Excitation spectra and electromagnetic transitions between low-lying nonrotational states of odd-proton nuclei with $Z = 97-109$

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Background: Calculations of the structure of the low-lying states of nuclei with $Z = 97–109$ play an important role in understanding the properties of nuclei belonging to the new region of the nuclide chart, which is available now for experimental study.
Purpose: To calculate quasiparticle-phonon structure and the reduced γ-transition probabilities for the excited states with excitation energies below 1 MeV for odd-proton nuclei with $Z = 97-109$.
Methods: The quasiparticle-phonon model, which takes into account the quasiparticle-phonon interaction of different multipolarities, is used as a basis for the calculations.
Results: The quasiparticle-phonon structure and the γ-reduced transition probabilities of odd-proton nuclei ^{263,265,267,269}Mt, ^{259,261,263,265}Bh, ^{255,257,259,261,263}Db, ^{251,253,255,257,259,261}Lr, ^{249,251,253,255}Md, ^{245,247,249,251}Es, and ^{243,245,247}Bk are calculated. The α-decay chains starting from ^{263,265,267,269}Mt are analyzed.
Conclusion: The structure of the nuclear states with excitation energies below 1 MeV in the considered nuclei is mainly exhausted by the one-quasiparticle component. However, in some isotopes the quasiparticle-phonon admixtures plays an important role to destroy the smooth isotopic dependence of energy of the states. The nuclei in the α-decay chains starting from ^{263,265,267,269}Mt have up to two α-decay lines. The number of α-decay lines could be different in the α-decay chain and in the direct production of the nucleus.

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I. INTRODUCTION

Recent experimental studies of superheavy nuclei [1–7] have led to the production of new nuclei, and provided us valuable information about their single-particle excitations and equilibrium deformations. To support the experimental investigations, the systematic calculations of excitation spectra of heaviest nuclei have been performed [8–13] within the model of independent quasiparticles, or in the model including quasiparticles and phonons with the quasiparticle-phonon coupling. The last one is important for correct description of the excitation spectra [14–16]. These calculations, along with experimental results, allowed us to assign quantum numbers to the states observed and analyze the α-decay chains. The models without quasiparticle-phonon coupling usually result in a smooth isotopic dependence of spectrum. However, there are some irregularities found in the experiments [17] which are waiting to be explained. We expect the quasiparticle-phonon coupling may be the cause of these irregularities.

The microscopic approaches, which are used to study the structure of heaviest nuclei, are either the self-consistent ones based on some variants of the energy-density functional [18–32] or the microscopic-macroscopic methods [10,11,33–39] and the quasiparticle-phonon model (QPM) [40–43].

In Refs. [12,13], the calculated low-lying spectra and the reduced γ-transition probabilities between the low-lying states of the odd neutron nuclei with $Z = 100$, 102, 104, 106, 108 have been presented. In the present work the results of similar calculations but for the odd-proton transcurie nuclei are given. The method applied has been used previously to describe the structure of the well-deformed rare-earth and actinides nuclei with $A > 228$ [44–52]. The calculations performed provide us a good basis for the investigation of appearance of the isomeric states in heaviest nuclei. The lifetimes of found isomeric states are estimated and possible α decays from these states are discussed. The α-decay chains starting from ^{263,265,267,269}Mt are analyzed to find possible α-decay energies. The number of α-decay lines may depend on whether the nucleus is directly obtained or after the α decay of the parent nucleus.

II. MODEL HAMILTONIAN

The QPM Hamiltonian has the following structure:

$$H = H_{sp} + H_{\text{pair}} + H_M + H_{SM}, \tag{1}$$

where H_{sp} is the single-particle part of the total Hamiltonian, H_{pair} describes the monopole pairing forces with strength set to reproduce the odd-even differences of the experimental nuclear masses. The terms H_M and H_{SM} in Eq. (1) take into account the multipole and spin-multipole residual forces. The mean field potential in H_{sp} contains the central potential in the Woods-Saxon (WS) form for neutrons and protons, the spin-orbit part, and the Coulomb field for protons where the

TABLE I. The values of the quadrupole β_2 and hexadecapole β_4 deformation parameters and the energies (in keV) of the most collective low-energy phonons used in the calculations. They are rounded up to 10 keV.

