Letter

Structures in the heavy-ion fusion excitation function at and above the Coulomb barrier

C. L. Jiang,* W. F. Henning, B. P. Kay, and N. Watwood

Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA

(Received 14 March 2023; revised 23 June 2023; accepted 12 October 2023; published 16 November 2023)

Contrary to descriptions from coupled-channels or other model calculations, heavy-ion fusion excitation functions are not smooth near and above the Coulomb barrier. There appear to be weak but noticeable oscillations or structures within the excitation functions that can be observed clearly in representation $d(\sigma E)/dE$ and in the comparison with theoretical calculations $\sigma(E) - \sigma_{\rm th}(E)$. A rather similar phenomenon has been studied before and can be explained in light symmetric systems as the influence of the centrifugal barrier penetration only on even angular momentum, but it cannot be extended to heavier, more asymmetric fusion systems. A more thorough investigation may be required to investigate this newly indicated behavior.

DOI: 10.1103/PhysRevC.108.L051604

The data for more than a thousand heavy-ion fusion excitation functions can be found in the literature [1], and cover nine orders of magnitude, from nanobarns to barns. This substantial range in cross sections stems from various aspects of nuclear reaction dynamics under the strong influence of the nuclear structure of both the two colliding nuclei and the compound nuclei [2–7].

In this Letter we present a common phenomenon of heavy ion fusion excitation functions which appears less known to the community, and was observed by the systematic reanalysis of previously measured fusion reaction data.

In a simplistic view, the Coulomb barrier between the two colliding heavy ions produces a fusion excitation function that is expected to be continuously increasing from low to high colliding energies. It is interesting to know to what extent these excitation functions are either monotonic or more complex with underlying structures.

In the early 1960s, oscillation or resonance peaks were observed for collisions of ^{12}C + ^{12}C by Almquest *et al.* [8] at first for elastic scattering, then for various reactions including fusion. This phenomenon, appearing mostly for α-conjugated-nuclei, has been studied for about two decades and was summarized in a set of *Treatise* books edited by Bromley [9].

The excitation function, $\sigma(E)$, is commonly depicted in either linear or logarithmic scales. However, because of the steepness of the excitation function, it is often difficult to recognize possible structures or deviations from theoretical curves, especially in logarithmic plots. To alleviate this problem, representations other than $\sigma(E)$ are used for comparison between measurements and theoretical calculations (e.g., see discussion in Ref. [7]). A particular representation may emphasize the detailed behavior of the excitation function for some energy range and a specific structure can be noticed more evidently.

It is well known that coupled channels (CC) calculations can well explain the heavy-ion fusion excitation function around and above the Coulomb barrier [10–12]. Especially after the paper contributed by Rowley *et al.* [13], the reaction dynamics involved have been acquainted in more detail by using the representation of $d^2(\sigma E)/dE^2$. Rowley found, under the classical approximation, the so-called barrier distribution $B(E)$ can be obtained from the representation of $d^2(\sigma E)/dE^2$, and evidence of structures has been seen in this representation as well. A few strong peaks in these structures at lower energies can be well reproduced with the different coupling effects between channels of the fusion and other reactions, e.g., excitation and transfer reactions. While evident in the $d^2(\sigma E)/dE^2$ representation these structures are often difficult to discern in either the $\sigma(E)$ or the $\sigma(E)E$ spectra.

Due to the double differentiation process used in Rowley's method to obtain the barrier distribution, there are weaknesses to this method, including an ambiguity problem (depending on the energy step size taken in the differentiation process) and large uncertainties at high energies for the obtained barrier distribution spectra.

Recently it was presented that barrier distributions of all heavy-ion fusion excitation functions can be described very well by a three-Gaussian spectrum (or a four-Gaussian spectrum for deformed systems, like ^{16}O + ^{154}Sm) [14,15]. The corresponding cross section $\sigma(E)$ is an analytic function, which reproduces the excitation function quite well, even better than CC calculations.

The new indicated behavior is the appearance of structures along the smooth excitation function which cannot be reproduced by CC and other calculations, and are most evident in the representation of $d(\sigma(E)E)/dE$. It is well known that the first derivative of σE is related with the transmission coefficient of the fundamental cross section expression [3].

