Momentum dependence of the spin alignment of the ϕ meson

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We study the rapidity and azimuthal angle dependences of the global spin alignment ρ_{00} for ϕ mesons with respect to the reaction plane in Au+Au collisions at the BNL Relativistic Heavy Ion Collider by the relativistic coalescence model in the spin transport theory. The global spin alignment of ϕ mesons arises from local fluctuations of strong force fields whose values are extracted from the STAR's data. The calculated results show that $\rho_{00} < 1/3$ at the rapidity $Y = 0$, and then it increases with rapidity and becomes $\rho_{00} > 1/3$ at $Y = 1$. Such a rapidity dependence is dominated by the relative motion of the ϕ meson in the bulk matter. We also give a prediction for the azimuthal angle dependence of ρ_{00} at different rapidities.

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I. INTRODUCTION

In noncentral heavy-ion collisions, the colliding nuclei carry a global orbital angular momentum (OAM). A small portion of the global OAM is transferred into the quark-gluon plasma (QGP) in the form of vorticity fields. Quarks are then polarized by vorticity fields through spin-orbit couplings and form polarized hadrons by coalescence or recombination [1–5], see Refs. [6–11] for recent reviews. The effect of the vorticity field is supported by the global spin polarization of Λ and $\overline{\Lambda}$ hyperons observed by the STAR collaboration in Au+Au collisions [12,13]. According to the spin-flavor wave functions of Λ and $\overline{\Lambda}$ in the quark model [1,14], the spins of Λ and $\overline{\Lambda}$ are carried by s and $\bar{s}$ quarks, respectively, indicating that strange quarks in the QGP are also globally polarized.

It was proposed by Liang and Wang in 2005 that the polarized quarks can also form vector mesons by recombination with nonvanishing spin alignment [15]. The spin of a vector meson is described by a 3×3 spin density matrix ρ with elements $\rho_{\lambda_1\lambda_2}$, where subscripts $\lambda_1, \lambda_2 = 0, \pm 1$ label the spin quantum number along a specific spin quantization direction. The spin alignment refers to the element ρ_{00}, which denotes the probability for the spin state with $\lambda = 0$. The polarization vector of the vector meson is preferably aligned in the spin quantization direction when $\rho_{00} > 1/3$, while it is preferably aligned in the perpendicular direction when $\rho_{00} < 1/3$. In experiments, the spin alignment can be measured through polar angle distributions of daughter particles in p-wave strong decays, such as $\phi \to K^+ + K^-$ [14–18], or dilepton decays, such as $J/\psi \to \mu^+ + \mu^-$ [19–22].

Recently, the STAR collaboration has measured the global spin alignment of ϕ mesons with respect to the reaction plane in Au+Au collisions [18], and the results show a significant positive deviation from $1/3$. Such a deviation is much larger than contributions from conventional mechanisms such as vorticity fields and magnetic fields [14,15,23–25]. Other possible contributions are proposed in [26–30] but without quantitative results that can be compared with experimental data. Up to now, the effect of local correlation or fluctuation of a kind of the strong force field called the ϕ field [31–33] was the only mechanism that could quantitatively explain the STAR data. According to the chiral quark model [34–39], the SU(3) octet vector fields in the form of a 3×3 matrix can be induced by vector currents of pseudo-Goldstone bosons that surround s and $\bar{s}$ quarks in the hadronization stage. The ϕ field is just the "33" component of the SU(3) octet vector fields that is coupled to s and $\bar{s}$.

In this paper, we study the momentum dependence of ρ_{00}^y in the relativistic coalescence model based on the spin Boltzmann equation in nonequilibrium transport theory [32,33]. Here, the superscript $i = x, y, z$ in ρ_{00}^i denotes the spin quantization direction. In this paper, we choose the beam direction along the z axis, the normal direction of the reaction plane along the y axis, and the impact parameter along the x axis. The parameters for the ϕ field's local fluctuations are extracted by fitting the STAR data for ρ_{00}^y and ρ_{00}^x as functions of collision energy [18], and the values of these parameters are the same as in Ref. [33]. Some recent reviews on the STAR experiment and theoretical models can be found in Refs. [40–42].

