Letter

Entanglement maximization in low-energy neutron-proton scattering

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The entanglement properties of neutron-proton scattering are investigated using a measure that counts the number of entangled pairs produced by the action of a scattering operator on a given initial neutron-proton state. All phase shifts relevant for scattering at laboratory energies up to 350 MeV are used. Entanglement is found to depend strongly on the initial state. Entanglement is maximized in very low energy scattering if the initial spin state is $|\uparrow\downarrow\rangle$, but not if the initial state is $|\uparrow\uparrow\rangle$. At such energies the Hamiltonian obeys Wigner SU(4) symmetry, and an entanglement maximum is a sign of that symmetry. At higher energies the angular dependence of entanglement is strong and the entanglement is large for many scattering angles. The tensor force is shown to play a significant role in producing entanglement at laboratory kinetic energies greater than about 50 MeV.

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Stimulated by the connection with quantum computing, resting on the possibility that entanglement may enhance capabilities, the implications of entanglement in quantum mechanics and quantum field theory have recently been studied in many papers. For a long list of recent references see Ref. [1]. Ideas related to quantum entanglement provide a new way of looking at old problems and may provide insights into deep connections with underlying symmetries. For example, Refs. [2,3] argued that a principle of *maximum* entanglement is responsible for the particular sets of coefficients that define quantum electrodynamics. Similarly Refs. [4–7] argue that high-energy interactions involve maximally entangled states. Maximum entanglement is a property of nucleon valence quark distributions [8], and large entanglement entropy is a property of the nucleon state vector [9].

On the other hand, Refs. [10,11]. proposed that nucleon-nucleon scattering is described by entanglement suppression that is correlated with Wigner SU(4) symmetry [12]. See also [13]. Wigner used this symmetry, based on the approximate spin-isospin invariance of the nucleon-nucleon strong interaction and on the analogy with electron shell structure, where the spin-orbit interaction is less than the spin-spin one, to describe the low-lying spectra of light nuclei. Interactions of the form $M_0 + M_1\sigma_1\cdot\sigma_2 + M_2\tau_1\cdot\tau_2 + M_3\tau_1\cdot\tau_2\sigma_1\cdot\sigma_2$ obey the symmetry. The group SU(4) is generated by 15 operators. See, e.g., Refs. [14–16] and Eq. (20) below.

Here, I aim to provide a more detailed study of entanglement entropy in neutron-proton scattering. To see why this is worthwhile let us begin with some basic issues. The textbook [17] definition of entropy, the von Neumann entropy, is given by $S = -\mathrm{Tr}[\rho\log\rho]$, where ρ is the density matrix. The operator ρ can be diagonalized, with eigenvalues designated as p_n and $\sum_n p_n = 1$. In this diagonal representation S is expressed as

$$S = -\sum_{n=1}^{d} p_n \log p_n, \tag{1}$$

where d is the dimension of the space.

The quantity S is maximized when all of the probabilities are equal: $p_n = 1/d$. In that case $S_{\max} = \log d$. The value of $d = 2$ for a particle of spin 1/2. This situation of maximum entropy is one of no entanglement. If all of the probability eigenvalues are the same, the density matrix is given by $\rho_{\max} = \frac{\hat{I}}{d}$ where $\hat{I}$ is the identity operator. This is known as the classical or “garbage state” [18].

Instead the amount of entanglement of a state, $|\phi\rangle$, of two spin = 1/2 particles is measured by computing the amount of overlap with completely entangled Bell states:

$$|e_1\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle), \tag{2}$$

$$|e_2\rangle = \frac{i}{\sqrt{2}}(|\uparrow\uparrow\rangle - |\downarrow\downarrow\rangle), \tag{3}$$

$$|e_3\rangle = \frac{i}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle), \tag{4}$$

$$|e_4\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle). \tag{5}$$

Expanding in this complete set of functions one has

$$|\phi\rangle = \sum_{j=1}^{4}\alpha_j|e_j\rangle. \tag{6}$$

The reduced density matrix is defined by taking the trace of the operator $|\phi\rangle\langle\phi|$ of either of the two particles. The entanglement, E, of $|\phi\rangle$ can then be computed as the von Neumann entropy of the reduced density matrix of either of the two particles. Reference [18] found that the entanglement of ϕ can be expressed in terms of the entanglement entropy,