Three representations, σE, $d(\sigma E)/dE$, and $[d^2(\sigma E)/dE^2]$ ($B(E)$) are shown in (a)–(c) of Figs. 1 and 2, for two systems, ^{40}Ca + ^{90}Zr [16] and ^{16}O + ^{154}Sm [17], respectively. In these figures, symbols are either experimental data or

*cjiang@anl.gov

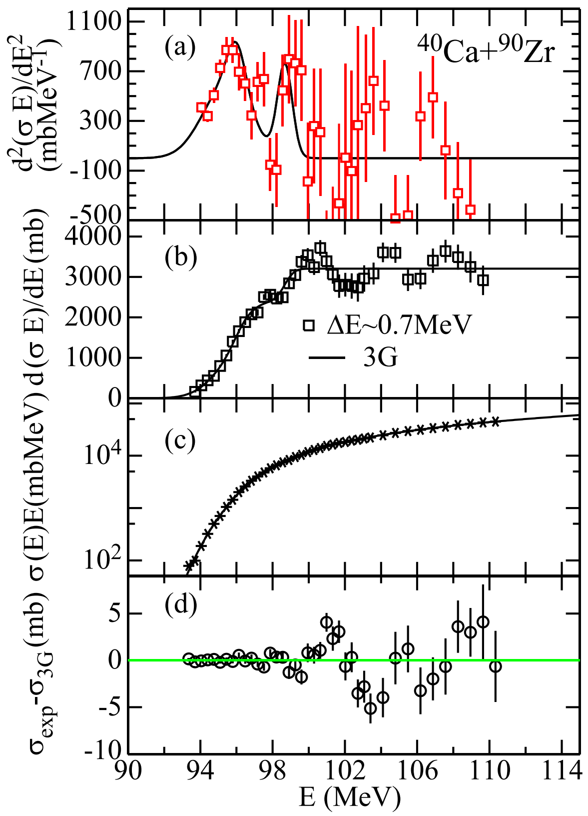

FIG. 1. Various representations vs E plots of the fusion cross sections for system $^{40}\mathrm{Ca} + {}^{90}\mathrm{Zr}$ [16]. (a) Barrier distribution $B(E)$, (b) $d(\sigma E)/dE$, (c) σE, and (d) $\sigma_{\mathrm{exp}} - \sigma_{\mathrm{th}}$. Curves are calculations from the three-Gaussian barrier distribution recipe.

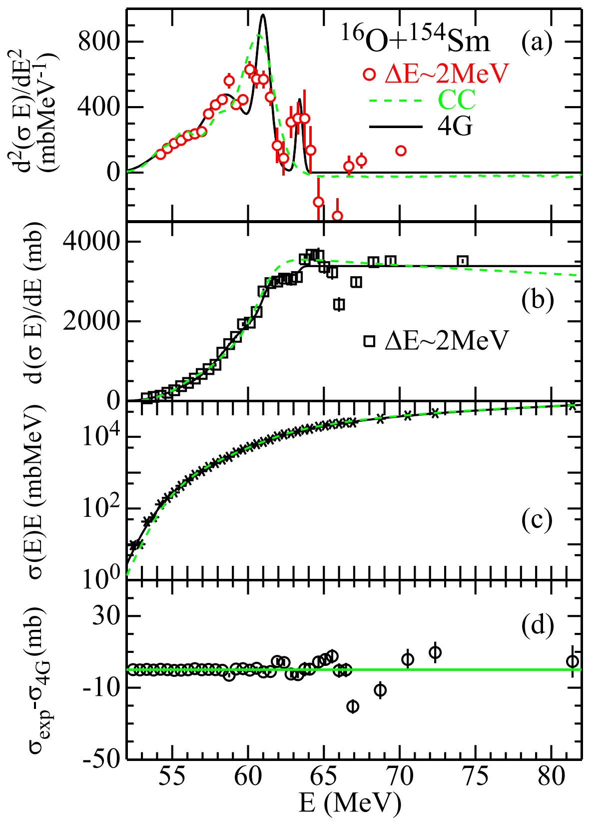

FIG. 2. Various representations vs E plots of fusion cross sections for system $^{16}\mathrm{O} + {}^{154}\mathrm{Sm}$ [17]. Others are the same as in Fig. 1.

