## Dense nuclear matter with phenomenological short distance repulsion

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The possibility of new short distance physics applicable inside the cores of neutron stars is incorporated into the equation of state generated by the quark-meson coupling model. The contribution of this new physics to the energy density is taken to be proportional to the amount of overlap between the quark cores of the baryons involved. With no change to the properties of symmetric nuclear matter at saturation density, including an incompressibility compatible with data on giant monopole resonances, one can sustain neutron stars with a maximum mass  $M_{\text{max}} > 2.1 M_{\odot}$ , even when hyperons are included.

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## I. INTRODUCTION

Understanding the equation of state (EoS) of dense nuclear matter, namely the pressure as a function of energy density [1–3],  $p = p(\epsilon)$ , is one of the key challenges being addressed in modern nuclear theory. The EoS is well known in the low density region [4,5] and is believed to be constrained by perturbative OCD in the high density region [6-9]. In the intermediate region, the description of nuclear matter is still a matter of considerable debate. Heavy ion reactions have provided important constraints [10,11] up to densities of order several times the saturation density of symmetric nuclear matter,  $n_0$ . However, the matter formed in these collisions exists only for a short time and in a small volume, which introduces a degree of model dependence in the interpretation. Intense interest has shifted to long lived, cold neutron stars (NSs) because their core densities are expected to be as large as 4-10 times  $n_0$ . At such large densities we cannot be sure whether the matter is hadronic, quark matter, or some hybrid form [6,12,13]. Furthermore, as the matter in a NS is stable long term, to the extent that it is hadronic, one must satisfy the conditions of  $\beta$  equilibrium and hyperons must be present.

Of the more than 40 NS mass measurements, most center around the canonical mass of  $1.4-1.5 M_{\odot}$  [13–16]. These rely on the detection of a NS in a binary system [15,17,18]. Given the compactness and the extraordinary distances of these stars from Earth, mass and radius measurements on the same star still carry considerable uncertainty, making them suboptimal for constraining the EoS. Much of the focus has shifted to the heaviest of these stars, of which three are known; PSR J0348+0432 has a mass of  $M = 2.01^{+0.04}_{-0.04} M_{\odot}$  [19], PSR J1614-2230 has a mass of  $M = 1.908^{+0.016}_{-0.016} M_{\odot}$  [20], and PSR J0740+6620 has a mass of  $M = 2.072^{+0.067}_{-0.066} M_{\odot}$  [21,22].

Measurements of NS radii are particularly difficult to narrow down [23]. However, the gravitational wave (GW) detection of GW170817 has placed new emphasis on the radii of NSs [24]. The NS merger event also allowed a totally new property, the tidal deformability, to be extracted from the waveform of the passing gravitational wave. For a  $1.1-1.5 M_{\odot}$ NS, the radius is largely independent of the mass. GW170817 involved component masses of  $m_1 \in [1.18, 1.36] M_{\odot}$  and  $m_2 \in [1.36, 1.58] M_{\odot}$ , with radii for either star  $R = 11.9 \pm$ 1.4 km, making it highly likely that the event involved a binary NS system [24] (GW190425 may contain NSs but there is a high probability that the merger event has at least one black hole component [25]). The tidal deformability measured for GW170817, which was reported to be  $\Lambda_{1.4} = 190^{+390}_{-120}$ , is a measure of the quadrupole deformation of a spherical object caused by an inhomogenous external gravitational field [24]. GW measurements have the further advantage that they may offer insights concerning the presence of exotic matter in stars [26, 27].

With the observation of NSs with masses  $M \approx 2M_{\odot}$ , many EoSs have been ruled out [19,21,28]. This is particularly true for those where hyperons are included. Under  $\beta$  equilibrium, hyperons are predicted to appear at or above  $3n_0$ , and, because they have low momentum near the threshold for their introduction, the pressure corresponding to a given energy density is lower than it would be without hyperons. Then the maximum mass of the star must be lower [13,17,18,29].

The quark-meson-coupling (QMC) model is unique amongst models for the binding of nuclear matter in that the change in the structure of any bound hadron, induced by the strong mean scalar field in medium [30,31], plays a key role. In particular, the resultant density dependent reduction in the coupling of baryons to the scalar field [32] is equivalent to introducing many-body forces [33]. The three-body force, in

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particular, acts between all combinations of baryons, *NNN*, *YNN*, *YYN*, and *YYY* (with *Y* a hyperon), without the introduction of any new parameters. As a result of this three-body force, the QMC model predicted the existence of NSs with masses up to  $2M_{\odot}$ , with hyperons [34], three years before the first was observed [28].

Following the discovery of NSs with masses as large as  $2M_{\odot}$ , many possible solutions to what was regarded as the "hyperon puzzle" have been proposed. These include density dependent couplings, many body forces, and a possible phase transition from hadronic to deconfined quark matter [35,36]. By interpolating between accurate low ( $n_B < 1.1n_0$ ) and high ( $n_B > 40n_0$ ) density EoSs, several researchers have derived phenomenological EoSs matching heavy NS observations [6–9]. The requirement that the EoS approaches that of perturbative QCD at high densities has a distinct imprint on the speed of sound ( $c_s^2$ ). Conformal matter has a limit of  $c_s^2 < 1/3$  approaching from below, which suggests deconfined quark matter [6], although the appearance of hyperons has a similar signature [37,38].

