Two- and three-photon fusion into charmonium in ultraperipheral nuclear collisions

R. Fariello,[1,2] D. Bhandari,[3] C. A. Bertulani,[3,4] and F. S. Navarra[3,2]

[1]*Departamento de Ciências da Computação, Universidade Estadual de Montes Claros, Avenida Rui Braga, sn, Vila Mauricéia, CEP 39401-089, Montes Claros, MG, Brazil*

[2]*Instituto de Física, Universidade de São Paulo, Rua do Matão 1371 - CEP 05508-090, Cidade Universitária, São Paulo, SP, Brazil*

[3]*Institut für Kernphysik, Technische Universität Darmstadt, 64289 Darmstadt, Germany*

[4]*Department of Physics and Astronomy, Texas A&M University-Commerce, Commerce, Texas 75429, USA*

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In this paper we investigate the production of charmonium states in two- and three-photon fusion processes in nucleus-nucleus collisions at the CERN Large Hadron Collider energies. Our results indicate that the experimental study of these processes is feasible and can be used to constrain the theoretical decay widths and give information on the non c-$\bar{c}$ components of these states.

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I. INTRODUCTION

During the last 20 years dozens of new charmonium states have been observed at the CERN Large Hadron Collider (LHC) [1–7]. Some of them are, beyond any doubt, multiquark (or exotic) states, i.e., states in which the minimum quark content is $c\bar{c}q\bar{q}$. This is the case of all charged exotic states [7]. Among the charge neutral states there are some which are, for several reasons, incompatible with the $c\bar{c}$ configuration. This is the case of the most famous exotic state, the $X(3872)$, which is now called $\chi_{c1}(3872)$. There are other charge neutral states, whose multiquark nature is still under debate, such as the $\psi(3770)$.

The central discussion in this field is about the internal structure of the multiquark states. The most often studied configurations are the meson molecule and the tetraquark. The main difference between a tetraquark and a meson molecule is that the former is compact and the interaction between the constituents occurs through color exchange forces whereas the latter is an extended object and the interaction between its constituents happens through meson exchange forces [1–7].

One aspect that is sometimes forgotten, is that, being quantum systems, these states can be mixtures. There may be charmonium-tetraquark, charmonium-molecule, or tetraquark-molecule mixtures. Here, again, different works which consider these multiquark states as mixtures do not reach a consensus. For example, in the case of the well studied $\chi_{c1}(3872)$, in Ref. [8] the mass and strong decay width were very well reproduced assuming that it has a $c\bar{c}$ component with a weight of 97% and a tetraquark component with 3% weight. On the other hand, in Ref. [9] it was shown that, in the case of production in proton-proton collisions, the best description of the data could be achieved with a charmonium-molecule combination, i.e., $\chi'_{c1} - D\bar{D}^*$, in which the $c\bar{c}$ component is of the order of 28–44 %. In spite of the discrepancies, it is remarkable that in both works a large $c\bar{c}$ component is required to explain data.

The study of exotic states started in B factories [10] and then went to hadron colliders. The hadronic production of exotic states became a new way to discriminate between different configurations. The production of $\chi_{c1}(3872)$ in proton-proton collisions in the pure molecular approach was studied in [11–13]. In [14], the analysis of recent data from the LHCb with the comover interaction model favored the compact tetraquark configuration. An attempt to use the pure tetraquark model to study $\chi_{c1}(3872)$ and T_{4c} [$X(6900)$] production in proton-proton collisions was presented in [15]. All these works have improved our understanding of these new states, but there are still important questions to be answered.

The very recent publication of the CMS collaboration [16] reporting the observation of the $\chi_{c1}(3872)$ in Pb-Pb collisions opened a new era in the study of exotics in heavy ion collisions. The main advantage of using heavy ion projectiles is the very large number of produced c-$\bar{c}$ pairs. In the case of central collisions, the main disadvantage is that the total number of produced particles is very large and it becomes difficult to search for the multiquark states.