$$H(x) \equiv -x\log_2(x) - (1-x)\log_2(1-x), \tag{7}$$

which has a maximum of unity at $x = 1/2$ and vanishes for $x = 0, 1$. One computes. the concurrence,

$$C = \left|\sum_j \alpha_j^2\right|, \tag{8}$$

where one squares the complex numbers α_j, and the result is that

$$E(C) = H\left(\frac{1}{2}(1+\sqrt{1-C^2})\right). \quad (9)$$

The state of maximum entropy has $C = 0$ and $E(C) = H(1) = 0$, and so has no entanglement. On the other hand, taking $|\phi\rangle = |e_i\rangle$, with i being any one of the numbers from 1 to 4, gives $C = 1$ and $E(C) = H(1/2) = 1$, the maximum entanglement.

Beane, Kaplan, Klco and Savage (BKKS) Ref. [10] defined the entanglement power of the S matrix in a two-particle spin space [19] by the action of the S matrix on an incoming two-particle tensor product state with randomly oriented spins,

$$|\psi_{\rm in}\rangle = \hat{R}(\Omega_1)|\uparrow\rangle_1 \otimes \hat{R}(\Omega_2)|\uparrow\rangle_2, \quad (10)$$

where $\hat{R}(\Omega_j)$ is the rotation operator acting in the jth spin-$\frac{1}{2}$ space. This initial state is achieved in experiments by having a polarized beam impinge on a polarized target with all possible orientations available. No present experimental setup can achieve that situation. The two-particle density matrix of the final state is then given by $\hat{\rho}_{12} = |\psi_{\rm out}\rangle\langle\psi_{\rm out}|$ with $|\psi_{\rm out}\rangle = \hat{\mathbf{S}}|\psi_{\rm in}\rangle$. The entanglement power, $\mathcal{E}$, of the S matrix, $\hat{\mathbf{S}}$, is then [10]

$$\mathcal{E}(\hat{\mathbf{S}}) = 1 - \int \frac{d\Omega_1}{4\pi}\frac{d\Omega_2}{4\pi}\,{\rm Tr}_1\big[\hat{\rho}_1^2\big], \quad (11)$$

where $\hat{\rho}_1 = {\rm Tr}_2[\,\hat{\rho}_{12}\,]$ is the reduced density matrix for particle 1 that acts in a space of dimension $d = 2$.

At sufficiently low energies the action of the S matrix changes the amplitudes of the two states with total spin $S = 0, 1$, in the ${}^1S_0$ and ${}^3S_1$ channels. BKKS studied the spin-space entanglement of two distinguishable particles, the proton (1) and neutron (2). Neglecting the tensor-force-induced mixing of the ${}^3S_1$ channel with the ${}^3D_1$ channel, the S matrix was expressed in terms of the ${}^1S_0$ and ${}^3S_1$ phase shifts $\delta_{0,1}$, the entanglement power of $\hat{\mathbf{S}}$ was calculated to be

$$\mathcal{E}(\hat{\mathbf{S}}) = \tfrac{1}{6}\,\sin^2(2(\delta_1-\delta_0)), \quad (12)$$

which vanishes when $\delta_1 - \delta_0 = m\frac{\pi}{2}$ for any integer m. But $\mathcal{E}(\hat{\mathbf{S}})$ is maximal when the difference in phase shifts is $\pi/4$. The triplet phase shift at 0 energy is π because of the presence of the deuteron bound state and decreases with increasing energy. The singlet phase shift vanishes at 0 energy and increases as the energy increases. Using the phase shifts of [20] the difference passes through $\pi/4$. at a laboratory energy of around 8.7 MeV and $\mathcal{E}(\hat{\mathbf{S}})$ is maximized at that energy, according to Eq. (12).