The QMC model has been extensively developed since the prediction of high mass NSs with hyperons. The introduction of an isovector scalar meson did not dramatically change the predictions for NS properties, with the most significant change being an increase in the NS radius, for a given mass [39].

The energy density functional derived in the model [32,40] has been applied to the properties of even-even nuclei across the periodic table, with considerable success [41–44]. However, studies of the giant monopole resonances (GMRs) required the introduction of a term cubic in the scalar potential, which also lowered the incompressibility of nuclear matter [41]. The same term included in calculations of the EoS of dense nuclear matter tends to lower the maximum neutron star mass.

The parameters of the QMC model, associated with the exchanged mesons, are determined by the properties of infinite nuclear matter at saturation density [45]. On the other hand, the cores of the highest mass NSs involve much higher densities, where the finite-size baryons may overlap; something not included in the model. In that region nonperturbative gluon exchanges may lead to additional repulsion not accounted for by longer-range meson exchange. With this in mind, here we extend the QMC model to include a new, phenomenological, short-ranged repulsive force, which only affects the EoS at densities well above saturation density.

The structure of this paper is as follows. In Sec. II we summarize the calculation of the EoS of dense matter within the QMC model and introduce the new short distance repulsive force, designed to affect the EoS only at densities well above nuclear matter density. The results and concluding remarks are presented in Secs. III and IV.

## **II. THEORETICAL FRAMEWORK**

In the MIT bag model, the quarks are confined in a spherical region, the interior of which is a strictly perturbative vacuum. In the QMC model the baryons retain their individual identity, even at higher densities [46,47]. One of the criticisms which Stone *et al.* [34] highlighted is the possibility that the QMC model would begin to breakdown at some unknown threshold density, where the hadrons start to overlap and eventually transition into quark matter. Actually, this may even be apparent at lower densities, of order  $4n_0$ , where nucleons geometrically touch [15]. Performing a lattice QCD calculation, Bissey *et al.* showed that three quarks in a baryon are connected by a Y-shaped flux tube [48]. Elsewhere a nonperturbative QCD vacuum exists, through which baryons are permitted to move freely. Thus a literal interpretation of the MIT bag model may not reflect the full physical picture of nuclear matter [34,40]. While this provides some justification for treating the baryons as nonoverlapping to higher densities than one might naively expect, one may anticipate that new interactions are likely to be needed eventually.

Berryman and Gardner proposed an extension to the standard model based upon gauging baryon number [49]. Their new short-ranged, repulsive quark-quark force was mediated by a vector boson coupling to baryon number *B*. Normally this would be suppressed by the highly repulsive short-range (0.4–0.6 fm) *NN*, *YN*, and *YY* potentials [50]. However, beyond saturation density, as the baryons start to *overlap*, the new interaction would have an affect on the resulting EoS [49,51]. To model this, a two-particle *NN* Yukawa potential was used, and Berryman and Gardner showed that the ensuing NS had a maximum mass of 2–2.2  $M_{\odot}$  [49].

In the QMC model the  $\omega$  meson generates the short distance repulsion between baryons. However, as explained earlier, as the baryons begin to overlap we expect that there may be additional repulsion arising from nonperturbative QCD in the multiquark environment. Indeed, quark model studies over the past 40 years have shown that the Pauli exclusion principle, in combination with the increase in energy associated with hidden color configurations, leads to strong short distance repulsion between nucleons (see, e.g., Harvey [52]). This has been confirmed more recently by, for example, lattice QCD studies by the HAL Collaboration [53]. Low energy nucleon-nucleon scattering, which is used to constrain phenomenological descriptions of this in terms of, say,  $\omega$  meson exchange, does not probe that short distance behavior [54] in detail and one might expect that this additional repulsion could be manifest in the much more dense systems at the core of a neutron star. To model these short distance interactions, we propose an additional, phenomenological term to be added to the QMC energy density, which only has appreciable effects at supranuclear densities. The functional form used to model the degree of overlap is taken to have a simple Gaussian form, given in Eq. (18), and is required to stiffen the EoS without changing the physics at saturation density.

Our summary of the key details of the QMC model follows closely the work of Motta *et al.* [39] and Guichon *et al.* [40]. The calculation is carried out in the Hartree-Fock approximation, with the inclusion of the isovector scalar meson,  $\delta$ , as introduced by Motta *et al.* [39].