It is also possible to study multiquark states in ultraperipheral collisions (UPCs). High energy hadrons are an intense source of photons (for a review see Refs. [17–21]). At large impact parameters ($b > R_{h_1} + R_{h_2}$), the photon-induced interactions become dominant with the final state being characterized by the multiquark state and the presence of two intact hadrons if the resonance was produced in two- or three-photon interactions. Experimental results at the LHC [22–25] have shown that the extraction of photon-induced interactions in hadronic collisions is under experimental possibilities and can be used to increase our understanding of quantum chromodynamics (QCD). The study of exotic meson production in UPCs was pioneered in [26], where production cross sections of various light and heavy mesons in nucleus-nucleus collisions were computed. Later, in Refs. [27,28], the same formalism was used to obtain the production cross sections of mesons and heavy exotic states in *pp*, *pA*, and *AA* collisions.

In this work we will focus on c-$\bar{c}$ states, giving special attention to the states, which are presently quoted by the Particle Data Group (PDG) [29] as c-$\bar{c}$, but whose nature is still under debate and which might still be multiquark states, or at least, might have a multiquark (either tetraquark or molecular) component. We will argue that we can use photon fusion processes in UPCs to confirm (or not) their c-$\bar{c}$ nature. This is possible because in these processes we only use quantum electrodynamics (QED) and a well-established method to project quark-antiquark pairs into bound states, avoiding some model dependence inherent to hadronic processes. We will revisit and update the calculations performed in [30] including new states. We will study the most recently observed particles using the quantum number assignments published in the most recent compilation made by the PDG [29]. We shall consider both two-photon and three-photon processes. As it will be shown, all the ingredients of the calculation are under control. The formalism developed in [30] applies to fermion-antifermion systems, being thus appropriate to the study of conventional quarkonium states. We will also apply it to the controversial cases, where the multiquark nature of the state is still under debate. A future experimental confirmation of our predictions would establish the quark-antiquark nature of these states.

We will consider Pb-Pb UPCs at $\sqrt{s_{NN}} = 5.02$ TeV. In this case, vector charmonium production from photon-Pomeron (pP) fusion has cross sections of the order of millibarns [31,32] whereas vector charmonium production from threephoton fusion (3pf) has a typical cross section of hundreds of nanobarns [30] which is of the order of the light-by-light (LBL) scattering cross section measured by ATLAS and CMS. LBL identification was possible after a careful background subtraction, which included several kinematical cuts and restriction of the acoplanarity of the two outgoing photons. Similar techniques and cuts could be used to discriminate between pP and 3pf vector charmonium production. The first step, addressed in the present work, is to determine the order of magnitude of the total cross section. The second step (work in progress) is to determine the region of the phase space where 3pf dominates. Similar considerations apply to the production of scalar and tensor charmonium states, where we need to distinguish photon-photon fusion from Pomeron-Pomeron fusion.

This paper is organized as follows. In Sec. II we provide a brief description of the formalism used for particle production in two-photon fusion at hadronic colliders. In Sec. III we discuss meson production in three-photon interactions. In all cases we present the update of the results obtained for the production of charmonium in Pb-Pb collisions, including new states and making predictions for the LHC. Finally, in Sec. IV we present a brief summary and discussion of the results.

II. TWO-PHOTON PROCESSES

The theoretical treatment of UPCs in relativistic heavy ion collisions is extensively covered in the literature [17–21]. In what follows we will briefly review the main formulas that make predictions for meson production in two and three photon interactions, which were derived in [30].

The differential cross section for the production of C-even mesons through two-photon fusion is given by [30]

$$\frac{d\sigma}{dP_z} = \frac{16(2J+1)}{\pi^2}\frac{Z^4\alpha^2}{M^3}\frac{\Gamma_{\gamma\gamma}}{E}\int d\mathbf{q}_{1t}d\mathbf{q}_{2t}\,(\mathbf{q}_{1t}\times\mathbf{q}_{2t})^2 \times \frac{\left[F_1(q_{1t}^2)F_2(q_{2t}^2)\right]^2}{\left(q_{1t}^2+\omega_1^2/\gamma^2\right)^2\left(q_{2t}^2+\omega_2^2/\gamma^2\right)^2}, \tag{1}$$

where P_z, E, M, and J are the longitudinal momentum, energy, mass, and spin of the produced meson, respectively; $\Gamma_{\gamma\gamma}$ is the two-photon decay width of the meson; Z, α, and γ are the atomic number, the fine structure constant, and the Lorentz factor. Finally, F_1 and F_2 are the projectile and target form factors. Following [30] it is easy to relate the meson variables with the photon energies ω_1 and ω_2:

$$E = \omega_1 + \omega_2\ , \quad \omega_1 - \omega_2 = P_z\ , \quad \text{and} \quad \omega_1\omega_2 = M^2/4.$$

The photon energies ω_1 and ω_2 are related to the mass M and the rapidity Y of the outgoing meson by $\omega_1 = \frac{M}{2}e^{Y}$ and $\omega_2 = \frac{M}{2}e^{-Y}$.