Nucleus	β_2	β_4	$\omega(K_i^\pi = 0_1^+)$	$\omega(K_i^\pi = 2_1^+)$	$\omega(K_i^\pi = 0_1^-)$	$\omega(K_i^\pi = 1_1^-)$	$\omega(K_i^\pi = 2_1^-)$
^{263}Mt	0.263	−0.034	770	900	890	1025	740
^{265}Mt	0.254	−0.041	1620	1580	1330	1740	1370
^{267}Mt	0.269	−0.054	900	700	1300	1200	950
^{269}Mt	0.250	−0.052	1250	1860	1800	1735	1860
^{259}Bh	0.262	−0.006	1350	1840	2300	1300	1980
^{261}Bh	0.266	−0.008	1600	1700	1210	1640	935
^{263}Bh	0.259	−0.031	1560	1530	940	1600	980
^{265}Bh	0.262	−0.034	1475	1520	1140	1790	1200
^{255}Db	0.253	0.002	1040	1750	1140	1040	1160
^{257}Db	0.262	0.023	1520	1960	2500	590	900
^{259}Db	0.251	−0.010	1530	1610	2310	1800	1435
^{261}Db	0.246	−0.020	1470	1440	920	1190	800
^{263}Db	0.256	−0.028	1350	1420	1050	1670	980
^{251}Lr	0.266	0.033	1380	2000	1915	1760	1570
^{253}Lr	0.266	0.033	1530	1950	1075	580	460
^{255}Lr	0.254	0.029	1200	1300	1300	1000	900
^{257}Lr	0.253	0.016	1200	1300	1200	1000	900
^{259}Lr	0.253	0.002	1440	1400	1760	1660	1145
^{261}Lr	0.251	−0.010	1390	1450	920	1420	1210
^{249}Md	0.277	0.037	1200	1100	1500	1100	590
^{251}Md	0.279	0.035	1200	1300	1300	1000	900
^{253}Md	0.266	0.033	1200	1300	1300	1000	900
^{255}Md	0.279	0.035	1200	1300	1310	1000	900
^{245}Es	0.275	0.039	1200	1300	800	1000	1400
^{247}Es	0.275	0.039	990	1080	1200	900	950
^{249}Es	0.277	0.037	1200	1300	1310	1000	900
^{251}Es	0.261	0.037	1160	1335	1030	1180	870
^{243}Bk	0.272	0.041	930	1290	1590	1220	1690
^{245}Bk	0.272	0.041	1430	1680	1570	1530	1680
^{247}Bk	0.272	0.041	1380	1530	1920	1210	1630

spin-orbit potential is defined as in Ref. [40] taking into account the isospin dependence of the central potential. The depth of the WS potential

$$V_{WS} = -V_0\{1 + \exp[(r - R(\theta, \varphi))/a]\} \qquad (2)$$

for protons and neutrons is set as $V_0 = 54.25 \pm 39.6(N - Z)/A$ MeV. Diffusion parameter is equal to $a_n = 0.72$ fm for neutrons and $a_p = 0.65$ fm for protons. Here, we assume an axially deformed shape of nuclei

$$R(\theta, \varphi) = R_0[1 + \beta_0 + \beta_2 Y_{20}(\theta, \varphi) + \beta_4 Y_{40}(\theta, \varphi)], \qquad (3)$$

where $R_0 = r_0 \mathrm{A}^{1/3}$ ($r_0 = 1.24$ fm for protons and 1.26 fm for neutrons).

The deformation parameters β_2 and β_4 for each nucleus are listed in Table I. The parameter β_0, which takes into account the volume conservation, is determined by the relation $\beta_0 = -\frac{1}{4\pi}(\beta_2^2 + \beta_4^2)$. For nuclei under consideration the values of β_2 are in the range 0.246–0.279. These values correspond to the well-deformed nuclei, only for Bk isotopes are the values of β_2 the same. Some regularities are not visible in dependence of β_2 on the number of neutrons in isotones. This suggests that variations of β_2 are related to the characteristics of the last occupied level. The value of β_4 changes a sign with increasing number of protons. Being positive in Bk, Es, and Md isotopes it takes negative value in Bh and Mt isotopes. The calculation of the equilibrium deformations are carried out using the microscopic-macroscopic method with the two-center shell model potential [34,37,38] taking into account pairing and Strutinsky shell corrections [53,54]. Note that with the parameters used, the energy spectra of the Woods-Saxon and the two-center potentials coincide with a good accuracy (see Fig. 1 in [12]).

To describe the long-range particle-hole residual interaction, the effective separable forces are used. All other information about the Hamiltonian is given in [12,13].

After transformation to quasiparticle and phonon operators the Hamiltonian takes the form

$$H = \sum_q \epsilon_q \alpha_q^+ \alpha_q + \sum_{\mu\pi i} \omega_{\mu^\pi i} Q_{\mu^\pi i}^+ Q_{\mu^\pi i} + \sum_{qq'\mu\pi i} \Gamma_{qq'\mu^\pi i} \alpha_q^+ \alpha_{q'} (Q_{\mu^\pi i}^+ + Q_{\mu^\pi i}), \qquad (4)$$

where α_q^+ is the creation operator of the quasiparticle in the state q with the energy ϵ_q and $Q_{\mu^\pi i}^+$ is the creation operator of the ith phonon with the energy $\omega_{\mu^\pi i}$ in the state with given angular momentum projection μ and parity π. Thus, in the random phase approximation the problem is reduced to determine the phonon energies in even-even nuclei and then the excitation energies of odd-A nuclei. Note, that the amplitudes $\Gamma_{qq'\mu^\pi i}$ of the quasiparticle-phonon interaction do not contain free parameters and are uniquely determined by the matrix elements of the residual interaction, phonon energy, and other phonon characteristics.

The Hamiltonian (4) is diagonalized in the configuration space including single quasiparticle and quasiparticle $\otimes$ phonon states. We confine ourselves in this paper by three lowest phonon states ($i = 1, 2, 3$) with $\mu = 0, 1, 2, 3$. As shown in our calculations, the contribution of the phonons with higher i and μ to the wave functions of the states with energies lower than 1 MeV is small.

