Influence of the spin cut-off parameter on the isomeric cross-section ratio of the $(n, 2n)$ reaction within the Huizenga-Vandenbosch method

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The influence of the spin cut-off parameter on the isomeric cross-section ratio of the $(n, 2n)$ reaction is studied within the Huizenga-Vandenbosch method, in which the neutron transmission coefficient is evaluated from the optical model, and the spin cut-off parameter is treated for the residual nucleus following the first neutron emission (σ_1) and that following the second neutron emission (σ_2) separately. It is found that the isomeric cross-section ratio is definitely sensitive to the spin cut-off parameters, especially the σ_2. Moreover, various formulas of the spin cut-off parameters are compared and applied to calculate the $(n, 2n)$ isomeric ratios of a series of nuclei from ^{45}Sc to ^{198}Hg at $E_n = 14$ MeV, and the results are compared with the evaluated values from JENDL-5 and JEFF-3.3, which shows that the method is more reliable in the light- and medium-mass region, and most of the calculated results are described well by the formulas giving the lower spin cut-off parameters than the rigid-body formula, indicating that the effective moment of inertia is less than the rigid body moment of inertia for most nuclei. Furthermore, with the above method the overall isomeric ratios as a function of the incident energy for ^{45}Sc$(n, 2n)^{44m,g}$Sc, ^{85}Rb$(n, 2n)^{84m,g}$Rb, and ^{120}Te$(n, 2n)^{119m,g}$Te reactions are well described, which shows the validity of the present method in calculating the isomeric ratio of the $(n, 2n)$ reaction below 20 MeV.

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I. INTRODUCTION

The isomeric cross-section ratio, especially its dependence on the incident energy, is of considerable significance in testing nuclear reaction models, and providing valuable information on the spin cut-off parameter characterizing the spin dependence of the level density [1,2]. The Huizenga-Vandenbosch (H-V) method [1,3] based on the Hauser-Feshbach statistical model [4] is a powerful method for calculating the isomeric ratio and is generally adopted in related works [5–11], including deducing the spin dependence of the level density according to the experimental isomeric ratios of the $(n, 2n)$ and (n, γ) reactions, which take place primarily via the compound nucleus mechanism. However, knowledge about the isomeric ratio as a function of the incident energy is significantly lacking, as well as knowledge about the influence of the spin cut-off parameter on the isomeric ratio.

The H-V method essentially focuses on the calculation of the spin distribution of the compound nucleus and the residual nucleus following the particle and γ-ray emission, and thus the isomeric ratio is determined from the final spin distribution and the spins of the isomeric states. Therefore, the spin cut-off parameter is one of the most important parameters within the H-V method. Up to now, there have been a few methods developed for calculating the energy and mass-dependent spin cut-off parameter, including the microscopic models [12,13], the rigid-body approximation [14,15], the formula derived by Ericson [16,17] and the newly proposed empirical formula by Egidy [18], most of which are applied in the present work to calculate the isomeric cross-section ratio and compared with the evaluated values from JENDL-5 [19] and JEFF-3.3 [20] in order to determine the most appropriate formula of the spin cut-off parameter within the H-V method.

In this work, the dependence of the isomeric cross-section ratio of the $(n, 2n)$ reaction on the spin cut-off parameter is investigated within the H-V method, taking the case of ^{45}Sc$(n, 2n)^{44m,g}$Sc at $E_n = 14$ MeV as example, in which the neutron transmission coefficient is evaluated from the optical model, and the spin cut-off parameter is treated for the residual nucleus following the first neutron emission (σ_1) and that following the second neutron emission (σ_2) separately, due to the large difference of the excitation energy. Moreover, the isomeric ratios of the $(n, 2n)$ reaction of 18 isotopes from ^{45}Sc to ^{198}Hg at $E_n = 14$ MeV are calculated with various formulas of the spin cut-off parameter and compared with the evaluated values to determine the most appropriate formula, based on which the isomeric ratios as a function of the incident energy are calculated for ^{45}Sc$(n, 2n)^{44m,g}$Sc, ^{85}Rb$(n, 2n)^{84m,g}$Rb, and ^{120}Te$(n, 2n)^{119m,g}$Te, and the results agree well with the evaluated values and the experimental data, which indicates the predictive power of the H-V method in calculating the isomeric ratio of the $(n, 2n)$ reaction below 20 MeV.

This paper is organized as follows. A detailed introduction of the method is presented in Sec. II. And in Sec. III, the

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calculated spin cut-off parameters with various formulas and the corresponding isomeric cross-section ratio of the $(n, 2n)$ reaction and its dependence on the incident energy are shown and the corresponding discussions are given. A summary of the present work and future prospects are presented in Sec. IV.

II. METHODS

The relative probability of forming the isomeric state by the compound-nuclear reaction is governed mainly by the spin difference between the states which decay to the isomer and the isomeric spin itself and the formation probabilities of these states with different spins [1]. Within the framework of the spin dependent statistical model of nuclear reaction, Huizenga and Vandenbosch proposed a method in which the isomeric cross-section ratio is almost quantitatively related to the spin dependence of the level density and the multiplicity of the γ-ray cascade, i.e., the H-V method, in order to calculate the isomeric cross-section ratio of the nuclear reaction taking place primarily via the compound nucleus mechanism. In the present work, the H-V method [1,3,21] is adopted to evaluate the $(n, 2n)$ isomeric ratios of a series of nuclei across the large mass range.

