




Normal single-spin asymmetries in electron-proton scattering: Two-photon exchange with intermediate-state resonances

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We calculate the beam (B_n) and target (A_n) normal single-spin asymmetries in electron-proton elastic scattering from two-photon exchange amplitudes with resonance intermediate states of spin-parity $1/2^\pm$ and $3/2^\pm$ and mass $W \lesssim 1.8$ GeV. The latest CLAS exclusive meson electroproduction data are used as input for the transition amplitudes from the proton to the excited resonance states. For B_n , the spin $3/2$ resonances dominate by an order of magnitude over the spin $1/2$ states. In general we observe cancellations between the negative contributions of the $\Delta(1232)$ and $N(1520)$ across both beam energy and scattering angle, and the positive contributions of the $\Delta(1700)$ and $N(1720)$, leading to a rather large overall uncertainty band in the total B_n . At forward angles and beam energies $E_{\text{lab}} < 1$ GeV, where the $\Delta(1232)$ dominates, the calculated B_n tend to overshoot the A4 and SAMPLE data. The calculated B_n compare well with the measured values from the A4 and Q_{weak} experiments with $E_{\text{lab}} > 1$ GeV.

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I. INTRODUCTION

Over the last two decades the role of two-photon exchange (TPE) in electron-proton elastic scattering has received considerable attention in both the theoretical and experimental nuclear physics communities in an effort to understand its impact on hadron structure-dependent observables [1–4]. Among these is the analysis of the proton's electric (G_E) to magnetic (G_M) form factor ratio, $\mu_p G_E/G_M$, where μ_p is the proton's magnetic moment, and the well-known discrepancy between determinations of this ratio by the longitudinal-transverse (LT) separation and polarization transfer (PT) methods. It has been realized for some time now that TPE can make large contributions to the LT separation while having a minimal effect on the PT results [5–7]. As a result, a consistent description is possible with the inclusion of TPE effects [8].

While the real (dispersive) part of the TPE amplitude can be accessed directly from the measurement of the ratio of the unpolarized e^+p to e^-p scattering cross sections, the imaginary (absorptive) part of TPE can be determined from beam and target-normal single-spin asymmetries (SSAs), denoted B_n and A_n , respectively. Two-photon exchange generates a SSA at leading order in the electromagnetic coupling α , with either the beam or target polarized normal (or transverse) to the scattering plane. Explicitly, the experimentally measured

asymmetry is defined by

$$\text{SSA} = \frac{\sigma^\uparrow - \sigma^\downarrow}{\sigma^\uparrow + \sigma^\downarrow}, \quad (1)$$

where σ^\uparrow (σ^\downarrow) is the cross section for ep elastic scattering with either beam or target spin polarized parallel (antiparallel) to the normal vector \mathbf{n} in the scattering plane. The normal vector \mathbf{n} is defined as

$$\mathbf{n} = \frac{\mathbf{k} \times \mathbf{k}'}{|\mathbf{k} \times \mathbf{k}'|}, \quad (2)$$

where \mathbf{k} and \mathbf{k}' are the three-momenta of the incident and scattered electrons, respectively.

The leading term of the SSA comes from the imaginary part of the TPE amplitude. It was first shown by de Rújula *et al.* [9] that time-reversal invariance implies no contribution to SSA from the single-photon exchange transition amplitude, \mathcal{M}_γ . The leading term of the beam or target-normal SSA arises from the absorptive part of the TPE transition amplitude $\mathcal{M}_{\gamma\gamma}$, denoted $\text{Abs}[\mathcal{M}_{\gamma\gamma}]$, according to the relation

$$\text{SSA} = \frac{\text{Im} \left(\sum_{\text{spins}} \mathcal{M}_\gamma^* \text{Abs}[\mathcal{M}_{\gamma\gamma}] \right)}{\sum_{\text{spins}} |\mathcal{M}_\gamma|^2}. \quad (3)$$

While there is some inconsistency with the notation used for this observable in the literature, in this work the convention A_n for target-normal SSA and B_n for beam-normal SSA will be used.

As defined in Eq. (3), the SSA is of order α . The beam-normal asymmetry B_n is further suppressed by the small factor m_e/E_{lab} , where m_e is the electron mass and E_{lab} is the beam

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energy in the laboratory frame, so that B_n is expected to be of order 10^{-6} – 10^{-5} for beam energies in the GeV range. For the target-normal SSA A_n there is no additional suppression, and hence it is anticipated to be of order 10^{-3} – 10^{-2} for the same beam energy. In addition to providing an avenue to the exploration of TPE effects, the beam-normal SSA B_n plays a particularly important role in parity-violating electron-scattering experiments that use longitudinally polarized lepton beams to measure the asymmetry due to the spin flip. A nonzero B_n , even if small numerically, could contribute to a false asymmetry due to a slow drift in the rapid flip of the beam polarization. As a requirement to control possible systematic errors, parity-violating experiments typically determine the beam-normal SSA as a byproduct. For example, the highly precise Q_{weak} experiment [10] at Jefferson Lab recently determined the weak charge of the proton in a search for physics beyond the standard model, which required knowledge of the systematic error from B_n at forward-scattering angles.

Following the initial measurement by the SAMPLE Collaboration [11] at a beam energy $E_{\text{lab}} = 0.2$ GeV and backward laboratory scattering angle, subsequent experiments from the G0 [12,13], HAPPEX [14], and Q_{weak} [10] Collaborations at Jefferson Lab, and the A4 collaboration at Mainz [15–17], measured B_n over a wide range of scattering angles. A trend observed in the data is the suppression of B_n with increasing energy, although the correlation between energy and scattering angle is less clear. For backward scattering at relatively low energies, Refs. [13,16] find B_n to be of order 10^{-5} , whereas the more recent measurement [17] at intermediate scattering angles finds B_n of order 10^{-6} over a similar range of beam energies. Note that the SAMPLE [11] result is in relative tension with the two other lower energy and backward angle measurements from G0 [13] and A4 [16], which may be related to the more restricted mass range of resonance states that can contribute at the lower SAMPLE energy. In contrast, the relatively higher energy ($1 \lesssim E_{\text{lab}} \lesssim 3$ GeV) experiments [10,12,14,17] correspond to small scattering angles (with the exception of the single datum of Ref. [17]), and are consistently in the range of ≈ -7 to -4 ppm. More recently, the beam-normal SSA has been measured at very forward angles by the PREX and CREX Collaborations at Jefferson Lab on spin-0 nuclei [18], with unexpected behavior found for ^{208}Pb .

In theoretical developments, following de Rújula *et al.* [9] several model estimates of B_n have been made in the literature [19–23]. The hadronic approximation with a doubly virtual Compton-scattering analogy of the imaginary part of the TPE correction was used by Pasquini and Vanderhaeghen [21,22], in which the πN intermediate state was considered along with the elastic nucleon, with input taken from the MAID electroproduction amplitudes [24]. Using a generalized parton distribution approach that is more applicable at high Q^2 , with a real Compton scattering (RCS) analogy, Gorchtein *et al.* [19] found rather different results, with even an opposite sign, compared with Refs. [21,22]. Subsequently, Gorchtein [23] used a quasireal Compton-scattering (QRCS) formalism to estimate both B_n and A_n , although the results are still not in agreement with that of Refs. [21,22]. The significant disagreement between the measured value of beam-normal SSA by the PREX collaboration [14] and the corresponding

theoretical estimate [25] for heavier target nucleus ^{208}Pb raised questions about the calculations in general. More recently, Koshchii *et al.* [26] calculated B_n for electron scattering from several spin-0 nuclei, accounting for inelastic intermediate-state contributions, in addition to several other improvements on the uncertainty calculation. However, the result does not resolve the discrepancy between the theoretical estimates and the PREX [14] data for a ^{208}Pb target nucleus.

In contrast with the beam-normal asymmetry, for the target-normal SSA, A_n , there are currently no available data for a proton target. An experiment to measure A_n in both e^-p and e^+p scattering has been proposed at Jefferson Lab Hall A for beam energies $E_{\text{lab}} = 2.2, 4.4, \text{ and } 6.6$ GeV using the Super Big-Bite Spectrometer [27]. Earlier, a nonzero value of A_n was found for the neutron, extracted from quasielastic electron scattering from ^3He [28], assuming the proton A_n is given by the TPE contribution with a nucleon intermediate state [29].

To better understand both the beam and target-normal SSAs originating from the spin-parity $1/2^\pm$ and $3/2^\pm$ resonance intermediate states associated with πN and $\pi\pi N$ channels, we revisit the imaginary part of the TPE amplitude in ep elastic scattering using the latest results for the electrocouplings extracted from recent CLAS data [30–32]. This is complementary to our work on the resonance contributions to the real (dispersive) part of the TPE amplitude, which is relevant for corrections to the ep elastic-scattering cross section [8,33]. We begin in Sec. II by reviewing the kinematics of electron-proton scattering at the one- and two-photon exchange level. In Sec. III we introduce the single-spin asymmetries for both beam and target polarization normal to the electron-proton scattering plane and discuss the calculation of spin $1/2$ and spin $3/2$ intermediate-state contributions. Numerical results for the beam SSAs B_n and the target SSAs A_n are presented in Sec. IV, including a discussion of uncertainties and comparisons with available data. A parity-violating transverse beam asymmetry, which we denote as B_x , can also arise from a transverse spin polarization in the scattering plane. As discussed in Sec. IV, this turns out to be negligibly small at the kinematics of interest. Finally, in Sec. V we conclude with a summary of the main results of this analysis and some discussion about future extensions of this work.

II. ELASTIC ELECTRON-PROTON SCATTERING

In this section, we define the general kinematic quantities needed for describing elastic electron-proton scattering. For convenience, the calculation of SSA quantities is performed in the center-of-mass (c.m.) frame, although the experimental kinematics are typically given in the laboratory (or target rest) frame. Where appropriate, we give the relevant expressions in both frames.

A. Kinematics and definitions

For the elastic-scattering process $e(k) + N(p) \rightarrow e(k') + N(p')$ (see Fig. 1), the four-momenta of the initial and final electrons (with mass m_e) are labeled by k and k' , with corresponding laboratory frame energies E_{lab} and E'_{lab} .

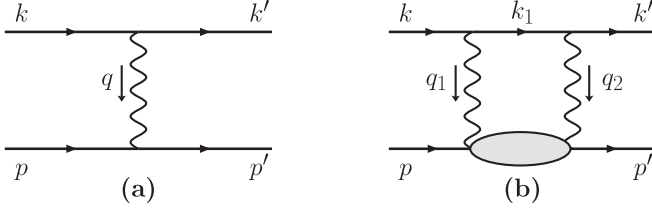


FIG. 1. Contributions to elastic electron–nucleon scattering from (a) one-photon exchange (OPE), and (b) two-photon exchange amplitudes, with particle momenta as indicated. The intermediate hadronic state is taken to be a resonance of invariant mass W . Only the s -channel box diagram is shown for the TPE process, since the imaginary part relevant for SSA originates solely from the box diagram, with the intermediate electron and hadronic states on-shell. The two virtual photons carry momenta q_1 and q_2 , giving the total momentum transfer $q = q_1 + q_2$.