## A. Energy density

In infinite nuclear matter, using the mean-field approximation, the effective mass of a baryon with flavor f is

given by

$$M_f^*(\bar{\sigma},\bar{\delta}) = \frac{\Omega_u N_u + \Omega_d N_d + \Omega_s N_s - z_0}{R_B} + BV_B + \Delta E_M.$$
(1)

 $R_B$  and  $V_B$  are the bag radius and volume respectively.  $\Omega_i$ expresses the lowest eigenvalue for the three valence quarks, calculated in the local scalar mean fields ( $N_i$  denotes the number of u, d, or s quarks). The input parameters, determined at zero density, are B,  $\alpha_s$ , and  $z_0$ . These are respectively the bag pressure, the strong coupling constant, which sets the size of the hyper-fine color interaction ( $\Delta E_M$ ), and the zero point gluon fluctuation terms. They are fitted to satisfy the mass of the free baryons and boundary conditions imposed by the MIT bag model [46,55]. Although the calculation of  $\Delta E_M$  is complicated, it is essentially given by a spin matrix element multiplied by  $\alpha_s$  divided by the bag eigenenergies of the pairs of quarks involved. It is the reduction in those eigenenergies in medium which leads to the enhancement of the hyperfine interaction there [56], which we include.

In the following, the baryon species with flavor f which we include are n, p,  $\Lambda$ , and  $\Xi^{0,-}$ , since previous calculations have shown that the  $\Sigma^{0,\pm}$  and  $\Delta$  baryons are absent for  $n_B < 1.2 \text{ fm}^{-3}$  [34,57] (further explanations may be found in the results Sec. III B). The total number density is given by  $n_B = \sum_f n_f$ . Note that each flavor baryon f has a definite strangeness (S), isospin projection (m), and total isospin (t).

At the deepest level the QMC model takes into account the coupling of the lightest scalar ( $\sigma$  and  $\delta$ ) and vector ( $\omega$  and  $\rho$ ) mesons to the *u* and *d* quarks. Zweig's rule suggests that the coupling of these nonstrange mesons to the s quark should be suppressed, and in order to reduce the number of parameters in the model we set it to zero. We defer discussion of the phenomenological success of this approach, which provides some a posteriori justification, to Sec. III B. Of course, one does expect that there will be some mesons which couple to the strange quarks but  $s\bar{s}$  mesons have masses in excess of 1 GeV and, in addition to the expected small coupling to nonstrange nucleons for such mesons, their large masses suggest that their effects will be strongly suppressed by short-range correlations. At the mean field level we solve self-consistently for the mass of the baryon using the MIT bag model but including the effect of the the  $\sigma$  and  $\delta$  mesons. The resulting energy of a baryon with momentum  $\vec{k}$  is then

$$E_f = \sqrt{M_f^*(\bar{\sigma}, \bar{\delta})^2 + \vec{k}^2} + g_\omega^f \bar{\omega} + g_\rho^f I_m^f \bar{\rho}.$$
 (2)

Here, the mean isoscalar and isovector scalar mean-fields are  $\bar{\sigma}$  and  $\bar{\delta}$ , respectively, while  $\bar{\omega}$  and  $\bar{\rho}$  are the time components of the isoscalar and isovector vector mean fields. In Eq. (2) the coupling strength of the  $\omega$  meson is dependent on the number of nonstrange quarks such that  $g_{\omega}^{f} = (1 + S/3)g_{\omega}$ , with S being the strangeness of f flavor baryon. The strange quark does

not carry isospin and as such  $g_{\rho}^{f} = g_{\rho}$  since the weighting is carried by  $I_{m}^{f}$ , the isospin projection.  $g_{\omega}$  and  $g_{\rho}$  are the coupling constants for the  $\omega - N$  and  $\rho - N$  respectively. A similar notation for the  $\sigma$  and  $\delta$  mesons will be used in Eq. (3). The effective mass in Eq. (1) may be expressed in terms of the scalar fields as follows:

$$M_{f}^{*}(\bar{\sigma},\bar{\delta}) = M_{f} - w_{\sigma}^{f}g_{\sigma}\bar{\sigma} + \tilde{w}_{\sigma}^{f}\frac{d}{2}(g_{\sigma}\bar{\sigma})^{2}$$
$$- t_{\delta}^{f}g_{\delta}I_{m}^{f}\bar{\delta} + \tilde{d}g_{\sigma}g_{\delta}\bar{\sigma}I_{m}^{f}\bar{\delta}$$
$$= M_{f} - g_{\sigma}^{f}(\bar{\sigma})\bar{\sigma} - g_{\delta}^{f}(\bar{\sigma})I_{m}^{f}\bar{\delta}.$$
(3)

The scalar polarizabilities (d and  $\tilde{d}$ ) and the flavor dependent weights ( $w_{\sigma}^{f}$ ,  $\tilde{w}_{\sigma}^{f}$ , and  $t_{\delta}^{f}$ ) are calculated using the underlying bag model over a range of densities and involve no new parameters. These reflect the self-consistent response of the internal structure of the bound baryons to the applied scalar fields. The scalar polarizabilities are the origin of the repulsive three-body forces which arise naturally within the model [33,40]. Finally the coupling strengths for the nucleon, calculated in free space using the MIT bag model, are denoted  $g_{\sigma}$  and  $g_{\delta}$  for the  $\sigma - N$  and  $\delta - N$ , respectively. The scalar fields change the effective mass [see Eq. (3)] and the vector fields shift the energy of the baryon [see Eq. (2)].