The paper is organized as follows. In Sec. II we give the formula for ρ_{00} as functions of the ϕ field's fluctuations. Assuming that the fluctuations are isotropic in the laboratory frame, we derive a compact formula for the momentum dependence of ρ_{00}. In Sec. III we compute the rapidity and azimuthal angle dependence of the global spin alignment ρ_{00}^y for ϕ mesons in Au+Au collisions. Finally we make a summary and a discussion on the our result in Sec. IV. As notational convention, we use boldface symbols for three-dimensional vectors, such as $\mathbf{p} = (p_x, p_y, p_z)$.

II. THEORETICAL MODEL

In nonrelativistic quark coalescence model, the spin alignment of the ϕ meson depends on the spin polarizations of its constituent quark and antiquark [14,15,31]. In Refs. [32,33], some of us constructed a relativistic quark coalescence model based on spin Boltzmann equation in transport theory to describe the spin phenomena of vector mesons. Considering the quark polarization induced by $F_\phi^{\mu\nu}$, the field strength tensor of the ϕ field, the spin alignment for the ϕ meson is given by [32,33]

$$\rho_{00}(x,\mathbf{p}) = \frac{1}{3} - \frac{4g_\phi^2}{m_\phi^2 T_\text{h}^2} C_1\left[\frac{1}{3}\mathbf{B}'_\phi\cdot\mathbf{B}'_\phi - (\boldsymbol{\epsilon}_0\cdot\mathbf{B}'_\phi)^2\right] - \frac{4g_\phi^2}{m_\phi^2 T_\text{h}^2} C_2\left[\frac{1}{3}\mathbf{E}'_\phi\cdot\mathbf{E}'_\phi - (\boldsymbol{\epsilon}_0\cdot\mathbf{E}'_\phi)^2\right], \quad (1)$$

where C_1 and C_2 are two coefficients depending on m_ϕ (the ϕ meson's mass) and m_s (mass of constituent strange quark), T_h is the local temperature at the hadronization time, and $\boldsymbol{\epsilon}_0$ denotes the unit vector along the measuring (spin quantization) direction which is also the ϕ meson's polarization vector for the spin state $\lambda = 0$. Here, $\mathbf{E}'_\phi$ and $\mathbf{B}'_\phi$ are electric and magnetic parts of the ϕ field in the meson's rest frame, which are functions of spacetime. When boosted to the laboratory frame, $\mathbf{E}'_\phi$ and $\mathbf{B}'_\phi$ also depend on the meson's momentum $\mathbf{p}$, i.e., they can be expressed in terms of fields in the laboratory frame $\mathbf{B}_\phi$ and $\mathbf{E}_\phi$ as

$$\mathbf{B}'_\phi = \gamma\mathbf{B}_\phi - \gamma\mathbf{v}\times\mathbf{E}_\phi + (1-\gamma)\frac{\mathbf{v}\cdot\mathbf{B}_\phi}{v^2}\mathbf{v},$$
$$\mathbf{E}'_\phi = \gamma\mathbf{E}_\phi + \gamma\mathbf{v}\times\mathbf{B}_\phi + (1-\gamma)\frac{\mathbf{v}\cdot\mathbf{E}_\phi}{v^2}\mathbf{v}, \quad (2)$$

where $\gamma = E_\mathbf{p}^\phi/m_\phi$ is the Lorentz contraction factor and $\mathbf{v} = \mathbf{p}/m_\phi$ is the ϕ meson's velocity. Substituting Eq. (2) into Eq. (1), we are able to express ρ_{00} in terms of $\mathbf{B}_\phi$ and $\mathbf{E}_\phi$ which depend only on spacetime but not on momentum.