In the H-V method, the most important factors are (1) the spins of the compound-nuclear states, (2) the number and types of steps in the deexcitation process, (3) the angular momentum carried away by the emitted particles and the γ ray, (4) the probability of forming states with different spins at each step of the cascade, and finally (5) the spins of the isomeric state and the ground state. It should be noted that the two assumptions are made in order to perform the detailed calculations, which are that the levels of both parities are present in equal number so that the parity changes are not followed in the deexcitation process, and the γ-ray cascade is believed to consist mostly of dipole radiations so that the pure dipole radiations are taken into account.

A. The spin distribution of the initial compound nucleus

In calculation of the isomeric ratio of the $(n, 2n)$ reaction, the first step is the calculation of the spin distribution of the formed compound nucleus. The cross section for the formation of a compound nucleus with spin J_c at a bombarding energy E is given as follows according to the Hauser-Feshbach model [4],

$$\sigma(J_c, E) = \pi\lambda^2 \sum_{S=|I-s|}^{I+s} \sum_{\ell=|J_c-S|}^{J_c+S} \frac{2J_c+1}{(2s+1)(2I+1)} T_\ell(E), \quad (1)$$

where λ is the de Broglie wavelength of the incoming projectile, s and I are the spins of the projectile and the target nucleus, respectively. T_ℓ is the barrier transmission coefficient of the incident particle with orbital angular momentum ℓ and energy E. Here in Eq. (1) and in the following equations the transmission coefficients are calculated with the TALYS 1.95 code [22] in which the optical model is incorporated with local optical model parameters for many nuclei. The normalized spin distribution of the initial compound nucleus is written as

$$P(J_c) = \frac{\sigma(J_c, E)}{\sum_{J_c} \sigma(J_c, E)}. \quad (2)$$

B. The spin distribution of the residual nucleus following successive neutron emission

The compound nucleus with a given excitation energy can decay by successive particle emission to the residual nucleus with a variety of spin values. The relative probability for a compound state with spin J_c to emit a neutron with orbital angular momentum ℓ leading to a final state with spin J_f is given by

$$P(J_f)_{J_c} \propto \rho(J_f) \sum_{S=|J_f-\frac{1}{2}|}^{J_f+\frac{1}{2}} \sum_{\ell'=|J_c-S|}^{J_c+S} T_\ell{}'(E_n), \quad (3)$$

where $T_{\ell'}(E_n)$ is the barrier transmission coefficient of the emitted neutron with angular momentum ℓ' and energy E_n. In principle, the evaporated neutron spectra can be subdivided into several energy bins and the associated transmission coefficients are obtained individually for each bin. It was found that using a single set of transmission coefficients which are associated with the averaged energy of the evaporated neutron was a surprisingly accurate approximation [3]. Thus, the transmission coefficients corresponding to the averaged neutron energy are used in the present work, and the averaged energy is evaluated from the evaporated neutron spectra calculated with the TALYS 1.95. The other important input quantity $\rho(J_f)$ in Eq. (3) denotes the spin dependence of the level density of the residual nucleus, which is given from the Fermi gas model by

$$\rho(J_f) = \frac{2J_f+1}{2\sigma^2} \exp\left[-\frac{\left(J_f+\frac{1}{2}\right)^2}{2\sigma^2}\right], \quad (4)$$

where σ is the spin cut-off parameter representing the width of the spin distribution of the level density.

The yield of the state with spin J_f coming from the initial spin J_c is obtained by multiplying the initial normalized yield P_{J_c}. To sum over all values of J_c, the normalized spin distribution of the residual nucleus following the first neutron emission is

$$P(J_f) = \sum_{J_c} P(J_c) \frac{\rho(J_f) \sum_{S=|J_f-\frac{1}{2}|}^{J_f+\frac{1}{2}} \sum_{\ell'=|J_c-S|}^{J_c+S} T_{\ell'}(E_n)}{\sum_{J_F} \rho(J_F) \sum_{S=|J_F-\frac{1}{2}|}^{J_F+\frac{1}{2}} \sum_{\ell'=|J_c-S|}^{J_c+S} T_{\ell'}(E_n)}, \quad (5)$$

based on which the spin distribution following the second neutron emission is calculated by repeating the above steps, in which the initial spin distribution is that following the first neutron emission, and the averaged neutron energy is recalculated based on the energy spectra of the second neutron as well as the associated transmission coefficients.

C. The spin distribution of the residual nucleus following successive γ-ray emission

After the second neutron is emitted, the residual nucleus will continue to undergo deexcitation by emitting one or a cascade of γ rays and finally reach the ground state or the isomeric state. The relative probability of decaying from a state J_i to J_f by γ-ray emission is assumed to be simply proportional to the density of final states with spin J_f. Thus, the normalized probability of J_f is given by the following formula:

$$F(J_f) = \sum_{J_i=|J_f-l|}^{J_f+l} \frac{F(J_i)\rho(J_f)\delta_{J_i,J_f}}{\sum_{J_F=|J_i-l|}^{J_i+l}\rho(J_F)}, \tag{6}$$

where l is the multiplicity of the emitted γ ray and $F(J_i)$ is the normalized probability of the initial state. For the first γ-ray emission, $F(J_i) = P(J_f)$, where $P(J_f)$ is the normalized spin distribution following the second neutron emission. Considering the pure dipole radiation during the deexcitation, the multiplicity l is taken to be one.