The initial and final nucleons (mass M) have four-momenta p and p' , respectively. The four-momentum transfer from the electron to the nucleon is given by $q = p' - p = k - k'$, with the photon virtuality $Q^2 \equiv -q^2 > 0$. For the TPE process, the two virtual photons transfer four-momenta q_1 and q_2 to the proton, so that $q = q_1 + q_2$.

One can express the elastic-scattering cross section in terms of any two of the Mandelstam invariants s (total electron–nucleon invariant mass squared), t , and u , where

$$\begin{aligned} s &= (k + p)^2 = (k' + p')^2, & t &= (k - k')^2 = q^2, \\ u &= (p - k')^2 = (p' - k)^2, \end{aligned} \quad (4)$$

with the constraint $s + t + u = 2M^2 + 2m_e^2$. For the OPE amplitude, and for the A_n SSA, the electron mass can be neglected at the kinematics of interest. However, for the B_n SSA the electron mass must be retained for two reasons. First, B_n has an overall factor of m_e , and second, B_n has a mass-dependent quasisingularity when the intermediate electron three-momentum $|\mathbf{k}_1| \rightarrow 0$.

For the imaginary part of the scattering amplitude, the intermediate states are on-shell. In the c.m. frame we have for the energies and three-momenta of the particles,

$$E_k = \frac{s - M^2 + m_e^2}{2\sqrt{s}}, \quad |\mathbf{p}| = |\mathbf{k}| = \sqrt{E_k^2 - m_e^2}, \quad (5a)$$

$$E_{k_1} = \frac{s - W^2 + m_e^2}{2\sqrt{s}}, \quad |\mathbf{k}_1| = \sqrt{E_{k_1}^2 - m_e^2}, \quad (5b)$$

$$E_p = \sqrt{E_k^2 - m_e^2 + M^2}, \quad (5c)$$

where $W^2 = (p + q_1)^2 = (p' - q_2)^2$ is the invariant squared mass of the intermediate state resonance. For the four-momentum transfer squared between the electron and nucleon, Q^2 , and the virtualities of the two exchanged photons, $Q_1^2 = -q_1^2$ and $Q_2^2 = -q_2^2$, we have

$$Q^2 = 2|\mathbf{k}|^2(1 - \cos \theta_{\text{c.m.}}), \quad (6a)$$

$$Q_1^2 = 2(E_k E_{k_1} - m_e^2 - |\mathbf{k}||\mathbf{k}_1| \cos \theta_1), \quad (6b)$$

$$Q_2^2 = 2(E_k E_{k_1} - m_e^2 - |\mathbf{k}||\mathbf{k}_1| \cos \theta_2), \quad (6c)$$

where $\theta_{\text{c.m.}}$ is the c.m. scattering angle, and $\cos \theta_2 = \cos \theta_{\text{c.m.}} \cos \theta_1 + \sin \theta_{\text{c.m.}} \sin \theta_1 \cos \phi_1$. The Mandelstam variable s is given in the laboratory frame as $s = M^2 + m_e^2 + 2ME_{\text{lab}}$, with E_{lab} the electron-beam energy in the laboratory frame. In the laboratory frame we also have

$$Q^2 = 2E_{\text{lab}}E'_{\text{lab}}(1 - \cos \theta_{\text{lab}}), \quad E'_{\text{lab}} = E_{\text{lab}} - \frac{Q^2}{2M}, \quad (7)$$

where E'_{lab} is the energy of the electron scattered by angle θ_{lab} .

For inelastic excitations the minimum value of W is taken to be the pion production threshold, $W_{\text{th}} = M + m_\pi$. For a given s , the maximum value of W corresponds to an intermediate electron at rest, $|\mathbf{k}_1| = 0$, so that

$$W_{\text{max}} = \sqrt{s} - m_e, \quad E_{k_1} = m_e. \quad (8)$$

At $W = W_{\text{max}}$ the four-momentum transfers of the two virtual photons become

$$Q_1^2 = Q_2^2 = m_e \frac{(W_{\text{max}}^2 - M^2)}{\sqrt{s}}, \quad (9)$$

so that the two photons are almost on-shell (i.e., real). This QRCS region [20–23] requires special attention to reliably compute the SSA numerically. We discuss this further in Sec. III C and in the Appendix.

B. One- and two-photon exchange amplitudes

The explicit expression for the one-photon exchange (OPE) or Born amplitude \mathcal{M}_γ of Fig. 1 can be written as [2]

$$\mathcal{M}_\gamma = e^2 \bar{u}_e(k') \gamma_\mu u_e(k) \frac{1}{Q^2} \bar{u}_N(p') \Gamma^\mu(q) u_N(p), \quad (10)$$

where e is the charge of the proton, and the hadronic current operator Γ^μ is parametrized in terms of the Dirac F_1 and Pauli F_2 form factors for on-shell particles,

$$\Gamma^\mu(q) = F_1(Q^2) \gamma^\mu + F_2(Q^2) \frac{i\sigma^{\mu\nu} q_\nu}{2M}. \quad (11)$$

The two-photon exchange amplitude $\mathcal{M}_{\gamma\gamma}$ contains contributions from the box diagram of Fig. 1 and the corresponding crossed-box diagram (not shown). However, since the crossed-box amplitude is purely real, we focus only on the box diagram contribution, $\mathcal{M}_{\gamma\gamma}^{\text{box}}$. The loop integral of the box diagram amplitude can be written as [2]

$$\mathcal{M}_{\gamma\gamma}^{\text{box}} = -ie^4 \int \frac{d^4 q_1}{(2\pi)^4} \frac{L_{\mu\nu} H^{\mu\nu}}{(q_1^2 - \lambda^2)(q_2^2 - \lambda^2)}, \quad (12)$$

where λ is an infinitesimal photon mass introduced to regulate infrared divergences. Such divergences are absent for normal single-spin asymmetries, but λ can be kept as an infinitesimal parameter to improve numerical stability. The leptonic and hadronic tensors, $L_{\mu\nu}$ and $H^{\mu\nu}$, respectively, are given by

$$L_{\mu\nu} = \bar{u}_e(k') \gamma_\mu \frac{(\not{k}_1 + m_e)}{k_1^2 - m_e^2 + i0^+} \gamma_\nu u_e(k), \quad (13a)$$

$$\begin{aligned} H^{\mu\nu} &= \bar{u}_N(p') \Gamma_{R \rightarrow \gamma N}^{\mu\alpha}(p_R, -q_2) S_{\alpha\beta}(p_R, W) \\ &\quad \times \Gamma_{\gamma N \rightarrow R}^{\beta\nu}(p_R, q_1) u_N(p), \end{aligned} \quad (13b)$$

where the intermediate lepton four-momentum is $k_1 = k - q_1$, and the four-momentum of the resonance R (with mass W) is $p_R = p + q_1 = p' - q_2$. The transition operators

$\Gamma_{\gamma N \rightarrow R}^{\beta\nu}(p_R, q_1)$ and $\Gamma_{R \rightarrow \gamma N}^{\mu\alpha}(p_R, -q_2)$ between the nucleon and intermediate-state resonance R can be expressed in terms of the three transition form factors G_1 , G_2 , and G_3 . These form factors can also be written in terms of the corresponding helicity amplitudes $A_{1/2}$, $A_{3/2}$, and $S_{1/2}$ (see Ref. [8]).

For spin 1/2 baryon intermediate states, the propagator $S_{\alpha\beta}(p_R, W)$ is simply the spin 1/2 Feynman propagator,

$$S_{\alpha\beta}(p_R, W) = \delta_{\alpha\beta} \frac{(\not{p}_R + W)}{p_R^2 - W^2 + i0^+} = \delta_{\alpha\beta} S_F(p_R, W). \quad (14)$$

For spin 3/2 intermediate states, on the other hand, the hadronic propagator has the more complicated form

$$S_{\alpha\beta}(p_R, W) = -\mathcal{P}_{\alpha\beta}^{3/2}(p_R) \frac{(\not{p}_R + W)}{p_R^2 - W^2 + i0^+}, \quad (15)$$

where

$$\mathcal{P}_{\alpha\beta}^{3/2}(p_R) = g_{\alpha\beta} - \frac{1}{3} \gamma_\alpha \gamma_\beta - \frac{1}{3 p_R^2} (\not{p}_R \gamma_\alpha (p_R)_\beta + (p_R)_\alpha \gamma_\beta \not{p}_R) \quad (16)$$

is the spin 3/2 projection operator for momentum p_R .

III. SINGLE-SPIN ASYMMETRIES IN ELECTRON-PROTON SCATTERING

In this section we discuss several technical aspects of the TPE amplitude, including the generalization of the calculation from point particles to the case of finite resonance widths (Sec. III B), and the quasisingular behavior of the asymmetry B_n (Sec. III C). We begin, however, with some general considerations about TPE amplitudes and their contributions to SSAs.

A. General features

In the definition of the beam or target-normal SSA in Eq. (3), the denominator is identical to the Born cross section for unpolarized elastic ep scattering, since the spin components (beam or target) have no impact at the Born level. Summing over final-state spins and averaging over initial-state spins, one can write the squared Born amplitude in terms of the invariant Mandelstam variables s and $Q^2 = -t$,

$$\sum_{\text{spins}} |\mathcal{M}_\gamma|^2 = \sum_{\text{spins}} \mathcal{M}_\gamma^\dagger \mathcal{M}_\gamma = \frac{e^4}{Q^4} D(s, Q^2), \quad (17)$$

where we define the factor

$$D(s, Q^2) = \frac{2}{4M^2 + Q^2} \{ G_E^2(Q^2) 8M^2 [(s - M^2)^2 - Q^2 s] + G_M^2(Q^2) Q^2 [2M^4 + Q^4 + 2(s - 2M^2)(s - Q^2)] \}, \quad (18)$$

with terms of order of m_e^2 neglected.

To derive the absorptive part of the TPE amplitude, one can exploit the Cutkosky cutting rules [34], which involve the replacements

$$\frac{1}{p_R^2 - W^2 + i0^+} \rightarrow -2\pi i \theta(p_R^0) \delta(p_R^2 - W^2), \quad (19a)$$

$$\frac{1}{k_1^2 - m_e^2 + i0^+} \rightarrow -2\pi i \theta(k_1^0) \delta(k_1^2 - m_e^2), \quad (19b)$$

which place the intermediate-state lepton and hadron on their mass shells. This substitution provides the discontinuity, $\text{Disc}(i\mathcal{M}_{\gamma\gamma}) = -2 \text{Im} \mathcal{M}_{\gamma\gamma}$, of the TPE box diagram of Fig. 1, and hence the absorptive part of TPE amplitude $-\text{Abs} \mathcal{M}_{\gamma\gamma}$. After applying the Cutkosky cutting rules, the absorptive part of the TPE amplitude in Eq. (3) can be written as

$$\text{Abs} \mathcal{M}_{\gamma\gamma} = e^4 \int \frac{d^3 \mathbf{k}_1}{(2\pi)^3 2E_{k_1}} \frac{\bar{u}_e(k') \gamma_\mu (\not{k}_1 + m_e) \gamma_\nu u_e(k)}{Q_1^2 Q_2^2} \mathcal{W}^{\mu\nu}. \quad (20)$$

The hadronic tensor $\mathcal{W}^{\mu\nu}$ in Eq. (20) contains all the information about the transition from the proton initial state to all possible intermediate hadronic states, including the elastic nucleon state and the inelastic transitions to the nucleon excited-state resonances. In practice, the SSAs are calculated including contributions from each of the three-star and four-star, spin 1/2 and 3/2 resonance intermediate states from the PDG [35] below mass $M_R = 1.8$ GeV, which are then added together with the elastic nucleon contribution to obtain the complete result.