However, the coupling to giant resonances should be discussed separately because of the strong collectivity of these modes. As mentioned in Ref. [55], the particle-vibration coupling to high energy excitations creates an additional contribution to the single-particle energies of the order of $\epsilon_F A^{-1}$, where ϵ_F is the Fermi energy. For nuclei under consideration it gives ~150 keV. At the same time, the particle-vibration coupling is not self-consistently treated in our approach that can introduce some uncertainties in the calculations and limit the accuracy. Therefore, we limit ourselves to the coupling only with the low-lying vibrational modes, which contribute the most to fluctuations in the average nuclear potential. The values of their energies are listed in Table I in keV, where they are rounded up to 10 keV. The calculations include single-particle states with the energies from the bottom of the potential well up to +5 MeV.

The basic set of the Woods-Saxon parameters and the spin-orbit strength were justified in our previous publications based on the numerous calculations of the one-quasiparticle spectra of well studied heavy nuclei. We use the same parameter set also for superheavy nuclei. To check a sensitivity of the results to the variation of the Woods-Saxon parameters, the calculations were performed not only with the basic set but also with the other sets of the single-particle potential parameters. As shown, the reasonable variations of the radius and diffuseness parameters do not cause large changes in the energy spectra. The variation of the spin-orbit strength produces larger deviations in energies. However, these variations do not exceed 300 keV, which is still acceptable to conclude the stability of the results obtained. Of course, a significant variation of the multipole strength constants, which leads to large changes of the phonon energies, influences strongly on the excitation spectra, however, only at energies larger than 400 keV where the admixture of the quasiparticle $\otimes$ phonon components can be essential. Other details of calculation scheme are given in Refs. [12,13].

III. EXCITATION SPECTRA AND γ TRANSITIONS IN ODD-PROTON NUCLEI WITH Z = 97–109

Using the theoretical approach formulated above and the basic set of the parameters given in Refs. [12,13] we calculate the excitation spectra and the reduced γ-transition probabilities in the odd-proton nuclei: ^{263,265,267,269}Mt, ^{259,261,263,265}Bh, ^{255,257,259,261,263}Db, ^{251,253,255,257,259,261}Lr, ^{249,251,253,255}Md, ^{245,247,249,251}Es, and ^{243,245,247}Bk. The calculations are restricted by the excitation energies below 1 MeV. With some exception the corresponding excited states have a one-quasiparticle structure. In many cases a weight of the one-quasiparticle component exceeds 90%. However, in several isotopes the structure of the high-lying states is determined by the quasiparticle$\otimes$phonon component. The phonons, which play a role, are mainly the octupole ones with $K = 1$ or 2, or β-vibrational one.

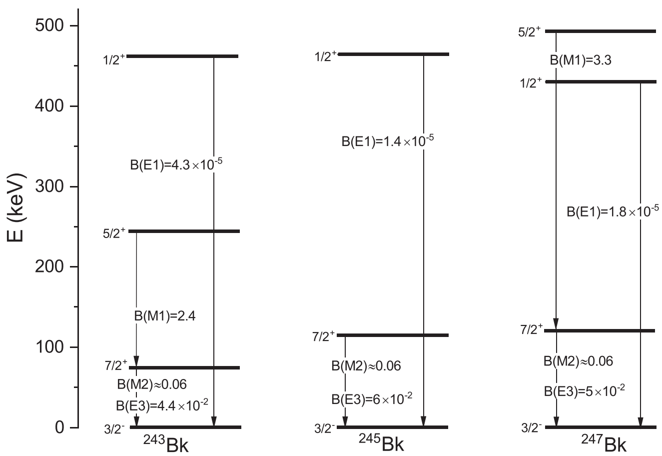

FIG. 1. Calculated spectra of ^{243,245,247}Bk. The structure of all indicated states is exhausted by one-quasiparticle component. Possible γ transitions are marked together with the corresponding reduced transition probabilities in Weisskopf units.

As follows from our calculations, the following single-particle proton Nilsson states determine the structure of the nuclei considered at excitation energies below 1 MeV: $3/2^-[521]$, $7/2^+[633]$, $7/2^-[514]$, $9/2^+[624]$, $1/2^-[521]$, $5/2^-[512]$, and $11/2^+[615]$. In this sequence they successively replace each other as the ground nuclear states with increasing number of protons. Being the ground states for nuclei with a certain number of protons, they begin to appear as the excited states with an increase or decrease in the number of protons, gradually moving away from the ground state. Since the calculated deformations of nuclei under consideration vary little with the number of nucleons, the above sequence of the single particle states corresponds to the gradual filling by protons of the single particle states of the mean field potential used. This means that experimental information on the spectrum of the excited states of nuclei under consideration will either confirm the correctness of the mean field potential used, or indicate the need to change it. Note that information about the mean field potential of the nucleus is of great importance in calculating the survival probability of superheavy nuclei.

A. Excitation spectra and γ transitions in Bk isotopes

The calculated excitation spectra of Bk isotopes together with the reduced γ-transition probabilities are shown in Fig. 1. In these nuclei the calculated ground state is $3/2^-[521]$, which coincides with the experimental assignment [56]. In all three isotopes the calculated first excited state is $7/2^+[633]$ as in the experimental data. This $7/2^+$ state decays in the considered nuclei by $M2$ transition to the ground state with $B(M2) \approx$

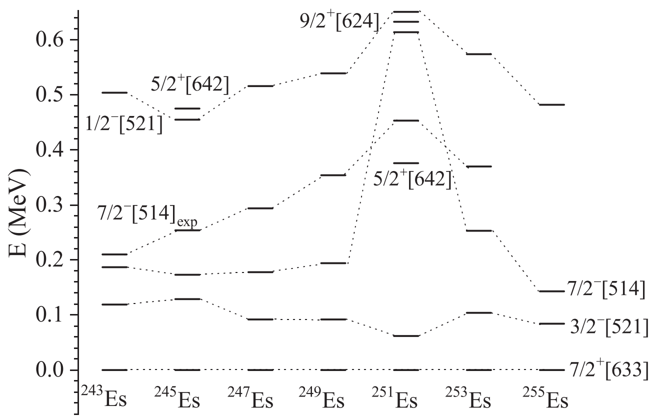

FIG. 2. Calculated spectra in isotopes of Es. Available experimental energies [56] of the $7/2^-$ state are indicated.

