets of partial widths were proposed:

Alternative-1: $\Gamma_{p0} = 1.0$ MeV, $\Gamma_{\alpha 0} = 10$ keV, $\Gamma_{\alpha 1} = 140$ keV, $\Gamma_{\gamma 0} = 44$ eV, $\Gamma_{\gamma 1} = 5$ eV;

Alternative-2: $\Gamma_{p0} = 150$ keV, $\Gamma_{\alpha 0} = 60$ keV, $\Gamma_{\alpha 1} = 940$ keV, $\Gamma_{\gamma 0} = 290$ eV, $\Gamma_{\gamma 1} = 35$ eV.

In Ref. [21], it was reported that $\Gamma_{p0}/\Gamma = 0.05$, which aligns more closely with Alternative-2 in their study. Conversely, in Ref. [10], this resonance was found to be more compatible with Alternative-1.

Numerous resonances above 17.23 MeV have been reported in the literature, but due to the mixing of resonances at these levels, analysis becomes challenging. Obtaining accurate resonance parameters is therefore more difficult than at lower energy levels. Although various studies have reported on the spin, parity, and partial width (Γ_{p0}, Γ_{p1}, $\Gamma_{\alpha 0}$, $\Gamma_{\alpha 1}$, $\Gamma_{\gamma 0}$, $\Gamma_{\gamma 1}$, Γ_{n0}) values of most of the resonances above these three levels [8–10], there is limited information available on the angular momentum (l) and channel spin (s) values of $\alpha_1 - {}^8\mathrm{Be}^*$, $\alpha_0 - {}^8\mathrm{Be}$, $p_0 - {}^{11}\mathrm{B}$, $p_1 - {}^{11}\mathrm{B}^*$, and $n_0 - {}^{11}\mathrm{C}$ pairs. To address this gap in knowledge, a comprehensive approach is needed, rather than studying resonances using a single reaction channel and treating them as isolated. The multichannel *R*-matrix method is a suitable approach for this purpose.

R-matrix theory was originally introduced by Wigner and Eisenbud [22], and subsequently elaborated upon by Lane and Thomas [23]. The strength of the *R*-matrix theory lies in its ability to extract information about the structure of a nucleus without requiring direct knowledge of its internal structure. Instead, unknown values related to the internal structure are treated as parameters, and the values obtained through calculations are then compared with experimental data to determine the correct values.

In *R*-matrix theory, the configuration space is divided into two regions: the internal region and the external region. The internal region is defined as the region where all nucleons are in close proximity to one another, forming a compound nucleus of nuclear dimensions. In this region, the total wave function at any given energy (E) is composed of eigenfunctions with eigenvalues E_λ [23], which have zero derivative at the boundary between the two regions. Conversely, in the external region, particle pairs (channels) are only affected by Coulomb interactions with one another. Here, the wave function has an analytical solution dependent solely on energy, masses, charges, and the intrinsic and relative angular momenta. The *R*-matrix theory is constructed at the boundary where the wave functions and their derivatives in the two regions are equal [24].

In the *R*-matrix theory, a key quantity is the *R* matrix, an element of which is defined as

$$R_{cc'} = \sum_\lambda \frac{\gamma_{\lambda c}\gamma_{\lambda c'}}{E_\lambda - E} . \tag{1}$$

λ is the energy level, $\gamma_{\lambda c} = (\hbar^2/2M_\alpha)^{1/2}\ u_{\lambda c}(a)$ is the reduced width amplitude of the level λ for entrance channel c, $\gamma_{\lambda c'} = (\hbar^2/2M_\alpha)^{1/2}\ u_{\lambda c'}(a)$ is the reduced width amplitude of the level λ for exit channel c', $u_\lambda(a)$ is the eigenfunction at boundary surface between internal and external region, E_λ is the energy eigenvalue of λ, and E is the energy of the relative motion of the particles.

For elastic scattering, where the entrance and exit channels are identical, the *R* function can be simplified to the following form:

$$R = \sum_\lambda \frac{\gamma_\lambda^2}{E_\lambda - E}, \tag{2}$$

Using the *R* function (R) the collision function (U) is calculated with

$$U_l = \frac{I_l}{O_l}\frac{1 - L_l^* R_l}{1 - L_l R_l}, \tag{3}$$

where I_l, O_l, and L_l refer to the incoming wave, outgoing wave, and logarithmic derivative of O_l, respectively, in the external region. Finally, the differential elastic scattering cross section is

$$\sigma(\theta) = \frac{1}{4}k^{-2}\left|\sum_l (2l+1)(1-U_l)P_l(\cos\theta)\right|^2 \tag{4}$$

and the total cross section is

$$\sigma_t = \int \sigma(\theta) d\Omega = \pi k^2 \sum_l (2l+1)|1 - U_l|^2, \tag{5}$$

where $P_l(\cos\theta)$ refers to the Legendre polynomial, k is the wave number, and l is the orbital angular momentum.

In the case of a many-channel system, the analysis becomes more complex. The collision matrix $\mathbf{U}$ is then given by

$$\mathbf{U} = \rho^{\frac{1}{2}}\ \mathbf{O}^{\frac{1}{2}}\ (1 - \mathbf{R}\mathbf{L}_0)^{-1}\ (1 - \mathbf{R}\mathbf{L}_0^*)\ \mathbf{I}\ \rho^{-\frac{1}{2}}. \tag{6}$$

Here ρ is the matrix with diagonal elements $\rho_c = k_\alpha r_\alpha$, k_α is the wave number, and r_α is the relative radial coordinate of the pair α.

With the relation of collision matrix, the transition matrix will be

$$T_{cc'} = e^{2i\omega_c}\delta_{cc'} - U_{cc'} . \tag{7}$$

Using the transition matrix, the angle-integrated cross section is obtained as

$$\sigma_{\alpha\alpha'} = \frac{\pi}{k_\alpha^2}\sum_{Jll'ss'} g_J \left|T_{cc'}^J\right|^2, \tag{8}$$

where ω_c is the Coulomb phase shift, *{α/α}'* is the entrance/exit pair, $\delta_{cc'}$ is the Kronicker delta, $g_j = \frac{2J+1}{(2I_{\alpha_1}+1)(2I_{\alpha_2}+1)}$ is the statistical spin factor, and $I\alpha_1$, $I_{\alpha 2}$, and J are the projectile spin, target spin, and the total angular momentum respectively [24].

Differential cross section calculation requires a more rigorous effort than angle integrated cross section.

In the case of many levels and few channels the inversion of the channel matrix $(1 - \mathbf{RL}_0)$ used to calculate collision matrix $\mathbf{U}$ is useful. However, if there are many channels and few levels, the collision matrix is expressed by A matrix $(A_{\lambda\lambda'})$ to save time in calculations.

Although *R*-matrix theory uses formal parameters which are reduced width amplitudes and pole energies, experimentally observed parameters are resonance energies and Breit-Wigner partial widths. Two techniques are used to convert these experimental parameters to formal parameters: Barker transformation [25] and Brune transformation [26].

We do not provide here a detailed explanation of the *R*-matrix theory due to its complexity. However, interested readers can consult the extensive literature on this theory, including works such as [23,24,26–28].

This study presents a novel attempt to perform an *R*-matrix analysis of ^{12}C resonance levels with multiple exit channels, utilizing numerous experimental data. The investigation includes all open reaction channels, enabling the measurement not only of the single-level resonance parameters of one or two specific reaction channels, but also the effect of competing parallel channels [29].

II. METHODS (MULTICHANNEL *R*-MATRIX ANALYSIS)

In the present work we used AZURE2 *R*-matrix program which is a multichannel, multilevel *R*-matrix computer code originally written in FORTRAN 77 [24] and later translated into the C++ language. The AZURE program has been developed to study low energy reactions which involve charged particles, gammas, and neutrons with light nucleons such as He, Li, Be, B, C, etc. The program is particularly useful for calculating resonance energies and amplitudes, partial widths and asymptotic normalization coefficients (ANCs). The Brune formalism was used for the transformation from experimental parameters (resonance energies and widths) to formal parameters (reduced width amplitudes and pole energies) and vice versa.

A. Fitting procedure

For fitting the calculated values to the experimentally obtained cross section data, the program uses MINUIT package [30]. Calculation of the chi-squared value (χ^2) is made using the method of [31]

$$\chi^2 = \sum_i \left[\sum_j \frac{(f(x_{i,j}) - c_i n_i y_{i,j})^2}{(c_i n_i \sigma_{i,j})^2} + \frac{((c_i - n_i)/n_i)^2}{\delta^2_{c_{\exp,i}}} \right] \quad (9)$$

where $f(x_{i,j})$ is the calculated quantity form the R matrix (i.e., cross section, S factor, phase shift, etc.), c_i is the normalization fit parameter, n_i is the starting normalization, $y_{i,j}$ is the data point value, $\sigma_{i,j}$ is the statistical uncertainty of the data point, and $\delta_{c_{\exp,i}}$ is the percent systematic uncertainty of the data set [24].