The number of γ rays to be emitted depends on the residual excitation energy following the second neutron emission and generally varies between one and four γ rays per cascade [3]. The averaged number N_γ of γ rays for the pure dipole emission in the γ cascade can be estimated by [6]

$$N_\gamma = \tfrac{1}{2}\sqrt{aE}, \tag{7}$$

where E is the excitation energy of the residual nucleus following the second neutron emission. And the averaged energy of the dipole γ ray emitted from a nucleus of initial excitation energy E is calculated by [23]

$$E_\gamma = 4(E/a - 5/a^2)^{\frac{1}{2}}. \tag{8}$$

The energy of each succeeding γ ray in the cascade is obtained by calculating the new initial excitation energy.

It is assumed that the last γ ray to be emitted leads the excited nucleus to the ground state or the isomeric state, and the last γ-ray transition depends on which transition has the smaller spin change. Thus, one can find a separate spin I_d from the spin values between the ground and isomeric state, which makes the state with the spin $I > I_d$ deexcite to the high-spin product and the state with the spin $I < I_d$ deexcite to the low-spin product. In this work, I_d is defined as $I_d = (I_g + I_m)/2$, in which I_g and I_m represent the spins at the ground and isomeric state, respectively.

If I_m is larger than I_g and I_d is not one of the possible values of the final states, the isomeric cross-section ratio can be expressed as

$$R = \frac{\sigma_m}{\sigma_g} = \frac{1 - \sum_{J_f<I_d} F(J_f)}{\sum_{J_f<I_d} F(J_f)}, \tag{9}$$

where σ_m and σ_g are the production cross section of the residual nucleus at the isomeric state and the ground state, respectively. Otherwise if I_d is one of the possible values of the final states, the isomeric cross-section ratio is expressed as

$$R = \frac{\sigma_m}{\sigma_g} = \frac{1 - \left[\sum_{J_f<I_d} F(J_f) + \frac{1}{2}F(J_f = I_d)\right]}{\sum_{J_f<I_d} F(J_f) + \frac{1}{2}F(J_f = I_d)}. \tag{10}$$

D. The spin cut-off parameter

The spin cut-off parameter governs the distribution of the spins on the nuclear levels according to the Fermi gas model [24] and has a significant influence on the isomeric cross-section ratio, which is obtained with various methods shown below.

One of the methods is that derived by Ericson [16], in which the spin cut-off parameter is defined by the following equation within the Fermi gas model,

$$\sigma^2 = \frac{6}{\pi^2} a\langle m_j^2\rangle t, \tag{11}$$

where $\langle m_j^2\rangle$ is the averaged value of the square of the projection of the total angular momentum for the fermion states around the Fermi level and is approximately expressed by $\langle m_j^2\rangle = 0.24A^{2/3}$ based on the statistical mechanical calculation [17]. Thus, the corresponding σ^2 is written by

$$\sigma^2 = 0.146A^{2/3}\sqrt{aU}, \tag{12}$$

with the level-density parameter a expressed as [25]

$$a = \tilde{a}\left(1 + \delta W \frac{1 - \exp(-\gamma U)}{U}\right), \tag{13}$$

in which the energy-dependent shell effect is taken into account, and $\tilde{a}$ is the asymptotic level-density parameter, and δW and γ are the shell correction energy and the shell damping parameter, respectively, and U is the effective excitation energy defined as the difference of the excitation energy E_x and the pairing correction energy Δ. The parameters in Eq. (13) are evaluated from TALYS 1.95.

Another frequently used formula is based on the assumption that the nucleus is a rigid sphere with the moment of inertia I_0, and one can relate σ^2 to the rigid moment of inertia I_0 and the thermodynamic temperature t through the expression $\sigma^2 = I_0 t$, which is expected to be valid at high excitation energies. Furthermore, the shell effect is taken into account according to the microscopic level-density studies that the quantity σ^2/t is not constant [26], and thus the spin cut-off parameter is written by the following expression [27]

$$\sigma_F^2 = I_0\frac{a}{\tilde{a}}t = 0.01389\frac{A^{5/3}}{\tilde{a}}\sqrt{aU}, \tag{14}$$

with $I_0 = \frac{2}{5}m_0R^2A/(\hbar c)^2$ and the radius $R = 1.2A^{1/3}$ fm.