deduced directly from the data. In panels (b) and (a), the deduced results are calculated from the single-differentiation and double-differentiation methods, respectively, where ΔE are the energy steps used in the differentiation process. In Figs. 1 and 2, black curves are obtained from the empirical recipe: three-Gaussian (3G) or four-Gaussian (4G) barrier height distribution. In Fig. 2(a)–2(c), the green curves are obtained from CC calculations. In Figs. 1–2(c) the calculated curves, either from CC calculations or from empirical, multi-Gaussian recipes, reproduce the experimental results rather well. In most cases, calculations from the empirical recipes and the CC calculations are nearly the same at energies near and above the Coulomb barrier which is the energy range discussed in this Letter. We do not give the comparison between CC calculations and the empirical fittings here, but concentrate on the detailed comparison between the calculations and the experimental data. Thus, since the multi-Gaussian fittings reproduce the data well, further comparison to experimental data will often be made with these fits.

In Figs. 1–2(b), it can be noted that in the low energy region the calculations reproduce the data well, but near and above the Coulomb barrier, at the high energy part of these plots, they only give an average behavior of the data. It seems that the experimental data of $d(\sigma E)/dE$ display oscillations or structures in the higher energy region.

It should be noted that the experimental data in Figs. 1–2(a) are obtained through double differentiation, yielding large uncertainties. However, the experimental data in Figs. 1–2(b) are obtained through single-differentiation with much less uncertainty, which makes the oscillations/structures believable. The shape and amplitudes of these oscillations are different, and seem to depend on the system.

Since the appearance of oscillations/structures is difficult to discern in a plot of the excitation function, as shown in Figs. 1 and 2(c), Figs. 1 and 2(d) show the comparison between the experiment and the multi-Gaussian fits, $\sigma_{\mathrm{exp}} - \sigma_{3G}$ or $\sigma_{\mathrm{exp}} - \sigma_{4G}$, respectively. Here, one can see distinct signs of the oscillation/structure behavior.

The oscillation/structure at high energies appears also in the representation of $d^2(\sigma E)/dE^2$, see Figs. 1 and 2(a). It must be indicated, similar $d^2(\sigma E)/dE^2$ spectra have already been shown in the original papers [16,17]. Esbensen and Alberto in a study of the fusion of $^{48}\mathrm{Ca} + {}^{90}\mathrm{Zr}$, already showed the spectra $d(\sigma E)/dE$ [18]. Due to the large uncertainties, negative barrier values given in the results, and CC calculations do not reproduce that part well, no detailed attention was given to behaviors of that energy region in these papers.

Upon further investigation, we found that this oscillation behavior indicated in the reactions $^{40}\mathrm{Ca} + {}^{90}\mathrm{Zr}$ and $^{16}\mathrm{O} + {}^{154}\mathrm{Sm}$ is not special but popular for heavy-ion fusion. At least 20 more experimental measurements potentially have the same behavior. They are: $^{40}\mathrm{Ca} + {}^{192}\mathrm{Os}$ [19], $^{40}\mathrm{Ca} + {}^{194}\mathrm{Pt}$ [19], $^{40}\mathrm{Ca} + {}^{94}\mathrm{Zr}$ [20], $^{40}\mathrm{Ca} + {}^{96}\mathrm{Zr}$ [21], $^{16}\mathrm{O} + {}^{208}\mathrm{Pb}$

[22], $^{40}Ca + ^{40}Ca$ [23], $^{58}Ni + ^{60}Ni$ [24], $^{32}S + ^{110}Pd$ [25], $^{36}S + ^{110}Pd$ [25], $^{34}S + ^{168}Er$ [26], $^{28}Si + ^{64}Ni$ [27], $^{40}Ca + ^{48}Ca$ [28], $^{48}Ca + ^{48}Ca$ [29], $^{16}O + ^{144}Sm$ [17], $^{16}O + ^{148}Sm$ [17], $^{17}O + ^{144}Sm$ [17], $^{16}O + ^{186}W$ [17], $^{12}C + ^{92}Zr$ [30], $^{32}S + ^{89}Y$ [31], and $^{58}Ni + ^{64}Ni$ [32]. All of these fusion excitation functions are measured and published after Rowley's paper in 1991, since then, many measurements of the excitation functions have been performed with fine energy steps in the studying of fusion barrier distributions.