In the zero-width approximation, for the elastic nucleon and inelastic spin 1/2 resonances of mass M_R the hadronic tensor $\mathcal{W}^{\mu\nu}$ takes the simplified form

$$\mathcal{W}^{\mu\nu} = 2\pi \delta(W^2 - M_R^2) \bar{u}_N(p') \Gamma_{R \rightarrow \gamma N}^\mu(p_R, -q_2) (\not{p}_R + W) \times \Gamma_{\gamma N \rightarrow R}^\nu(p_R, q_1) u_N(p). \quad (21)$$

To assess the validity of this approximation, we also examine the effect of replacing the zero-width result by a finite-width distribution in W^2 , centered around $W = M_R$. For spin 3/2 resonances, the hadronic tensor uses the Rarita-Schwinger spinors for each intermediate state and can be written as

$$\mathcal{W}^{\mu\nu} = -2\pi \delta(W^2 - M_R^2) \times \bar{u}_N(p') \Gamma_{R \rightarrow \gamma N}^{\mu\alpha}(p_R, -q_2) \mathcal{P}_{\alpha\beta}^{3/2}(p_R) (\not{p}_R + W) \times \Gamma_{\gamma N \rightarrow R}^{\beta\nu}(p_R, q_1) u_N(p). \quad (22)$$

Using Eqs. (10), (17), and (20) one can write the SSA as

$$\text{SSA} = \frac{\alpha Q^2}{2\pi^2 D(s, Q^2)} \sum_{\text{spins}} \int \frac{d^3 \mathbf{k}_1}{2E_{k_1}} \frac{\bar{u}_e(k) \gamma_\rho u_e(k') \bar{u}_e(k') \gamma_\mu (\not{k}_1 + m_e) \gamma_\nu u_e(k) \bar{u}_N(p) \Gamma^\rho(-q) u_N(p')}{Q_1^2 Q_2^2} \mathcal{W}^{\mu\nu}. \quad (23)$$

For the two different cases of beam and target-normal SSA, the spin sum will lead to different expressions for the SSAs. Taking the spin sum, one can express Eq. (23) in a concise form in terms of the leptonic and hadronic tensors, $L_{\rho\mu\nu}$ and $H^{\rho\mu\nu}$, respectively, as

$$\text{SSA} = \frac{\alpha Q^2}{\pi D(s, Q^2)} \int \frac{d^3\mathbf{k}_1}{2E_{k_1}} \frac{\text{Im} L_{\rho\mu\nu} H^{\rho\mu\nu}}{Q_1^2 Q_2^2}. \quad (24)$$

For the beam polarized parallel or antiparallel to the normal \mathbf{n} to the scattering plane defined in Eq. (2), the leptonic tensor $L_{\rho\mu\nu}$ contains the lepton spin polarization vector $s_n^\mu \equiv (0; \mathbf{s}_n)$, and takes the form

$$L_{\rho\mu\nu}^B = \frac{1}{2} \text{Tr}[(1 + \gamma_5 \not{s}_n)(\not{k} + m_e)\gamma_\rho(\not{k}' + m_e)\gamma_\mu(\not{k}_1 + m_e)\gamma_\nu], \quad (25)$$

where the superscript ‘‘B’’ denotes the fact that the lepton tensor corresponds to the beam-normal case. Note that the imaginary part in Eq. (3) for B_n comes solely from this spin-polarization-dependent term. However, the corresponding hadronic tensor for the beam-normal case, $H_B^{\rho\mu\nu}$, remains independent of the polarization of the target hadron and is equivalent to the hadronic tensor for the case of unpolarized ep scattering.

For spin 1/2 intermediate states, the hadronic tensor becomes

$$H_B^{\rho\mu\nu} = \frac{1}{2} \text{Tr}[(\not{p} + M)\Gamma_\rho(-q)(\not{p}' + M)\Gamma_R^\mu \Gamma_{R \rightarrow \gamma N}(p_R, -q_2) \times (\not{p}_R + W)\Gamma_{\gamma N \rightarrow R}^\nu(p_R, q_1)]\delta(W^2 - M_R^2). \quad (26)$$

For spin 3/2 resonances, on the other hand, the hadronic tensor is given by

$$H_B^{\rho\mu\nu} = -\frac{1}{2} \text{Tr}[(\not{p} + M)\Gamma_\rho(-q)(\not{p}' + M) \times \Gamma_{R \rightarrow \gamma N}^{\mu\alpha}(p_R, -q_2)\mathcal{P}_{\alpha\beta}^{3/2}(p_R) \times (\not{p}_R + W)\Gamma_{\gamma N \rightarrow R}^{\beta\nu}(p_R, q_1)]\delta(W^2 - M_R^2). \quad (27)$$

For the target-normal SSA, A_n , the corresponding leptonic tensor, $L_{\rho\mu\nu}^A$, is identical to that for unpolarized ep scattering and can be written as

$$L_{\rho\mu\nu}^A = \frac{1}{2} \text{Tr}[(\not{k} + m_e)\gamma_\rho(\not{k}' + m_e)\gamma_\mu(\not{k}_1 + m_e)\gamma_\nu]. \quad (28)$$

Unlike for B_n , the hadronic tensor $H_A^{\rho\mu\nu}$ for the target-normal SSA A_n depends on the target polarization vector, S_n^μ . For spin-1/2 resonances, $H_A^{\rho\mu\nu}$ becomes

$$H_A^{\rho\mu\nu} = \frac{1}{2} \text{Tr}[(1 + \gamma_5 \not{S}_n)(\not{p} + M)\Gamma_\rho(-q)(\not{p}' + M) \times \Gamma_{R \rightarrow \gamma N}^\mu(p_R, -q_2) \times (\not{p}_R + W)\Gamma_{\gamma N \rightarrow R}^\nu(p_R, q_1)]\delta(W^2 - M_R^2), \quad (29)$$

while for spin 3/2 resonances it is given by

$$H_A^{\rho\mu\nu} = -\frac{1}{2} \text{Tr}[(1 + \gamma_5 \not{S}_n)(\not{p} + M) \times \Gamma_\rho(-q)(\not{p}' + M)\Gamma_{R \rightarrow \gamma N}^{\mu\alpha}(p_R, -q_2)\mathcal{P}_{\alpha\beta}^{3/2}(p_R) \times (\not{p}_R + W)\Gamma_{\gamma N \rightarrow R}^{\beta\nu}(p_R, q_1)]\delta(W^2 - M_R^2). \quad (30)$$

For the numerical calculation, it will be convenient to transform the phase-space integral over the intermediate electron

momentum \mathbf{k}_1 of Eq. (24) in terms of the Lorentz-invariant Mandelstam variable s . Defining the kinematics in the c.m. frame, the integration over $d^3\mathbf{k}_1 \rightarrow k_1^2 d|\mathbf{k}_1| d(\cos\theta_{k_1}) d\phi_{k_1}$ can be written as

$$\int \frac{d^3\mathbf{k}_1}{2E_{k_1}} \rightarrow - \int_{M^2}^{W_{\max}^2} dW^2 \frac{|\mathbf{k}_1|}{4\sqrt{s}} \int_{-1}^1 d\cos\theta_{k_1} \int_0^{2\pi} d\phi_{k_1}, \quad (31)$$

with $W_{\max} = \sqrt{s} - m_e$. Here we have utilized the c.m. frame relation for the intermediate electron three-momentum given in Eq. (5b).

$$\text{SSA} = - \frac{\alpha Q^2}{\pi D(s, Q^2)} \int_{M^2}^{W_{\max}^2} dW^2 \frac{|\mathbf{k}_1|}{4\sqrt{s}} \int_{-1}^1 d\cos\theta_{k_1} \int_0^{2\pi} d\phi_{k_1} \frac{\text{Im} L_{\rho\mu\nu} H^{\rho\mu\nu}}{Q_1^2 Q_2^2}. \quad (32)$$

The tensor product $L_{\rho\mu\nu} H^{\rho\mu\nu}$ in Eq. (32) depends on the totally antisymmetric Levi-Civita tensor, $\epsilon_{\alpha\beta\gamma\delta}$, which is defined following the FeynCalc [36] convention $\epsilon_{0123} = -1 = -\epsilon^{0123}$. In the following we use the shorthand notation $\epsilon(abcd) \equiv \epsilon_{\alpha\beta\gamma\delta} a^\alpha b^\beta c^\gamma d^\delta$. For the beam-normal spin asymmetry B_n there are four independent antisymmetric tensors that be constructed from the beam-normal spin four-vector s_n and three of the four-momenta k , p , q , and q_1 . For the target-normal spin asymmetry A_n there is one antisymmetric tensor needed. In the c.m. frame these can be written as

$$\epsilon(kpq s_n) = -(E_k + E_p)|\mathbf{k}|^2 \sin\theta_{\text{c.m.}}, \quad (33a)$$

$$\epsilon(kpq_1 s_n) = -(E_k + E_p)|\mathbf{k}||\mathbf{k}_1| \sin\theta_{k_1} \cos\phi_{k_1}, \quad (33b)$$

$$\epsilon(kqq_1 s_n) = |\mathbf{k}|\{[E_k|\mathbf{k}_1| \cos\theta_{k_1} - E_{k_1}|\mathbf{k}|] \sin\theta_{\text{c.m.}} + E_k|\mathbf{k}_1|(1 - \cos\theta_{\text{c.m.}}) \sin\theta_{k_1} \cos\phi_{k_1}\}, \quad (33c)$$

$$\epsilon(pqq_1 s_n) = |\mathbf{k}|\{[E_p|\mathbf{k}_1| \cos\theta_{k_1} - (E_k - E_{k_1} + E_p)|\mathbf{k}|] \sin\theta_{\text{c.m.}} + E_p|\mathbf{k}_1|(1 - \cos\theta_{\text{c.m.}}) \sin\theta_{k_1} \cos\phi_{k_1}\}, \quad (33d)$$

$$\epsilon(kppq_1) = (E_k + E_p)|\mathbf{k}||\mathbf{k}|^2 \sin\theta_{\text{c.m.}} \sin\theta_{k_1} \sin\phi_{k_1}. \quad (33e)$$

B. Finite-width effect

A finite resonance width is usually accommodated by using the well-known relativistic Breit-Wigner distribution in W^2 . In this analysis we use a closely related alternative distribution, denoted as a Sill distribution by Giacosa *et al.* [37], which avoids the problem of normalization inherent in the Breit-Wigner expression. In this approach the δ -function distribution $\delta(W^2 - M_R^2)$ that appears in Eqs. (29) and (30) is replaced by the function

$$\delta_{\text{Sill}}(W^2) = \frac{\theta(W^2 - W_{\text{th}}^2)}{\pi} \frac{\sqrt{W^2 - W_{\text{th}}^2} \tilde{\Gamma}}{(W^2 - M_R^2)^2 + (W^2 - W_{\text{th}}^2) \tilde{\Gamma}^2}, \quad (34)$$

where

$$\tilde{\Gamma} = \Gamma \frac{M_R}{\sqrt{M_R^2 - W_{\text{th}}^2}}, \quad (35)$$

and Γ is the usual resonance width. The Sill distribution has the desirable property that

$$\int_{W_{\text{th}}^2}^{\infty} dW^2 \delta_{\text{Sill}}(W^2) = 1 \quad (36)$$

for any threshold $W_{\text{th}}^2 < M_R^2$. It vanishes as $W \rightarrow W_{\text{th}}$, but is otherwise very similar to the conventional Breit-Wigner distribution.