As it was already mentioned, $F(q^2)$ is the nuclear form factor and it plays a crucial role in this formalism. The precise form of the form factor is the main source of uncertainties in our calculations. The Woods-Saxon distribution, with central density ρ_0, size R, and diffuseness a gives an excellent description of the densities of stable heavy nuclei. Fortunately, the Woods-Saxon distribution is extremely well described by the convolution of a hard sphere and a Yukawa function [33]. The form factors can be calculated analytically as

$$F(q^2) = \frac{4\,\pi\,\rho_0}{A\,q^3}\,[\sin(qR) - qR\cos(qR)]\left[\frac{1}{1+q^2a^2}\right]. \tag{2}$$

For Pb we use $R = 6.63$ fm and $a = 0.549$ fm, with ρ_0 normalized so that $\int d^3r\rho(r) = 208$ [34]. With the above expressions it is easy to compute the total cross sections with an adequate form factor [33].

During the derivation of the above formula for the cross section, we had to use a prescription to bind together the produced quark and antiquark into a bound state. We did this using the projection operators [30]

$$\bar{u}\cdots v \longrightarrow \frac{\Psi(0)}{2\sqrt{M}}\,\mathrm{tr}[\cdots(\not{P}+M)i\gamma^5],$$
$$\bar{u}\cdots v \longrightarrow \frac{\Psi(0)}{2\sqrt{M}}\,\mathrm{tr}[\cdots(\not{P}+M)i\,\not{\hat{e}}^*], \tag{3}$$

where $\cdots$ denotes any matrix operator. The first equation describes the production of spin 0 and the second describes the production of spin 1 particles, respectively. The function $\Psi(\mathbf{r})$ denotes the bound state wave function calculated at the origin, $\mathbf{r} = 0$, and $\hat{e}^*$ is the polarization vector of the outgoing vector meson. Squaring the corresponding amplitude yields the

TABLE I. Cross sections for production of C-even mesons in Pb-Pb ultraperipheral collisions at $\sqrt{s_{NN}} = 5.02$ TeV. The decay widths are taken from the PDG [29].

State	Mass	$\Gamma_{\gamma\gamma}$[keV]	$\sigma^{\rm LHC}$[mb]
π_0, 0^{-+}	134	0.0078	38.0
η, 0^{-+}	547	0.46	17.3
η', 0^{-+}	958	4.2	21.8
f_2, 2^{++}	1275	2.4	22.4
a_2, 2^{++}	1318	1.0	8.3
η_c, 0^{-+}	2984	5.38	0.45
χ_{0c}, 0^{++}	3415	2.2	0.11
χ_{2c}, 2^{++}	3556	0.56	0.12
$\eta_c(2S)$, 0^{-+}	3637	2.14	0.09

factor $|\Psi(0)|^2$, which is then related to the decay width $\Gamma_{\gamma\gamma}$ through the formula derived by Van Royen and Weisskopf in Ref. [35] (see the discussion in [30]) for fermion-antifermion annihilation. Hence, because of the hadronization prescription, the cross section formulas derived in [30] apply to quark-antiquark states. Nevertheless, in order to obtain a first estimate we shall use the Van Royen–Weisskopf formula also for states, which are very likely multiquark states, such as the $X(6900)$.

In what follows we will compute the production cross sections for conventional c-$\bar{c}$ and also to states whose status is still under debate. Therefore our results will serve as baseline for the experimental search in UPCs. If our predictions are confirmed this will be an argument in favor of the quark-antiquark assignment. If there are large discrepancies between data and our numbers, this will indicate the existence of a molecular or tetraquark component. As mentioned in the Introduction, charge neutral states can always be mixtures and in the existing calculations involving mixtures, the c-$\bar{c}$ component is always large. Hence our calculations will be relevant. Our strategy is conservative. Instead of creating a model for the production of multiquark states, we stick to the well know QED amplitudes complemented by experimental information.