In another example, five systems, $^{16}O + ^{208}Pb$ [22], $^{34}S + ^{168}W$ [26], $^{40}Ca + ^{192}Os$ [19], $^{40}Ca + ^{94}Zr$ [16], and $^{40}Ca + ^{96}Zr$ [21] are shown in Figs. 3 and 4, respectively. Experimental results (black symbols), obtained from the single-differentiation method, are compared with the 3G or 4G descriptions (black curve). It can be immediately observed that this aforementioned behavior also appears and cannot be reproduced by the multi-Gaussian recipe or CC calculations (not shown). Also importantly, the structures in the $d(\sigma E)/dE$ spectra are different in detail from system to system. (In Fig. 4, system $^{40}Ca + ^{90}Zr$ is shown repeatedly. One may notice the interesting changes between the three systems $^{40}Ca + ^{90,94,96}Zr$ [20]. But that is beyond the main point of the present article and we do not want to discuss more here.)

The newly indicated behavior in heavy-ion fusion collisions, though shown often in experiments already, seems unfamiliar and has not been explored and explained yet. One possible reason is that the value of barrier distribution, $B(E)$, must be positive, but in the $d^2(\sigma E)/dE^2$ spectra, negative heights appear in the high energy region, where multi-Gaussian model and also CC calculations (e.g., [16] for system $^{40}Ca + ^{90}Zr$) cannot reproduce the structures in the $d^2(\sigma E)/dE^2$ spectra. Is it possible that in this region, $d^2(\sigma E)/dE^2$ contains not only the information of barrier distribution, but also additional information of reaction dynamics. To our knowledge, it may need to be explored further.

All systems discussed above, either mentioned or shown explicitly in Figs. 1–4 are for medium or heavy mass. Oscillation behavior appears for light-mass systems as well and has been studied, but that behavior is well known and can be attributed to the resonance phenomenon or by the penetration of centrifugal barriers of successive angular momentum which are well separated in energy.

Poffe *et al.* [33] and Esbensen [34] discussed structures in the excitation functions for $^{20}Ne + ^{20}Ne$ and $^{28}Si + ^{28}Si$, and attributed them to the penetration of centrifugal barriers. Since these reactions are symmetric, only incident waves with even angular momentum can contribute to the fusion reactions, and the separations in energy between successive angular momentum barriers are relatively large. The structure for $^{28}Si + ^{28}Si$ was also reproduced with CC calculations by Esbensen. But he emphasized, "Since many reaction channels are expected to open up at high angular momentum and high energies in heavier systems, the effect of couplings to these channels may smear out the peak structure."

The new phenomena for the medium- and heavy- mass fusion reactions are all asymmetric systems, excluding $^{40}Ca + ^{40}Ca$, with some systems like $^{16}O + ^{208}Pb$, being