C. Quasisingular behavior in B_n

As discussed at the end of Sec. II A, the beam-normal SSA B_n is sensitive to the quasisingular behavior of the integrand in Eq. (32) when the intermediate-state electron three-momentum $|\mathbf{k}_1| \rightarrow 0$. This is the QRCS region, where $W \rightarrow W_{\text{max}}$ and the two virtual photons have four-momenta Q_1^2 and Q_2^2 of order m_e [see Eq. (9)]. In this region of W , the integrand of Eq. (32) is characterized by a slowly varying numerator and a rapidly varying denominator. This behavior does not affect the target-normal SSA A_n because for this asymmetry the numerator in Eq. (32) vanishes as $W \rightarrow W_{\text{max}}$.

To address this behavior in the numerical calculations in a practical way, we have devised the following strategy in the QRCS region with W just below W_{max} . The slowly varying numerator of the integrand in Eq. (32) is evaluated at $Q_1^2 = Q_2^2 = 0$, which is then a constant independent of θ_{k_1} and ϕ_{k_1} . We keep the mild W dependence, but make no further approximation and leave the denominator intact. Thus we are left with an integral over W in this region that is proportional to the angular integral

$$J(W) = \frac{|\mathbf{k}_1|}{4\sqrt{s}} \int d\Omega_{k_1} \frac{1}{Q_1^2 Q_2^2}. \quad (37)$$

This integral can be done analytically, as discussed in Refs. [20,23,33]. Unlike Refs. [20,23] however, we only apply the analytic expression using $J(W)$ to the tail region, $W_{\text{max}} - 5m_e \leq W \leq W_{\text{max}}$, and use the full three-dimensional numerical quadrature of Eq. (32) elsewhere. Details of the matching procedure at $W = W_{\text{max}} - 5m_e$ and the analytic expression for $J(W)$ are given in the Appendix.

IV. NUMERICAL SINGLE-SPIN ASYMMETRY RESULTS

In this section we present the results of our calculation of single-spin asymmetries for both beam (Sec. IV C) and target (Sec. IV D) spin normal to the scattering plane, at the kinematics of several previous experiments. Before discussing the

results for B_n and A_n , we will illustrate the input parameters used in the evaluation of the integral in Eq. (32).

A. Resonance parameters

In our numerical calculations, for the proton elastic electric (G_E) and magnetic (G_M) form factors we use the parametrization from Ref. [38]. For the hadronic transition currents $\Gamma_{R \rightarrow \gamma N}$ and $\Gamma_{\gamma N \rightarrow R}$ in Eq. (32), we use the CLAS parametrization [30] of the input resonance electrocouplings $A_h(Q^2)$ at the resonance points, where A_h represents the longitudinal electrocoupling, $S_{1/2}$, and the two transverse electrocouplings, $A_{1/2}$ and $A_{3/2}$. The dependence of the electrocouplings A_h on the invariant mass W is given in Ref. [8].

For the inelastic intermediate states in Fig. 1(b), in this work we include the contributions of four spin-parity $3/2^\pm$ nucleon (isospin 1/2) and Δ (isospin 3/2) resonances $\{\Delta(1232)3/2^+, N(1520)3/2^-, \Delta(1700)3/2^-, \text{ and } N(1720)3/2^+\}$, and five spin-parity $1/2^\pm$ resonances $\{N(1440)1/2^+, N(1535)1/2^-, \Delta(1620)1/2^-, N(1650)1/2^-, \text{ and } N(1710)1/2^+\}$. (In the following, for ease of notation we will drop the spin-parity suffix from the resonance state labels.) The Breit-Wigner mass M_R and the constant decay width Γ of the nine excited-state resonances are set to those used in the CLAS parametrization [30] of the resonance electrocouplings A_h , and their numerical values are listed in the second and third columns of Table I.

Laboratory threshold energies $E_{\text{lab}}^{\text{th}} = (M_R^2 - M^2)/2M$ for the excitation of resonances R in the zero-width limit are shown in Table I. Values range between 0.34 GeV for the first-excited state $\Delta(1232)$ to 1.11 GeV for the highest-mass state $N(1720)$. It is evident from the threshold energy values that in the zero-width approximation the states beyond the $N(1650)$ do not contribute to the total SSA for beam energies below 1 GeV, where most of the experiments to measure B_n have taken data. In practice, the unstable resonances have a finite-decay width with a distribution in the squared invariant mass W^2 , starting from the threshold, W_{th}^2 , of the prominent $n\pi^+$ decay channel of most resonances. Accounting for the finite-width effect for each resonance, using the Sill distribution of Eq. (34), gives a nonzero contribution from the higher-mass resonances even at beam energies $E_{\text{lab}} \lesssim 1.0$ GeV. The effect of such a nonzero width on the beam and target SSAs B_n and A_n will be discussed in more detail below.

B. Uncertainty estimation

Apart from the dependence on the width, we also propagate the uncertainty on the input resonance electrocouplings, ΔA_h , into the estimation of the uncertainties on the beam-normal SSA B_n , using

$$\Delta B_n = \sqrt{\left(\frac{\partial B_n}{\partial A_{1/2}}\right)^2 (\Delta A_{1/2})^2 + \left(\frac{\partial B_n}{\partial A_{3/2}}\right)^2 (\Delta A_{3/2})^2 + \left(\frac{\partial B_n}{\partial S_{1/2}}\right)^2 (\Delta S_{1/2})^2}, \quad (38)$$

and similarly for the uncertainty, ΔA_n , on the target-normal asymmetry, A_n . A constant, Q^2 -independent uncertainty on

the electrocouplings was assumed for each of the resonances in the range of $0 \leq Q^2 \leq 5$ GeV², with the exception of the

TABLE I. Mass M_R , width Γ , and threshold energy $E_{\text{lab}}^{\text{th}}$ of each N or Δ resonance of spin-parity J^\pm . The uncertainty bands $\Delta A_{1/2}$, $\Delta A_{3/2}$, and $\Delta S_{1/2}$ on the respective electrocouplings $A_{1/2}$, $A_{3/2}$, and $S_{1/2}$, used in estimating the uncertainty in B_n and A_n , are given in the last three columns. The uncertainties ΔA_h are given as a percentage of the maximum absolute value of the corresponding electrocouplings, except for the $\Delta(1232)$, where $\Delta A_{1/2}$ and $\Delta A_{3/2}$ are given as a percentage of $A_{1/2}$ and $A_{3/2}$, respectively.

Resonance	M_R (GeV)	Γ (GeV)	$E_{\text{lab}}^{\text{th}}$ (GeV)	$\Delta A_{1/2}$ (%)	$\Delta A_{3/2}$ (%)	$\Delta S_{1/2}$ (%)
$\Delta(1232)3/2^+$	1.232	0.117	0.34	3.0	4.5	3.6
$N(1440)1/2^+$	1.430	0.350	0.64	10.0		15.9
$N(1520)3/2^-$	1.515	0.115	0.75	6.1	5.3	8.9
$N(1535)1/2^-$	1.535	0.150	0.78	5.0		22.1
$\Delta(1620)1/2^-$	1.630	0.140	0.91	21.2		12.1
$N(1650)1/2^-$	1.655	0.140	0.98	15.8		23.6
$\Delta(1700)3/2^-$	1.700	0.293	1.09	5.0	9.1	12.9
$N(1710)1/2^+$	1.710	0.100	1.09	15.0		49.2
$N(1720)3/2^+$	1.748	0.114	1.11	4.5	10.7	13.8

$\Delta(1232)$, for which there is more empirical information. The uncertainties on the transverse $A_{1/2}$ and $A_{3/2}$ electrocouplings of the $\Delta(1232)$ transition display some Q^2 dependence, and decrease with Q^2 , following the magnitudes of the respective electrocouplings [30]. As shown in Table I, the uncertainties $\Delta A_{1/2}$ and $\Delta A_{3/2}$ on the two transverse electrocouplings are assumed to be $\approx 3\%$ and 4.5% of the corresponding electrocouplings, respectively. For the longitudinal electrocoupling $S_{1/2}$, the uncertainty $\Delta S_{1/2}$ for the $\Delta(1232)$ transition (similar to all other resonances) can be approximated by a constant $\approx 3.6\%$ of the maximum value of $S_{1/2}$ [30], which occurs at $Q^2 = 0.127 \text{ GeV}^2$. The constant uncertainties ΔA_h for the remaining states are given in Table I as a percentage of the maximum value of the corresponding electrocouplings A_h over the range $0 \leq Q^2 \leq 5 \text{ GeV}^2$.

C. Beam-normal single-spin asymmetry B_n

In this section we present the results for the beam-normal SSA B_n , computed at beam energies relevant for existing experiments. To analyze the role of the resonances on the total SSA, in Fig. 2 we illustrate the contributions to B_n from the individual resonances at beam energies between $\approx 0.5 \text{ GeV}$ and $\approx 3 \text{ GeV}$ as function of the laboratory scattering angle θ_{lab} .

Among the resonances considered, the four spin-3/2 states $\Delta(1232)$, $N(1520)$, $\Delta(1700)$, and $N(1720)$ have sizable effects, with some partial cancellation observed between them. Contributions from resonances with spin 1/2 are smaller by at least an order of magnitude. However, both the lower-mass spin-3/2 resonances $\Delta(1232)$ and $N(1520)$ give negative contributions to B_n , even though these states have different isospin and parity. On the other hand, the two higher-mass spin-3/2 states $\Delta(1700)$ and $N(1720)$, with opposite parity and different isospin, make positive contributions to the total B_n . No definite correlation between the isospin and parity is therefore observed in the imaginary part of the TPE amplitude for the case of normally polarized electrons elastically scattering from unpolarized protons.