In Table I we show the cross sections for the production of C even mesons in Pb-Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV using the formalism described above. For the decay width of the η_c we have used the PDG value but it should be mentioned that the value of this quantity is still under debate [36,37]. We have also included the results for $\eta_c(2S)$, which was not so well measured when we wrote our previous work on this subject [30].

In Table II we show the update of the results obtained in [28] for the production cross section of the $J = 0$ and $J = 2$ particles. In the PDG compilation, the quantum numbers and the nature of the $X(3940)$ are still undefined and its two-photon decay width was not measured. We have used the theoretical values calculated in [38,39]. The states $\chi_{c0}(3915)$ and $\chi_{c2}(3930)$ are treated as conventional c-$\bar{c}$ scalar and tensor states, respectively. However these assignments have been questioned (see, for example, [40]). In Table II we included results for the very recently measured $X(6900)$ state [41].

TABLE II. Cross sections for production of C-even charmonium states in Pb-Pb ultraperipheral collisions at different energies. The highest energy might be relevant for collisions at the FCC [44].

			$\sigma\,[\mu$b]		
State	Mass	$\Gamma_{\gamma\gamma}$[keV]	2.76 TeV	5.02 TeV	39 TeV
$X(3940)$, 0^{++}	3943	0.33 [38,39]	5.5	9.7	32.5
$X(3940)$, 2^{++}	3943	0.27 [38,39]	22.5	39.6	133.0
$\chi_{c2}(3930)$, 2^{++}	3922	0.08 [29]	7.1	12.4	41.7
$\chi_{c0}(3915)$, 0^{++}	3919	0.20 [29]	3.4	6.0	20.1
$X(6900)$, (I)	6900	67 [43]	120.5	238	912
$X(6900)$, (NI)	6900	45 [43]	81	160	612

This state was seen in the J/ψ-J/ψ invariant mass spectrum and therefore it could be a $c\bar{c}c\bar{c}$ state. After the observation there was a series of works discussing its structure and hadronic production. Among them, Ref. [42] is of special relevance to us. The authors have used the equivalent photon approximation (EPA) and the convolution formula included in the Appendix for the sake of discussion. The formalism described there is quite similar to the one described above and the use of the Low formula for the $\gamma\gamma \to R$ reaction is equivalent to using the Van Royen–Weisskopf formula. Since in Ref. [42] the authors did not know the two-photon decay width of the $X(6900)$, they could not be very precise in their estimate. Later, this information was extracted from the analysis of light-by-light scattering in Ref. [43]. The main observation was that the fit of the measured $\gamma\gamma$ invariant mass spectrum becomes much better when one adds a resonance of mass $\simeq$6900 MeV. Using different assumptions in the analysis they arrive at the values of $\Gamma_{\gamma\gamma}$ quoted in Table II, where I stands for interference and NI for no interference (for more details see [43]). These values are surprisingly large and when inserted into our formulas yield very large production cross sections.

III. THREE-PHOTON PROCESSES

The formalism described in the previous section is analogous to the equivalent photon approach and the cross section could be rewritten as the well known convolution EPA formula (given in the Appendix) for the process $\gamma\gamma \to R$, where R is any integer spin resonance. However, this formula can only be used for the production of $J = 0$ or 2 states. For the case of vector meson production we need the three-photon fusion process. In Ref. [30] we derived the expression for the cross section of three-photon fusion into a C-odd meson. In differential form it reads

$$\frac{d\sigma}{dP_z} = 1024\,\pi\,|\Psi(0)|^2(Z\alpha)^6\frac{1}{M^3E}\int\frac{dq_{1t}\,q_{1t}^3\,\left[F\left(q_{1t}^2\right)\right]^2}{\left(q_{1t}^2+\omega_2^2/\gamma^2\right)^2}$$
$$\times\int\frac{dq_{2t}\,q_{2t}\,\left[F\left(q_{2t}^2\right)\right]^2}{\left[q_{2t}^2+(2\omega_1-\omega_2)^2/\gamma^2\right]^2}$$
$$\times\left[\int\frac{dk_t\,k_t\,F\left(k_t^2\right)}{\left(k_t^2+(\omega_1-\omega_2)^2/\gamma^2\right)}\right]^2. \qquad (4)$$

TABLE III. Cross sections for production of C-odd mesons in Pb-Pb ultraperipheral collisions at $\sqrt{s_{NN}} = 5.02$ TeV. The decay widths are taken from the PDG [29].