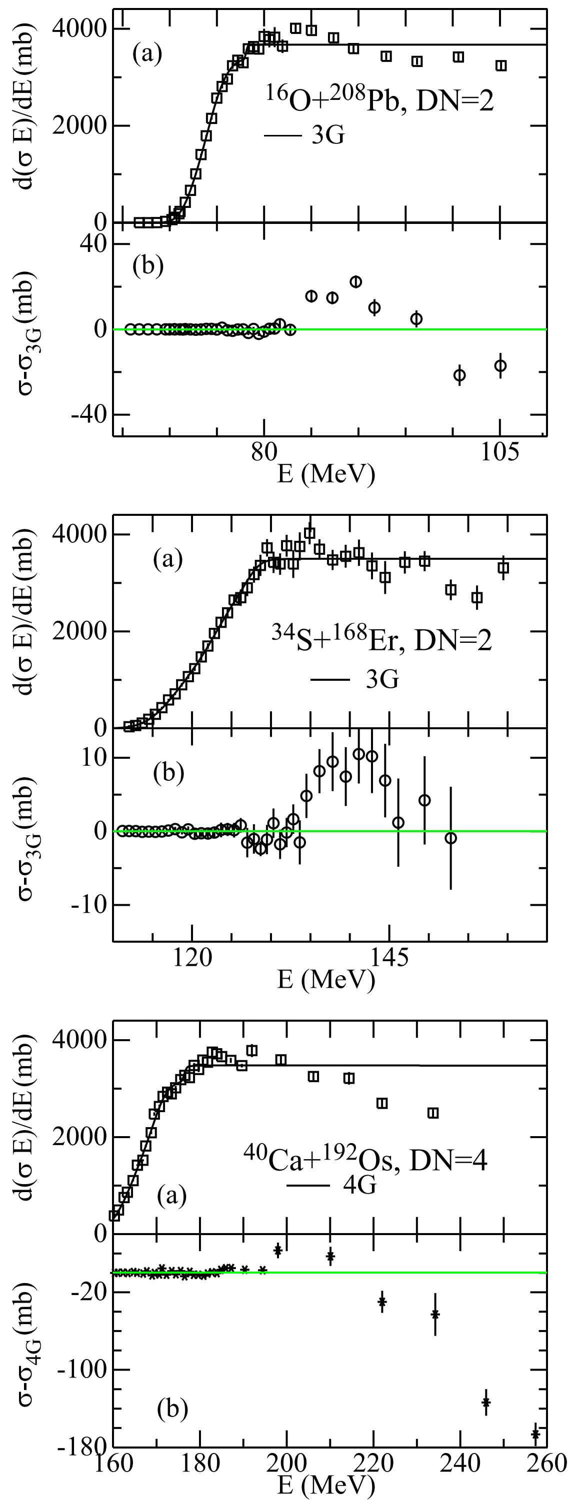

FIG. 3. $d(\sigma E)/dE$ and $\sigma - \sigma_{\text{th}}$ spectra for fusion systems $^{16}O + ^{208}Pb$ [22] in upper part, $^{34}S + ^{168}Er$ [26] in middle part, and $^{40}Ca + ^{192}Os$ [19] in lower part, respectively. DN is the step number used in the differentiation process.

extremely so, therefore the arguments given by Poffe and Esbensen cannot be applicable.

CC calculations developed a great deal during the period of studying fusion enhancement, which appears significant in