In this study, we focused primarily on ensuring that the fit curve resulting from the applied iterations showed a behavior resembling the data curve, particularly in resonance peaks. During the iterations, we fixed most of the parameters to values close to the literature. In addition, not all of the other parameters were released simultaneously. Instead, for most of the iterations, we released only one parameter at a time, while fixing all the remaining parameters. We continued the iterations by releasing the next parameter after fitting the released parameter. In some iterations, we released a few parameters together. The number of parameters released in each run varied from 1 to 5.

The parameters used in the calculations—which are resonance energies, spins and parities of the resonances, partial widths, ANCs of the channels, and orbital angular momenta and spins of the particle pairs—were carefully chosen to ensure compatibility with values reported in the literature. The background poles [15.44 (2^+), 20.6 (3^-), 21.6 (2^+), 22.65 (1^-)] and their associated widths were selected from known resonances reported in the literature [10], except for the 23.0 (3^-) resonance. If the parameters are not reported in the literature they were obtained by numerous trials with the chi-squared (χ^2) minimization technique. For each parameter, trials were conducted with small values and progressed to larger ones.

B. Experimental data

The present study included all open proton-particle channels, namely ^{11}B$(p,\alpha_1)^8$Be*, ^{11}B$(p,\alpha_0)^8$Be, ^{11}B$(p,p_0)^{11}$B, ^{11}B$(p,p_1)^{11}$B*, ^{11}B$(p,n_0)^{11}$C, as well as the most significant gamma channels, ^{11}B$(p,\gamma_0)^{12}$C, ^{11}B$(p,\gamma_1)^{12}$C*. However, due to insufficient data, the other gamma channels were not included in the analysis.

We used approximately 60 data sets in this study, all obtained from the Experimental Nuclear Reaction Database (EXFOR) [32]. Table I provides a summary of the data used. Some datasets lacked reported error values, while others had error values that were significantly lower than the other data sets. To address this issue and ensure that all data sets were treated equally by the AZURE algorithm, we assigned a uniform error value of 10% to all data sets in the study.

To ensure consistency among the data sets, we applied a normalization coefficient (NC) to certain data sets expressed with arbitrary units, such as the α_0 channel data from Ref. [43], γ_1 and γ_0 channel data from Ref. [45], and n_0 channel data from Ref. [54]. In addition, p_1 channel data provided by Ref. [50] that disagreed with the data set of Ref. [49] were normalized using an NC value of 4.8. To represent the 16.57 (2^-) resonance, the energy values of the Ref. [45] γ_1 data set were reduced by 0.2 MeV. All the applied normalization coefficients and changes in energy values can be found in Table II.

We used some of the CS data sets by converting them to astrophysical S factor. Throughout the work, for the transformation from the cross section to S factor we used the standard expression

$$\sigma(E) = S(E) \exp\left(\sqrt{\frac{E_G}{E}}\right), \quad (10)$$

TABLE I. Data sets used in this work for the analysis of the reaction channels.

Reaction	References
^{11}B(p, p_0)^{11}B	Kokkoris *et al.*, 2010 [33]
	Segel *et al.*, 1965 [8]
	Chiari *et al.*, 2001a [34]
	Mayer *et al.*, 1998 [35]
^{11}B(p, α_1)^{8}Be*	Becker *et al.*, 1987 [19]
	Segel *et al.*, 1965 [8]
	Beckman *et al.*, 1953 [36]
	Ligeon and Bontemps, 1972 [37]
	Liu *et al.*, 2002 [38]
^{11}B(p, α_0)^{8}Be	Kokkoris *et al.*, 2010 [33]
	Davidson *et al.*, 1979 [39]
	Munch *et al.*, 2020 [40]
	Beckman *et al.*, 1953 [36]
	Spitaleri *et al.*, 2004 [41]
	Symons and Treacy, 1963 [42]
	Becker *et al.*, 1987 [19]
	Holland *et al.*, 1955 [43]
	Segel *et al.*, 1965 [8]
^{11}B(p, γ_0)^{12}C	Segel *et al.*, 1965 [8]
	Wright *et al.*, 1982 [44]
	Gove and Paul, 1955 [45]
^{11}B(p, γ_1)^{12}C*	Segel *et al.*, 1965 [8]
	He *et al.*, 2016 [46]
	Wright *et al.*, 1982 [44]
	Generalov *et al.* 2005 [47]
	Gove and Paul, 1955 [45]
^{11}B(p, p_1)^{11}B*	Huus and Day, 1953 [48]
	Segel *et al.*, 1965 [8]
	Preketes-Sigalas *et al.*, 2016 [49]
	Boni *et al.*, 1988 [50]
^{11}B(p, n_0)^{11}C	Blaser *et al.*, 1951 [51]
	Gibbons and Macklin, 1959 [52]
	Ramavataram *et al.*, 1980 [53]
	Blair *et al.*, 1955 [54]

where $\sigma(E)$ is the cross section (barn), $S(E)$ is the astrophysical S factor (barn/sr), E is the energy of the center-of-mass (CM) frame, and

$$E_G = \left(\frac{2\pi e^2 Z_1 Z_2}{\hbar c}\right)^2 \frac{M_r c^2}{2}, \tag{11}$$

where E_G is the Gamow energy, Z_1, Z_2 are the atomic numbers of projectile (p) and target (^{11}B), $M_r = m_1 m_2/(m_1 + m_2)$ is the reduced mass, and m_1, m_2 are masses of the projectile and target.

C. Photon channels

The R-matrix theory described above works truly for particle-particle reactions only. In the intrinsic region, photon channels can be included in the calculation together with particle channels. On the other hand, if there is a radiative capture in the external region, the situation may change. In this case, the scattering state will receive contributions from both resonant and hard-sphere phase shifts. In AZURE, these external capture and resonant contributions are used together consistently and, following Ref. [55], external capture is expressed in terms of the asymptotic normalization coefficient (ANC), which can be written as

$$C_{\alpha slf} = \sqrt{\frac{2}{a_{\alpha slf}}}\theta_{\alpha slg}\frac{N_f^{1/2}}{W_{-\eta_\alpha,\ l_f+1/2}(2l_{\alpha f}a_c)} \tag{12}$$

where $\theta_{\alpha slg}$ is the dimensionless reduced width amplitude, N_f is the normalization factor, and $W_{-\eta_\alpha,\ l_f+1/2}(2l_{\alpha f}a_c)$ is the Whittaker function [24].

III. RESULTS AND DISCUSSION

We used a significant amount of data for all channels in this analysis. As a result we have found that most data sets have a high correlation with analysis results. The multichannel approach has proved to be successful in analyzing the resonance values and decay parameters of the ^{12}C nucleus. We have obtained a high correlation of almost all resonance energies and spin-parities with the values reported in the literature (Table III). However, there is no agreement in the 19.5 MeV region. In the vicinity of the 19.5 MeV resonance, numerous resonances have been reported in the literature, such as 19.5 (2^-), 19.55 (4^-), 19.69 (1^+), 19.55 (2^-), and 19.59 (4^-) (Table III). Our proposed value of 19.64 (2^-) is somewhere in between.

In the present analysis, we have observed that the total widths of the resonances to be generally compatible with the literature values (Table III). However, we have found that the total widths of 18.38 (3^-), 19.0 (1^-), 19.2 (2^-), and 19.64 (2^-) resonances differ slightly from the previous values (Table III). According to the literature [10], the total width of the 15.44 (2^+) resonance was reported to be 1770 keV. In our analysis, the width of this resonance which is the only bound state has been found to be 565 keV, with an asymptotic normalization coefficient (ANC) of 0.63 fm$^{-1/2}$ for the $p_0 - {}^{11}$B channel.

The resonance partial widths obtained are generally compatible with the literature values, although there are some differences in the case of the 19.00 (1^-) and 19.44 (2^+) resonances (Table IV). This can be attributed to the interference of a few broad resonances [19.0 (1^-), 19.2 (2^-), 19.44 (2^+), 19.64 (2^-)] affecting the analysis. To compare with literature, the partial widths of the 18.35 (2^-) resonance reported by Ref. [15] in percent (%) have been converted to keV and presented in (Table IV). The present work showed that, for the partial widths of the 17.23 (1^-) resonance, Alternative-2 suggested by Ref. [8] provides a better fit to the experimental data sets than Alternative-1.

In the present analysis, we have obtained the values of orbital angular momentum (l) and channel spin (s) for channels in which resonance levels decay, which are given together with the literature values (Table V). They are generally compatible, with a few exceptions. Notably, some of the l and s values could not be found in the literature and have been calculated for the first time. For the 15.44 MeV (2^+) resonance, which decays via the ^{11}B(p, α_1)^{8}Be* channel, two different

TABLE II. Normalization coefficients and the amount of energy increase/reduction used in this study for some data sets.