In the mean field approximation each of the meson field operators is set to its expectation value, while the corrections are treated as a small perturbation,  $\sigma \rightarrow \langle \bar{\sigma} \rangle + \Delta \sigma$ . The integrals range from 0 to the relevant Fermi momentum

$$k_f(n_f) = \sqrt[3]{3\pi^2 n_f}.$$
 (4)

Omitting the arguments for the Fermi momentum and the effective mass, the meson fields are then

$$m_{\sigma}^{2}\bar{\sigma} = -2\sum_{f} \frac{\partial M_{f}^{*}}{\partial\bar{\sigma}} \int^{k_{f}} \frac{d^{3}k}{(2\pi)^{3}} \frac{M_{f}^{*}}{\sqrt{\vec{k}^{2} + M_{f}^{*2}}}, \quad (5)$$

$$m_{\omega}^{2}\bar{\omega} = \sum_{f} g_{\omega}^{f} n_{f}, \qquad (6)$$

$$m_{\rho}{}^{2}\bar{\rho} = \sum_{f} g_{\rho} I_{m}^{f} n_{f}, \qquad (7)$$

$$m_{\delta}^{2}\bar{\delta} = -2\sum_{f} \frac{\partial M_{f}^{*}}{\partial\bar{\delta}} \int^{k_{f}} \frac{d^{3}k}{(2\pi)^{3}} \frac{M_{f}^{*}}{\sqrt{\vec{k}^{2} + M_{f}^{*2}}}.$$
 (8)

The contribution to the energy density from baryons and the meson fields,  $\epsilon_B$ , is

$$\epsilon_B = \frac{\langle \mathcal{H}_B + \mathcal{V}_\sigma + \mathcal{V}_\omega + \mathcal{V}_\rho + \mathcal{V}_\delta + \mathcal{V}_\pi \rangle}{V}.$$
 (9)

In the Hartree-Fock approximation, all meson exchanges are evaluated as follows:

$$\frac{\langle \mathcal{H}_B \rangle}{V} = 2 \sum_f \int^{k_f} \frac{d^3k}{(2\pi)^3} \sqrt{\vec{k}^2 + M_f^{*2}},$$
(10)
$$\frac{\langle \mathcal{V}_\sigma \rangle}{V_f} = \frac{m_\sigma^2 \bar{\sigma}^2}{\sigma^2} + \frac{\lambda_3}{\sigma^3} g^3 \bar{\sigma}^3 + \sum_{j=1}^{\infty} \left(\frac{\partial M_f^*}{\partial j}\right)^2 \int^{k_f} \int^{k_f} \int^{k_f} \frac{d^3k_1 d^3k_2}{\sigma^3} \frac{1}{\sigma^3} \frac{M_f^{*2}}{\sigma^3}$$
(11)

$$\frac{\lambda_{\sigma}}{V} = \frac{m_{\sigma}^2 \bar{\sigma}^2}{2} + \frac{\lambda_3}{3!} g_{\sigma}^3 \bar{\sigma}^3 + \sum_f \left(\frac{\partial M_f^*}{\partial \bar{\sigma}}\right)^2 \int^{k_f} \int^{k_f} \frac{d^3 k_1 d^3 k_2}{(2\pi)^6} \frac{1}{(\vec{k_1} - \vec{k_2})^2 + m_{\sigma}^2} \frac{M_f^{*2}}{\sqrt{\vec{k_1}^2 + M_f^{*2}}} \sqrt{\vec{k_2}^2 + M_f^{*2}}, \tag{11}$$

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$$\frac{\langle \mathcal{V}_{\omega} \rangle}{V} = \frac{m_{\omega}^2 \bar{\omega}^2}{2} - \sum_f g_{\omega}^{f^2} \int^{k_f} \int^{k_f} \frac{d^3 k_1 d^3 k_2}{(2\pi)^6} \frac{1}{(\vec{k_1} - \vec{k_2})^2 + m_{\omega}^2},\tag{12}$$

$$\frac{\langle \mathcal{V}_{\rho} \rangle}{V} = \frac{m_{\rho}^{2} \bar{\rho}^{2}}{2} - \sum_{f,f'} g_{\rho}^{2} \int^{k_{f}} \int^{k_{f}} \frac{d^{3}k_{1} d^{3}k_{2}}{(2\pi)^{6}} \frac{C_{m,m'} \delta_{S,S'}}{(\vec{k_{1}} - \vec{k_{2}})^{2} + m_{\rho}^{2}},\tag{13}$$

$$\frac{\langle \mathcal{V}_{\delta} \rangle}{V} = \frac{m_{\delta}^2 \bar{\delta}^2}{2} + \sum_{f,f'} \left( g_{\delta}^f(\bar{\sigma}) g_{\delta}^{f'}(\bar{\sigma}) \right) \int^{k_f} \int^{k_f} \frac{d^3 k_1 d^3 k_2}{(2\pi)^6} \frac{C_{m,m'} \delta_{S,S'}}{(\vec{k}_1 - \vec{k}_2)^2 + m_{\delta}^2} \frac{M_f^*}{\sqrt{\vec{k}_1^2 + M_f^{*2}}} \frac{M_{f'}^*}{\sqrt{\vec{k}_2^2 + M_{f'}^{*2}}}.$$
(14)