State	Mass	$\Gamma_{e^+e^-}$[keV]	σ[nb]
ρ^0	770	6.77	2466.9
ω	782	0.6	215.3
J/ψ	3097	5.3	476.5
$\psi(2S)$	3686	2.1	161.4
$\psi(3770)$	3770	0.26	19.5
$\psi(4040)$	4040	0.86	59.7
$\psi(4160)$	4160	0.48	32.4
$\psi(4230)$	4230	1.53	101.5
$\psi(4415)$	4415	0.58	36.9

The definition of the variables are as in Eq. (1). However, in the present case, the wave function $|\Psi(0)|^2$ can no longer be related to the $\gamma\gamma$ decay width. On the other hand, vector mesons can decay into e^+e^- pairs and the corresponding decay widths are well known experimentally. Using a similar derivation as for the $\gamma\gamma$ decay, the e^+e^- decay width of the vector mesons found to be proportional to the wave function squared [35], i.e., $\Gamma_{e^+e^-} \propto |\Psi(0)|^2$. Using the relation derived in [35] we arrive at [30]

$$\sigma = \int d\omega\, 96\,\pi\, \frac{\Gamma_{e^+e^-}}{M^3}\, \frac{n(\omega)}{\omega}\, H(M,\omega), \tag{5}$$

where $n(\omega)$ is given by

$$n(\omega) = \frac{2}{\pi}\, Z^2\alpha \int \frac{dq\, q^3\, [F(q^2)]^2}{(q^2+\omega^2/\gamma^2)^2} \tag{6}$$

and

$$H(M,\omega) = Z^4\alpha^3 M^2 \int \frac{dq_{2t}\, q_{2t}\, \left[F\left(q_{2t}^2\right)\right]^2}{\left[q_{2t}^2+(M^2/2\omega-\omega)^2/\gamma^2\right]^2} \times \left[\int \frac{dk_t\, k_t\, F\left(k_t^2\right)}{k_t^2+(M^2/4\omega-\omega)^2/\gamma^2}\right]^2. \tag{7}$$

In Table III we present the cross sections for vector charmonium production. The first four lines are just an update of the results found in [30]. The other lines present states which may be exotic. A common feature shared by all these ψ states (with the exception of $\psi(3770)$) is that they are all above a $D\bar{D}$ threshold and yet this decay mode is not a dominant one. This fact (among other things) raises the suspicion that these are not conventional $c\bar{c}$ states.

The nature of the $\psi(3770)$ resonance is still a subject of debate. Conventionally, it has been regarded as the lowest-mass D-wave charmonium state above the $D\bar{D}$ threshold, i.e., a pure $c\bar{c}$ meson in the quark model. However, in Ref. [45] it was suggested that the $\psi(3770)$ may contain a considerable tetraquark component. In that work it was also suggested that the tetraquark nature of the state would reveal itself in the decay $\psi(3770) \rightarrow \eta J/\psi$ and a prediction of the decay width in this channel was given. Very recently, this decay was observed by the BESIII collaboration [46] and the measured width was close to the prediction made in [45], giving support to the possible tetraquark component of the $\psi(3770)$. In our formalism, we treat the vector mesons as $c\bar{c}$ bound states. So our predicted cross section refers to the production of a conventional charmonium or to the charmonium component of the mixed charmonium-tetraquark state.