The above two methods are less appropriate when the nucleus locates at low excitation energies ($E_x < \Delta$). For this case, the spins of the low-lying discrete levels are used to determine the spin cut-off parameter σ_d, which is determined by the equation [28]

$$\sigma_d^2 = \frac{1}{3\sum_{i=N_L}^{N_U}(2J_i+1)}\sum_{i=N_L}^{N_U} J_i(J_i+1)(2J_i+1), \tag{15}$$

in the energy range from a lower discrete level N_L with energy E_L to an upper level N_U with energy E_U, where J_i is the spin of the discrete level i. This formula is usually applied in combination with Eq. (14) to describe the whole excitation energy range, and the final functional from Ref. [15] is expressed as

$$\sigma^2(E_x) = \begin{cases} \sigma_d^2 & (0 < E_x < E_d) \\ \sigma_d^2 + \frac{E_x - E_d}{S_n - E_d}\left[\sigma_F^2(E_x) - \sigma_d^2\right] & (E_d \leqslant E_x \leqslant S_n) \\ \sigma_F^2(E_x) & (E_x > S_n), \end{cases} \tag{16}$$

where E_d is defined as the energy in the middle of the $N_L - N_U$ region, i.e., $E_d = \frac{1}{2}(E_L + E_U)$. σ_d^2 is assumed to be constant up to the energy E_d and linearly interpolated to σ_F^2 given by Eq. (14) and the matching point is chosen to be the neutron separation energy S_n.

As for the nucleus at the low excitation energy, the systematical formula could also give a reasonable estimate for energies in the order of 1–2 MeV and is written by

$$\sigma^2 = (0.83A^{0.26})^2. \tag{17}$$

In addition, a new empirical formula [18] was proposed by Egidy based on the comparison of various experimental and calculated momenta in the energy-spin plane using a total of 7202 levels with spin assignment in 224 nuclei between F and Cf, in order to the describe the mass and energy dependence of the spin cut-off parameter in the large energy range, which is expressed as

$$\sigma^2 = 0.391A^{0.675}(E - 0.5Pa')^{0.312}, \tag{18}$$

where Pa' is calculated from the mass value [29] with the formula:

$$Pa' = \frac{1}{2}[M(A+2, Z+1) - 2M(A, Z) + M(A-2, Z-1)], \tag{19}$$

In this work, the above methods for the spin cut-off parameter σ are adopted to evaluate the isomeric cross-section ratio, and a comparison is made in order to determine the most appropriate expression for σ within the H-V method.

III. CALCULATION RESULTS

The isomeric cross-section ratio is determined from the final spin distribution of the residual nucleus within the H-V method, which is obtained from the modification of the spin distribution during the deexcitation process where the particles and γ rays are successively emitted and take away some energy and angular momentum. Figure 1 shows the spin distributions corresponding to the successive stages in the $(n, 2n)$ reaction, taking the case of 14 MeV $n + {}^{45}$Sc as an example. One can see that all of the spin distributions are the Gaussian-like types, and the spin distribution is obviously shifted to the left side with the narrower peak width during the deexcitation process, which shows that the neutron and γ-ray emission lead to a large modification of the spin distribution. Thus, the isomeric ratio is significantly influenced by the spin dependence of the level density of the residual nucleus, i.e., the spin cut-off parameter.

We further investigate the dependence of the isomeric cross-section ratio on the spin cut-off parameter in the $(n, 2n)$ reaction. The spin cut-off parameter σ is generally dependent on the excitation energy and the mass of the nucleus, so the σ is handled for the residual nucleus following the first neutron emission (σ_1) and that following the second neutron emission (σ_2) separately due to the large difference of the excitation energy between them. In addition, the spin cut-off parameter of the residual nucleus following γ-ray emission is set to be equal to σ_2 in the present work, owing to the quite low excitation energy after the second neutron emission. Figure 2 shows the dependence of the calculated isomeric

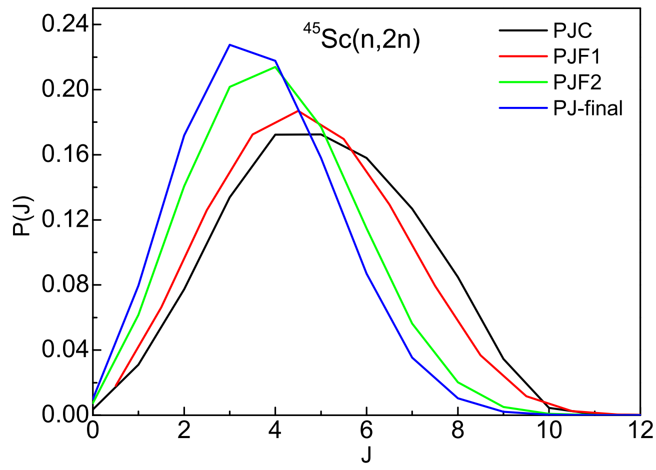

FIG. 1. The spin distributions corresponding to the successive stages in the $(n, 2n)$ reaction of 14 MeV $n + {}^{45}$Sc, including that of the compound nucleus (PJC), the residual nucleus after emitting the first neutron (PJF1), the second neutron (PJF2), and γ ray (PJ final, $N_\gamma = 1$).