The quantity $\mathcal{E}(\hat{\mathbf{S}})$ was evaluated as a function of the center-of-mass nucleon momentum, p, (up to a laboratory energy of 350 MeV) using a phase shift analysis [20] and results of nucleon-nucleon potentials. BKKS focused on values of p between about 250 and 350 MeV/c, there finding that the $\mathcal{E}(\hat{\mathbf{S}}) \approx 0.05$ and thus suppressed. However, the maximum value of $\mathcal{E}(\hat{\mathbf{S}})$ is only 1/6 so that $\mathcal{E}/\mathcal{E}_{\rm max} \approx 0.3$, which is not very small. Moreover, including s-wave scattering is not sufficient because all of the measured phase shifts are needed to describe scattering at those values of p. Nevertheless, BKKS concluded that "Entanglement suppression in the strong-interaction S matrix is shown to be correlated with ... the Wigner $SU(4)$ symmetry for two flavors."

However, there is a problem with using Eq. (11) to determine entanglement. Suppose the density matrix is that of maximum entropy, $\rho_{\rm max}$. Then

$${\rm Tr}_2\rho_{\rm max} = \frac{\hat{I}_1}{2}, \quad (13)$$

where $\hat{I}_1$ is the identity operator of the subspace of particle 1. On the other hand, defining $\rho_i \equiv |e_i\rangle\langle e_i|$ and taking ${\rm Tr}_2$ yields also

$${\rm Tr}_2\rho_i = \frac{\hat{I}_1}{2}, \quad (14)$$

which is the same as that of the state of maximum entropy and zero entanglement. The use of either $\rho_{\rm max}$ or ρ_i in Eq. (11) would yield the same value, namely $\mathcal{E} = 1/2$.

Here, I present an alternative analysis using the precise measurement of entanglement power of Ref. [18]. This is done by starting with an initial pure state of 0 entanglement:

$$|\phi_i\rangle = |\uparrow\downarrow\rangle = -i|e_3\rangle + |e_4\rangle. \quad (15)$$

Here, $C = 0$ and $H = 0$ from Eqs. (8) and (9).

The action of scattering produces a normalized density matrix of the form

$$\rho_f = \frac{M|\phi\rangle\langle\phi|M^\dagger}{{\rm Tr}[M|\phi\rangle\langle\phi|M^\dagger]}, \quad (16)$$

where $M(\mathbf{p}_f, \mathbf{p}_i)$ is the neutron-proton scattering operator acting in the two-nucleon spin space. This expression has been used ubiquitously to analyze nucleon-nucleon, nucleon-nucleus, pion-nucleon scattering, and many other reactions involving nuclei. It is the text-book [17] method to compute the density matrix, the present method for analyzing quantum entanglement [3] and the time-honored method to analyze nucleon-nucleon scattering data [21].

Use of invariance principles (parity, time reversal, and isospin) [22] shows there are five independent amplitudes needed to capture the scattering amplitude. In particular [21],

$$\begin{aligned} M(\mathbf{p}_f, \mathbf{p}_i) = {} & a + c\,(\hat{\boldsymbol{\sigma}}_1 + \hat{\boldsymbol{\sigma}}_2)\cdot\hat{\mathbf{n}} + m\,\hat{\boldsymbol{\sigma}}_1\cdot\hat{\mathbf{n}}\hat{\boldsymbol{\sigma}}_2\cdot\hat{\mathbf{n}} \\ & + g[\hat{\boldsymbol{\sigma}}_1\cdot\hat{\mathbf{P}}\hat{\boldsymbol{\sigma}}_2\cdot\hat{\mathbf{P}} + \hat{\boldsymbol{\sigma}}_1\cdot\hat{\mathbf{K}}\hat{\boldsymbol{\sigma}}_2\cdot\hat{\mathbf{K}}] \\ & + h[\hat{\boldsymbol{\sigma}}_1\cdot\hat{\mathbf{P}}\hat{\boldsymbol{\sigma}}_2\cdot\hat{\mathbf{P}} - \hat{\boldsymbol{\sigma}}_1\cdot\hat{\mathbf{K}}\hat{\boldsymbol{\sigma}}_2\cdot\hat{\mathbf{K}}]. \end{aligned} \quad (17)$$

The Hoshizaki coordinate system is used: $\hat{\mathbf{P}} = (\sin\theta/2, 0, \cos\theta/2)$, $\hat{\mathbf{n}} = (0, 1, 0)$, $\hat{\mathbf{K}} = (\cos\theta/2, 0, -\sin\theta/2)$, with θ as the c.m. scattering angle. The results presented here use the amplitudes from the NN online website: Ref. [23] that are computed from the measured phase shifts of Ref. [20].