Data set	Reaction/Figure	Normalization coefficient (NC)	Energy increase/reduction
Holland *et al.*, 1955 [43]	^{11}B$(p, \alpha_0)\,^8$Be/Fig. 2	0.00035	
Gove and Paul, 1955 [45]	^{11}B$(p, \gamma_0)\,^{12}$C/Fig. 4(c)	3.0×10^{-9}	
Blair *et al.*, 1955 [54]	^{11}B$(p, n_0)\,^{11}$C/Fig. 6(b)	0.0013	
Boni *et al.*, 1988 [50]	^{11}B$(p, p_1)^{11}$B*/Fig. 8(d)	4.8	
Gove and Paul 1955, [45]	^{11}B$(p, \gamma_1)^{12}$C*/Fig. 5(c)	2.0×10^{-9}	0.15 MeV (energy reduction)

channels with (l, s) values (2.0) and (2.2) were included in the calculations. The orbital angular momentum (l) of the 16.57 MeV (2^-) resonance, which is the subject of discussion in the literature for the ^{11}B$(p, \alpha_1)\,^8$Be* reaction channel, has been found to be 3.

The study presents the light and heavy particle spin and parity, excitation energy, separation energy, and channel radius values for all reaction channels, along with their literature values (Table VI). Most of the values obtained in this analysis are compatible with the literature, with a few exceptions. In particular, the energy value for the first excitation level of the ^{11}B nucleus, which has a spin-parity value of $1/2^-$, has been found to be 1.00 MeV in this study, whereas it was previously reported as 2.14 MeV in the litera-

TABLE III. Comparison of the resonance levels of the ^{12}C nucleus used in the present work with the values given in the literature. The *R*-matrix parameters released during the fitting procedure are shown in bold.

	Present work		References	Present work	References
Level no.	J^π	Energy (MeV)	Energy (J^π)	Γ (keV) ANC (fm$^{-1/2}$)	Γ (keV)
1[a]	2^+	15.44	15.44(2^+) [10] 15.44(0^+) [56]	**565 keV** (**0.63 fm$^{-1/2}$**) [b]	1770 [10]
2	2^+	16.1058	16.106 [10]	**6.625**	6.7 [57] 5.3 [10]
3	2^-	16.57	16.576 [10] 16.56 [8]	300	300 [8,10]
4	1^-	17.23	17.23 [10] 17.26 [8]	1150.3	1150 [8,10]
5	0^+	17.77	17.77 [8,42] 17.79 [10] 17.80 [57]	92	92 [8], 96 [10,57] 110 [42]
6	1^+	18.13	18.13 [10] 18.16(2^-) [58]	600	600 [10] 240 [58]
7	2^-	18.35	18.35 [10], 18.34(2^+) [42] 18.2 [8]	350	350 [10], 320 [42] 380 [15]
8	3^-	18.38	18.36 [8,15] 18.38 [10,57]	**288.4**	310 [8] 400 [57]
9	0^-	18.4	18.4 [8] 18.39 [10]	42	42 [8] 43 [10]
10	2^+	**18.83**	18.81 [8,10]	**101.9**	100 [8,10]
11	1^-	**19.0**	19.2 [8,10]	**960**	1100 [8,10]
12	2^-	**19.2**	19.4 [10,15] 19.3 [8]	**449.5**	490 [10]
13	2^+	**19.44**	19.4 [8], 19.39 [10]	**1086**	1100 [8,10]
14	2^-	**19.64**	19.5(2^-) [8], 19.55(4^-) [10], 19.69(1^+) [10], 19.55(2^-) [10,15], 19.59(4^-) [9]	**445.5**	485 [10] 230 [10]
15 [a]	3^-	20.6	20.6 [10]	**280.2**	280 [10]
16[a]	2^+	21.6	21.6 [10]	1200	1200 [10]
17[a]	1^-	22.65	22.65 [10]	**3206**	3200 [10]
18[a]	3^-	23.5		**10000**	

[a]Background pole.
[b]Asymptotic normalization coefficient (ANC).

TABLE IV. Comparison of the partial widths (Γ) of the resonances used in the analysis with the values given in the literature. The parameters released during the fitting procedure are shown in bold. P.W. indicates present work.

Partial width resonance		Γ_{p0} (keV), ANC ($\text{fm}^{-1/2}$)		$\Gamma_{\alpha 1}$ (keV)		$\Gamma_{\alpha 0}$ (keV)		$\Gamma_{\gamma 0}$ (eV)		$\Gamma_{\gamma 1}$ (eV)		Γ_{p1} (keV)		Γ_{n0} (keV)	
E_x (MeV)	J^π	P.W.	Ref.	P.W.	Ref.	P.W.	Ref.	P.W.	Ref.	P.W.	Ref.	P.W.	Ref.	P.W.	Ref.
15.44	2^+	**0.63** [a]		**125** **160**		**280**				**73**					
16.1058	2^+	**0.0217**	0.0215 [10] 0.069 [14]	6.3	5 [10] 6.3 [14]	0.29	0.26 [10] 0.38 [13] 0.29 [14]	0.6	0.59 [10] 0.22 [14]	12.8	12.8 [10] 6.8 [14]				
16.57	2^-	**125**	150 [8,10]	**175**	150 [8,10]	**0**	<0.27 [8,10],	**0.027**	<0.4 [8,10]	8	8 [8,10]				
17.23	1^-	150	150 [8]	940	940 [8]	60	60 [8]	290	290 [8]	35	35 [8]				
17.77	0^+	76	76 [8]	11.4	11.4 [8]	4.6	4.6 [8]	**0**	<0.5 [8]	**0.4**	<0.5 [8]				
18.13	1^+	600	600 [10] 240 [10,58],												
18.35	2^-	**350**	285 [15]	0	95 [15]										
18.38	3^-	**60**	68 [8]	177	177 [8]	**50**	65 [8]	**0.002**	<1.5 [8] 0.002 [10]	3.2	3.2 [8]	**1.4**	<1.5 [8]		
18.4	0^-	33	33 [8]	**0**	<5 [8]	**0**	<1 [8]	**0**	<0.5 [8]	**0.4**	<0.5 [8]	9	9 [8]		
18.83	2^+	97	97 [8]	**1.4**	<1.5 [8]	**0.2**	<0.2 [8]	0.4	0.4 [8]	2	2 [8]	2	2 [8]	**1.3**	1.1 [8]
19.00	1^-	300	300 [8]	200	200 [8]	0	50 [8]	**2**	25 [8]	0	10 [8]	**300**	400 [8]	**160**	150 [8]
19.2	2^-	**400**		**35**				**8.75**		**7**				**14.5**	
19.44	2^+	**430**	450 [8]	**470**	450 [8]	**70**	20 [8]	**0.6**	<3 [8]	0	3 [8]	11	**50** [8]	**105**	100 [8]
19.64	2^-	**420**		**20**										5.5	
20.6	3^-	**280**								**212**					
21.6	2^+	**1200**													
22.65	1^-	**2200**		**1000**				**6000**							
23.5	3^-	**10000**													

[a]ANC.

TABLE V. Comparison of orbital angular momentum (l) and total angular momentum (s) values used in calculations with literature values. P.W. indicates present work.

Channel Resonance		p_0		α_1		α_0		γ_0		γ_1		p_1		n_0	
E_x (MeV)	J^π	P.W. (s, l)	Ref. (l)	P.W. (s, l)	Ref. (l)	P.W. (s, l)	Ref. (l)	P.W. $(s, \pi l)$	Ref. (πl)	P.W. $(s, \pi l)$	Ref. (πl)	P.W. (s, l)	Ref. (l)	P.W. (s, l)	Ref. (l)
15.44	2^+	(2, 1)		(2, 0) (2, 2)		(0,2)				(2,$M1$)					
16.1058	2^+	(1, 1)	1 [19,59],	(2, 2)	2 [16,60] 0 [19]	(0, 2)	2 [16]	(0, $E2$)	$E2$ [10,14],	(2, $M1$)	$M1$ [10,14]				
16.57	2^-	(2, 0)	0 [8,19,42],	(2, 3)	1 [19], 3 [3,12,60]			(0, M2)	$M2$ [10]	(2, $E1$)	$E1$ [8,10,14]				
17.23	1^-	(1, 0)	0 [19,21]	(2, 1)	1 [60] 3 [60]	(0, 1)		(0, $E1$)	$E1$ [42]	(2, $M2$)					
17.77	0^+	(1, 1)		(2, 2)	2 [60]	(0, 0)				(2, $E2$)					
18.13	1^+	(2, 1)													
18.35	2^-	(2, 2)													
18.38	3^-	(1, 2)	2 [8]	(2, 1)	1 [3,60]	(0, 3)	3 [19]	(0, $E3$)		(2, $E1$)	$E1$ [8]	(1, 2)			
18.4	0^-	(2, 2)	2 [8]							(2, $M2$)	$M2$ [8]	(0, 0)			
18.83	2^+	(2, 1)	1 [8]	(2, 0)		(0,2)		(0, $E2$)	$E2$ [8]	(2, $E2$)	$M1$ [8]	(1, 1)	1 [8]	(1, 1)	1 [8]
19.00	1^-	(1, 2)		(2, 3)				(0, $E1$)				(1, 0)		(1, 0)	
19.2	2^-	(2, 2)		(2, 1)				(0, $M2$)		(2, $E1$)				(2, 0)	
19.44	2^+	(1, 1)		(2,2)	2 [60]	(0,2)		(0, $E2$)				(1, 1)		(1, 1)	
19.64	2^-	(1, 2)		(2, 1)										(2, 0)	
20.6	3^-	(1, 2)								(2, $E1$)					
21.6	2^+	(1, 3)													
22.65	1^-	(1, 2)		(2, 1)				(0, $E1$)							
23.5	3^-	(1, 2)													

TABLE VI. Comparison of the parameters belonging to the particle pairs used in this study with literature values.