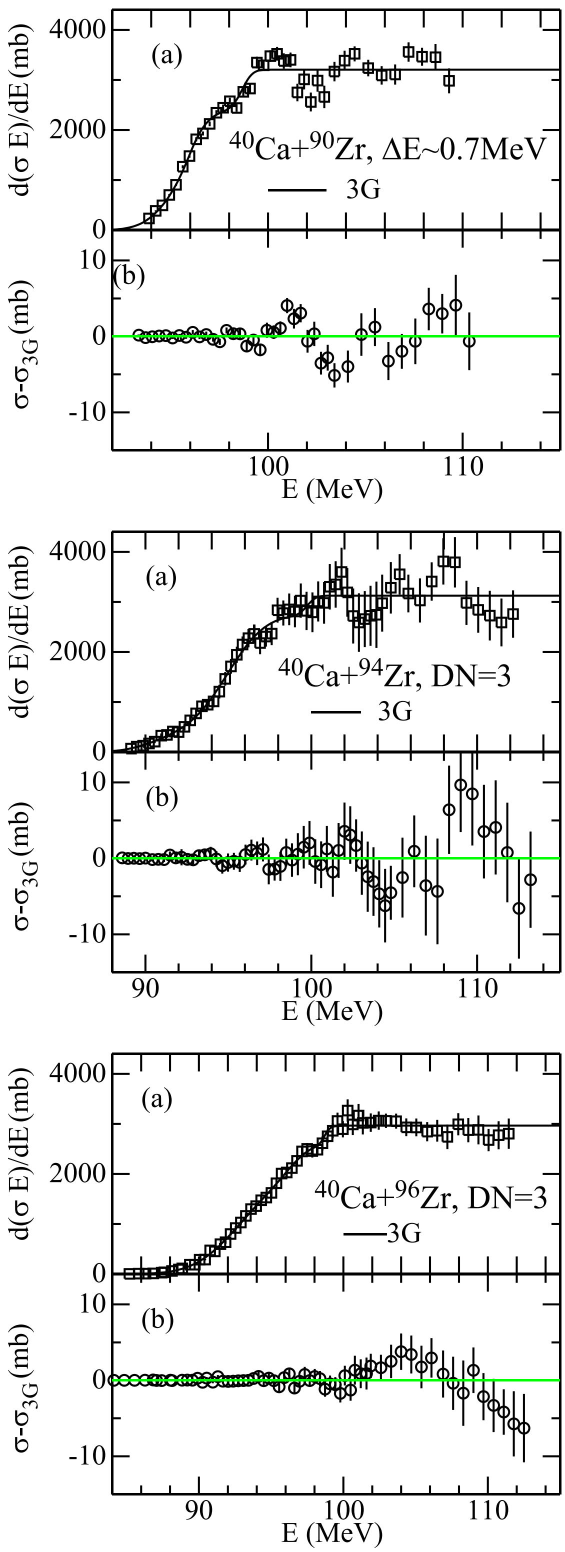

FIG. 4. $d(\sigma E)/dE$ and $\sigma - \sigma_{\text{th}}$ spectra for fusion systems ^{40}Ca + ^{90}Zr [16] in upper part, ^{40}Ca + ^{94}Zr [20] in middle part, and ^{40}Ca + ^{96}Zr [21] in lower part, respectively. DN is the step number used in the differentiation process.

the sub-barrier energy region, where the cross sections of excitation reactions and transfer reactions are all much stronger

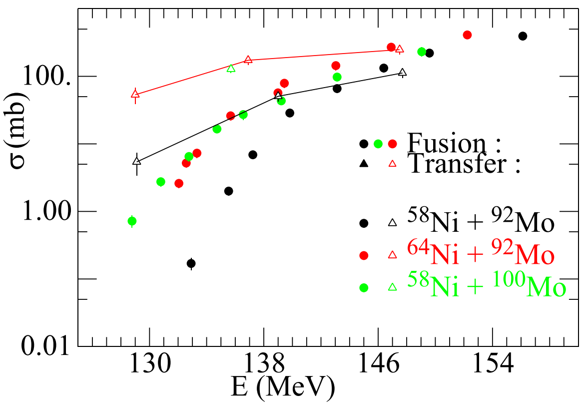

FIG. 5. Excitation functions of fusions and transfer reactions for some systems of Ni + Mo. Data come from Refs. [35–37], respectively.

than the ones for fusion, the channel coupling induces the enhancement. The unknown behavior discussed above is located around and above the energies of the Coulomb barrier, where cross sections of fusion, transfers, and excitation are all strong. In this region competitions may become more important than enhancement. In addition, some new reaction mechanisms may open successively, like deep inelastic, incomplete fusion, or breakup, etc. It may be worthwhile to mention, in the fusion systems discussed in Fig. 1, the entrance channel ^{40}Ca + ^{90}Zr, does not have strong transfer reactions involved (there is no transfer channel with positive Q value). Therefore it may be more favorable to exhibit something else.

At present, CC calculations treat transfer reactions rather approximately and the competition between reaction channels implicitly. Due to the approximations taken, we do not expect the present CC model calculations to treat the competition effect well at high energies. We do not know yet, whether competition has important relations with the present 'oscillation/structure' discussions, or if there are other factors that influence CC calculations, e.g., the application of IWBC (incoming wave boundary condition) in the CC model eliminates possibilities to see structures.

Heavy-ion transfer reactions have been measured for a long time. Most of these experimental measurements were performed only at one or two colliding energies. For illustrating the competition between fusion and transfer reactions, Fig. 5 shows an example for collisions between Ni and Mo [35–37]. One can see that, for collisions of ^{58}Ni + ^{92}Mo and ^{64}Ni + ^{92}Mo, for increasing bombarding energy, the total cross sections of transfer reaction become gradually weaker than the fusion reactions. The influences of transfer on fusion must be treated better than in the present CC calculations.