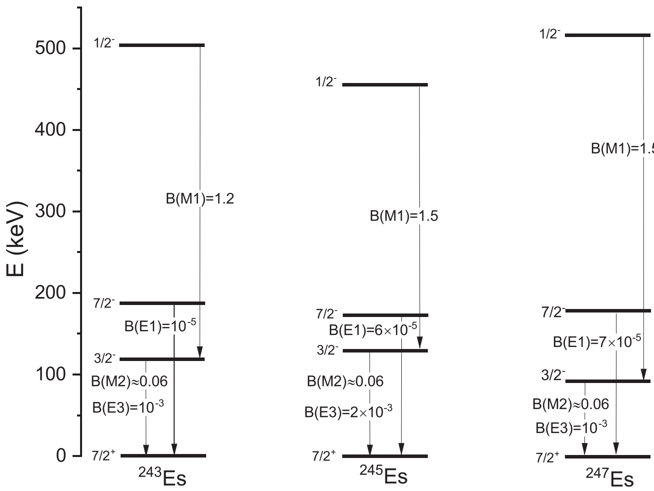

FIG. 3. The same as in Fig. 1, but for ^{243,245,247}Es.

0.06 W.u. and by $E3$ transition with $B(E3) = 0.04-0.06$ W.u. The values of $B(M2)$ correspond to γ-decay lifetime $\tau_\gamma \approx 0.1-10$ ms. Thus, the first excited state $7/2^+$ at an energy of about 100 keV can be treated as an isomeric one.

The higher lying group of the excited states $1/2^+$, $5/2^+$, and $7/2^-$ is located around 500 keV. These states decay mainly to the ground or to the first excited state. Note, that in ^{243}Bk the $5/2^+[642]$ state is lying 250 keV lower than in ^{245,247}Bk because of the more complicated structure of the $5/2^+$ in ^{243}Bk where the weight of the one-quasiparticle component is 87%. In ^{245,247}Bk the corresponding weights are 98% and 100%. The calculated quadrupole and hexadecapole deformations are the same in all Bk isotopes considered.

The next excited state $3/2^+[402]$ of ^{243}Bk (626 keV) has an excitation energy by 200 keV lower than in ^{245,247}Bk, where its excitation energy is around 800 keV. The weight of the one-quasiparticle component of this state in ^{243}Bk is 76% and there is a one-quasiparticle$\otimes$phonon component with 0^+ phonon. The main component of the $3/2^+$ state in ^{245,247}Bk is 3/2[651].

The lowest $1/2^+$ and $5/2^+$ have as the main components 1/2[660] and 5/2[642] Nilsson states lying below Fermi level in the considered nuclei.

B. Excitation spectra and γ transitions in Es isotopes

In Figs. 2–4, the calculated excitation spectra and the reduced γ-transition probabilities are shown in the Es isotopes. In all considered isotopes the calculated ground state is $7/2^+[633]$ and the first excited state is $3/2^-[521]$ at about 50–100 keV. In the experimental spectra of ^{245,247,251}Es, the ground state is $3/2^-$ and only in ^{249}Es the ground state is $7/2^+$. However, the experimental first excited state $7/2^+$ is at very small excitation energy.

In all considered isotopes of Es, the first excited state decays into the ground state by $M2$ and $E3$ transitions. In ^{243,245,247,249}Es the value of the $B(E3; 3/2^- \rightarrow 7/2^+)$ is about 10^{-3} W.u. and B(M2; $3/2^- \rightarrow 7/2^+$) is about 0.06 W.u. The estimated lifetime is $\tau_\gamma \approx 1$–3 ms, i.e., the calculated $3/2^-$ states can be treated as the isomeric ones.

The calculated second excited state in considered Es isotopes is $7/2^-[514]$, excluding ^{251}Es. Its excitation energy takes the value between 100–200 keV, although in ^{251}Es the excitation energy of the $7/2^-$ state increases up to 600 keV. According to our calculations this state decays mainly to the ground state by $E1$ transition with $B(E1; 7/2^- \rightarrow 7/2^+) \approx 10^{-4}$ W.u. with corresponding lifetime $\tau_\gamma \approx 1$ ns.