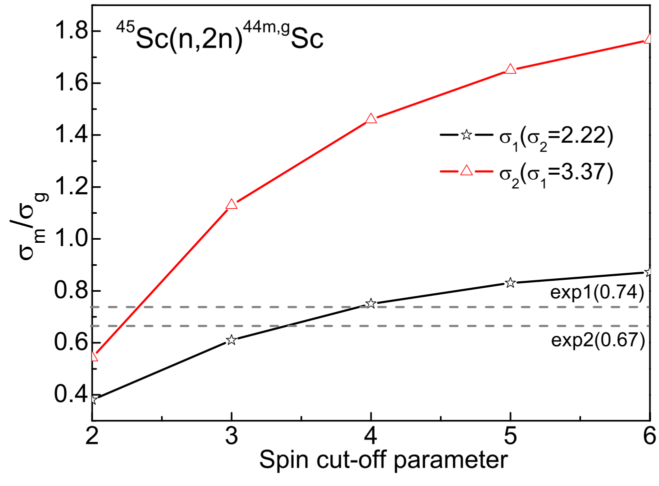

FIG. 2. The dependence of the calculated isomeric cross-section ratio in the $(n, 2n)$ reaction of 14 MeV $n + {}^{45}$Sc on the spin cut-off parameter σ_1 of the residual nucleus following the first neutron emission (black curve) and σ_2 of the residual nucleus following the second neutron emission (red curve), in which the averaged number of γ-ray emissions is obtained from the empirical formula and set to be 1.0. The dashed lines show the corresponding experimental values [30,31].

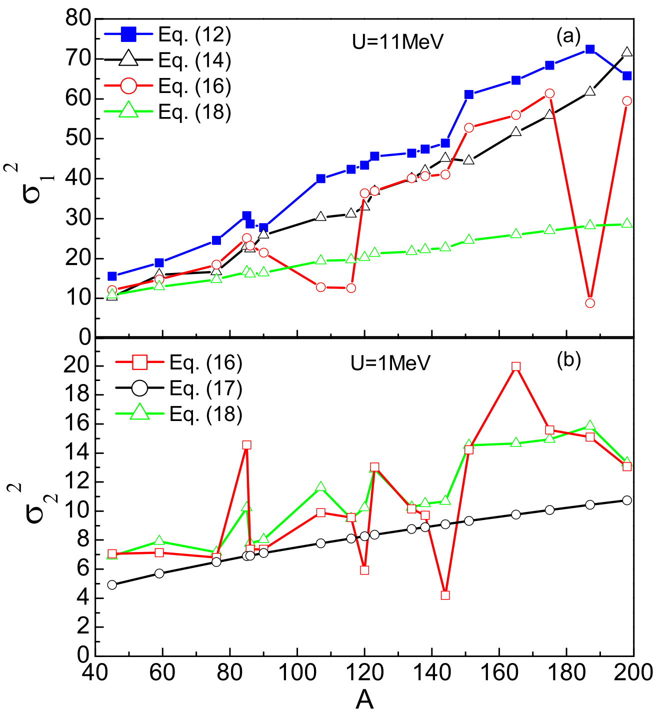

FIG. 3. Comparison of the spin cut-off parameters calculated with various formulas shown in Sec. II for a series of nuclei at the excitation energy $U = 11$ MeV (a) and at the excitation energy $U = 1$ MeV (b).

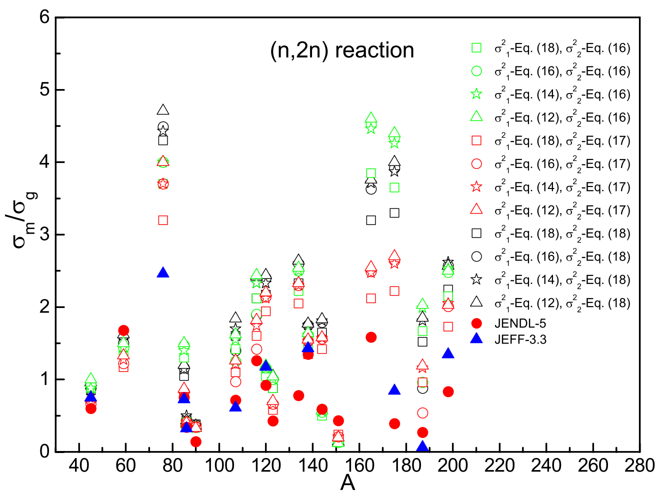

FIG. 4. The calculated isomeric cross-section ratios of the $(n, 2n)$ reactions at $E_n = 14$ MeV for a series of nuclei compared with the evaluated results from JENDL-5 and JEFF-3.3, in which the spin cut-off parameter σ_1 and σ_2 are that of the residual nucleus following the first and the second neutron emission, respectively, and calculated with various formulas introduced in Sec. II.

cross-section ratio in ^{45}Sc$(n, 2n)^{44m,g}$Sc at $E_n = 14$ MeV on σ_1 and σ_2, respectively, with the other spin cut-off parameter fixed as an empirical value, which is obtained from the formulas in Sec. II, i.e., $\sigma_1 = 3.37\hbar$ from Eq. (18) and $\sigma_2 = 2.22\hbar$ from Eq. (17). It can be obviously seen that the isomeric cross-section ratio increases with the σ_1 increasing, and increases more rapidly with the σ_2 increasing, which means that the isomeric cross-section ratio is definitely sensitive to the spin cut-off parameters, especially the σ_2. The experimental values are also shown in Fig. 2 by the dashed lines. According to the experimental data, the corresponding fitted value of σ_1 is located around $3.6\hbar$ and σ_2 is located around $2.2\hbar$, which are close to the empirical values, indicating the reliability of the present method and the spin cut-off parameter calculation.