We learn from Eq. (1) that the deviation from 1/3 for ρ_{00} is caused by the anisotropy of local field fluctuations in the meson's rest frame. The deviation is positive (negative) when the fluctuation of $\mathbf{B}'_\phi$ or $\mathbf{E}'_\phi$ in the measuring direction $\boldsymbol{\epsilon}_0$ is larger (smaller) than the average fluctuation in directions perpendicular to $\boldsymbol{\epsilon}_0$. We assume that the fluctuations in the laboratory frame are parametrized in an anisotropic form,

$$\langle g_\phi^2 \mathbf{B}_\phi^i \mathbf{B}_\phi^j/T_\text{h}^2\rangle = \langle g_\phi^2 \mathbf{E}_\phi^i \mathbf{E}_\phi^j/T_\text{h}^2\rangle = F^2\delta^{ij} + \Delta\hat{\mathbf{a}}^i\hat{\mathbf{a}}^j, \quad (3)$$

while the correlation between $\mathbf{B}_\phi$ and $\mathbf{E}_\phi$ is neglected, $\langle g_\phi^2 \mathbf{B}_\phi^i \mathbf{E}_\phi^j/T_\text{h}^2\rangle = 0$. Here, $\langle\cdots\rangle$ denotes the spacetime average, $\hat{\mathbf{a}}$ denotes the direction of anisotropy, F^2 is the isotropic part of the fluctuation, and Δ denotes the difference between the fluctuation in $\hat{\mathbf{a}}$ and the average fluctuation in directions perpendicular to $\hat{\mathbf{a}}$. The field configuration in Eq. (3) becomes isotropic when $\Delta = 0$. The field fluctuations in the meson's rest frame can be obtained using Eqs. (2) and (3).

By substituting Eq. (2) into Eq. (1), averaging over spacetime, and then applying Eq. (3) with $\Delta = 0$, we obtain the spin alignment in the y direction, the direction of the global OAM with $\boldsymbol{\epsilon}_0 = (0, 1, 0)$,

$$\langle\delta\rho_{00}^y\rangle(\mathbf{p}) = \frac{8}{3m_\phi^4}(C_1+C_2)F^2\left(\frac{p_x^2+p_z^2}{2} - p_y^2\right), \quad (4)$$

where $\delta\rho_{00}^y \equiv \rho_{00}^y - 1/3$ and $(C_1 + C_2)F^2$ is a positive number. Obviously, for static ϕ mesons with $\mathbf{p} = 0$ we should have $\rho_{00}^y = 1/3$ because field fluctuations are isotropic in the laboratory frame. However, the motion of the ϕ meson will break the symmetry. According to the Lorentz transformation of fields in Eq. (2), field components perpendicular to the motion direction are enhanced by the γ factor, while the component in the motion direction is not. Therefore when observing in the meson's rest frame, the fluctuation in the direction of motion will be smaller than fluctuations in perpendicular directions. For a meson with $p_x = p_z = 0$ and $p_y \neq 0$, we obtain the relation $\langle(\mathbf{B}'_{\phi,y})^2\rangle < \langle(\mathbf{B}'_{\phi,x})^2\rangle = \langle(\mathbf{B}'_{\phi,z})^2\rangle$, leading to $\rho_{00}^y < 1/3$ according to Eq. (1) or (4). Similarly, motions along x and z directions lead to $\rho_{00}^y > 1/3$.

We can also rewrite the result in Eq. (4) in terms of the transverse momentum p_T, the azimuthal angle φ, and the rapidity Y,

$$\langle\delta\rho_{00}^y\rangle(\mathbf{p}) \propto \frac{1}{2}p_T^2[3\cos(2\varphi) - 1] + \sqrt{m_\phi^2 + p_T^2}\sinh^2 Y. \quad (5)$$

The azimuthal angle dependence shows a $\cos(2\varphi)$ structure, which was first derived in Ref. [33]. We also find that the spin alignment increases with rapidity. This is because the anisotropy of field fluctuations in the meson's rest frame becomes more significant in the direction of the meson's larger momentum.