Equation (11) includes the term cubic in the  $\sigma$  field required to lower the incompressibility of symmetric nuclear matter sufficiently to yield acceptable GMR energies. The strength of the self-interaction is governed by the coefficient  $\lambda_3$ . The solutions of Eqs. (12) and (13) are simple to evaluate, because they correspond to two-body interactions. Equations (11) and (14) are more complicated, because the  $\sigma$  and  $\delta$  mean fields are functions of  $M_{f}^{*}$ , which in turn depend on  $\sigma$  and  $\delta$ . These are solved self-consistently at  $\beta$  equilibrium for each given number density (see Stone et al. [34]). The notation for  $C_{m,m'}$  is given in Eq. (15) and is expressed in terms of the Kronecker delta  $(\delta_{m,m'})$ , the isospin  $(I_t^f)$ , and its projection  $(I_m^f)$ , consistent with Ref. [39]. Each baryon flavour f has an associated S, t, and m corresponding to the strangeness, isospin, and projection respectively. Equations (13) and (14) involve double summation and are only nonzero when S = S' $(\delta_{S,S'})$ . In Eq. (14) we have used the fact that  $\frac{\partial M_f^*}{\partial \bar{\delta}} = -g_{\delta}^f(\bar{\sigma})I_m^f$  and allow  $C_{m,m'}$  to carry the isospin dependence,

$$C_{m,m'} = \delta_{m,m'} I_m^{f^2} + (\delta_{m,m'+1} + \delta_{m+1,m'}) I_t^f.$$
(15)

While we have used a nonrelativistic approximation to the Fock terms in Eqs. (11) to (14), rather than the relativistic form used in earlier work [57], this has no bearing on the effect of the overlap term which is the focus of the present study.

Finally the long-range pion Fock terms [34,58] are

$$\frac{\langle \mathcal{V}_{\pi} \rangle}{V} = \left(\frac{g_A}{2f_{\pi}}\right)^2 \left\{ J_{pp} + 4J_{pn} + J_{nn} + \frac{1}{25} (J_{\Xi^-\Xi^-} + 4J_{\Xi^-\Xi^0} + J_{\Xi^0\Xi^0}) \right\}, (16)$$

where

$$J_{ff'} = \int^{k_f} \int^{k'_f} \frac{d^3k_1 d^3k_2}{(2\pi)^6} \left[ 1 - \frac{m_{\pi}^2}{(\vec{k_1} - \vec{k_2})^2 + m_{\pi}^2} \right]$$
(17)

and the  $\delta$  function associated with the first term in the square brackets is dropped because of short distance repulsion. Note that parameters in Eqs. (16) and (17) are set to  $m_{\pi} = 139$  MeV,  $g_A = 1.26$ , and  $f_{\pi} = 93$  MeV [40].

The phenomenological bag overlap term, denoted  $\mathcal{H}_O$ , is approximated by a simple Gaussian wave function, with the overall strength  $E_0$  and range b treated as free parameters, subject to the constraint that this term must not change nuclear matter properties at saturation density:

$$\frac{\langle \mathcal{H}_O \rangle}{V} = E_0 n_B \exp\left\{-\left(\frac{n_B^{-1/3}}{b}\right)^2\right\}.$$
 (18)

We note that  $n_B^{-1/3}$  is roughly the average distance between baryons in a Fermi gas. Given that *b* characterizes the size of the quark cores of the baryons, we expect it to be of order 0.5 fm. This additional term is added to Eq. (9). The overlap term is assumed independent of the quark content of the baryon and is repulsive. This concludes the construction of the energy density for nuclear matter within the QMC framework.

### B. $\beta$ equilibrium

The material in the NS is assumed to be cold and in  $\beta$  equilibrium. Any particulate species which exist longer than the timescale of the system then participate to minimize the energy density. This includes the hyperons which are stable at high densities, because of Pauli blocking. The total energy density of the system, including the leptons (electron and muon), is minimized under the condition of  $\beta$  equilibrium. Thus the total energy density is

$$\epsilon_{\text{total}} = \epsilon_B + \epsilon_e(n_e) + \epsilon_\mu(n_\mu), \tag{19}$$

where  $\epsilon_B$  is given by Eqs. (9) and (18). The energy density of the electrons and muons is described by a free gas of leptons:

$$\epsilon_l(n_l) = 2 \int^{k_l(n_l)} \frac{d^3k}{(2\pi)^3} \sqrt{\vec{k}^2 + m_l^2},$$
 (20)

where  $k_l(n_l)$  is given by Eq. (4).

Charge neutrality is imposed along with conservation of baryon number. The equilibrium condition is then given by Eq. (21), shown below [34]:

$$\delta \left\{ \epsilon_B(n_n, n_p, n_\Lambda, n_{\Xi^0}, n_{\Xi^-}) + \epsilon_e(n_e) + \epsilon_\mu(n_\mu) + \Lambda_1 \sum_i n_i q_i + \Lambda_2 \left( \sum_f n_f - n_B \right) \right\} = 0.$$
(21)

Here  $\Lambda_i$  are Lagrange multipliers. The electrical charge is denoted by  $q_i$  with the summation over the baryons and the leptons.