Channel no.	Light particle	Spin	Heavy particle	Spin	Excitation energy[a] (MeV)		Separation energy[b] (MeV)		Channel radius (fm)	
					Present	Ref.	Present	Ref	Present	Ref.
1	p	$1/2^+$	^{11}B	$3/2^-$	0	0	15.9572	15.96 [19] 15.957 [10]	4.5	4.67 [61]
2	A	0^+	^{8}Be	2^+	3.03	2.994 [65]	10.3966	10.31 [15]	4.3	5.09 [16] 4.31 [10]
3	A	0^+	^{8}Be	0^+	0	0	7.3666	7.367 [10]	4.3	
4	Γ	1^+	^{12}C	0^+	0	0	0	0	0	
5	Γ	1^+	^{12}C	2^+	4.43	4.44 [15]	0	0	0	
6	p	$1/2^+$	^{11}B	$1/2^-$	1	2.14 [65] 2.12 [10]	16.9572	18.096 [65] 18.081 [10]	5.2	
7	n	$1/2^+$	^{11}C	$3/2^-$	0	0	18.711	18.722 [10]	5.2	

[a]Excitation energy: the energy required to bring a nucleus from its ground state to a higher energy level.
[b]Separation energy: the energy required to separate a nucleon, or a particle composed of nucleons, from a nucleus.

ture (Table VI). No explanation for this discrepancy could be found.

The following subsections provide more details on the fit results for each channel.

A. ^{11}B$(p, \alpha_1)^8$Be*

The low energy cross section values for this reaction channel, particularly at 16.10 MeV (2^+) and 16.57 MeV (2^-) resonance levels, have been extensively studied as it is the key reaction of aneutronic fusion reactor designs [5,62]. In this study, we used integral cross section (CS) data from Refs. [8,19,36] as well as differential cross section (DCS) data from Refs. [37,38] [Figs. 1(a) and 1(b)]. The channel radius has been determined to be 4.3 fm, which is different from the literature value of 5.09 fm [16]. The partial widths of the channel for 16.10 (2^+), 16.57 (2^-), and 19.44 (2^+) resonances have been calculated as 6.3, 175, and 470 keV respectively (Table IV). These values are different from the literature values of 5, 150, and 450 keV [10]. Additionally, the present study has found partial widths of 285, 35, and 20 keV for the α_1 channel of the resonances 15.44 (2^+), 19.2 (2^-), and 19.64 (2^-) respectively, for which there is no information in the literature. The best fit for the ^{11}B$(p, \alpha_1)^8$Be* decay of 15.44 MeV (2^+) resonance has been obtained by including two different channels with (l, s) values (2, 0) and (2, 2) in the calculations (Table V).

The literature presents different values for the orbital angular momentum (l) of the 16.57 (2^-) resonance, leading to controversy. Reference [19] calculated this value as $l = 1$, while Refs. [3,12] found it to be $l = 3$. In contrast, Refs. [18,63] claimed a mixture of $l = 1$ and $l = 3$. In this study, both l values were considered. Although no significant difference was observed in terms of their impact on the analysis results around the 16.57 (2^-) resonance, the $l = 3$ value gives better fits to the data in the higher energy region.

In the low energy region, the calculated CS values for the α_1 channel, which includes the 16.10 (2^+) and 16.57 (2^-) resonances, are noticeably lower than most of the datasets, indicating a strong contribution from a background pole or direct process. Reference [56] previously used the 15.44 MeV resonance as a background pole with a spin-parity assignment of 0^+. In the present work, this resonance was also used with both 0^+ and 2^+ spin-parity assignments, but no significant contribution to the α_1 channel could be obtained.

Other authors in the past have suggested that there is an interference of direct process amplitude with the 16.57 (2^-) resonance amplitude. Reference [19] stated in their comments that at the 16.57 (2^-) resonance with $l_p = 0$, a direct process contribution must involve $l_p = 0$ and suggested with a

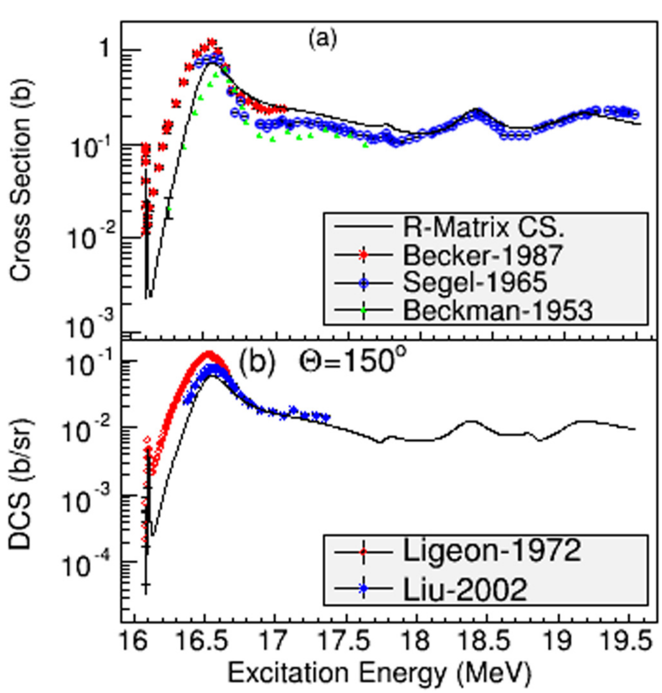

FIG. 1. Comparison of the CS and DCS values of ^{11}B$(p, \alpha_1)^8$Be reaction calculated by R-matrix analysis with (a) CS data of Refs. [8,19,36] (b) DCS data of Refs. [37,38] at 150°.

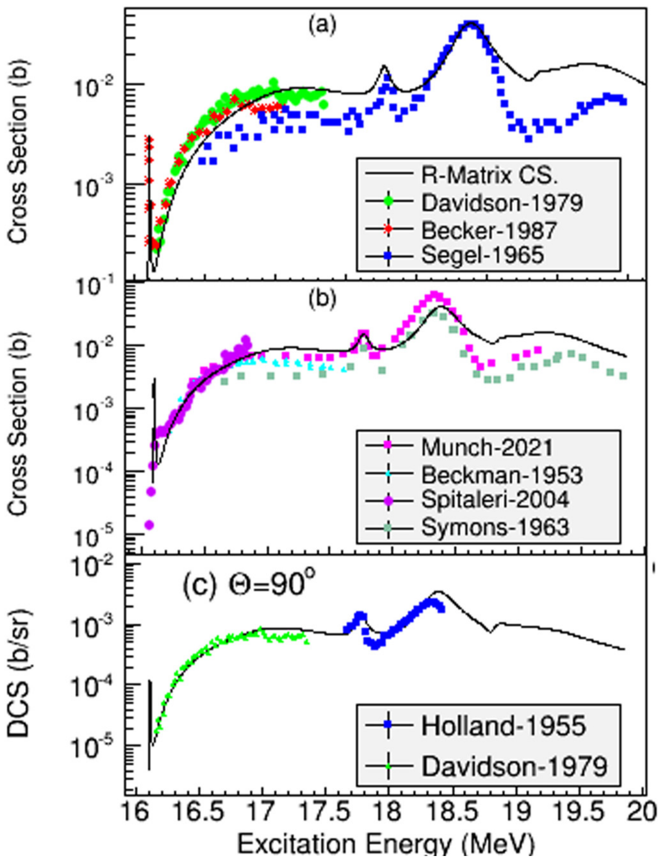

FIG. 2. Comparison of the CS and DCS values of $^{11}B(p,\alpha_0)^8Be$ reaction calculated by R-matrix analysis with (a) CS data of Refs. [8,19,39], (b) CS data of Refs. [36,40–42] and (c) DCS data of Ref. [43] with normalization constant (NC) = 0.00035 and DCS data of Ref. [39] at 90°.

polynomial fit that there must be an $l_p = 1$ direct reaction contribution in the vicinity of the 16.10 (2^+) resonance. Reference [59] similarly assumed an interference at this region with s-wave protons and found a resonance energy of 596 keV and a width of $\Gamma = 383$ keV through their fit. Reference [56] also claimed that there must be indirect contributions from both channel spins of $s = 1$ and $s = 2$, but only channel spin $s = 2$ interferes with the 16.57 (2^-) resonance.