As the spin cut-off parameter σ has an important influence on the isomeric cross-section ratio, the appropriate calculation method of σ is studied within the H-V method in this work in order to improve the predictive power of the H-V method. First, using various formulas of σ introduced in Sec. II, we calculate and compare the spin cut-off parameters of a series of nuclei at the excitation energy $U = 11$ and 1 MeV, which are approximate to the excitation energy of the residual nucleus following the first neutron emission and that following the second neutron emission, respectively, in most of the $(n, 2n)$ reactions at $E_n = 14$ MeV. Equations (12), (14), (16), and (18) are applicable in calculating σ at the higher excitation energies, and with these formulas the corresponding spin cut-off parameter σ_1 at $U = 11$ MeV are calculated and shown in Fig. 3(a) for these nuclei including ^{45}Sc, ^{59}Co, ^{76}Ge, ^{85}Rb, ^{86}Sr, ^{90}Zr, ^{107}Ag, ^{116}Cd, ^{120}Te, ^{123}Sb, ^{134}Ba, ^{138}Ce, ^{144}Sm, ^{151}Eu, ^{165}Ho, ^{175}Lu, ^{187}Re, ^{198}Hg, which are important target nuclei of the isomeric $(n, 2n)$ reactions. On the whole, the spin cut-off parameters from the above four types of formulas increase nearly linearly with the mass of the nucleus increasing, although there are more or less fluctuations due to the shell effects. However, there are significant differences among the absolute values of the spin cut-off parameter, and the overall values of σ_1 from Eq. (12) are the highest and those from Eq. (18) are the lowest. In addition, the values from Eq. (14) are a little similar with those from Eq. (16), which is understandable because the expression of Eq. (14) is adopted for the case of the higher excitation energy in Eq. (16), and meanwhile there are quite large differences for several nuclei due to the contribution of the discrete levels in Eq. (16) leading to the vast fluctuations. Figure 3(b) shows the calculated spin cut-off parameters σ_2 of the neighboring nuclei $(A - 1, Z)$ of the above nuclei (A, Z) at $U = 1$ MeV with Eqs. (16)–(18) which are applicable at the lower excitation energy, and with the increasing mass of the nucleus the σ_2 increases more slowly than that at the higher excitation energy shown in Fig. 3(a), accompanying the strong fluctuations except that obtained from Eq. (17). The values calculated with Eq. (17) seem to be an average of those with Eq. (16) when the mass of the nucleus is less than 160, indicating the rationality of the systematical formula from Eq. (17).

The isomeric cross-section ratios of the $(n, 2n)$ reactions bombarded by 14 MeV neutron are calculated based on the above formulas of the spin cut-off parameter within the H-V method, and the calculated results are shown in Fig. 4 for the same target nuclei as those shown in Fig. 3, together with the evaluated values from JENDL-5 and JEFF-3.3. There are totally 12 types of combinations of appropriate formulas for calculating the spin cut-off parameter σ_1 and σ_2 in the present work, and the corresponding calculated results are relatively concentrated when the mass number A_t of the target nucleus is less than 160, and the divergences become much larger when $A_t > 160$, indicating the method is more reliable in describing the light- and medium-mass region ($A_t < 160$), and the result is consistent with Ref. [7] which shows a reduction of the nuclear moment of inertia from the rigid-body value by up to 70% in the heavy-mass region within the H-V method and the deviation between the theoretical spin cut-off parameters and those deduced from the experimental isomeric ratios increases with increasing mass number. It also can be seen from Fig. 4 that the calculated results with Eq. (17) (red symbols) for σ_2 are overall closer to the evaluated values whichever formula is applied for σ_1, indicating that the σ_2 has a more significant influence on the isomeric cross-section ratio of the $(n, 2n)$ reactions within the H-V method. And meanwhile, with Eq. (17) the calculated spin cut-off parameter σ_2 almost locates the lowest across the whole mass region shown in Fig. 3, except ^{120}Te and ^{144}Sm for which the lowest σ_2 are those from Eq. (16) and the corresponding calculated results are more agreeable with the evaluated values, thus, it sees that the isomeric cross-section ratios calculated with the lowest σ_2 values shown in Fig. 3 agree better with the evaluated data. And it is also applicable for the case of σ_1. One can see that there are better agreements between the evaluated values and the calculated results with the lowest σ_1 from Eq. (18) for most nuclei and those with Eq. (16) for the other nuclei ^{107}Ag, ^{116}Cd, and ^{187}Re than those with the higher σ_1 calculated with the rigid-body formula from Eq. (14), which means that the effective moment of inertia should be less than the rigid-body moment of inertia I_0 from Eq. (14) in certain extent. Similar results have been previously reported from other theoretical calculations [32–35]. On the whole, the formula proposed by Egidy [Eq. (18)] is more appropriate for the σ_1 and the systematical formula from Eq. (17) is more appropriate for the σ_2 within the H-V method for most nuclei.