The first result, for laboratory kinetic energy of 1 MeV is shown in Fig. 1 finds that entanglement is maximized at all scattering angles. This result can be understood by assuming that only s-waves contribute, approximately true at 1 MeV. In that case, $c = 0$, $h = 0$, and $m = g$, which means that M can be expressed as $M_L = a + m\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$. Then using Eq(13.2) of Ref. [21] the operator M can be expressed in terms of Bell

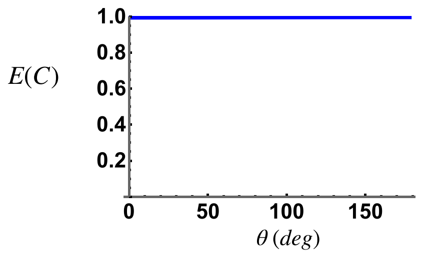

FIG. 1. Entanglement at 1 MeV Computations use the phase shifts of Ref. [20]. The state is $M|\uparrow\downarrow\rangle$.

states as

$$M|\phi_i\rangle = \frac{-i}{\sqrt{2}}(a+m)|e_3\rangle + \frac{1}{\sqrt{2}}(a-3m)|e_4\rangle. \tag{18}$$

At very low energies $a+m \propto e^{i\delta_1}\sin\delta_1$, and $a-3m \propto e^{i\delta_0}\sin\delta_0$. Then a direct computation leads to the result

$$1-C^2 = \frac{4\sin^2\delta_1\sin^2\delta_0\cos^2(\delta_1-\delta_0)}{\sin^2\delta_0+\sin^2\delta_1}, \tag{19}$$

so that $C=1$ and $H=1$ when the phase shifts differ by $\pi/2$. The triplet phase shift is π at 0 energy because of the deuteron bound state in that channel. It drops rapidly with increasing laboratory energy. The singlet phase shifts vanishes at 0 energy and increases rapidly with energy. Thus a phase shift difference of $\pi/2$ is inevitable and occurs at about 1 MeV as shown in Fig. 2.

The result of Fig. 1 can be interpreted in terms of Wigner SU(4) symmetry [12,14]. A nuclear Hamiltonian consistent with SU(4) symmetry obeys

$$\left[H,\sum_i \vec{\tau}_i\right] = \left[H,\sum_i \vec{\sigma}_i\right] = \left[H,\sum_i \vec{\tau}_i\vec{\sigma}_i\right] = 0. \tag{20}$$

At sufficiently low energies for which the scattering is described using s-wave phase shifts as the matrix M_L, and the two-nucleon potential can be expressed in the same way [24]. In that case, the Hamiltonian satisfies SU(4) symmetry and that symmetry is consistent with maximum entanglement. However at higher energies, all of the terms of Eq. (17) enter into the two-nucleon potential and SU(4) symmetry is broken. Then one expects to different values of $E(C)$.

The results for laboratory kinetic energies up to 50 MeV are shown in Fig. 3. Observe that the angular dependence varies rapidly as the laboratory kinetic energy is increased

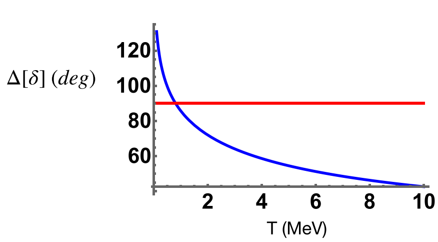

FIG. 2. The phase shift difference $\Delta\delta \equiv \delta_1 - \delta_0$ varies with energy. The line at $\pi/2$ is shown for comparison. The phase shifts of Ref. [20] are used.