The Particle Data Group (PDG) has been updating the parameters of vector charmonium states, like $\psi(4160)$ and $\psi(4230)$, following an improved data analysis and higher statistics of the data. The study reported by BES yields higher Breit-Wigner (BW) mass for $\psi(4160)$: $M = 4191.7 \pm 6.5$ MeV [47]. In spite of the fact that the updated mass and width parameters of these two states are closer to each other, they are still regarded as different states with the same quantum numbers whose underlying nature remains elusive. $\psi(4160)$ is considered as a 2^3D_1 $c\bar{c}$ state due to its consistency with the predictions of the quark potential model [48]. The $\psi(4160)$ and $\psi(4230)$ have the same quantum numbers with a mass difference approximately equal to 40 MeV but can hardly be described within the quark model at the same time [48]. Furthermore, while the $\psi(4160)$ appears in the open charm channels, it is not present in the hidden-charm channels, and the decay channels of $\psi(4230)$ appearing in the PDG table are mostly due to hidden-charm channels. Clearly, these states deserve further studies. In [49] it has been argued that the $\psi(4160)$ and $\psi(4230)$ are in fact the same state. The measurement of the production cross sections of these two states in the three photon fusion may help elucidating their nature.

Before closing this section, it is important to mention that we found some numerical discrepancies when we compared our present results with the older ones, reported in [30]. Unfortunately, the 20 year old codes could not be recovered and a careful comparison could not be done. Since the discrepancies are important only for some light mesons (π^0, η, ρ^0, and ω), we suspect that they are related to the technique used to perform the integrations in the expressions for the cross sections, Eqs. (1) and (5). In these equations the masses appear in the denominators. Although the integrals are not divergent, the integrands show peaks for smaller values of the momenta and masses. This is where different integration algorithms might lead to different results. We feel confident about our numbers because we are now a bigger team than 20 years ago and we could check our results more carefully. Most importantly, the focus of the paper is on the heavy meson sector, where the discrepancies are very small.

IV. SUMMARY

In this work we have studied the charmonium production in UPCs at LHC energies due to two- and three-photon fusion processes. These are clean processes where the particles of the initial state remain intact in the final state and can be detected with the appearance of two rapidity gaps between the heavy ions and the produced particle. We have used the QED formulas (derived in [30]) complemented with the experimental data on decay widths. We predict sizable values for the cross sections in Pb-Pb collisions. Our conclusion is that experimental studies using UPCs are worth pursuing, they will be valuable to constrain decay widths calculated theoretically and, ultimately, they will help determining the structure of the considered states, confirming or not their quark-antiquark nature.

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APPENDIX: EQUIVALENT PHOTON APPROXIMATION FOR TWO-PHOTON PROCESSES

Using the equivalent photon approximation for UPC of two hadrons, h_1 and h_2, we obtain the cross section for the production of a generic charmonium state, R, given by (see, e.g., [17,20])

$$\sigma(h_1h_2 \to h_1 \otimes R \otimes h_2; s) = \int \hat{\sigma}(\gamma\gamma \to R; W) N(\omega_1, \mathbf{b}_1) \times N(\omega_2, \mathbf{b}_2) S^2_{abs}(\mathbf{b}) \mathrm{d}^2\mathbf{b}_1 \mathrm{d}^2\mathbf{b}_2 \mathrm{d}\omega_1 \mathrm{d}\omega_2 \,, \quad (A1)$$

where $\sqrt{s}$ is center-of-mass energy for the h_1h_2 collision ($h_i = p, A$). The cross product symbol $\otimes$ indicates a rapidity gap in the final state and $W = \sqrt{4\omega_1\omega_2}$ is the invariant mass of the $\gamma\gamma$ system. The quantity $N(\omega_i, b_i)$ is known as the equivalent photon spectrum generated by hadron (nucleus) i, and $\sigma_{\gamma\gamma\to R}(\omega_1, \omega_2)$ is the cross section for the production of a state R by means of real photons with energies ω_1 and ω_2. Besides, in Eq. (A1), ω_i denotes the energy of the photon emitted by the hadron (nucleus) h_i at an impact parameter, or distance, b_i from h_i. Finally, in the above formula $S^2_{abs}(\mathbf{b})$ is the survival probability written as the square of the scattering matrix, introduced here to enforce the absorption at small impact parameters $b \lesssim R_{h_1} + R_{h_2}$ [50]. We adopt the equivalent photon flux expression

$$N(\omega, b) = \frac{Z^2\alpha_{em}}{\pi^2}\frac{1}{b^2\omega}\left[\int u^2 J_1(u) F\left(\sqrt{\frac{(b\omega/\gamma)^2 + u^2}{b^2}}\right) \times \frac{1}{(b\omega/\gamma)^2 + u^2}\mathrm{d}u\right]^2, \quad (A2)$$