B. $^{11}B(p,\alpha_0)^8Be$

In nuclear reaction analysis, the α_0 channel is of considerable importance due to the determination of the boron ratio in various thin films [35,64]. In this study, the analysis was performed using data sets given in Refs. [8,19,33,36,39–43] (Figs. 2 and 3). The partial widths of this channel for the resonances 16.10 (2^+), 18.38 (3^-), 19 (1^-), and 19.44 (2^+) have been determined as 0.29, 50, 0, and 70 keV, respectively, while Ref. [10] tabulated these values as 0.26, 65, 50, and 20 keV respectively (Table IV).

In this study, it has been found that the partial width of the α_0 channel for the state 15.44 (2^+) is 280 keV, for which there is no information available in the literature. While it has been observed that the 15.44 (2^+) resonance does not have a

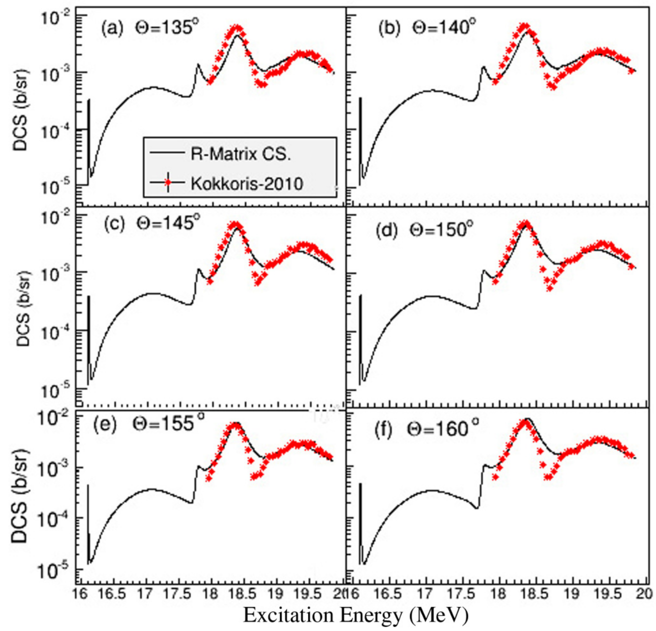

FIG. 3. Comparison of the DCS values calculated for the $^{11}B(p,\alpha_0)^8Be$ reaction via the R-Matrix method with the DCS data of Ref. [33] at $\theta_{Lab} = 135$, 140, 145, 150, 155, 160 [labeled (a) through (f), respectively].

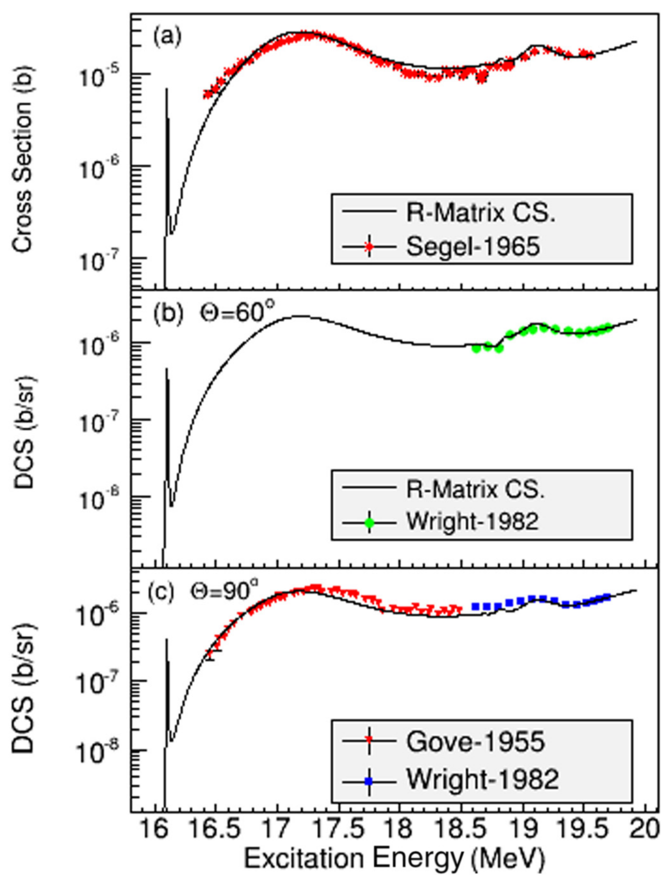

FIG. 4. Comparison of the values calculated by the R-Matrix method for the $^{11}B(p,\gamma_0)^{12}C$ reaction with (a) CS data of Ref. [8], (b) DCS data of Ref. [44] at 60° and (c) DCS data of Ref. [45] with NC = 3.0×10^{-9} and Ref. [44] at 90°.

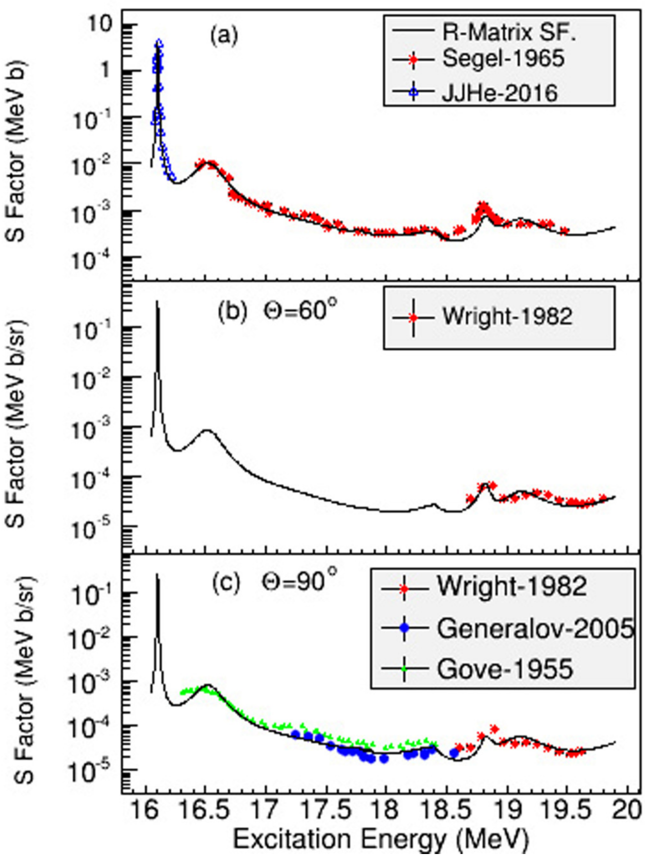

FIG. 5. Comparison of the values calculated by the R-Matrix method for the ^{11}B(p, γ_1)^{12}C reaction with (a) CS data of Refs. [8,46] (b) DCS data of Ref. [44] at 60° and (c) DCS data of Refs. [44,45,47] at 90° with NC $= 2.0 \times 10^{-9}$ and an energy reduction of 0.15 MeV for Ref [45]. data. The CS data and DCS data have been converted to S-Factor.

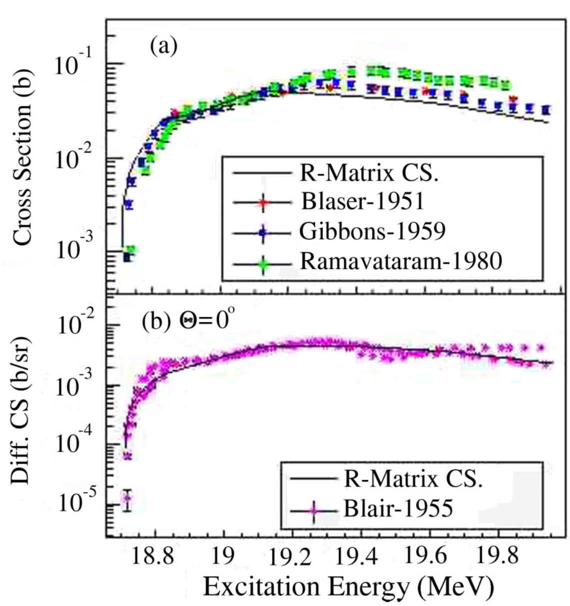

FIG. 6. Comparison of the values calculated by the R-Matrix method for the ^{11}B(p, n$_0$)^{11}C reaction with (a) CS data of Refs. [51–53] and (b) DCS data of Ref. [54] at 0° with NC = 0.0013.