We note that Eqs. (4) and (5) are based on the assumption (3) with vanishing anisotropy $\Delta = 0$ in the laboratory frame. A nonzero Δ will contribute to ρ_{00}^y, but the analytical formula is too complicated to be given here. Our numerical results for nonzero Δ in the next section will show that its contribution is small compared with the results for $\Delta = 0$. So the spin alignment is dominated by the isotropic part of the field fluctuations.

III. NUMERICAL RESULTS

Similar to the previous work [33] by some of us, we consider the anisotropy with respect to the z direction or the beam direction in heavy-ion collisions. So we assume that transverse and longitudinal fluctuations are different $F_T^2 = F^2$ and $F_z^2 = F^2 + \Delta$, which are regarded as parameters that can be extracted from the STAR's data on momentum integrated ρ_{00}^y [18]. At collision energies $\sqrt{s_{NN}} = 11.5$, 19.6, 27, 39, 62.4, and 200 GeV, the extracted values of F^2 and Δ are $F^2/m_\pi^2 = 16.5$, 3.74, 3.42, 1.02, 2.85, 0.359 and $\Delta/m_\pi^2 = 3.33$, -0.468, -0.9, 0.218, 0.336, 0.128, respectively.

In Fig. 1, we present ρ_{00}^y as functions of the ϕ meson's rapidity Y at collision energies $\sqrt{s_{NN}} = 11.5$–200 GeV. At each rapidity and energy, we assume that the spacetime averages of fluctuations follow Eq. (3). We take averages over the meson's transverse momentum in the range 1.2 GeV $< p_T <$ 5.4 GeV and the azimuthal angle in the range $0 < \varphi < 2\pi$, weighted by the ϕ meson's momentum spectra

$$E_\mathbf{p}\frac{d^3N}{d^3\mathbf{p}} = \frac{d^2N}{2\pi p_T dp_T dY}[1 + 2v_2(p_T)\cos(2\varphi)], \quad (6)$$

where the transverse momentum spectra and $v_2(p_T)$ are taken from STAR's data [43–46]. At all energies, the derivation

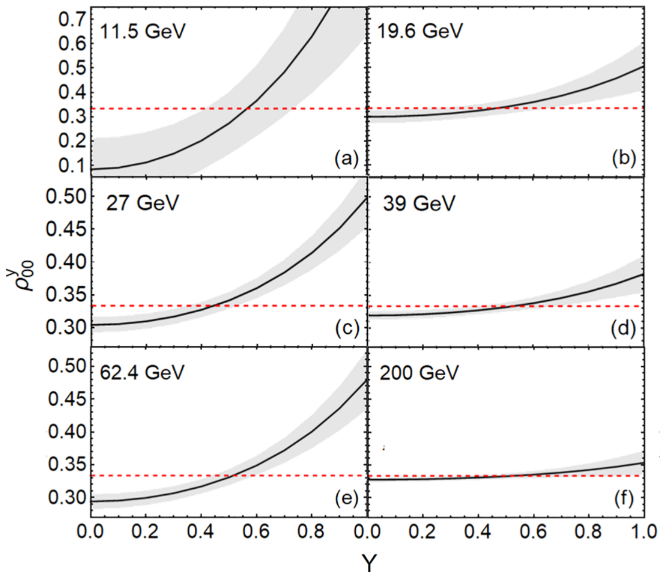

FIG. 1. The global spin alignment ρ_{00}^{y} (solid lines) as functions of rapidity at collision energies $\sqrt{s_{NN}} = 11.5, 19.6, 27, 39, 62.4$, and 200 GeV. The shaded areas are error bands from fitting parameters for local fluctuations in strong force fields. The dashed lines indicate the value of 1/3 without spin alignment.