At low beam energies, $E_{\text{lab}} \lesssim 0.5 \text{ GeV}$, the $\Delta(1232)$ state gives the dominant contribution to B_n [Fig. 2(a)]. As the energy increases, the higher-mass resonances start playing a more significant role. At $E_{\text{lab}} = 0.855 \text{ GeV}$, for example

[Fig. 2(b)], the effect from the $N(1520)$, which has threshold energy $E_{\text{lab}}^{\text{th}} = 0.75 \text{ GeV}$, becomes comparable to that of the $\Delta(1232)$. It is interesting to note that the higher-mass resonance states $\Delta(1700)$ and $N(1720)$ show non-negligible effects even at beam energies below their excitation threshold [see Fig. 2(b)]. Such contributions, originating from the tail of the W^2 distribution for the nonzero-width case, are not accounted for in the more approximate zero-width calculations. However, at energies above the threshold, the $\Delta(1700)$ and $N(1720)$ begin to dominate, as Figs. 2(c)–2(e) demonstrate. The dependence of these major resonances on the energy for fixed scattering angles will be further discussed below. The overall magnitudes of the peak points of B_n decrease with increasing beam energies for each of the resonances above their threshold, as evident from the scale of the panels in Fig. 2.

It is also important to note that, at forward laboratory scattering angles θ_{lab} , where most of the experimental data exist, the $\Delta(1232)$ contribution alone is a good approximation of the total, with the small effects from other resonances largely canceling in this region. Furthermore, the elastic nucleon intermediate state gives a negligibly small effect in B_n , unlike the real part of the TPE amplitude in unpolarized ep elastic scattering [8].

The combined effect of all nine resonances listed in Table I, along with the nucleon elastic contribution, on the total B_n is illustrated in Fig. 3, at the same kinematics as in Fig. 2. The full results with the finite resonance decay widths are contrasted with the approximate results computed in the zero-width approximation over the entire range of scattering angles θ_{lab} . Overall, the finite-width effect is small in the forward limit for all the considered beam energies, but the results of the two width approximations deviate in the far forward and backward angles. We believe this may be attributable to a non-negligible contribution from the QRCS region with W above or below the threshold value $W = M_R$, which is the only value of W in the zero-width case.

At the lower beam energies, $E_{\text{lab}} = 0.5102 \text{ GeV}$ and 0.855 GeV , the overall B_n , including the effects of all elastic and resonance intermediate states, can be approximated by the $\Delta(1232)$ state alone. Over the entire range of scattering angles θ_{lab} studied, the total B_n remains negative, with peak

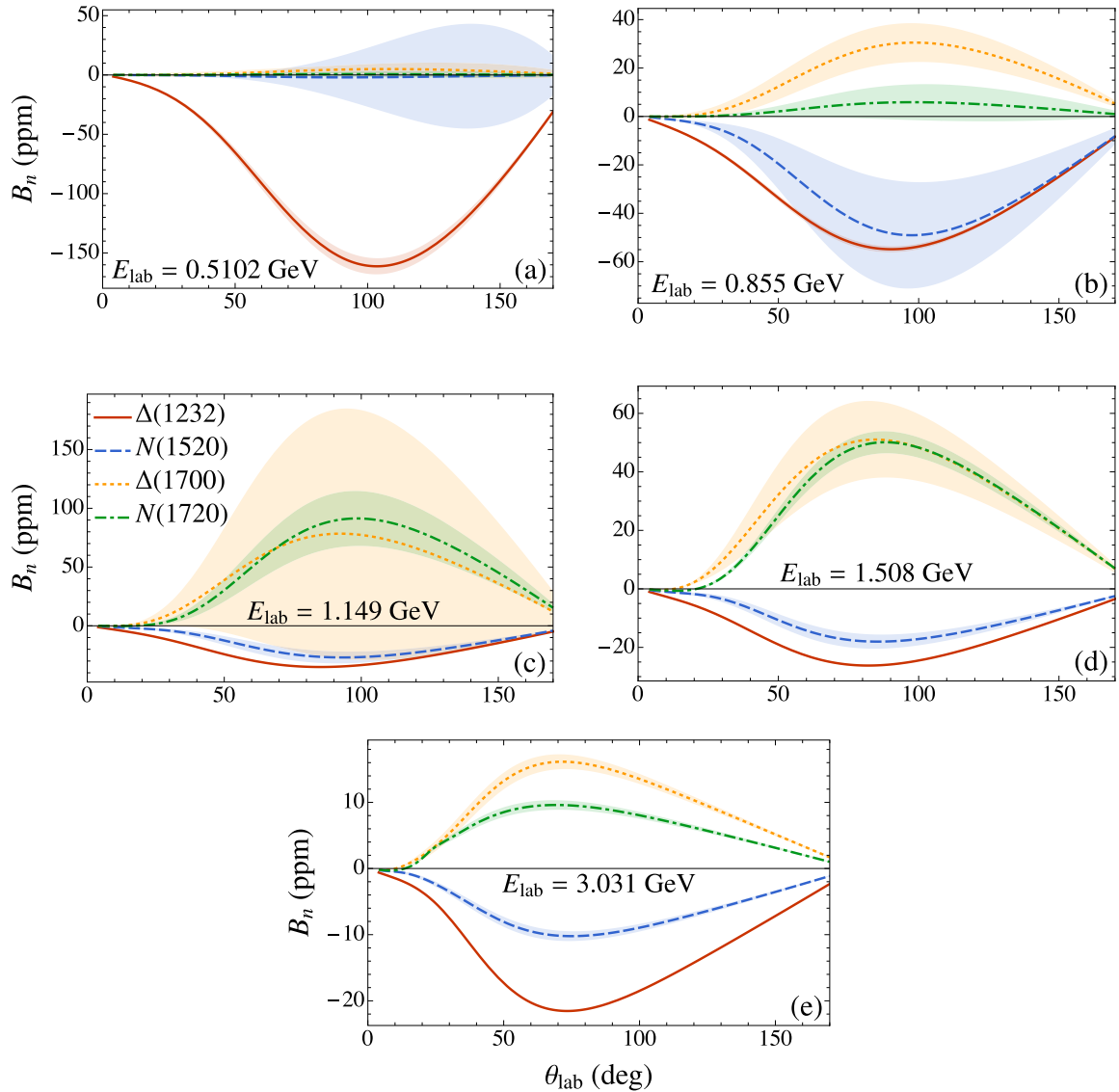


FIG. 2. Resonance contributions to the beam-normal SSA B_n (in parts per million) as a function of scattering angle θ_{lab} at five representative beam energies E_{lab} equal to (a) 0.5102 GeV, (b) 0.855 GeV, (c) 1.149 GeV, (d) 1.508 GeV, and (e) 3.031 GeV. Only the four largest contributors are shown, with the bands reflecting uncertainties arising from the input electrocouplings A_h .

magnitude of ≈ 160 ppm and ≈ 70 ppm for $E_{\text{lab}} = 0.5102$ GeV and 0.855 GeV, respectively. Compared with the experimental values, the calculated B_n overshoots the asymmetries measured by the A4 Collaboration at MAMI at $\theta_{\text{lab}} \approx 35^\circ$ [15, 17] [Figs. 3(a) and 3(b)]. On the other hand, the calculated B_n is in good agreement with the high-precision Q_{weak} measurement [10] at $E_{\text{lab}} = 1.149$ GeV and $\theta_{\text{lab}} = 7.9^\circ$, within uncertainties [Figs. 3(c) and 3(d)]. The effect of the finite width at the Q_{weak} energy is relatively small at forward angles [zoomed-in plot in Fig. 3(c)], but results in a significantly reduced asymmetry at less forward angles, $\theta_{\text{lab}} \gg 20^\circ$ – 30° , compared with the zero-width approximation.

Interestingly, the recent measurement of the asymmetry by the A4 Collaboration [17] at the larger beam energy $E_{\text{lab}} = 1.508$ GeV and angle $\theta_{\text{lab}} = 34.1^\circ$ shows excellent agreement with the calculation, especially for the finite-width model. As seen in Figs. 3(d) and 3(e), the asymmetry changes sign

to become positive at intermediate and backward-scattering angles, $\theta_{\text{lab}} \gtrsim 40^\circ$ in the $E_{\text{lab}} \approx 1$ – 1.5 GeV range (see also Fig. 4 below). At beam energy $E_{\text{lab}} \approx 3$ GeV, three data points are available from the G0 [12] and HAPPEX [14] Collaborations in the forward-angle region, $6^\circ \leq \theta_{\text{lab}} \lesssim 10^\circ$. The calculated value of B_n agrees with the sign of the measured asymmetry within the uncertainty range but has slightly smaller magnitude for the HAPPEX data point in particular. A complete list of experimental and calculated B_n values is presented in Table II, including also the early SAMPLE Collaboration result [11] at $E_{\text{lab}} = 0.2$ GeV.

To further illustrate the energy dependence of the total B_n and its individual resonance contributions, we show in Fig. 4 the asymmetry as a function of E_{lab} up to 1.5 GeV at the two representative scattering angles $\theta_{\text{lab}} = 35^\circ$ and 145° that are close to the experimental values. The results illustrate again