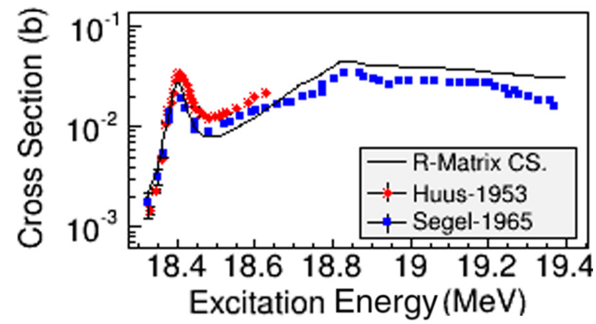

FIG. 7. Comparison of the values calculated by the R-Matrix method for the ^{11}B(p,p$_1$)^{11}B* reaction with CS data of Refs. [8,48].

significant contribution to the $\alpha_1 - {}^8$Be* channel, it has been found to be useful in achieving a better fit for the α_0 channel above 16.5 MeV.

C. ^{11}B(p, γ_0)^{12}C

In this study, the partial width of the γ_0 channel for the 19.0 (1^-) state has been found to be 2 eV, which contrasts with the value of 25 eV given in Ref. [8] (Table IV). On the other hand, the present analysis has revealed an 8.75 eV partial width for the 19.2(2^-) state, which was not included in Ref. [8]. The datasets from Refs. [8,44,45] were used in the analysis. The CS data from Ref. [8] and DCS datasets from Refs. [44,45] at 60 ° and 90 ° represent the analysis result well overall (Fig. 4). However, at the 17.23 (1^-) resonance, a slight disagreement has been observed between the peak value resulting from the analysis and the peak value revealed by the data set [45], with

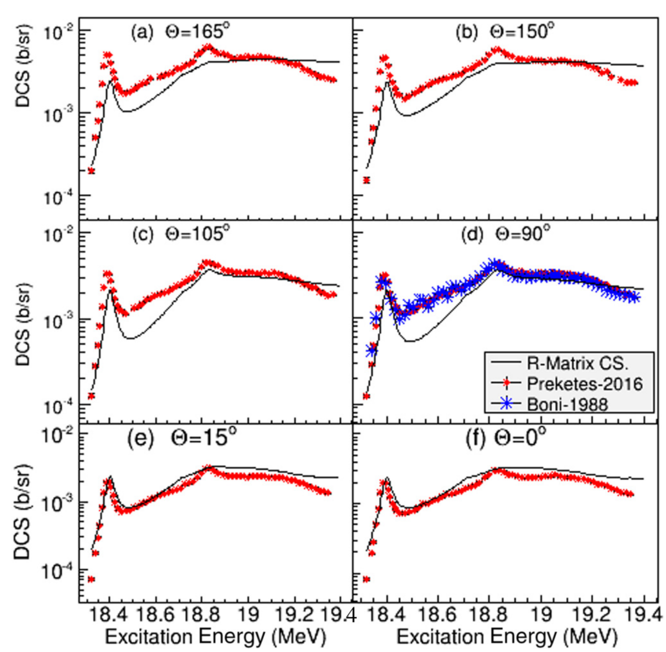

FIG. 8. Comparison of the values calculated by the R-Matrix method for the ^{11}B(p,p$_1$)^{11}B* reaction with DCS data of Ref. [49] at $\theta_{\rm Lab} = 165°$, 150°, 105°, 90°, 15° [labeled (a) through (e) respectively] and (d) DCS data of Ref. [50] at 90° with NC = 4.8.

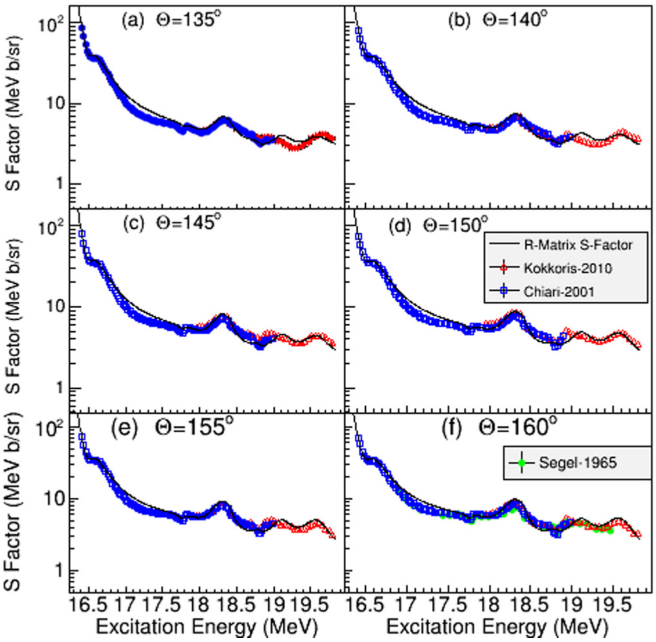

FIG. 9. Comparison of the values calculated by the R-Matrix method for the ^{11}B(p, p$_0$)^{11}B reaction with DCS data of Refs. [8,33,34] at $\theta_{\rm Lab}$ = 135°, 140°, 145°, 150°, 155° and 160° [labeled (a) through (f), respectively]. DCS data have been converted to S-Factor.

an energy difference of approximately 90 keV [Fig. 4(c)]. EXFOR [32] has reported the energy error for this data set [45] as 30 keV, which is only the digitizing error. However, the authors of Ref. [45] explained the target thickness to be nonuniform and to vary from 0 to 200 keV, which, together with the digitizing error, accounts for the disagreement in the energy values.

D. ^{11}B(p, γ_1)^{12}C

The present work has found a partial width of 7 eV for the 19.2 (2^-) state in contrast to Ref. [8]'s claim of 10 eV for the 19.2 (1^-) state and 3 eV for the 19.4 (2^+) state (Table IV). The analysis results of the present study are highly compatible with the datasets of Refs. [8,46]. Resonances 16.10 (2^-), 16.57 (2^+), and 18.81 (2^+) have been prominently represented in the analysis (Fig. 5). The 60° dataset of Ref. [44] shows general agreement with the analysis result. Datasets from Refs. [44,47] used for 90° are in good agreement with the analysis results.

Reference [45]'s dataset for this channel has been given with arbitrary units, and thus the fit has been obtained by a normalization constant of 2.5×10^{-9} (Table II). Moreover, Ref. [45] dataset has high disagreement with the analysis results in terms of energy values. To address this issue, the energy values of this dataset were reduced by 0.15 MeV and included in the calculation, resulting in representation of the 16.57 MeV resonance with a shift of about 0.15 MeV [Fig. 5(c)]. A similar energy shift of the data set for γ_0 channel was explained in the previous section. Since both data sets

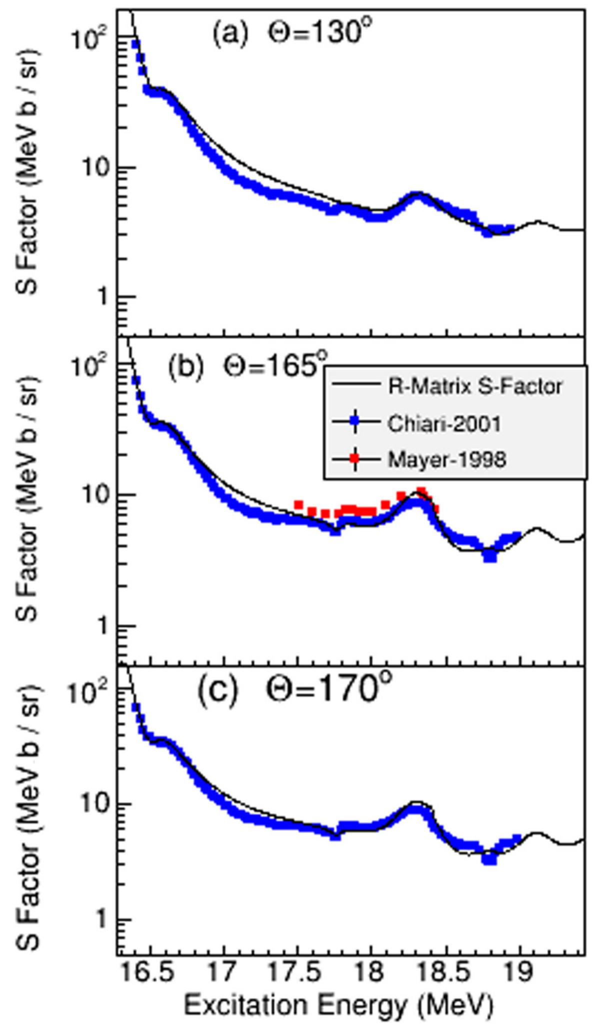

FIG. 10. Comparison of the values calculated by the R-Matrix method for the ^{11}B(p, p$_0$)^{11}B reaction with the DCS data of Refs. [34,35] at $\theta_{\rm Lab}$ = 130°, 165° and 170° [labeled (a) through (c), respectively]. DCS data have been converted to S-Factor.

were taken from Ref. [45], the same explanation for the energy shift applies here as well.