The calculated spectrum of the excited states of ^{251}Es shows an interesting feature associated with a sharp decrease in the calculated energy of the octupole phonon excitation of the even-even core of this nucleus compared to neighboring ones. As a result, the structure of the lowest excited states is changed. If in ^{249}Es and ^{253}Es $7/2^+$ (gs), $3/2^-$ and $5/2^+$ (581 keV and 675 keV, respectively) are almost one-quasiparticle states, in ^{251}Es they have noticeable components of the quasiparticle$\otimes$phonon type, while the weight of the one-quasiparticle component decreases to 87% in the $7/2^+$ state, to 71% in the $3/2^-$ state, and to 68% in the $5/2^+$ state. At the same time, the structure of the $7/2^-$ state is almost exhausted by the one-quasiparticle component (93%). As a result, the $7/2^+$, $3/2^-$, and $5/2^+$ states drop sharply in energy relative to the $7/2^-$ state, due to the effect of increased

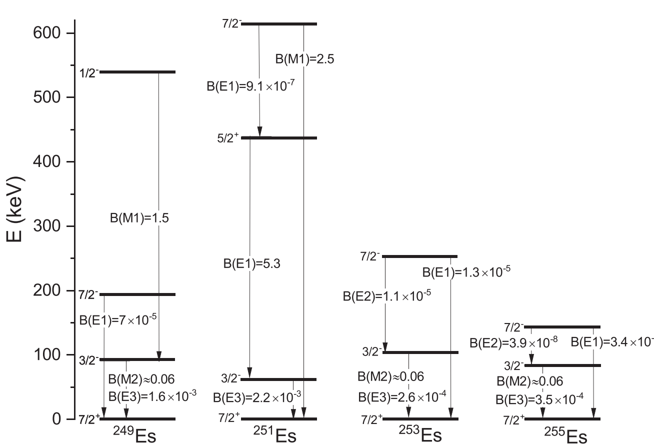

FIG. 4. The same as in Fig. 1, but for ^{249,251,253,255}Es.

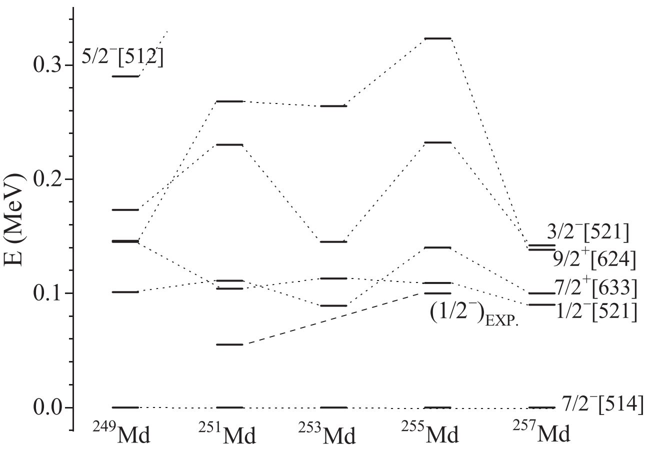

FIG. 5. The same as in Fig. 2, but for the isotopes of Md. Available experimental energies of the $1/2^-$ state are indicated.

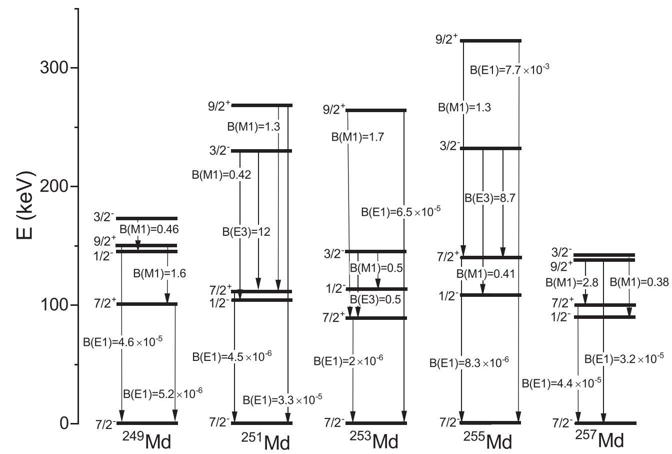

FIG. 6. The same as in Fig. 1, but for ^{249,251,253,255,257}Md.

quasiparticle-phonon interaction, which manifests itself in the spectrum as a sharp increase in the excitation energy of the $7/2^-$ state.

The next group of the excited states in the isotopes of Es consists of $1/2^-$, $5/2^+$, and $9/2^+$. Their excitation energies take the values in the interval 400–800 keV. The $1/2^-$ state decays by $M1$ transition to the first excited $3/2^-$ state with $B(M1) \approx 1$ W.u. Only in ^{251}Es the calculated value of $B(M1)$ for this transition is 0.14 W.u. This value corresponds to the lifetime $\tau_\gamma = 3$ ps.

Above 600 keV the excitation spectrum of ^{251}Es is much more dense than the excitation spectra of ^{243,245,247,249}Es. The reason is a presence of the low-lying $K^\pi = 2^-$ octupole phonon in the calculated spectra of the even-even neighbors of ^{251}Es. Collectivity of this octupole phonon decreases with decreasing number of neutrons.

C. Excitation spectra and γ transitions in Md isotopes

The calculated ground state of ^{249,251,253,255,257}Md is a one-quasiparticle $7/2^-[514]$ state (Fig. 5). This result is in agreement with the experimental information. The calculated first excited state in ^{251,255,257}Md is $1/2^-[521]$ one-quasiparticle state that is in agreement with the experimental data [56]. The one-quasiparticle state $1/2^-$ can decay only to either $7/2^-$ or $7/2^+$ state and, thus, it is an isomeric one (Fig. 6) because of large ΔK in transition. The excited state $7/2^+[633]$ at 100–150 keV decays by $E1$ transition to the ground state with $B(E1) \approx 10^{-6}$ W.u. and $\tau_\gamma \approx 10$ ns.