In addition, the isomeric cross-section ratio of the $(n, 2n)$ reaction as a function of the incident energy is calculated with the H-V method and the most appropriate formula for the spin cut-off parameter is adopted in the whole energy range. The upper panel of Fig. 5 shows the final spin distribution of the residual nucleus in ^{45}Sc$(n, 2n)^{44m,g}$Sc ($I_g = 2^+$, $I_m = 6^+$), which dominates the isomeric cross-section ratio based on the spins of the ground state and the isomeric state of the residual nucleus, and the calculated isomeric ratio as a function of the incident energy is shown in the bottom panel, compared with the evaluated values from JEFF-3.3 and JENDL-5 and the experimental data. It can be seen from Fig. 5 that with the incident energy increasing, the final spin distribution is shifted to the left side, leading to the decrease of the isomeric cross-section ratio shown in the bottom panel, and the decreasing tendency is almost similar to the evaluated values and the calculated results agree well with the experimental data around $E_n = 14$ MeV. And one also can see that the isomeric ratio decreases much more rapidly around $E_n = 17$ MeV, and then becomes flatter when E_n is larger than 17 MeV, although the calculated results locate between the evaluated values from JEFF-3.3 and those from JENDL-5 throughout

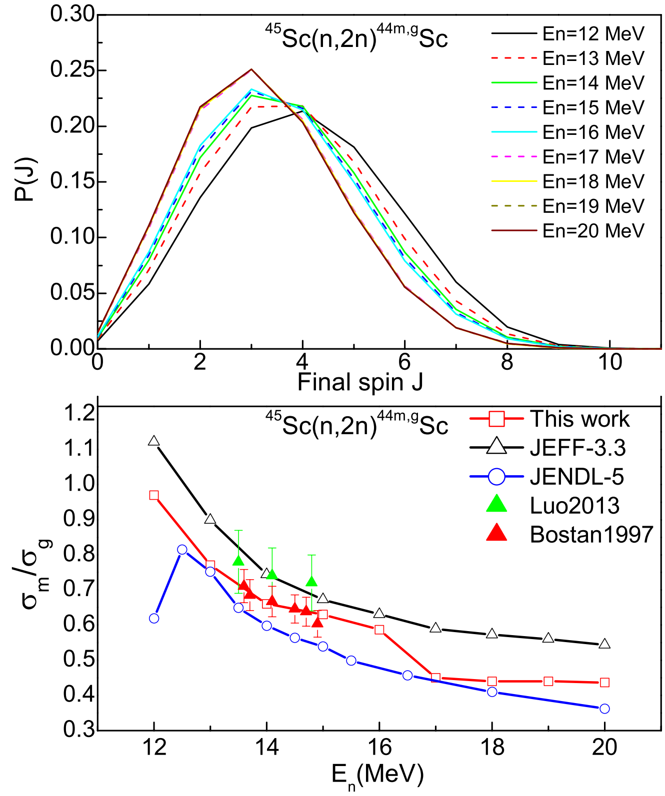

FIG. 5. The spin distribution of the final residual nucleus in ^{45}Sc$(n, 2n)^{44m,g}$Sc reaction for a series of neutron energies below 20 MeV (upper panel). The calculated isomeric cross-section ratio of ^{45}Sc$(n, 2n)$ reaction as a function of the incident neutron energy, compared with the evaluated data from JEFF-3.3 and JENDL-5, and the experimental data [30,31] (bottom panel).

the energy range. It seems that the rapid decrease around $E_n = 17$ MeV results from the increase of the averaged number N_γ of γ rays, which also influences the final spin distribution in certain extent. Moreover, the flatter tendency compared with the evaluated data around the energies larger than 17 MeV is possibly because that when the incident energy is larger enough, the contribution of the pre-equilibrium reaction becomes non-negligible, which is not taken into account due to the limitation of the method used in the present work. It should be noted that the present method is based on the conventional Hauser-Feshbach model with the transmission coefficient independent of K, i.e., the projection of the angular momentum on the symmetry axis, however, the deformed nuclei should in principle be treated with a deformed Hauser-Feshbach model which gives different results [39], and the present results would be further improved if the deformed nuclei are treated based on the deformed Hauser-Feshbach model.

The calculated results of ^{85}Rb$(n, 2n)^{84m,g}$Rb ($I_g = 2^-$, $I_m = 6^-$) and ^{120}Te$(n, 2n)^{119m,g}$Te ($I_g = 1/2^+$, $I_m = 11/2^-$) are shown in Figs. 6 and 7, respectively, together with the evaluated values and the experimental data. It can be seen from Fig. 6 that the final spin distribution of the residual nucleus in ^{85}Rb$(n, 2n)$ is insensitive to the incident energy,