E. ^{11}B(p,n)^{11}C

In the present study, two types of datasets, total cross section [51–53] and differential cross section at 0° [54], were used for the ^{11}B(p, n_0)^{11}C reaction channel (Fig. 6). The partial widths for the n_0 channel have been found to be 14.5 and 5.5 eV for the 19.2 (2^-) and 19.64 (2^-) states, respectively, while Ref. [8] did not include these states (Table IV).

F. ^{11}B(p, p_1)^{11}B*

For the analysis of this reaction channel, we have used CS and DCS datasets from Refs. [8,48–50] at 0°, 15°, 90°, 105°, 150°, and 165° (Figs. 7 and 8). We have calculated the partial widths of the p_1 channel for the 19.0 (1^-) and 19.44 (2^+) resonances as 300 and 11 keV, respectively (Table IV). In contrast, Refs. [8,10] have tabulated these values as 400 and 11 keV, respectively.

G. ^{11}B(p, p_0)^{11}B

For the analysis of this reaction channel, we have used the DCS data sets of Refs. [8,33,34] with angles at 135°, 140°, 145°, 150°, 155°, 160° (Fig. 9) and datasets of Refs. [34,35] at 130°, 165°, 170° (Fig. 10). We have calculated partial widths of 125, 60, 430, and 400 keV for the p_0 channel of 16.57 (2^-), 18.38 (3^-), 19.4 (2^+), and 19.2 (2^-) resonances, respectively, whereas Refs. [8,10] proposed values of 150, 68, 450 keV for 16.57 (2^-), 18.38 (3^-), 19.4 (2^+) states respectively. On the other hand, although 19.2 (2^-) resonance was tabulated in Ref. [10], no partial width was assigned for this resonance.

IV. CONCLUSION

We have calculated the cross sections (CS) and astrophysical S factors for the reactions of ^{11}B(p, α_1)^{8}Be*, ^{11}B(p, α_0)^{8}Be, ^{11}B(p, p_0)^{11}B, ^{11}B(p, p_1)^{11}B*, ^{11}B(p, γ_0)^{12}C, ^{11}B(p, γ_1)^{12}C*, and ^{11}B(p, n_0)^{11}C through a multichannel-multilevel R-matrix analysis. The parameters used for these calculations have been compared with the literature.

The results show that most of the partial widths of the resonances are mostly compatible with the values in the literature. However, there is a clear difference for some of the channel widths of the 19.00 (1^-) and 19.44 (2^+) resonances, which is attributed to the interference of the broad resonances in this energy region.

This study has found the energy value of the first excitation level of the ^{11}B nucleus, with a spin-parity value of $1/2^-$, to be 1.00 MeV, which is significantly different from the literature value of 2.14 MeV.

The analysis of the 15.44 (2^+) resonance included two channels with (l, s) values of (2, 0) and (2, 2) for the ^{11}B(p, α_1)^{8}Be* reaction, and one channel for ^{11}B(p, α_0)^{8}Be reaction with (l, s) value of (0, 2). The calculations have resulted in a large improvement for the α_0 channel. However, no significant improvement has been observed for the α_1 channel.

The present work has found the orbital angular momentum (l) of the ^{11}B(p, α_1)^{8}Be* reaction channel for the 16.57 (2^-) resonance to be 3, which has been a subject of discussion in the literature.

The analysis results for α_1 channel in the vicinity of the 16.57 (2^-) and 16.1 (2^+) resonances are lower than some of the datasets, leading to speculation that direct contributions interfere with resonance values in this region.

The analysis of ^{12}C resonance energy values, spin-parities, and resonance decay parameters utilized a significant amount of data. As a result, it has been revealed that most of the data sets exhibit a high correlation with the analysis results. The application of multichannel R-matrix analysis has proved to be successful in analyzing the resonance values and decay parameters of the ^{12}C nucleus.

ACKNOWLEDGMENT

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[1] M. Hjorth-Jensen, The carbon challenge, Physics **4**, 38 (2011).

[2] J. H. Kelley *et al.*, The ^{11}B($p \rightarrow, \gamma$)^{12}C reaction below 100 keV, Phys. Rev. C **62**, 025803 (2000).

[3] M. C. Spraker *et al.*, The ^{11}B(p,α) ^{8}Be $\rightarrow \alpha + \alpha$ and the ^{11}B(α,α)^{11}B reactions at energies below 5.4 MeV, J. Fusion Energy **31**, 357 (2012).

[4] S. V. Putvinski, D. D. Ryutov, and P. N. Yushmanov, Fusion reactivity of the pB^{11} plasma revisited, Nucl. Fusion **59**, 076018 (2019).

[5] V. S. Belyaev, V. P. Krainov, A. P. Matafonov, and B. V. Zagreev, The new possibility of the fusion p + ^{11}B chain reaction being induced by intense laser pulses, Laser Phys. Lett. **12**, 96001 (2015).

[6] F. Belloni, D. Margarone, A. Picciotto, F. Schillaci, and L. Giuffrida, On the enhancement of p-^{11}B fusion reaction rate in laser-driven plasma by $\alpha \rightarrow$ p collisional energy transfer, Phys. Plasmas **25**, 020701 (2018).

[7] S. Eliezer and J. M. Martinez-Val, A novel fusion reactor with chain reactions for proton-boron11, Laser Part. Beams **38**, 39 (2020).

[8] R. E. Segel, S. S. Hanna, and R. G. Allas, States in C^{12} between 16.4 and 19.6 MeV, Phys. Rev. **139**, B818 (1965).

[9] F. Ajzenberg-Selove, Energy levels of light nuclei A = 11 – 12, Nucl. Phys. A **506**, 1 (1990).

[10] J. H. Kelley, J. E. Purcell, and C. G. Sheu, Energy levels of light nuclei A = 12, Nucl. Phys. A. **968**, 71 (2017).

[11] M. H. Sikora and H. R. Weller, A new evaluation of the ^{11}B(p,α)$\alpha\alpha$ reaction rates, J. Fusion Energy **35**, 538 (2016).

[12] S. Stave *et al.*, Understanding the B^{11}(p, α)$\alpha\alpha$ reaction at the 0.675 MeV resonance, Phys. Lett. B **696**, 26 (2011).

[13] M. Alcorta *et al.*, Properties of ^{12}C resonances determined from the ^{10}B (^{3}He, $p\alpha\alpha\alpha$) and ^{11}B (^{3}He, $d\alpha\alpha\alpha$) reactions studied in complete kinematics, Phys. Rev. C **86**, 064306 (2012).

[14] R. E. Segel and M. J. Bina, Partial widths of the 16.11-Mev state in C^{12}, Phys. Rev. **124**, 814 (1961).

[15] G. H. Neuschaefer and S. L. Tabor, Decays of high-lying states in ^{12}C, Phys. Rev. C **31**, 334 (1985).

[16] K. L. Laursen, H. O. U. Fynbo, O. S. Kirsebom, K. S. Madsbøll, and K. Riisager, Complete kinematical study of the 3α breakup of the 16.11 MeV state in ^{12}C, Eur. Phys. J. A **52**, 271 (2016).

[17] K. L. Laursen, H. O. U. Fynbo, O. S. Kirsebom, K. S. Madsbøll, and K. Riisager, Unbound states in ^{12}C populated by γ-decay of the (J^π, T) =(2 +,1) 16.11 MeV state, Eur. Phys. J. A **52**, 1 (2016).

[18] P. A. Treado, J. M. Lambert, V. E. Alessi, and R. J. Kane, Angular correlation and reaction mechanism for ^{11}B (p, α)^{8}Be$_{2.9}$(α)^{4}He with $E_p < 1.4$ MeV, Nucl. Phys. A **198**, 21 (1972).