In ^{253}Md, the $3/2^-[521]$ state decays by $M1$ transition with $B(M1) = 0.5$ W.u. to the $1/2^-[521]$ state and $9/2^+[624]$ state decays by $M1$ transition with $B(M1) = 1.4$ W.u. to the $7/2^+$(224 keV) state. In the other considered isotopes of Md the calculated $M1$ transition strengths are almost the same.

D. Excitation spectra and γ transitions in Lr isotopes

In ^{253,259,261}Lr the calculated ground state is $9/2^+[624]$. The calculated first excited state in these isotopes is $1/2^-[521]$ at 104, 28, and 17 keV, respectively. In ^{251,255,257}Lr the calculated ground state is $1/2^-[521]$ and the first excited state is $9/2^+[624]$ with excitation energy below 100 keV. Since γ transitions between these states are weak because of large ΔK and small energy differences, the first excited state in Lr isotopes is an isomeric one (Fig. 7). The experimentally known ground state of ^{255}Lr is determined, preliminary, as $1/2^-$. In ^{253}Lr the observed ground state is determined, preliminarily, as $7/2^-$. In the calculated spectra of ^{253}Lr the $7/2^-[514]$ state has the excitation energy of 475 keV (Fig. 7). However, in other considered isotopes of Lr the calculated $7/2^-[514]$ is the second excited state which decays to the $9/2^+$ state with $B(E1) \approx 10^{-6}$ W.u. corresponding to $\tau_\gamma \approx 30$ ns.

In the calculated spectra of ^{251,255,257,259,261}Lr the next group of the excited states consists of $5/2^-[512]$, $3/2^-[521]$, and $7/2^+[633]$. The reason of the difference between the calculated spectra of ^{253}Lr and other Lr isotopes is a strong admixture of a quasiparticle$\otimes$phonon component with $K^\pi =$

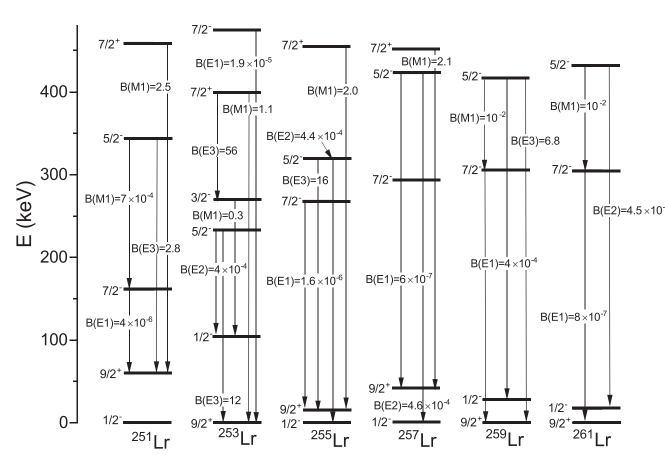

FIG. 7. The same as in Fig. 1, but for ^{251,253,255,257,259,261}Lr.

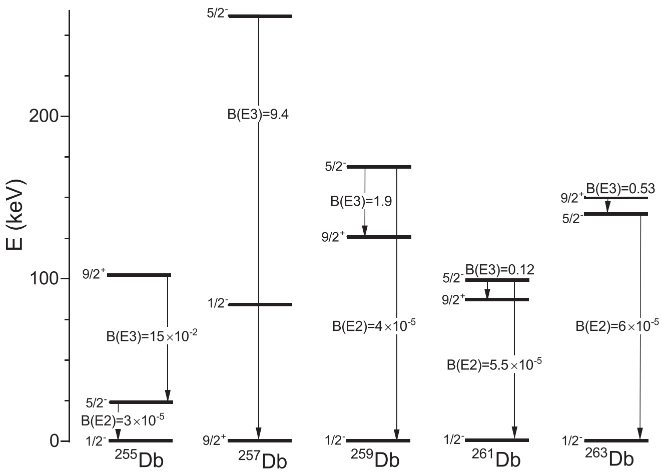

FIG. 8. The same as in Fig. 1, but for ^{255,257,259,261,263}Db.

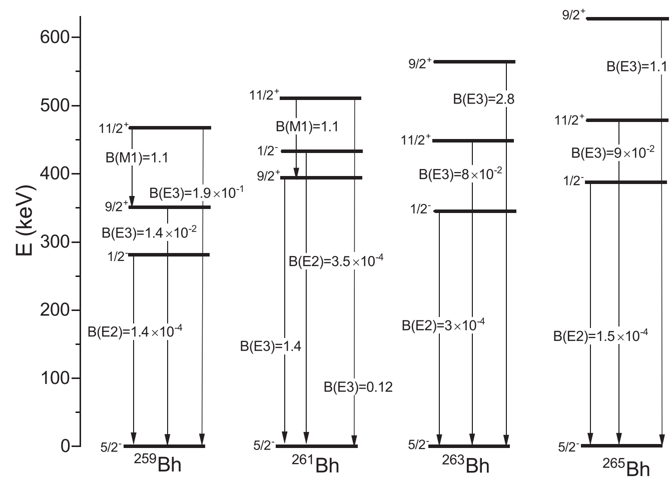

FIG. 9. The same as in Fig. 1, but for ^{259,261,263,265}Bh.