### **III. RESULTS**

The EoS generated by the QMC model supplemented by the phenomenological overlap term [Eq. (18)] will be studied either with or without the term cubic in the  $\sigma$  field. The strength of  $\sigma^3$  is determined by the coefficient  $\lambda_3$ , which is chosen to be  $\lambda_3 = 0.02$  fm<sup>-1</sup> or  $\lambda_3 = 0$  fm<sup>-1</sup>. The choice  $\lambda_3 = 0.02$  fm<sup>-1</sup> is motivated by the study of the energies of GMR by Martinez *et al.* [41], where it was found that this was the smallest value of  $\lambda_3$  capable of producing a value for the nuclear incompressibility compatible with that data. Where the overlap term is explicitly included, the EoS will be denoted by *overlap*. When it is not included the EoS will be denoted *no overlap*. If the derived EoS includes hyperons, then it will be denoted *F-QMC*, while the case where only nucleons are included will be denoted *N-QMC* as a comparison.

Given that the inclusion of  $\lambda_3 = 0.02 \text{ fm}^{-1}$  leads to a maximum NS mass that is unacceptably low when hyperons are included, various overlap parameters ( $E_0$  and b) were explored:

- (i) Overlap energy:  $E_0 = 3500, 4500, 5500$  MeV.
- (ii) Range parameter: b = 0.4, 0.5 fm.

The upper limits on both the overlap energy and range parameter were set by the requirement that there be no significant change in the properties of symmetric nuclear matter at saturation density. Where the parameters are left unspecified in what follows, they were chosen to be  $E_0 = 5500$  MeV and b = 0.5 fm. These are the preferred choice, in that they lead to an acceptable maximum mass while not altering the nuclear matter parameters.

The bulk properties of the NS are used to test the viability of the model at high density. The properties of interest are the mass-radius relationship and the tidal deformability. For brevity, only PSR J0740+6620 is shown when constraining the QMC EoS. The mass is taken to be  $M = 2.072^{+0.067}_{-0.066} M_{\odot}$ , with a 68% interval around the median [21,22]. GW170817 serves as a constraint for the tidal deformability,  $\Lambda_{\rm M}$  [24].

#### A. The choice of parameters

The mass of the  $\sigma$  meson is set at 700 MeV, while the masses of the other mesons are taken from their experimental values ( $m_{\delta} = 983$  MeV,  $m_{\rho} = 770$  MeV,  $m_{\omega} = 783$  MeV). In terms of these masses the meson-nucleon coupling strengths in free space are often written as

$$G_{\sigma} = \frac{g_{\sigma}^2}{m_{\sigma}^2}, \quad G_{\omega} = \frac{g_{\omega}^2}{m_{\omega}^2}, \quad G_{\rho} = \frac{g_{\rho}^2}{m_{\rho}^2}.$$
 (22)

 $G_{\delta} = 3 \text{ fm}^2$  is chosen as the preferred value for the coupling of the  $\delta$  field [39,59] but, for comparison, in the Appendix all calculations are repeated for  $G_{\delta} = 0 \text{ fm}^2$ .

In all cases (with and without  $\lambda_3$  and  $G_{\delta}$ ) we require that the bag overlap terms do not alter the properties of nuclear matter at saturation density. The couplings are fixed at the typical values  $n_0 = 0.16 \text{ fm}^{-3}$ , the binding energy per nucleon at  $n_0$  is taken to be  $E_B/A = -15.8 \text{ MeV}$ , while the symmetry energy is S = 30 MeV [1,2,60]. The couplings are specified without the overlap term and held fixed when the overlap is introduced.

The incompressibility and the slope of the symmetry energy are typically taken to lie in the ranges  $K_{\infty} = 250 \pm 50$  MeV [61,62] and  $L_0 = 60 \pm 20$  MeV [60]. We choose to use  $K_{\infty} =$ 260 MeV and  $L_0 = 62$  MeV respectively, because, while

the relation between the incompressibility and the energies of the GMR is somewhat complicated [63,64], calculations of the GMR using the QMC EDF tend to favor values of  $K_{\infty}$  at the lower end of this range [61].

As reported by Guichon et al. [40], the inclusion of  $\lambda_3 = 0.02 \text{ fm}^{-1}$  lowers  $K_{\infty}$  by about 10%, leading to the value 260 MeV noted earlier. This remains unchanged for b = 0.4fm for  $E_0$  ranging from 3500 to 5500 MeV, while it increases slightly (from 262 to 264 MeV) over this range of  $E_0$  for b =0.5 fm. So long as b < 0.6 fm then  $K_{\infty} < 300$  MeV is within the acceptable limits. On the other hand, for  $\lambda_3 = 0$  fm<sup>-1</sup> the incompressibility is 295 MeV (rising to 298 MeV for b = 0.5fm). Physically the range parameter sets the scale at which the extra repulsive force acts in medium. Since saturation density is relatively low, the overlap term has essentially no influence there and thus does not affect the properties of finite nuclei. In NSs, the gravitational force compacts the baryonic matter well past saturation, allowing them to eventually overlap [15]. The extra repulsion induced by baryon overlap stiffens the EoS at supranuclear densities. This is explored in Sec. III C.