where F is the nuclear form factor of the equivalent photon source. In order to estimate the $h_1h_2 \to h_1 \otimes R \otimes h_2$ cross section one needs the $\gamma\gamma \to R$ production cross section as input. Usually one uses the Low formula [51], where the cross section for the production of the R state in two-photon fusion reactions is given in terms of the two-photon decay width of R,

$$\sigma_{\gamma\gamma\to R}(\omega_1, \omega_2) = 8\pi^2(2J+1)\frac{\Gamma_{R\to\gamma\gamma}}{M_R}\delta\left(4\omega_1\omega_2 - M_R^2\right), \quad (A3)$$

where the decay widths $\Gamma_{R\to\gamma\gamma}$ are either taken from experiment or estimated theoretically. In the above formula, M_R and J are the mass and spin of the produced state, respectively.

[1] N. Brambilla, S. Eidelman, C. Hanhart, A. Nefediev, C.-P. Shen, C. E. Thomas, A. Vairo, and C.-Z. Yuan, Phys. Rep. **873**, 1 (2020).
[2] R. M. Albuquerque, J. M. Dias, K. P. Khemchandani, A. Martinez Torres, F. S. Navarra, M. Nielsen, and C. M. Zanetti, J. Phys. G **46**, 093002 (2019).
[3] Y. R. Liu, H. X. Chen, W. Chen, X. Liu, and S. L. Zhu, Prog. Part. Nucl. Phys. **107**, 237 (2019).
[4] F.-K. Guo, C. Hanhart, Ulf-G. Meißner, Q. Wang, Q. Zhao, and B.-S. Zou, Rev. Mod. Phys. **90**, 015004 (2018).
[5] A. Esposito, A. Pilloni, and A. D. Polosa, Phys. Rep. **668**, 1 (2017).
[6] A. Hosaka, T. Iijima, K. Miyabayashi, Y. Sakai, and S. Yasui, Prog. Theor. Exp. Phys. **2016**, 062C01 (2016).
[7] M. Nielsen and F. S. Navarra, Mod. Phys. Lett. A **29**, 1430005 (2014).
[8] R. D. Matheus, F. S. Navarra, M. Nielsen, and C. M. Zanetti, Phys. Rev. D **80**, 056002 (2009).
[9] C. Meng, H. Han, and K. T. Chao, Phys. Rev. D **96**, 074014 (2017).
[10] S. J. Brodsky and F. S. Navarra, Phys. Lett. B **411**, 152 (1997).
[11] C. Bignamini, B. Grinstein, F. Piccinini, A. D. Polosa, and C. Sabelli, Phys. Rev. Lett. **103**, 162001 (2009).
[12] A. Esposito, A. L. Guerrieri, L. Maiani, F. Piccinini, A. Pilloni, A. D. Polosa, and V. Riquer, Phys. Rev. D **92**, 034028 (2015); J. High Energy Phys. 07 (2013) 116.
[13] A. Esposito, B. Grinstein, L. Maiani, F. Piccinini, A. Pilloni, A. D. Polosa, and V. Riquer, Chin. Phys. C **42**, 114107 (2018); P. Artoisenet and E. Braaten, Phys. Rev. D **83**, 014019 (2011).
[14] A. Esposito, E. G. Ferreiro, A. Pilloni, A. D. Polosa, and C. A. Salgado, Eur. Phys. J. C **81**, 669 (2021).
[15] F. Carvalho, E. R. Cazaroto, V. P. Gonçalves, and F. S. Navarra, Phys. Rev. D **93**, 034004 (2016).
[16] A. M. Sirunyan *et al.* (CMS Collaboration), Phys. Rev. Lett. **128**, 032001 (2022).
[17] C. A. Bertulani and G. Baur, Phys. Rep. **163**, 299 (1988).
[18] C. A. Bertulani, S. Klein, and J. Nystrand, Annu. Rev. Nucl. Part. Sci. **55**, 271 (2005).
[19] A. Baltz *et al.*, Phys. Rep. **458**, 1 (2008).
[20] G. Baur, K. Hencken, and D. Trautmann, J. Phys. G **24**, 1657 (1998).
[21] G. Baur, Kai Hencken, D. Trautmann, S. Sadovsky, and Y. Kharlov, Phys. Rep. **364**, 359 (2002)
[22] B. Abelev *et al.* (ALICE Collaboration), Phys. Lett. B **718**, 1273 (2013).
[23] E. Abbas *et al.* (ALICE Collaboration), Eur. Phys. J. C **73**, 2617 (2013).
[24] R. Aaij *et al.* (LHCb Collaboration), J. Phys. G **40**, 045001 (2013).
[25] R. Aaij *et al.* (LHCb Collaboration), J. Phys. G **41**, 055002 (2014).
[26] C. A. Bertulani, Phys. Rev. C **79**, 047901 (2009).