from 1/3 is negative at the midrapidity $Y = 0$, while it increases at larger Y. To see the influence of the anisotropy in the field fluctuation, we isolate the contribution from Δ to ρ_{00}^{y} by taking the difference between results in Fig. 1 with nonvanishing Δ and those obtained with the same F^2 but $\Delta = 0$. As shown in Fig. 2, the effects of nonvanishing Δ are one order of magnitude smaller than $\rho_{00}^{y} - 1/3$ in Fig. 1. This indicates that the rapidity dependence in Fig. 1 is dominated by the isotropic part F^2 in Eq. (3).

The theoretical model in Sec. II provides a clear picture for the rapidity dependence of the spin alignment. The bulk matter in which ϕ mesons are produced can be treated as a nearly isotropic medium with a small anisotropy along the z direction in fluctuations of strong force fields described by Eq. (3), i.e., $F^2 \gg \Delta$. The ϕ meson's motion relative to the bulk matter breaks the rotational symmetry in meson's rest frame, leading to a larger probability for spin ± 1 states than

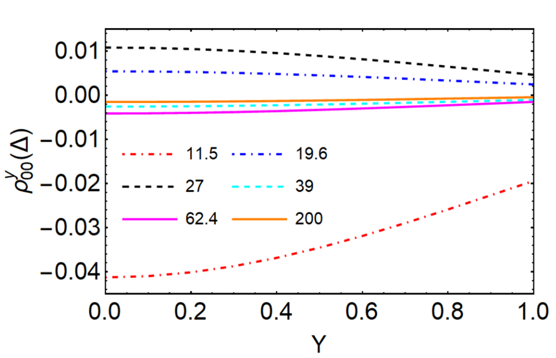

FIG. 2. Contributions from the anisotropy parameter Δ in the laboratory frame to the ρ_{00}^{y} versus rapidity at different collision energies.

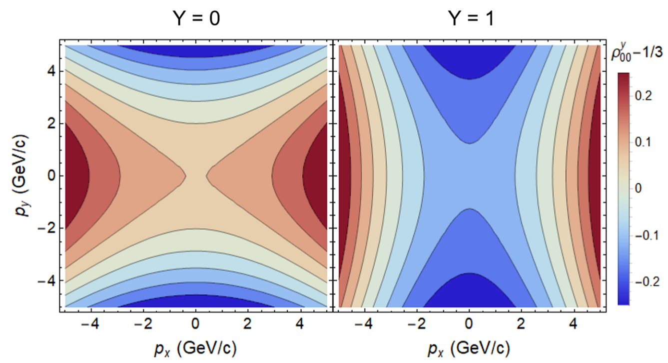

FIG. 3. The contour plots for $\delta\rho_{00}^{y}$ in the transverse momentum plane (p_x, p_y) for ϕ mesons at $Y = 0$ (left panel) and $Y = 1$ (right panel) in Au+Au collisions at 200 GeV.

the spin 0 state with respect to the motion direction. For example, if mesons move in the z direction, we have $\rho_{00}^{z} < 1/3$, or equivalently $\rho_{00}^{x} = \rho_{00}^{y} > 1/3$ because of the normalization condition $\rho_{00}^{x} + \rho_{00}^{y} + \rho_{00}^{z} = 1$.

In order to study the transverse momentum dependence of ρ_{00}^{y} in different rapidity regions, we present in Fig. 3 the contour plot for the deviation $\delta\rho_{00}^{y}$ in (p_x, p_y) plane at $Y = 0$ (left panel) and $Y = 1$ (right panel) at 200 GeV. We observe a significant quadrupole structure: the mesons with $|p_x| \gg |p_y|$ have $\delta\rho_{00}^{y} > 0$, while those with $|p_x| \ll |p_y|$ have $\delta\rho_{00}^{y} < 0$. Such a structure is the result of Eq. (4) or (5).