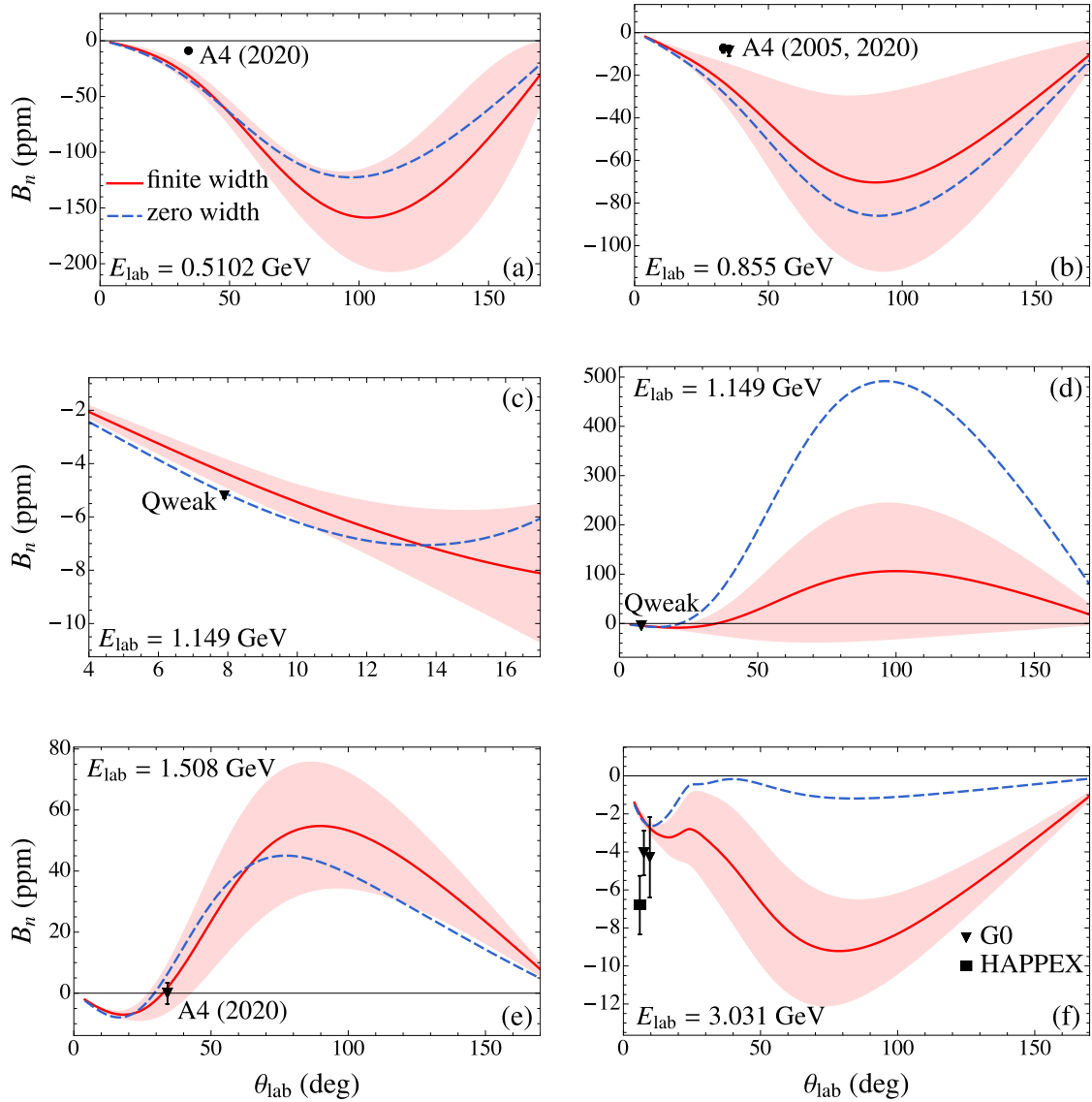


FIG. 3. The total contribution (red solid lines) from the nucleon elastic and all nine resonance states in Table I to the beam-normal SSA B_n as a function of θ_{lab} for fixed beam energies corresponding to the A4 [15,17], Q_{weak} [10], G0 [12], and HAPPEX [14] experiments (black symbols). The results in the zero-width approximation are shown for comparison (blue dashed lines).

the dominance at low energies of the total asymmetry by the $\Delta(1232)$ state. As expected, the higher mass resonances grow with increasing E_{lab} , reaching their peak values at the threshold energies of the corresponding excited states, shown in Table I. After reaching the threshold limit, the positive contributions from the two heavier states $\Delta(1700)$ and $N(1720)$ outweigh the combined negative effects of the lower-mass states $\Delta(1232)$ and $N(1520)$, yielding a net positive value of B_n at larger E_{lab} . Compared with the experimental data from the SAMPLE experiment [11] and the series of measurements by the A4 Collaboration [15–17], the calculations give the same sign as the data in Fig. 4 in the measured region. At the smaller scattering angle the calculation generally gives a larger magnitude for B_n than that observed, while at the larger scattering angles the agreement between experiment and theory is reasonable, within uncertainties. The results suggest that, while the spin 1/2 and spin 3/2 resonances give

contributions to B_n that have the correct sign and order of magnitude, there may still be room for higher spin states, such as spin 5/2 resonances, as well as nonresonant contributions to play some role.

D. Target-normal single-spin asymmetry A_n

For the target-normal SSA A_n , we consider four different beam energies, $E_{\text{lab}} = 1.245, 2.2, 3.605,$ and 6.6 GeV, corresponding to selected kinematics from the electron- ^3He scattering experiment in Jefferson Lab Hall A [28,39], and the proposed determination of the asymmetry in Ref. [27]. The contributions from the nucleon plus five largest excited-state resonances, $\{\Delta(1232), N(1520), N(1535), \Delta(1700),$ and $N(1720)\}$, to the total A_n are shown in Fig. 5 as function of the scattering angle θ_{lab} at the chosen beam energies. For the highest energy $E_{\text{lab}} = 6.6$ GeV, the asymmetry is shown up to

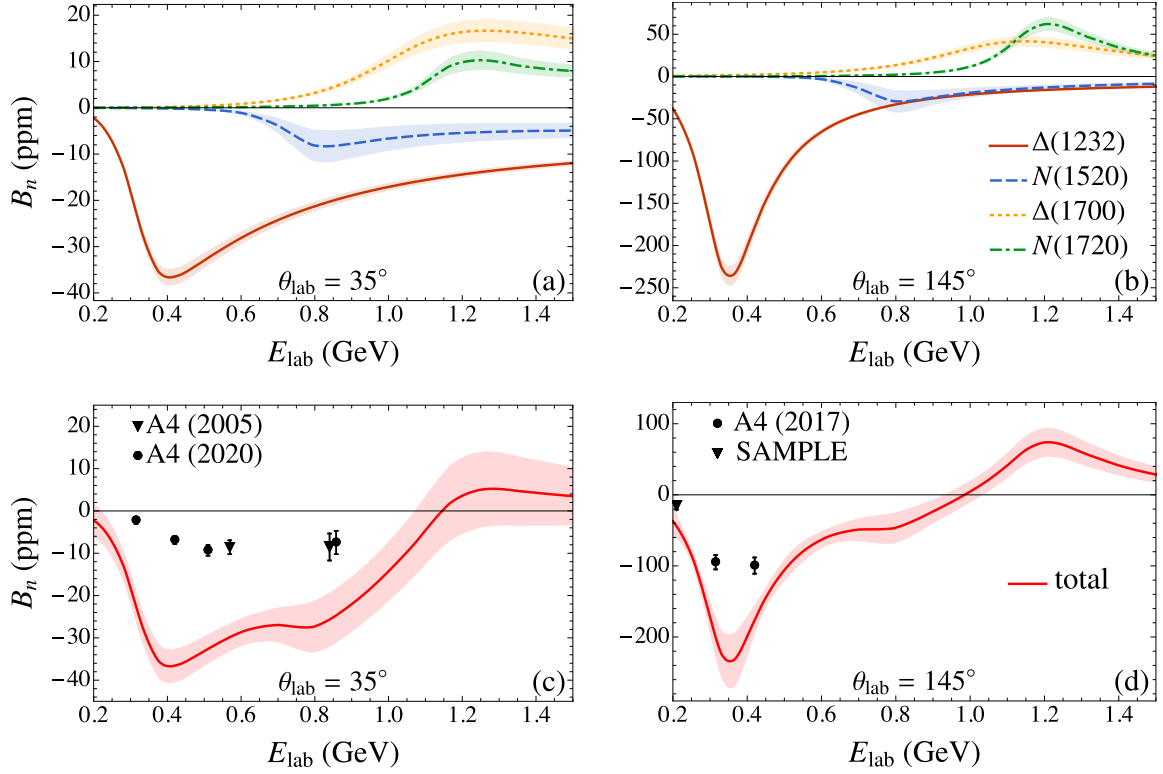


FIG. 4. Beam-normal SSA B_n as a function of beam energy E_{lab} in the laboratory frame at representative scattering angles $\theta_{\text{lab}} = 35^\circ$ and 145° . Contributions from the four largest resonance contributors are shown in the top row, while the bottom row represents the total B_n from the nucleon plus all nine resonances. The experimental data points in the forward angle region are from A4 experiments [15,17], and in the backward angle region from the SAMPLE [11] and A4 [16] experiments.

a scattering angle $\theta_{\text{lab}} \approx 25^\circ$, corresponding to $Q^2 = 5 \text{ GeV}^2$, beyond which the hadronic approximation and the input electrocouplings parametrization used in the calculation are not expected to be reliable.

As anticipated, A_n is in the subpercent to percent range and keeps increasing with beam energy in the far-forward to backward directions, in contrast with the beam-normal SSA B_n . To further compare with B_n , we observe that the nucleon

TABLE II. Experimental and calculated beam-normal SSA B_n from various experiments, along with the corresponding kinematics in the laboratory frame. The calculated results include uncertainty estimates from the input helicity amplitudes, while the experimental results give both statistical and systematic uncertainties.

Experiment	E_{lab} (GeV)	θ_{lab} ($^\circ$)	Q^2 (GeV^2)	Calculated B_n (ppm)	Experimental B_n (ppm)
SAMPLE (2001) [11]	0.2	146.1	0.1	-40.5 ± 4.5	-15.4 ± 5.4
A4 (2005) [15]	0.855	35.0	0.230	-25.1 ± 10.1	$-8.52 \pm 2.31 \pm 0.87$
	0.569	35.0	0.106	-29.9 ± 6.8	$-8.59 \pm 0.89 \pm 0.75$
G0 (2007) [12]	3.031	7.5	0.15	-2.36 ± 0.31	$-4.06 \pm 0.99 \pm 0.63$
	3.031	9.6	0.25	-2.73 ± 0.35	$-4.28 \pm 1.87 \pm 0.98$
G0 (2011) [13]	0.362	108.0	0.22	-320 ± 80	-176.5 ± 9.4
	0.687	108.0	0.63	-87 ± 60	-21 ± 24
HAPPEX (2012) [14]	3.026	6.0	0.099	-2.01 ± 0.27	-6.80 ± 1.54
A4 (2017) [16]	0.315	145.0	0.22	-201 ± 88	$-94.83 \pm 6.02 \pm 4.07$
	0.420	145.0	0.350	-176 ± 44	$-99.55 \pm 6.73 \pm 4.63$
A4 (2020) [17]	0.315	34.1	0.032	-21 ± 11	$-2.22 \pm 0.40 \pm 0.43$
	0.42	34.1	0.057	-34.5 ± 9.4	$-6.88 \pm 0.53 \pm 0.42$
	0.510	34.1	0.082	-31.0 ± 7.3	$-9.32 \pm 0.63 \pm 0.62$
	0.855	34.1	0.218	-24 ± 10	$-7.46 \pm 1.22 \pm 1.55$
	1.508	34.1	0.613	1.7 ± 8.2	$-0.06 \pm 2.89 \pm 1.90$
Q_{weak} (2020) [10]	1.149	7.9	0.0248	-4.34 ± 0.54	$-5.194 \pm 0.067 \pm 0.082$