[19] H. W. Becker, C. Rolfs, and H. P. Trautvetter, Low-energy cross sections for ^{11}B(p,3α), Z. Phys. A **327**, 341 (1987).
[20] W. von Witsch, M. Ivanovich, D. Rendić, V. Valković, G. C. Phillips, and K. Schäfer, Decay of ^{12}C via the ^{11}B(p,2α)α reaction, Nucl. Phys. A **180**, 402 (1972).
[21] G. Dearnaley, G. A. Dissanaike, A. P. French, and G. L. Jones, Study of the $B^{11} + p$ reactions, Phys. Rev. **108**, 743 (1957).
[22] E. P. Wigner and L. Eisenbud, Higher angular momenta and long range interaction in resonance reactions, Phys. Rev. **72**, 29 (1947).
[23] A. M. Lane and R. G. Thomas, *R*-matrix theory of nuclear reactions, Rev. Mod. Phys. **30**, 257 (1958).
[24] R. E. Azuma *et al.*, AZURE: An *R*-matrix code for nuclear astrophysics, Phys. Rev. C **81**, 045805 (2010).
[25] F. C. Barker, The boundary condition parameter in *R*-matrix theory, Aust. J. Phys. **25**, 341 (1972).
[26] C. R. Brune, Alternative parametrization of *R*-matrix theory, Phys. Rev. C **66**, 044611 (2002).
[27] P. Descouvemont and D. Baye, The *R*-matrix theory, Rep. Prog. Phys. **73**, 036301 (2010).
[28] R. J. Deboer *et al.*, The ^{12}C(α,γ) ^{16}O reaction and its implications for stellar helium burning, Rev. Mod. Phys. **89**, 035007 (2017).
[29] R. J. Deboer, J. Görres, G. Imbriani, P. J. Leblanc, E. Uberseder, and M. Wiescher, *R*-matrix analysis of ^{16}O compound nucleus reactions, Phys. Rev. C **87**, 015802 (2013).
[30] F. James, Minuit: Function minimization and error analysis reference manual, CERN Report No. CERNLIB-D5, 1998 (unpublished), https://cds.cern.ch/record/2296388/files/minuit.pdf.
[31] G. D'Agostini, On the use of the covariance matrix to fit correlated data, Nucl. Instrum. Methods Phys. Res. A **346**, 306 (1994).
[32] V. Zerkin, Experimental Nuclear Reaction Data (EXFOR), https://www-nds.iaea.org/exfor/.
[33] M. Kokkoris, A. Kafkarkou, V. Paneta, R. Vlastou, P. Misaelides, and A. Lagoyannis, Differential cross sections for the ^{11}B(p,α_o)^{8}Be and ^{11}B(p,p_0)^{11}B reactions, suitable for ion beam analysis, Nucl. Instrum. Methods Phys. Res., Sect. B **268**, 3539 (2010).
[34] M. Chiari, L. Giuntini, P. A. Mandò, and N. Taccetti, Proton elastic scattering cross-section on boron from 0.5 to 3.3 MeV, Nucl. Instrum. Methods Phys. Res., Sect. B **184**, 309 (2001).
[35] M. Mayer, A. Annen, W. Jacob, and S. Grigull, The ^{11}B(p,α_0)^{8}Be nuclear reaction and ^{11}B(p,p)^{11}B backscattering cross sections for analytical purposes, Nucl. Instrum. Methods Phys. Res., Sect. B **143**, 244 (1998).
[36] O. Beckman, T. Huus, and A. Zupančič, Excitation curves for α particles from B^{11} bombarded with protons, Phys. Rev. **91**, 606 (1953).
[37] E. Ligeon and A. Bontemps, Nuclear reaction analysis of boron and oxygen in silicon, J. Radioanal. Chem. **12**, 335 (1972).
[38] J. Liu, X. Lu, X. Wang, and W. K. Chu, Cross-sections of ^{11}B(p,α)^{8}Be reaction for boron analysis, Nucl. Instrum. Methods Phys. Res., Sect. B **190**, 107 (2002).
[39] J. M. Davidson, H. L. Berg, M. M. Lowry, M. R. Dwarakanath, A. J. Sierk, and P. Batay-Csorba, Low energy cross sections for ^{11}B(p,3α), Nucl. Phys. A **315**, 253 (1979).
[40] M. Munch, O. S. Kirsebom, J. A. Swartz, and H. O. U. Fynbo, Resolving the ^{11}B(p,α) cross-section discrepancies between 0.5 and 3.5 MeV, Eur. Phys. J. A **56**, 9 (2020).
[41] C. Spitaleri *et al.*, The ^{11}B(p,α_0)^{8}Be reaction at sub-Coulomb energies via the Trojan-horse method, Phys. Rev. C **69**, 055806 (2004).
[42] G. D. Symons and P. B. Treacy, The B^{11}(p, α)Be^{8} reaction and C^{12} states between 15 and 20 MeV, Nucl. Phys. **46**, 93 (1963).
[43] R. E. Holland, D. R. Inglis, R. E. Malm, and F. P. Mooring, Scarcity of low-energy levels of Be^{8} appearing in two boron reactions, Phys. Rev. **99**, 92 (1955).
[44] M. C. Wright, N. R. Roberson, H. R. Weller, D. R. Tilley, and D. Halderson, Proton capture in the 19 MeV region of ^{12}C, Phys. Rev. C **25**, 2823 (1982).
[45] H. E. Gove and E. B. Paul, Angular distribution of gamma rays from proton capture in B^{11}, Phys. Rev. **97**, 104 (1955).
[46] J. J. He *et al.*, Direct measurement of ^{11}B(p,γ)^{12}C astrophysical *S* factors at low energies, Phys. Rev. C **93**, 055804 (2016).
[47] L. N. Generalov, B. L. Lebedev, A. V. Livke, A. B. Modenov, and V. A. Chirkin, Experimental-computation studies of absolute γ-quantum detection efficiency of GC 5019 coaxial HPGe detector at $E_\gamma = 0.24 - 18.565$ MeV, Bull. Russ. Acad. Sci. Phys. **69**, 89 (2005).
[48] T. Huus and R. B. Day, The gamma radiation from ^{11}B bombarded by protons, Phys. Rev. **91**, 599 (1953).
[49] K. Preketes-Sigalas *et al.*, Study of the ^{11}B(p,p$'\gamma$)^{11}B reaction for PIGE applications, Nucl. Instrum. Methods Phys. Res., Sect. B **368**, 71 (2016).
[50] C. Boni, E. Cereda, G. M. B. Marcazzan, and V. De Tomasi, Prompt gamma emission excitation functions for PIGE analysis, Nucl. Instrum. Methods Phys. Res., Sect. B **35**, 80 (1988).
[51] J. P. Blaser, F. Boehm, and S. P. Marmier, Excitation functions of the (p,n) reaction for light elements, Helv. Phys. Acta **24**, 465 (1951).
[52] J. H. Gibbons and R. L. Macklin, Total neutron yields from light elements under proton and alpha bombardment, Phys. Rev. **114**, 571 (1959).
[53] K. Ramavataram, R. Larue, V. Turcotte, C. St-Pierre, and S. Ramavataram, Study of the ^{11}B(p,n)^{11}C reaction below the giant dipole resonance in ^{12}C, Nuovo Cimento A **58**, 342 (1980).
[54] J. K. Blair, J. D. Kington, and H. B. Willard, Gamma-ray and neutron yields from the proton bombardment of boron, Phys. Rev. **100**, 21 (1955).
[55] C. Angulo and P. Descouvemont, The ^{14}N $(p,\gamma)_{15}$O low-energy S-factor, Nucl. Phys. A. **690**, 755 (2001).
[56] F. C. Barker, Electron screening in reactions between light nuclei, Nucl. Phys. A. **707**, 277 (2002).
[57] S. S. Hanna, W. Feldman, M. Suffert, and D. Kurath, High-lying 0+ and 3- levels in ^{12}C, Phys. Rev. C **25**, 1179 (1982).
[58] P. R. Lewis, G. G. Shute, B. M. Spicer, V. C. Officer, P. R. Andrews, and S. M. Banks, The ^{13}C(p,d)^{12}C reaction at 119 MeV, Nucl. Phys. A. **474**, 499 (1987).
[59] C. Angulo, S. Engstler, G. Raimann, C. Rolfs, W. H. Schulte, and E. Somorjai, The effects of electron screening and resonances in (p,α) reactions on ^{10}B and ^{11}B at thermal energies, Z. Phys. A **345**, 231 (1993).
[60] J. L. Qebert and L. Marquez, Effects des resonances de ^{12}C sur l'emission de particules alpha dans la reaction ^{11}B(p,3a), Nucl. Phys. A **126**, 646 (1969).
[61] F. C. Barker and P. B. Treacy, Nuclear levels near thresholds, Nucl. Phys. **38**, 33 (1962).
[62] F. J. Wessel, M. W. Binderbauer, N. Rostoker, H. U. Rahman, and J. O'Toole, Colliding beam fusion reactor space propulsion system, AIP Conf. Proc. **504**, 1425 (2000).

[63] M. Kuhlwein *et al.*, Exclusive decay study of the 16.62 MeV ($2^-, T = 1$) resonance in ^{12}C, Phys. Lett. B **825**, 136857 (2022).

[64] E. Pitthan, M. V. Moro, S. A. Corrêa, and D. Primetzhofer, Assessing boron quantification and depth profiling of different boride materials using ion beams, Surf. Coatings Technol. **417**, 127188 (2021).

[65] J. D. Bronson, W. D. Simpson, W. R. Jackson, and G. C. Phillips, Three α-particle decay of C^{12}, Nucl. Phys. **68**, 241 (1965).