We also calculated the azimuthal angle dependence of $\delta\rho_{00}^{y}$ at a fixed transverse momentum $p_T = 2$ GeV, see Fig. 4. For mesons at $Y = 0$, $\delta\rho_{00}^{y}$ is positive at $\varphi = 0$ and decreases to a negative minimum value at $\varphi = \pi/2$. The curve shows a $\cos(2\varphi)$ behavior, as expected from Eq. (5). At a more forward rapidity $Y = 1$, $\delta\rho_{00}^{y}$ is shifted by a positive value relative to the $Y = 0$ curve, which is also described by Eq. (5). The effects of the anisotropy in strong force field fluctuations in the laboratory frame, which are quantified by Δ in Eq. (3), are also shown in Fig. 4. We see that the effects of nonvanishing Δ are small, implying that the azimuthal angle dependence is dominated by the isotropic part F^2 in Eq. (3).

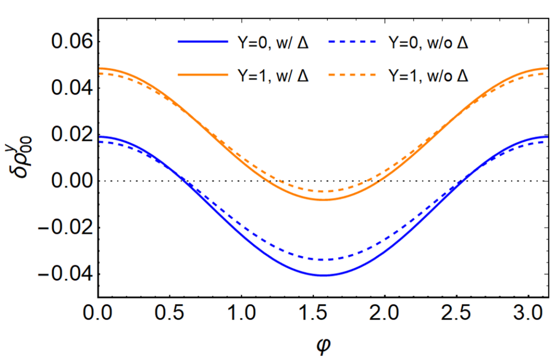

FIG. 4. Azimuthal angle φ dependence of $\delta\rho_{00}^{y}$ at rapidity $Y = 0$ (blue lines) and $Y = 1$ (orange lines) at 200 GeV. Solid lines are calculated using fluctuations parameters F^2 and Δ given in the first paragraph of Sec. III and dashed lines are calculated using the same F^2 but $\Delta = 0$.

IV. SUMMARY

The momentum dependence of the ϕ meson's global spin alignment in heavy-ion collisions is studied. According to the relativistic quark coalescence model in the spin transport theory for vector mesons [32,33], the derivation from 1/3 for ρ_{00} for the ϕ meson is driven by the anisotropy of local fluctuations of strong force fields in the vector meson's rest frame, which can be related to fluctuations in the laboratory frame through Lorentz transformation. From the geometry of the quark-gluon plasma produced in heavy-ion collisions, it is natural to assume that fluctuations are nearly isotropic in the laboratory frame, with tiny anisotropy in the z direction or beam direction. By neglecting the anisotropy part, we derive an analytical expression for $\rho_{00}^{y} - 1/3$ which is proportional to $(p_x^2 + p_z^2)/2 - p_y^2$, indicating that the meson's motion along x and z directions will enhance ρ_{00}^{y}, while the motion along y direction will decrease ρ_{00}^{y}. We then predict the rapidity dependence of ρ_{00}^{y} using fluctuation parameters that are extracted from the STAR experiment data on momentum-integrated ρ_{00}^{y} [18]. Our results show that ρ_{00}^{y} has a negative derivation from 1/3 at midrapidity $Y = 0$ and a positive derivation at slightly forward rapidity $Y = 1$. Predictions for the azimuthal angle dependence of ρ_{00}^{y} at these two rapidities have also been made.

Although we assume that fluctuations in the laboratory frame contain an isotropic part F^2 and an anisotropic part Δ, we find in our calculation that the contribution from the anisotropic part is negligible compared with the isotropic part, as manifested in Fig. 4. This means that we may safely set $\Delta = 0$ and still obtain nearly the same results as Fig. 1. The momentum dependence is mainly caused by the broken rotational symmetry due to the motion of the ϕ meson relative to the bulk matter. Such a mechanism can be tested in future experiments.

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