#### **B.** NS composition under $\beta$ equilibrium

Within the QMC model it was previously demonstrated [38,57] that among the hyperons only the  $\Lambda$  and  $\Xi^{0,-}$  appear. It is worth noting at this point that the omission of any coupling of the mesons to the s quark produces surprisingly good results for  $\Lambda$  hypernuclei [56] as well as the extremely limited data on  $\Xi$  hypernuclei [65]. It has long been established that  $\Sigma$ hyperons experience a repulsive interaction in nuclear matter [66], and this is indeed found in the QMC model. It is then not surprising that  $\Sigma$  hyperons were not found in earlier work on NS using the QMC model [67], which is why they are simply omitted here. The physical reason underlying this result is the enhancement of the color hyperfine interaction in medium [56] and the repulsive three-body force generated by the scalar polarizability. This means that the  $\Sigma^{\pm,0}$  baryons experience significant repulsion and are not energetically allowed at densities  $n_B \leq 1.2 \text{ fm}^{-3}$ .

The species fractions inside a NS, as predicted by F-QMC, are shown in Fig. 1. F-QMC predicts no hyperons below  $3n_0$ . The overlap term has no bearing on the species fraction because the repulsion introduced by the overlap is independent of quark content and hence minimization of the energy density is not affected. There is a difference in the appearance of hyperons with (solid) and without (dashed)  $\lambda_3$ . The  $\Xi^{0,-}$  appears slightly later when the term in  $\sigma^3$  is present. The relative abundances are also modified.

Table I lists the chemical potentials of the baryon species, with and without overlap ( $E_0 = 5500$  MeV and b = 0.5 fm), calculated at saturation density. The overlap term increases chemical potentials for all baryon species, but, since Table I is measured at low densities, the effect is negligible. We see that the  $\Lambda$  experiences an attractive potential of 35–40 MeV, while the attraction felt by the  $\Xi^0$  is considerably smaller. These

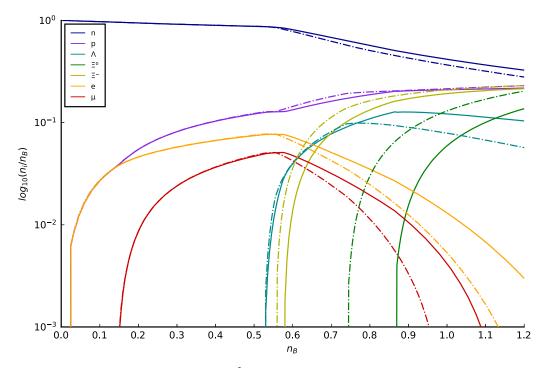


FIG. 1.  $\beta$ -equilibrium was calculated for  $0 < n_B < 1.2 \text{ fm}^{-3}$ . Species fractions of the nucleons, leptons, and hyperons are shown relative to the total baryon number,  $n_B$ , for F-QMC with overlap. Solid lines indicate  $\lambda_3 = 0.02 \text{ fm}^{-1}$ , while the dashed lines are for the case  $\lambda_3 = 0 \text{ fm}^{-1}$ . Not shown are the curves for F-QMC without overlap, as they have identical species fractions.

values are consistent with the fact that the  $\Lambda$  is bound in the 1*s* state in Pb by around 26 MeV [68], as well as with the recent observation of a  $\Xi$  weakly bound to <sup>14</sup>N [56,65,69].

### C. QMC EoS

The low density crustal region enveloping the core of a NS is populated by nuclei with increasing neutron excess [70–72]. Here the QMC EoS, which is an appropriate description of nuclear matter, is matched onto the low density EoS provided by Hempel and Schaffner-Bielich [73,74]. The Hempel and Schaffner-Bielich model is a relativistic mean field model for interacting nucleons which takes into account excluded volume effects. In what follows the QMC EoS is matched to the crust region at  $n \approx 0.7n_0$ , which is appropriate in describing the transition of nuclei to nuclear matter at the crust-core boundary.

The derived F-QMC (solid) and N-QMC (dashed) EoSs, with crust, are shown in Fig. 2. Note that in all figures, unless

TABLE I. The chemical potentials,  $\mu_i$  in MeV, for each baryon at saturation density, with and without overlap.

F-QMC	п	р	Λ	$\Xi^0$	$\Xi^-$	
	$\lambda_3 = 0.02~\mathrm{fm}^{-1}$					
Overlap	970	857	1076	1300	1326	
No overlap	970	857	1076	1300	1326	
	$\lambda_3 = 0 \text{ fm}^{-1}$					
Overlap	970	856	1080	1298	1333	
No overlap	970	856	1080	1298	1333	

otherwise stated, the overlap case corresponds to b = 0.5 fm and  $E_0 = 5500$  MeV. As expected, the hyperons soften the EoS when compared to that for nucleons only. While the inclusion of the overlap term has no influence on the nuclear matter parameters at saturation density, it is clear that the EoS is significantly stiffer at high density. In Fig. 2 we see that the effect of the overlap term becomes considerable at energy densities of order 250–350 MeV fm<sup>-3</sup>, or  $n_B > 2n_0$ . Furthermore, the degree of softening induced by the hyperons is reduced at higher densities. Comparing the two panels in Fig. 2, the cubic term in  $V(\sigma)$  [see Fig. 2(a)], acts to soften the EoS, whether hyperons are included or not.