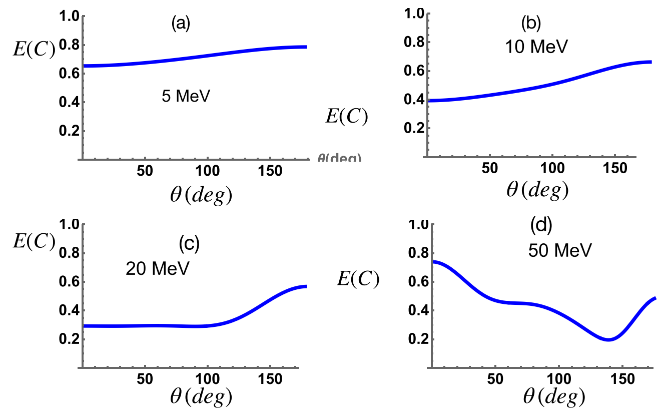

FIG. 3. $E(C)$ of Eq. (9) for several laboratory kinetic energies (5, 10, 20, 50 MeV) as a function of center of momentum angles. The state is $M|\uparrow\downarrow\rangle$.

from 1 to 50 MeV. This is due to the rapid dependence of the s-wave phase shifts on energy and the increasing importance of d, p, and f waves.

The results for laboratory kinetic energies between 100 and 350 MeV are shown in Fig. 4. Observe the persistent prominent peak at around 90°. It is useful to interpret this peak in terms of the underlying interaction. It has long been known [25] that one-pion exchange is important for these energies. Forward-angle charge exchange allows n-p scattering to peak at backward angles and thus provide a signature.

I therefore compute the entanglement effect of the tensor operator in a qualitative effort to interpret the persistent peak. This operator is given by the expression

$$S_{12} = 3\boldsymbol{\sigma}_1\cdot\hat{\mathbf{K}}\boldsymbol{\sigma}_2\cdot\hat{\mathbf{K}} - \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 \tag{21}$$

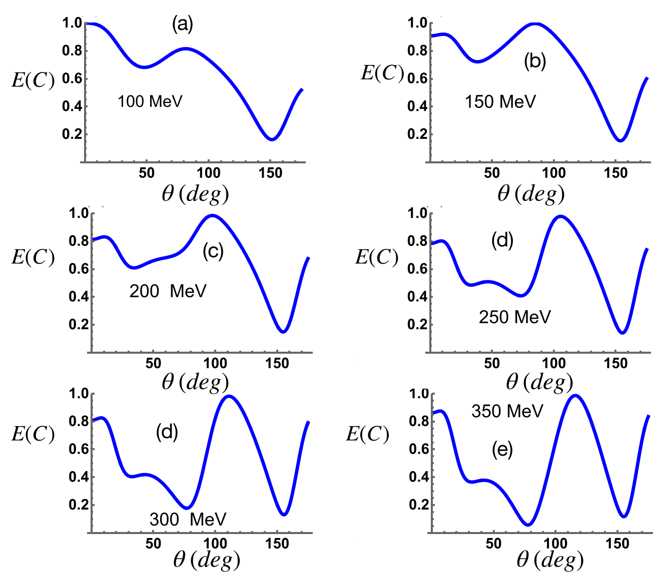

FIG. 4. $E(C)$ of Eq. (9) for several laboratory kinetic energies (100, 150, 200, 250, 300 MeV) as a function of center of momentum angles. The state is $M|\uparrow\downarrow\rangle$.

on the state $|\uparrow\downarrow\rangle$. The operator S_{12} acts only on triplet states, so the state $|\uparrow\downarrow\rangle$ is projected to the triplet state with magnetic quantum number 0, $|\chi_0\rangle/\sqrt{2} = -i|e_3\rangle/\sqrt{2}$. Then a calculation yields

$$S_{12}|e_3\rangle = \frac{i}{\sqrt{2}}[(3\cos\theta - 1)|e_3\rangle + 3\sin\theta|e_2\rangle], \tag{22}$$

a completely entangled state that has $E(C) = 1$. Thus it is reasonable to suggest that the large values of $E(C)$ seen in Fig. 4 for nonzero values of θ result from the tensor force in combination with the other components of the nuclear force. The effects of the tensor force are significant but not dominant. The expression (22) is only a qualitative reproduction of the full calculation.

One could also start with the state $|\uparrow\uparrow\rangle = \frac{1}{\sqrt{2}}(|e_1\rangle - i|e_2\rangle)$. This is also a direct product state with $C = 0$ and 0 entanglement entropy. In the s-wave limit the action of the scattering operator leaves the state invariant because this state is a spin eigenstate. The computed values of $E(C)$ essentially vanish for laboratory kinetic energies below about 50 MeV. This is because orbital angular momentum must be involved to for an operator to change the state $|\uparrow\uparrow\rangle$ to another $S = 1$ state with the same total angular momentum. For this reason the entanglement must vanish at $\theta = 0, \pi$. For higher energies there is an interesting angular dependence that displays significant entanglement. The results, obtained using all terms of the scattering matrix, are shown in Fig. 5.