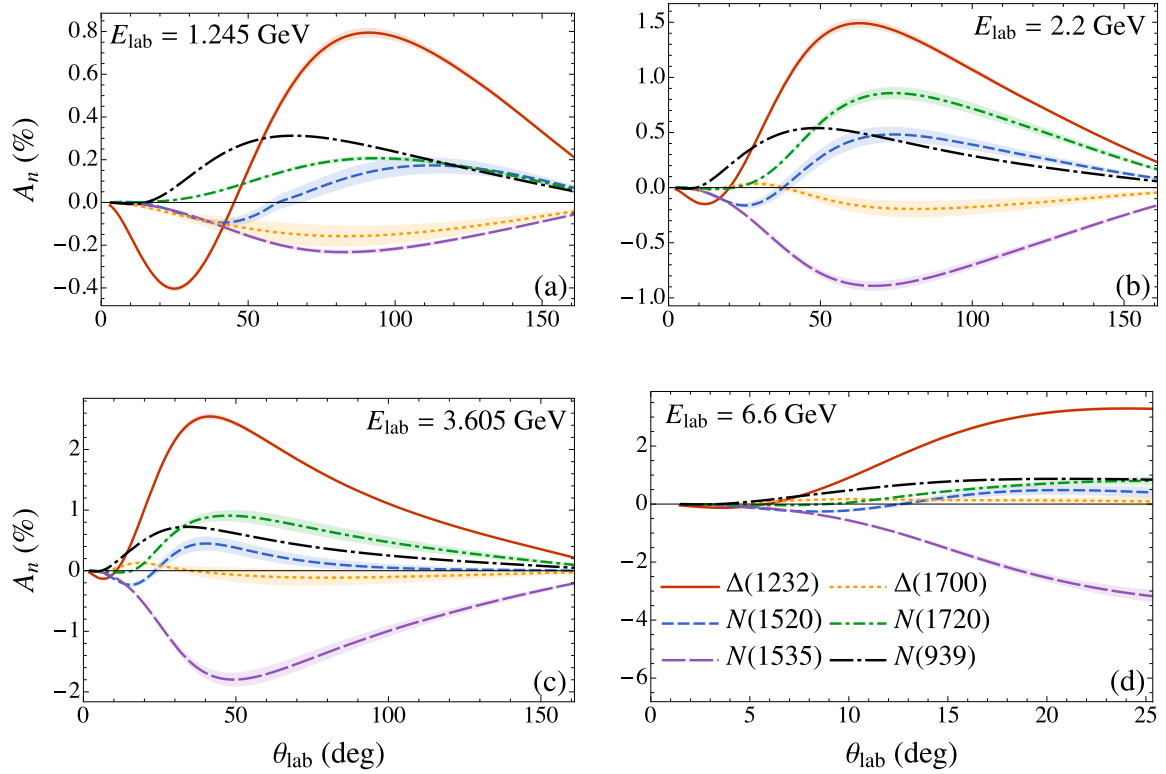


FIG. 5. The six largest contributions to the target-normal SSA A_n (in percent) as a function of the scattering angle θ_{lab} at four representative beam energies E_{lab} given by (a) 1.245 GeV, (b) 2.2 GeV, (c) 3.605 GeV, and (d) 6.6 GeV.

intermediate state alone has significant impact on the total A_n for any value of E_{lab} . Among the resonances, the $\Delta(1232)$ is again the dominant contributor over the entire range of θ_{lab} and for all beam energies considered. Particularly at forward angles, $\theta_{\text{lab}} \lesssim 20^\circ$, the only sizable contribution is that from the $\Delta(1232)$ state. The effect of other resonances becomes comparable with the $\Delta(1232)$ at relatively larger scattering angles.

Interestingly, unlike for B_n and the real part of the TPE correction [8], the contribution to the target-normal SSA A_n from the spin 3/2 nucleon state $N(1520)$ is relatively less significant for all beam energies E_{lab} considered over the entire range of θ_{lab} . The two other spin 3/2 resonances, the $\Delta(1700)$ and $N(1720)$, have noticeable contributions at the lower beam energies, $E_{\text{lab}} = 1.245$ and 2.2 GeV, but are of opposite sign, as shown in Fig. 5. At higher beam energies, the contribution to A_n from these two states becomes negligible [Figs. 5(c) and 5(d)]. On the other hand, the only spin 1/2 state, $N(1535)$, is found to be a significant contributor to the total A_n . As shown in Fig. 5(a), for $E_{\text{lab}} = 1.245$ GeV the A_n from $N(1535)$ outweighs the contribution from all other states, with the exception of $\Delta(1232)$. With increasing E_{lab} , the contribution from $N(1535)$ rises even faster, almost negating the $\Delta(1232)$ contribution alone at the highest beam energy in Fig. 5(d). Considering all such partial cancellations, however, the sum of the elastic nucleon and $\Delta(1232)$ resonance contributions appears to be a good approximation to the total A_n .

The total target-normal SSA A_n , including contributions from the nucleon elastic and the nine spin 1/2 and 3/2 resonances is illustrated in Fig. 6 as a function of the scattering angle θ_{lab} at the same four fixed beam energies. The results of the finite-width calculation, using a Sill distribution as in Eq. (34), are compared with the zero-width approximation. The finite-width results are qualitatively similar to the approximated ones, but quantitatively there are clear differences in some kinematic regions.

In general, the zero-width results have a smaller magnitude for the total A_n than the finite-width case. However, as observed above, the net A_n from the elastic nucleon and the resonances resembles the trend of the $\Delta(1232)$ state alone. The overall magnitude of the asymmetry can also be well approximated by the sum of the elastic nucleon and $\Delta(1232)$ contributions. As for the beam-normal SSA B_n , contribution from higher spin states, with spin $\geq 5/2$, as well as nonresonant backgrounds may need to be considered in future.

Unfortunately, to date there have not been any direct measurements of A_n in electron-proton scattering. However, there has been a measurement of A_n for electron scattering from polarized ^3He in the quasi-elastic region at Jefferson Lab Hall A [28], from which the electron-neutron asymmetry was extracted assuming an input ep asymmetry. The experiment scattered unpolarized electrons with energies $E_{\text{lab}} = 1.245$, 2.425, and 3.605 GeV from a ^3He target polarized normal to the scattering plane, with the scattered electrons detected at angle $\theta_{\text{lab}} = 17^\circ$, corresponding to three different c.m.

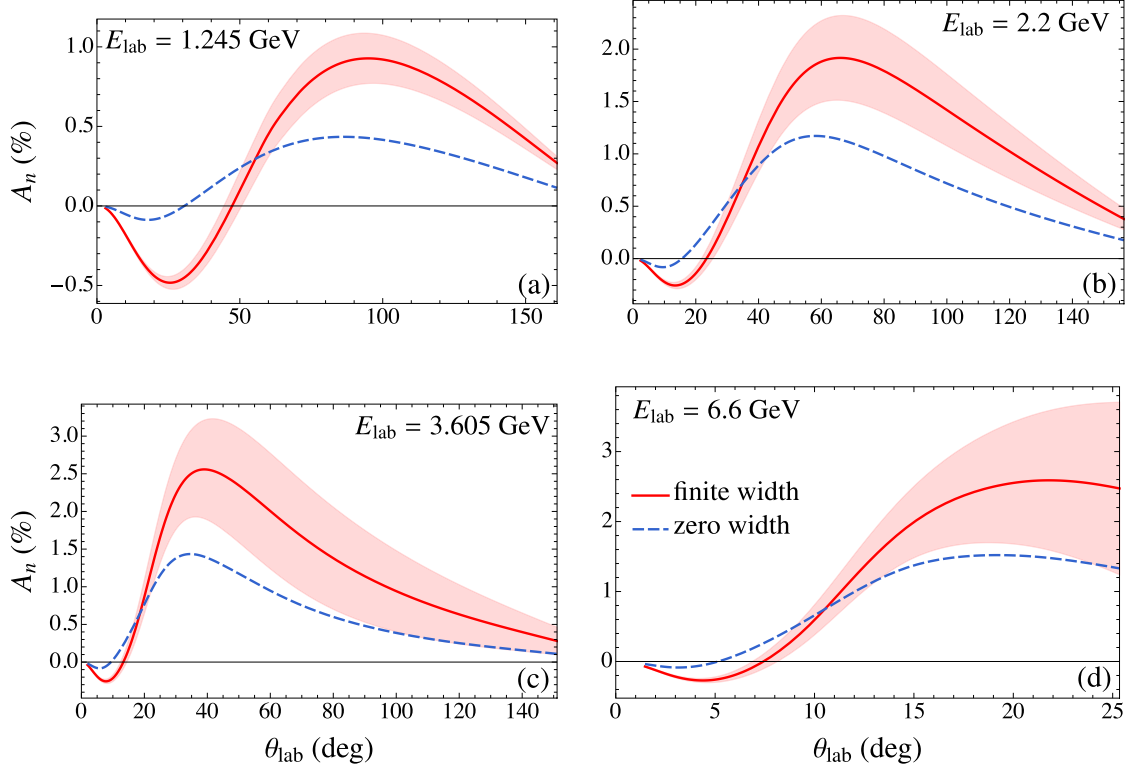


FIG. 6. The total contribution (red solid lines) from the nucleon plus all nine resonances to the target-normal SSA A_n as a function of θ_{lab} at fixed beam energies E_{lab} of (a) 1.245 GeV, (b) 2.2 GeV, (c) 3.605 GeV, and (d) 6.6 GeV. The zero-width results (blue dashed lines) are also shown for comparison.

angles $\theta_{\text{c.m.}} = 32^\circ$, 41° , and 48° for the three respective beam energies. For the input proton SSA A_n , the elastic proton intermediate-state contribution to the TPE amplitude from Ref. [29], giving $(0.01 \pm 0.22)\%$, $(0.24 \pm 2.96)\%$, and $(0.62 \pm 1.09)\%$ at the three beam energies shown in Table III, respectively, was used to extract the neutron asymmetry from the measured ^3He SSA.

In contrast, in this work we find a total contribution to A_n from the nucleon (N) elastic state and the nine resonances of $(-0.381 \pm 0.018)\%$, $(-0.173 \pm 0.049)\%$, and $(0.414 \pm 0.145)\%$, at $\theta_{\text{lab}} = 17^\circ$ and beam energies $E_{\text{lab}} = 1.245$, 2.425, and 3.605 GeV, respectively, illustrated in the central column of Table III. Overall, the input proton asymmetry A_n from Ref. [29] is larger than our calculated result for the nucleon elastic state only, although consistent within the uncertainty. The nucleon resonant contribution is sizable

TABLE III. Comparison of the target-normal SSA A_n calculated in this work with that used as input in the data analysis of electron- ^3He scattering, at the beam energies $E_{\text{lab}} = 1.245$, 2.425, and 3.605 GeV, and scattering angle $\theta_{\text{lab}} = 17^\circ$ [28].

E_{lab} (GeV)	A_n (this work)		Input A_n in Ref. [28]
	N	$N + \text{resonances}$	N only
1.245	0.008	-0.381 ± 0.018	0.01 ± 0.22
2.425	0.173	-0.173 ± 0.049	0.24 ± 2.96
3.605	0.400	0.414 ± 0.145	0.62 ± 1.09

at smaller beam energies but is negligible at the highest energy $E_{\text{lab}} = 3.605$ GeV.

E. Beam-transverse single-spin asymmetry B_x

A general electron-spin vector transverse to the beam direction \hat{z} is given by

$$\mathbf{S} = \cos \phi_s \hat{x} + \sin \phi_s \hat{y}, \quad (39)$$

where ϕ_s is the azimuthal angle with respect to the scattering (\hat{x} - \hat{z}) plane. As discussed in Sec. I above, interference between the OPE amplitude and the imaginary part of the TPE amplitude produces a beam-normal SSA B_n which depends only on the normal component of the spin.