In light of this phenomenon, there are steps that can be taken to further the investigation. First, there are some measurements that might need to be refined. There are differences between what appear in ^{16}O + ^{144}Sm and ^{16}O + ^{154}Sm, ^{40}Ca + ^{90}Zr and ^{40}Ca + ^{96}Zr, ^{40}Ca + ^{192}Os and ^{40}Ca + ^{194}Pt, etc. Repeating these experiments in finer detail is needed to determine if these differences are real and understandable. Second, for some colliding systems mentioned above, measurements should be extended to higher energies and performed in smaller energy steps. Third, there is an experimental gap in the mass range between $^{28}Si + ^{28}Si$ and $^{40}Ca + ^{40}Ca$. Investigation of this area will determine if similar behaviors appear in between these systems. It would be preferable that measurements are done with the same experimental setup and analysis for a more systematic comparison.

In summary, there is an unfamiliar phenomenon observed in fusion excitation functions above the barrier that has not been formally addressed. Currently there is no substantial explanation for this phenomenon and calculations like CC cannot reproduce it. This behavior newly indicated for heavy systems is somewhat similar with the previous known resonance phenomenon observed for fusions between nuclei ^{12}C and ^{16}O. We suggest further investigations for more experiments, and for developments of more complete and exact theoretical calculations of the fusion excitation function above the barrier, which can incorporate contributions from all competing channels, including both the entrance and compound channels, etc. The choice of fusion systems might also address simultaneously a second recent observation [38], namely an unexpected, near perfect overlap of scaled fusion excitations at the energies above the barrier including oscillations/structures behavior for systems leading to the same compound nucleus, but not to different ones.

This work was supported by the US Department of Energy, Office of Nuclear Physics under Contract No. DE-Ac02-06CH11357. Authors want to thank B. B. Back, K. Hagino, C. Hoffman, G. Montagnoli, K. E. Rehm, and A. M. Stefanini for interesting discussions.