2^- in the wave functions of $5/2^-_1$, $3/2^-_1$, and $7/2^+_1$ states of ^{253}Lr. In ^{253}Lr $5/2^-$(233 keV) state decays by $E2$ transition to the $1/2^-$(104 keV) state with the corresponding lifetime $\tau_\gamma \approx 20\,\mu$s. The $3/2^-$(270 keV) state decays by $M1$ transition to the $1/2^-$(104 keV) state with $B(M1) \approx$ 0.3 W.u. The $7/2^+_1$(399 keV) state decays by the collective $E3$ transition with $B(E3) = 56$ W.u. to the $3/2^-$(270 keV) state and by $M1$ transition with $B(M1) = 1.1$ W.u. to the ground state. Thus, there is only one isomeric state in Lr isotopes.

E. Excitation spectra and γ transitions in Db isotopes

The calculated ground state of ^{255,259,261,263}Db is $1/2^-[521]$ (Fig. 8). In ^{257}Db, the calculated ground state is $9/2^+[624]$. In this nucleus $1/2^-[521]$ is the first excited state with with an energy of 84 keV. According to the preliminary experimental data the ground state of ^{257,259}Db is $9/2^+$ [56]. In ^{259,261,263}Db the calculated low-lying $9/2^+$ state is the isomeric one (Fig. 8). The one-quasiparticle state $9/2^+[624]$ is the second excited state in ^{255,263}Db with an excitation energy of 102 and 147 keV, respectively, and the first excited state in ^{259,261}Db with an energy of 126 and 87 keV, respectively. Its lifetime with respect to the $M2$ transition to the first excited one-quasiparticle $5/2^-[512]$ state is larger than 6 ms. Thus, this state is predicted as an isomeric one.

F. Excitation spectra and γ transitions in Bh isotopes

In ^{259,261,263,265}Bh, the calculated ground state is $5/2^-[512]$ (Fig. 9). The next group of the excited nonrotational states in these isotopes which contains $1/2^-[521]$, $11/2^+[615]$, and $9/2^+[624]$ one-quasiparticle states is located between 300 keV–600 keV. In the calculated spectra of ^{261,263,265}Bh there are no other excited states below 1 MeV. As follows from our calculations, the low-lying isomeric states are not expected in Bh isotopes.

G. Excitation spectra and γ transitions in Mt isotopes

In ^{263,265,267,269}Mt isotopes, the calculated ground state is $11/2^+[615]$ and the first excited state $5/2^-$ is the isomeric one (Fig. 10). The α-decay lifetime from this isomeric state is estimated as at least two orders shorter than the $E3$ γ-decay lifetime. So, the α decay of Mt isotopes considered could have two α-decay lines.

IV. α-DECAY CHAINS OF MT ISOTOPES

α-decay chains of Mt isotopes considered are presented in Figs. 11 and 12. If these isotopes are produced in complete fusion reactions, the ground $11/2^+$ as well as isomeric $5/2^-$ states are populated in them. The lifetime of the $5/2^-$ state with respect to the γ decay to the ground state is about 2.5 ms, whereas its lifetime with respect to the α decay to the ground state of Bh is about 0.25 ms. So, the α decay from the isomeric state in Mt can occur. The α decay of the $5/2^-$ state populates the ground state of Bh. The ground state of Mt isotopes decay to the $11/2^+$ excited state in Bh. This excited

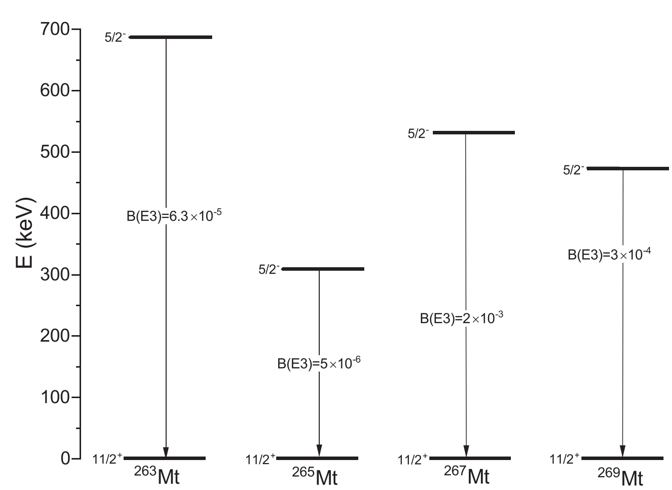

FIG. 10. The same as in Fig. 1, but for ^{263,265,267,269}Mt.