In principle, a beam-transverse SSA can also arise from the x component of \mathbf{S} due to a parity-violating interaction. At lowest order this involves the interference of the OPE amplitude \mathcal{M}_γ and the Z-exchange amplitude \mathcal{M}_Z . This same interference gives the usual lowest order parity-violating asymmetry A^{PV} for a longitudinally polarized beam,

$$A^{\text{PV}} = -\frac{G_F}{\sqrt{2}} \frac{Q^2}{4\pi\alpha} \frac{1}{\sigma_R} \left\{ g_V^e G_M G_A^Z \nu (1 - \epsilon) - g_A^e [G_E^Z \epsilon + G_M^Z \tau] \right\}, \quad (40)$$

where g_V^e and g_A^e are the vector and axial-vector eZ couplings, G_A^Z , G_E^Z , and G_M^Z are the proton weak form factors, G_F is the Fermi constant, and the reduced cross section is $\sigma_R = G_E^2 \epsilon + G_M^2 \tau$. The kinematic variables in Eq. (40) are dimensionless

quantities that can be expressed in terms of the Mandelstam variables s , t , and u as

$$\tau = -\frac{t}{4M^2}, \quad \nu = \frac{s-u}{4M^2}, \quad \epsilon = \frac{\nu^2 - \tau(\tau+1)}{\nu^2 + \tau(\tau+1)}. \quad (41)$$

The SSA for a purely transverse in-plane beam, denoted B_x , is given by

$$B_x = -\frac{G_F}{\sqrt{2}} \frac{Q^2}{4\pi\alpha} \frac{1}{\sigma_R} \frac{m_e}{E_{\text{lab}}} \sqrt{\frac{2\epsilon(1-\epsilon)}{\tau+1}} \left\{ g_V^e G_M G_M^Z 2\tau(\tau+1) - g_A^e [G_E G_E^Z (\nu + \tau + 1) + G_M G_M^Z \tau(\nu - \tau - 1)] \right\}. \quad (42)$$

In combination with B_n , this results in a general beam asymmetry of the form

$$B_n \sin \phi_s + B_x \cos \phi_s = \sqrt{B_n^2 + B_x^2} \sin(\phi_s + \delta_s), \quad (43)$$

$$\delta_s = \tan^{-1} \left(\frac{B_x}{B_n} \right).$$

The general beam asymmetry then retains a sinusoidal dependence on ϕ_s , but with a phase shift δ_s relative to the pure beam-normal SSA.

To obtain an order of magnitude estimate of the various asymmetries, we can write

$$A^{\text{PV}} \approx \frac{Q^2}{M_Z^2} \approx Q^2 \times 10^{-4}, \quad (44a)$$

$$B_n \approx \alpha \frac{m_e}{M} \approx 5 \times 10^{-6}, \quad (44b)$$

$$B_x \approx \frac{Q^2}{M_Z^2} \frac{m_e}{M} \approx Q^2 \times (5 \times 10^{-8}), \quad (44c)$$

with Q^2 in units of GeV^2 . Aside from using muons instead of electrons, there seems to be no natural way to enhance the ratio B_x/A^{PV} over the naive estimate of 5×10^{-4} . For the kinematics given in Table II, the largest value of this ratio is 2×10^{-4} for the A4 (2020) kinematics, suggesting that the transverse parity-violating asymmetry is indeed negligible compared with the longitudinal asymmetry. Measuring a phase shift $\delta_s \approx B_x/B_n$ seems equally unlikely, although the ratio could potentially be enhanced at higher Q^2 .

V. CONCLUSIONS

In this study we have calculated beam and target-normal single-spin asymmetries in elastic electron-proton scattering using the imaginary part of two-photon exchange amplitudes, including contributions from $J^P = 1/2^\pm$ and $3/2^\pm$ excited-state resonances with mass below 1.8 GeV. For the resonance electrocouplings at the hadronic vertices we employed helicity amplitudes from the latest analysis of CLAS meson electroproduction data at $Q^2 \lesssim 5 \text{ GeV}^2$.

The effect of finite resonance widths on the beam-normal SSA B_n has been carefully investigated and found to be negligible in the forward-angle region, becoming more noticeable at larger scattering angles. We believe this may be attributable to a non-negligible contribution from the QRCS region above the nominal threshold excitation energy.

Among the various intermediate-state contributions to B_n , the elastic nucleon and spin 1/2 resonances are suppressed by an order of magnitude or more compared with the spin 3/2 resonances. The $\Delta(1232)$ resonance alone is a good approximation at forward angles for all beam energies. The $N(1520)$ contribution is noticeably smaller than the $\Delta(1232)$, but both are negative across the range of energies and angles considered. The $\Delta(1700)$ and $N(1720)$ are major contributors in the far-forward and -backward angle regions above their threshold excitation energies, both having positive contributions across energy and angle. As a result, the total B_n is somewhat sensitive to cancellations between the resonance contributions, changing from negative to positive with increasing energy and angle. Uncertainties in the input electrocouplings are also significant for the $N(1520)$, $\Delta(1700)$, and $N(1720)$ states, leading to a rather large overall uncertainty band in the total B_n .

The results given in this work tend to overshoot the experimental B_n data at lower beam energies $E_{\text{lab}} < 1 \text{ GeV}$ at both forward and backward angles. This is the region in which the $\Delta(1232)$ dominates, with relatively small uncertainties in its input parameters. There is good agreement between theory and the high-precision Q_{weak} measurement at $E_{\text{lab}} = 1.149 \text{ GeV}$, and modest agreement at the highest available energy $E_{\text{lab}} \approx 3 \text{ GeV}$ and very forward angles, where the experimental uncertainties from the G0 and HAPPEX data are rather large.

For the target-normal SSA A_n , the higher resonances beyond the $\Delta(1232)$ have almost no net effect. Unlike B_n , the elastic nucleon intermediate state makes a significant contribution over the entire range of energy, E_{lab} from 0.5 to 6.6 GeV, considered in this work. The sum of nucleon and $\Delta(1232)$ contributions account for most of the total A_n . The spin 3/2 state $N(1520)$ is less significant for A_n than it is for B_n , but the spin 1/2 state $N(1535)$ becomes a major contributor. Also, unlike B_n , the peak magnitude of A_n versus θ_{lab} increases with energy in the range from $E_{\text{lab}} = 0.5$ to 6.6 GeV.

For future work, given the significant uncertainties in the parameters of the higher mass resonances, better data to constrain electrocouplings for the higher mass excitations, such as the $\Delta(1700)$, would be helpful. Effects of higher spin states, with spin $\geq 5/2$, can also be investigated, although uncertainties in the electrocouplings would limit the predictive power of such calculations. Carlson *et al.* [40] also extended the calculation of beam SSAs from excited-state resonance contributions to inelastic channels, such as the $ep \rightarrow e\Delta(1232)$ production process. Finally, we note that an interesting quark level study [19] of beam-normal SSAs, applicable at high- $Q^2 \gg M^2$ region, was performed in terms of a convolution of quark amplitudes and generalized parton distributions, which could be viewed as complementary to the resonance dominated region discussed in our analysis.

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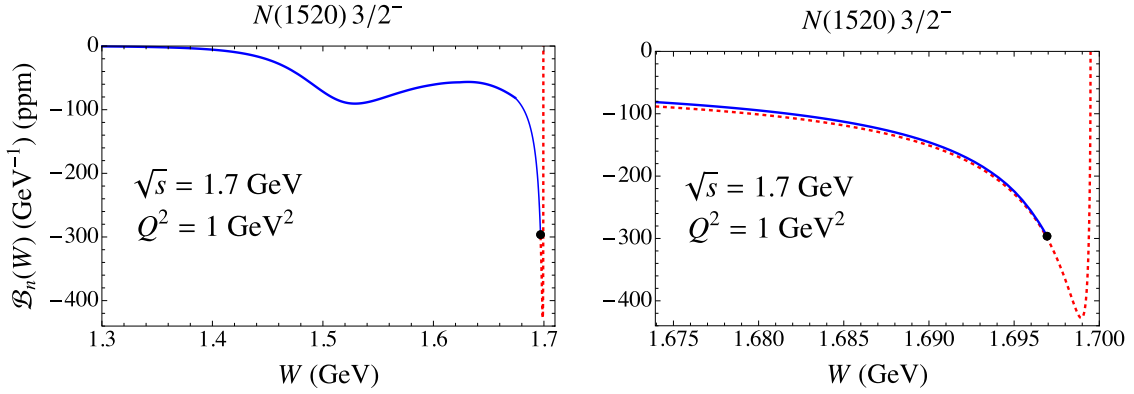


FIG. 7. The integrand $\mathcal{B}_n(W)$ of Eq. (A1) as a function of W for the $N(1520)$ resonance at a sample kinematics of $\sqrt{s} = 1.7$ GeV and $Q^2 = 1$ GeV². The right panel is magnified to show the quasisingular behavior of $\mathcal{B}_n(W)$ as $W \rightarrow W_{\max} = \sqrt{s} - m_e$. The dashed red line makes use of the analytic result of Eq. (A2), while the solid blue line is the fully numerical evaluation. The dot indicates our chosen matching point at $W = W_{\max} - 5m_e$.

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APPENDIX: NUMERICAL EVALUATION OF B_n IN THE QUASIREAL COMPTON-SCATTERING REGION

We elaborate here on our semi-analytic method of evaluating the integral of Eq. (32) in the QRCS region $W \approx W_{\max} = \sqrt{s} - m_e$. We define $B_n(W)$ via

$$B_n = \int_{W_{\text{th}}}^{W_{\max}} dW \mathcal{B}_n(W), \quad (\text{A1})$$

so that $\mathcal{B}_n(W)$ includes the angular integrals of Eq. (32). As discussed in Sec. III C, in the QRCS region the slowly varying tensor product $L_{\rho\mu\nu}H^{\rho\mu\nu}$ for B_n in Eq. (32) is evaluated at $Q_1^2 = Q_2^2 = 0$, leaving a numerator independent of θ_{k_1} and ϕ_{k_1} . The resulting expression is proportional to $J(W)$ as defined in Eq. (37), which can be evaluated analytically. Applying

Eqs. (36) and (37) of Ref. [33] to the present case, we find in agreement with Ref. [23] that

$$J(W) = \frac{\pi}{2\sqrt{s}Q^2 E_k x_1} \log\left(\frac{x_1 + x_2}{x_1 - x_2}\right),$$

$$x_1 = \sqrt{x_2^2 + \frac{4m_e^2}{Q^2}(1-z)^2}, \quad x_2 = \frac{|\mathbf{k}_1|}{E_k}, \quad (\text{A2})$$

where $z = E_{k_1}/E_k$.

Figure 7 shows $\mathcal{B}_n(W)$ for the $N(1520)$ resonance at the sample kinematics of $\sqrt{s} = 1.7$ GeV ($E_{\text{lab}} = 1.071$ GeV) and $Q^2 = 1$ GeV². This is above the nominal threshold energy of $E_{\text{lab}}^{\text{th}} = 0.75$ GeV for excitation of a zero-width resonance (see Table I). As shown in the left panel of Fig. 7, due to the behavior of $J(W)$, $\mathcal{B}_n(W)$ increases in magnitude with W above threshold, and has an extremum near $W = W_{\max} - m_e$ before falling sharply to zero at $W = W_{\max}$. The right panel is magnified to show the matching between the full numerical and semi-analytical regions. The dot indicates our chosen matching point at $W = W_{\max} - 5m_e$.

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