[27] V. P. Goncalves, D. T. da Silva, and W. K. Sauter, Phys. Rev. C **87**, 028201 (2013).
[28] B. D. Moreira, C. A. Bertulani, V. P. Goncalves, and F. S. Navarra, Phys. Rev. D **94**, 094024 (2016).
[29] R. L. Workman *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. **2022**, 083C01 (2022).
[30] C. A. Bertulani and F. Navarra, Nucl. Phys. A **703**, 861 (2002).
[31] For recent experimental reviews, see X. Wang (LHCB Collaboration), Acta Phys. Pol. Supp. **16**, 15 (2023); A. Matyja (ALICE Collaboration), Nucl. Part. Phys. Proc. **324**, 22 (2023).
[32] B. Z. Kopeliovich, M. Krelina, J. Nemchik, and I. K. Potashnikova, Phys. Rev. D **107**, 054005 (2023); A. Łuszczak and W. Schäfer, Phys. Rev. C **99**, 044905 (2019); V. P. Goncalves, B. D. Moreira, and F. S. Navarra, *ibid.* **90**, 015203 (2014)
[33] K. T. R. Davies and J. R. Nix, Phys. Rev. C **14**, 1977 (1976).
[34] C. W. de Jager, H. de Vries, and C. de Vries, At. Data Nucl. Data Tables **14**, 479 (1974).
[35] R. P. Royen and V. F. Weisskopf, Nuovo Cimento A **50**, 617 (1967).
[36] B. Colquhoun, L. J. Cooper, C. T. H. Davies, and G. P. Lepage (Particle Data Group and HPQCD Collaboration), Phys. Rev. D **108**, 014513 (2023).
[37] I. Babiarz, V. P. Goncalves, W. Schäfer, and A. Szczurek, Phys. Lett. B **843**, 138046 (2023).
[38] T. Branz, T. Gutsche, and V. E. Lyubovitskij, Phys. Rev. D **80**, 054019 (2009); **82**, 054010 (2010).
[39] T. Branz, R. Molina, and E. Oset, Phys. Rev. D **83**, 114015 (2011).
[40] A. M. Badalian and Y. A. Simonov, Eur. Phys. J. C **83**, 410 (2023).
[41] A. Hayrapetyan *et al.* (CMS Collaboration), arXiv:2306.07164; R. Aaij *et al.*, Sci. Bull. **65**, 1983 (2020); K. Yi and J. Zhang, PoS **(ICHEP2022)** 775; Y. Xu, Acta Phys. Polon. Supp. **16**, 21 (2023).
[42] V. P. Gonçalves and B. D. Moreira, Phys. Lett. B **816**, 136249 (2021).
[43] V. Biloshytskyi, V. Pascalutsa, L. Harland-Lang, B. Malaescu, K. Schmieden, and M. Schott, Phys. Rev. D **106**, L111902 (2022).
[44] N. Arkani-Hamed, T. Han, M. Mangano, and L. T. Wang, Phys. Rep. **652**, 1 (2016).
[45] M. B. Voloshin, Phys. Rev. D **71**, 114003 (2005).
[46] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. D **107**, L091101 (2023).
[47] M. Ablikim *et al.* (BES Collaboration), Phys. Lett. B **660**, 315 (2008); eConf **C070805**, 02 (2007).
[48] S. Godfrey and N. Isgur, Phys. Rev. D **32**, 189 (1985)
[49] Z. Y. Zhou, C. Y. Li, and Z. Xiao, arXiv:2304.07052.
[50] C. A. Bertulani and A. M. Nathan, Nucl. Phys. A **554**, 158 (1993).
[51] F. E. Low, Phys. Rev. **120**, 582 (1960).