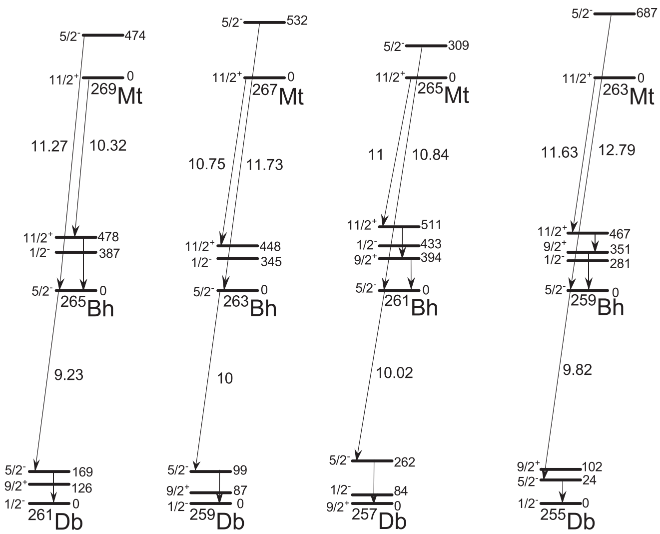

FIG. 11. Possible α-decay chains starting from ^{263,265,267,269}Mt. The calculated values of Q_α are given in MeV and the γ transitions following α decay are marked.

state decays to the ground state of Bh either directly by $E3$ transition with $\tau_\gamma = 0.8$ ms or through $9/2^+$ state. Its α-decay lifetime to Db is much longer, about a few seconds. Thus, the α decay of Bh isotopes considered occurs only from the ground state and populates the $5/2^-$ state in the corresponding isotopes of Db. The last state decays by $E2$ transition to the ground state of ^{255,259,261}Db ($1/2^-$[521]) with corresponding $\tau_\gamma \approx 1$ ms, whereas the allowed α decay from the $5/2^-$ state to corresponding state in Lr requires about 9.3 sec. So, the α decay of Bh mainly leads to the population of the ground state of Db, while the direct production of Db leads to the populations of two states ($1/2^-$ and $9/2^+$) from which α decays are possible. In the last case the α decay of Db has two lines.

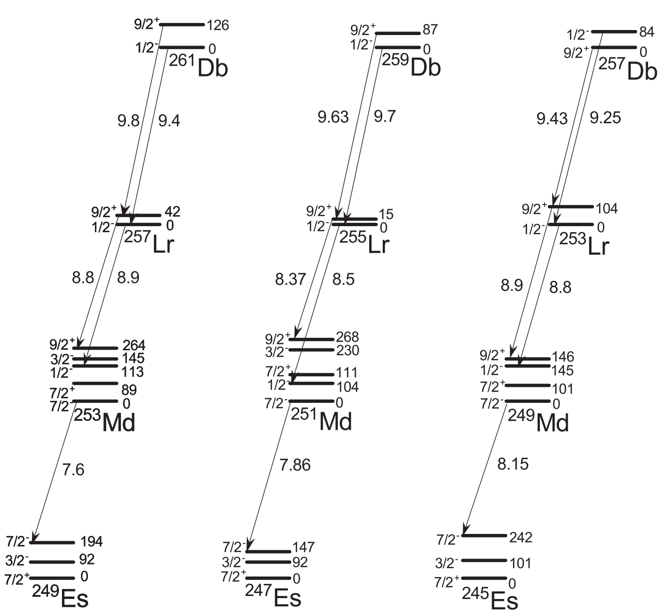

FIG. 12. Possible α-decay chains starting from ^{257,259,261}Db. The calculated values of Q_α are given in MeV and the γ-transitions following α-decay are marked.

The α decays from the ground and isomeric states of Db populate the corresponding states in Lr where they are very close in energy. Thus, the α decay of Lr can occur from the ground and isomeric states regardless of whether Lr is formed directly or in the α decay of Db. However, in the α-decay chains of Mt isotopes the Lr likely has only one α-decay line because in these chains only one state is populated in Db. The α decay of the ground state of Lr populates the isomeric $1/2^-$ state in Md. The α decay of this isomer looks unlikely, since it takes more than 30 s, and the transition of Md to its ground state is more likely. Thus, the α decay of Md has only one line.

In Ref. [57], the ground and isomeric states of ^{253,255}Lr are either $7/2^-$ or $1/2^-$. In our calculation there is $9/2^+$ state instead of $7/2^-$. The α-decay energies, which are calculated as in Ref. [38], are in good agreement with the experimental ones. So, based on the existing experimental data we cannot prove or disprove the calculated spectra of Lr isotopes.

V. SUMMARY

The systematic calculations of the excitation spectra, structure of the wave functions, and the γ-transition probabilities of odd-proton nuclei with $Z = 97$–109 were performed. The QPM was used to take into account the monopole pairing and the quasiparticle-phonon interaction. This model is improved by finding out the ground-state deformations for each nucleus using the microscopic-macroscopic approach. It is shown that the structure of nuclear states in the considered nuclei is mainly exhausted by the one-quasiparticle components. However, in some isotopes the quasiparticle$\otimes$phonon admixtures can play an important role. The quasiparticle-phonon interaction influences the ordering of the levels at the excitation energies characterized by quite dense excitation spectra and destroys the smooth isotopic dependence of state energies. Note, that for all considered nuclei the calculations are performed with the fixed parameters of the Hamiltonian which seem to be reliable in the wide region of the nuclide chart including heaviest nuclei. The calculated γ-transition probabilities allow us to find the isomeric states in the spectra of nuclear excitations and estimate the lifetimes of isomers.

The α-decay chains starting from ^{263,265,267,269}Mt were analyzed. The nuclei in these α-decay chains have either one or two α-decay lines. The number of α-decay lines can depend on whether the nucleus is produced directly or after the α decay of the parent nucleus. This fact deserves experimental study.

The calculated excitation spectra of the considered nuclei being compared with the experimental ones will give us information on the mean field potentials of heavy nuclei. This information is of great importance for calculations of the survival probabilities of superheavy nuclei.

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