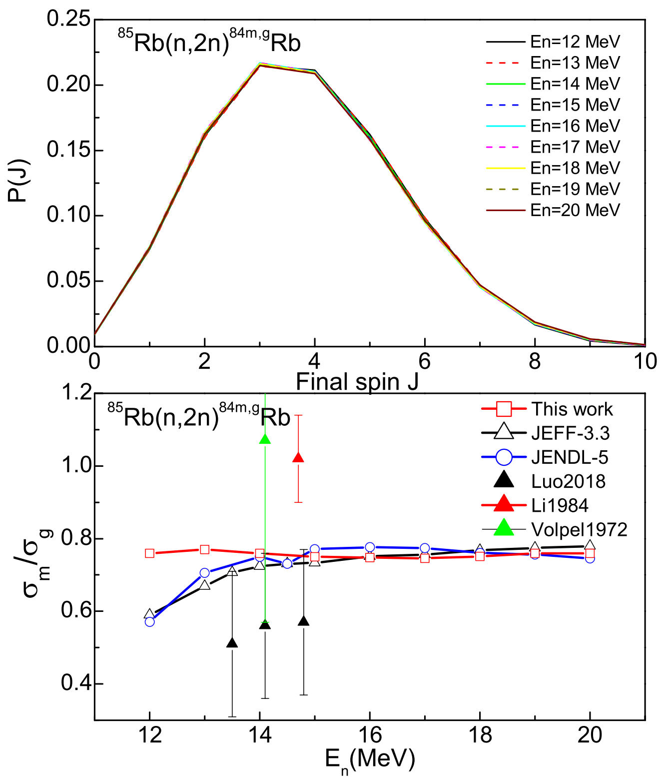

FIG. 6. Similar to Fig. 5 for the ^{85}Rb$(n, 2n)^{84m,g}$Rb reaction. The experimental data are from Refs. [36–38].

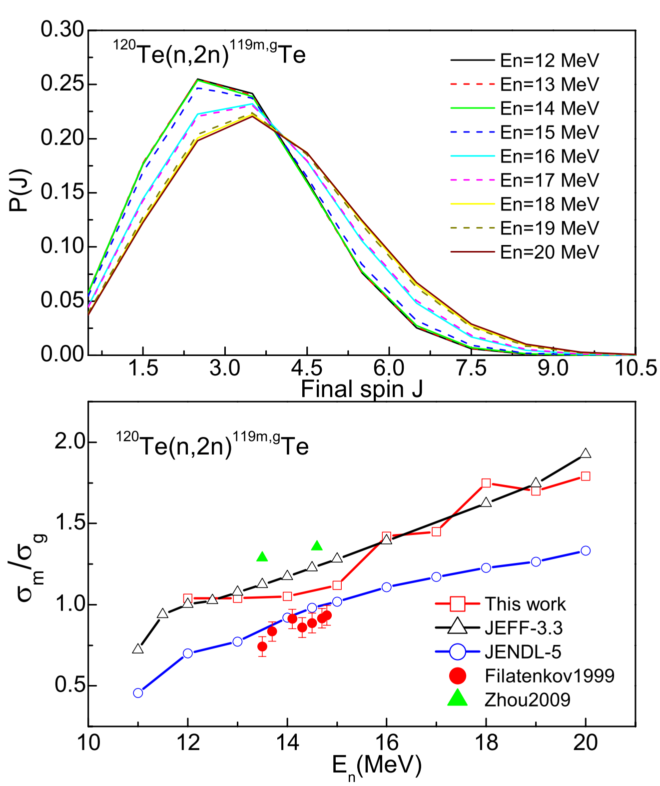

FIG. 7. Similar to Fig. 5 for ^{120}Te$(n, 2n)^{119m,g}$Te reaction. The experimental data are from Refs. [40,41].

and thus the calculated isomeric ratio is almost unchanged across the whole energy region, which is consistent with the evaluated values except that around $E_n = 12$ MeV. One can see from Fig. 7 that, with the incident energy increasing, the final spin distribution of ^{120}Te$(n, 2n)$ is shifted to the right side and the corresponding isomeric cross-section ratio increases, and the calculated results agree with the evaluated data from JEFF-3.3, although there are slight fluctuations around the larger energies. The overall agreement between the calculated results and the evaluated data indicates the validity of the present method in calculating the isomeric ratio of the $(n, 2n)$ reaction below 20 MeV. In the near future, the method will be further improved by introducing the contribution of the pre-equilibrium reaction and extended to describing more reaction channels including that of the charged particle emission.

IV. SUMMARY

In this work, we have studied the influence of the spin cut-off parameter on the isomeric cross-section ratio of the $(n, 2n)$ reaction and the isomeric ratio as a function of the incident energy within the Huizenga-Vandenbosch method.

We first investigate the dependence of the isomeric cross-section ratio on the spin cut-off parameter σ in the $(n, 2n)$ reaction, and the σ is treated for the residual nucleus following the first neutron emission (σ_1) and that following the second neutron emission (σ_2) separately, due to the large difference of the excitation energy between them, which shows that the isomeric cross-section ratio is definitely sensitive to the spin cut-off parameters, especially the σ_2. Moreover, various formulas of the spin cut-off parameters at the lower and higher excitation energies are compared and applied to calculate the $(n, 2n)$ isomeric ratios of a series of nuclei from ^{45}Sc to ^{198}Hg at $E_n = 14$ MeV, compared with the evaluated values from JENDL-5 and JEFF-3.3. It is found that the method is more reliable in the light- and medium-mass region ($A_t < 160$), and most of the calculated results are described well by the formulas giving the lower spin cut-off parameters than the rigid-body formula, which supports the previous investigation results in which the effective moment of inertia is less than the rigid body moment of inertia I_0 in Eq. (14) for most nuclei.

In addition, with the H-V method and the most appropriate formulas for the spin cut-off parameter, the isomeric ratios as a function of the incident energy in ^{45}Sc$(n, 2n)^{44m,g}$Sc, ^{85}Rb$(n, 2n)^{84m,g}$Rb, and ^{120}Te$(n, 2n)^{119m,g}$Te are calculated and compared with the evaluated values and experimental data, and the overall agreement is obtained. However, the deviation becomes larger around the higher excitation energy, which will be left to the improvement of the method in the future work.

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