[1] V. I. Zagrebaev, A. S. Denikin, A. V. Karpov, A. P. Alekseev, M. A. Naumenko, V. A. Rachkov, and V. V. Samarin, http://nrv.jinr.ru/, website accessed during 2020.
[2] M. Dasgupta, D. J. Hinde, N. Rowley, and A. M. Stefanini, Annu. Rev. Nucl. Part. Sci. **48**, 401 (1998).
[3] A. B. Balantekin and N. Takigawa, Rev. Mod. Phys. **70**, 77 (1998).
[4] K. Hagino and N. Takigawa, Prog. Theor. Phys. **128**, 1061 (2012).
[5] B. B. Back, H. Esbensen, C. L. Jiang, and K. E. Rehm, Rev. Mod. Phys. **86**, 317 (2014).
[6] G. Montagnoli and A. M. Stefanini, Eur. Phys. J. A **53**, 169 (2017).
[7] C. L. Jiang, B. B. Back, K. E. Rehm, K. Hagino, G. Montagnoli, and A. M. Stefanini, Eur. Phys. J. A **57**, 235 (2021).
[8] E. Almqvist, D. A. Bomley, and J. A. Kucher, Phys. Rev. Lett. **4**, 515 (1960).
[9] *Treatise on Heavy-ion Science*, edited by D. A. Bomley, (Plenum, New York, 1984), Vol. 3.
[10] C. H. Dasso, S. Landowne, and A. Winther, Nucl. Phys. A **405**, 381 (1983).
[11] C. H. Dasso, S. Landowne, and A. Winther, Nucl. Phys. A **407**, 221 (1983).
[12] K. Hagino, N. Rowley, and A. T. Kruppa, Comput. Phys. Commun. **123**, 143 (1999).
[13] N. Rowley, G. R. Satchler, and P. H. Stelson, Phys. Lett. B **254**, 25 (1991).
[14] C. L. Jiang and B. P. Kay, Phys. Rev. C **105**, 064601 (2022).
[15] C. L. Jiang, K. E. Rehm, B. B. Back, A. M. Stefanini, and G. Montagnoli, Eur. Phys. J. A **54**, 218 (2018).
[16] H. Timmers *et al.*, Phys. Lett. B **399**, 35 (1997).
[17] J. R. Leigh, N. Rowley, R. C. Lemmon, D. J. Hinde, J. O. Newton, J. X. Wei, J. C. Mein, C. R. Morton, S. Kuyucak, and A. T. Kruppa, Phys. Rev. C **47**, R437(R) (1993); J. R. Leigh, M. Dasgupta, D. J. Hinde, J. C. Mein, C. R. Morton, R. C. Lemmon, J. P. Lestone, J. O. Newton, H. Timmers, J. X. Wei, and N. Rowley *ibid.* **52**, 3151 (1995).
[18] H. Esbensen and A. M. Stefanini, Phys. Rev. C **89**, 044616 (2014).
[19] J. D. Bierman, P. Chan, J. F. Liang, M. P. Kelly, A. A. Sonzogni, and R. Vandenbosch, Phys. Rev. Lett. **76**, 1587 (1996); Phys. Rev. C **54**, 3068 (1996).
[20] A. M. Stefanini *et al.*, Phys. Rev. C **76**, 014610 (2007).
[21] A. M. Stefanini *et al.*, Phys. Lett. B **728**, 639 (2014).
[22] M. Dasgupta, D. J. Hinde, A. Diaz-Torres, B. Bouriquet, C. I. Low, G. J. Milburn, and J. O. Newton, Phys. Rev. Lett. **99**, 192701 (2007); C. R. Morton, A. C. Berriman, M. Dasgupta, D. J. Hinde, J. O. Newton, K. Hagino, and I. J. Thompson, Phys. Rev. C **60**, 044608 (1999).
[23] M. Montagnoli, A. M. Stefanini, C. L. Jiang, H. Esbensen, L. Corradi, S. Courtin, E. Fioretto, A. Goasduff, F. Haas, A. F. Kifle, C. Michelagnoli, D. Montanari, T. Mijatovic, K. E. Rehm, R. Silvestri, P. P. Singh, F. Scarlassara, S. Szilner, X. D. Tang, and C. A. Ur, Phys. Rev. C **85**, 024607 (2012).
[24] A. M. Stefanini *et al.*, Phys. Rev. Lett. **74**, 864 (1995).
[25] G. M. Zeng *et al.*, Phys. Rev. C **57**, 1727 (1998).
[26] C. R. Morton, A. C. Berriman, R. D. Butt, M. Dasgupta, A. Godley, D. J. Hinde, and J. O. Newton, Phys. Rev. C **62**, 024607 (2000).
[27] C. L. Jiang *et al.*, Phys. Lett. B **640**, 18 (2006).
[28] C. L. Jiang *et al.*, Phys. Rev. C **82**, 041601(R) (2010).
[29] A. M. Stefanini *et al.*, Phys. Lett. B **679**, 95 (2009)
[30] J. O. Newton, C. R. Morton, M. Dasgupta, J. R. Leigh, J. C. Mein, D. J. Hinde, H. Timmers, and K. Hagino, Phys. Rev. C **64**, 064608 (2001).
[31] A. Mukherjee, M. Dasgupta, D. J. Hinde, K. Hagino, J. R. Leigh, J. C. Mein, C. R. Morton, J. O. Newton, and H. Timmers, Phys. Rev. C **66**, 034607 (2002).
[32] A. M. Stefanini *et al.*, Phys. Rev. C **100**, 044619 (2019).
[33] N. Poffe, N. Rowley, and R. Lindsay, Nucl. Phys. A **410**, 498 (1983).
[34] H. Esbensen, Phys. Rev. C **85**, 064611 (2012).
[35] K. E. Rehm *et al.*, Phys. Lett. B **317**, 31 (1993).
[36] C. L. Jiang *et al.*, Phys. Lett. B **337**, 59 (1994).
[37] C. L. Jiang *et al.*, Annual Report of Physics Division, Argonne National Laboratory (1994/1995).
[38] C. L. Jiang, W. F. Henning, C. R. Hoffmann, B. P. Kay, and N. Watwood, following paper, Phys. Rev. C **108**, L051605 (2023).