Observe that the entanglement is generally large. The effects of the tensor force again seem to be prominent because

$$\sqrt{2}S_{12}|\uparrow\uparrow\rangle = 2|e_1\rangle + i(3\cos\theta + 1)|e_2\rangle) + 3i\sin\theta|e_3\rangle, \tag{23}$$

a state that by itself has $C = 3\cos^2\theta/2/(2 + 3\cos^2\theta/2)$. This expression shows that the tensor force matters, but it is not complete because all terms of M are needed to obtain the 0's at forward and backward angles.

A summary is in order. Entanglement is computed here using a technique [18] that literally counts the number of entangled pairs produced by the neutron-proton interaction. Simply taking the trace of the two-particle density matrix on particle 2 to obtain a one-body density matrix and computing the resultant entropy does not yield the entanglement entropy because very completely entangled and completely unentangled two-nucleon density matrices can yield the same one-particle density matrix.

Computations of $E(C)$ of Eq. (9) show that entanglement is large for low-energy neutron-proton scattering. At such energies the nuclear potential satisfies Wigner SU(4) symmetry,

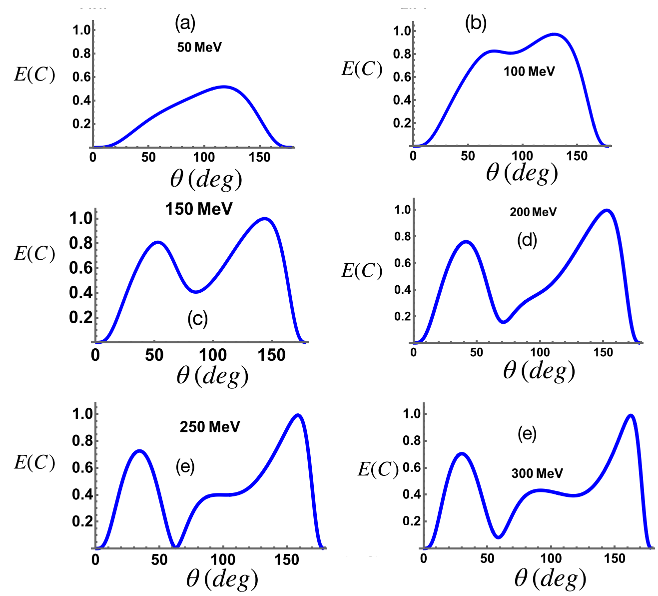

FIG. 5. $E(C)$ of Eq. (9) laboratory kinetic energies (50, 100, 150, 200, 250, 300 MeV) as a function of center of momentum angles. The state is $M|\uparrow\uparrow\rangle$.

so entanglement maximization is a sign of that symmetry. At higher energies the angular dependence of entanglement is strong and is generally not suppressed. The tensor force is shown to play a significant role in producing entanglement.

Additional commenting is worthwhile. The operators of Eq. (17) are symmetric under the interchange $(1, 2) \to (2, 1)$ and therefore conserve the spin quantum number. Violations of other symmetries would lead to additional operators that connect singlet and triplet states, potentially changing the angular dependence of the entanglement entropy. For example, charge symmetry breaking, a violation of isospin invariance of high order in chiral power counting [26], leads to (class IV) operators of the form $(\tau_1 - \tau_2)_z(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2)\cdot\hat{\mathbf{n}}$ [27]. Violations of parity would lead to operators of the form, for example, $(\tau_1 - \tau_2)_z(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2)\cdot(\mathbf{p}_i + \mathbf{p}_f)$ [28] and time reversal violation would allow terms of the form $(\tau_1 - \tau_2)_z(\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2)\cdot\hat{\mathbf{n}}$ [29]. Study of the effects of such operators is a subject for future investigation because there could be a strong connection between entanglement and the fundamental symmetries of the standard model.

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