In order to make the QMC EoS generated here (without crust) widely available, an analytic function has been fitted to the F-QMC with overlap EoS for  $\lambda_3 = 0.02$  fm<sup>-1</sup> and  $\lambda_3 = 0$  fm<sup>-1</sup>. This is valid for the energy density range 0–1600 MeV fm<sup>-3</sup> but must be matched to a crust EoS for  $n < 0.7n_0$ , corresponding to an energy density  $\epsilon \approx 105$ MeV fm<sup>-3</sup>. Equation (23) takes the argument for energy density in MeV fm<sup>-3</sup> and gives the pressure in MeV fm<sup>-3</sup>:

$$P(\epsilon) = N_1 \epsilon^{p_1} + N_2 \epsilon^{-p_2}.$$
(23)

The error computed is given by

RMSE (%) = 
$$\sqrt{\frac{1}{N} \sum_{i}^{N} \frac{(x_i - y_i)^2}{{y_i}^2} \times 100},$$
 (24)

and the parameters are summarized in Table II, with the energy density split into different regions. The relative mean squared error (RMSE) estimates the difference between the model's prediction  $(x_i)$ , as cited in Eq. (23), and that theorized by QMC  $(y_i)$ . Equation (23) is not suitable for

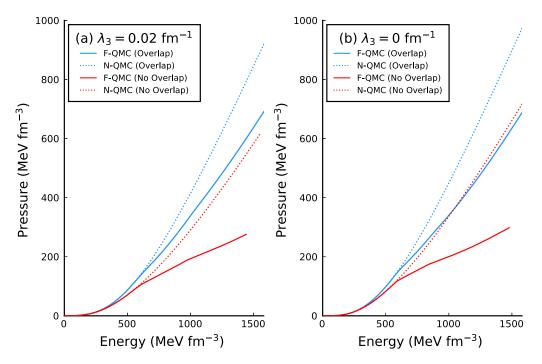


FIG. 2. The EoS for the models considered here, where N-QMC includes only nucleons and F-QMC includes hyperons. (a) corresponds to  $\lambda_3 = 0.02 \text{ fm}^{-1}$ , whilst the right panel (b) is  $\lambda_3 = 0 \text{ fm}^{-1}$ .

computing the speed of sound [Eq. (29)] as the domain boundaries do not precisely correspond to the appearance of new species.

# D. NS bulk properties

The Tolman-Oppenheimer-Volkoff (TOV) equation was used to compute the mass and radius of the NS. Assuming that the NS is nonrotating and spherically symmetric, the TOV equation is

$$\frac{dp}{dr} = -\frac{[p(r) + \epsilon(r)][M(r) + 4\pi r^3 p(r)]}{r[r - 2M(r)]},$$
(25)

where

$$M(r) = 4\pi \int_0^r \epsilon(r')(r')^2 dr'.$$
 (26)

TABLE II. Parameters for Eq. (23), corresponding to the F-QMC with overlap EoS for  $\lambda_3 = 0.02 \text{ fm}^{-1}$  and  $\lambda_3 = 0 \text{ fm}^{-1}$ . The domains for the energy density,  $\epsilon$  (MeV fm<sup>-3</sup>), have been split as denoted by the left-hand column.

$\epsilon$	$N_1$	$p_1$	$N_2$	$p_2$	RMSE	
	F-QMC with $\lambda_3 = 0.02 \text{ fm}^{-1}$					
0–34	$7.733 \times 10^{-4}$	1.203			1.49%	
35-90	$6.171 \times 10^{-7}$	3.043	1.578	1.128	0.64%	
91–133	$3.309 \times 10^{-7}$	3.186			0.10%	
134-298	$1.260 \times 10^{-6}$	2.921			1.09%	
299-550	$5.884 \times 10^{-6}$	2.656			1.16%	
551-620	$8.387 \times 10^{-5}$	2.234			0.13%	
621-1021	$1.730 \times 10^{-3}$	1.764			0.10%	
1022-1600	$8.269 \times 10^{-3}$	1.539			0.11%	
			F-QMC with $\lambda_3 = 0 \text{ fm}^{-1}$			
0–24	$9.725 \times 10^{-4}$	1.183			4.04%	
25-90	$6.234 \times 10^{-8}$	3.498	$1.549 \times 10^{-2}$	-0.3064	1.10%	
91-162	$1.376 \times 10^{-7}$	3.355			0.66%	
163-299	$6.243 \times 10^{-7}$	3.064			1.02%	
300-549	$7.150 \times 10^{-6}$	2.645			1.59%	
550-595	$1.143 \times 10^{-4}$	2.204			0.06%	
596-861	$4.517 \times 10^{-3}$	1.629			0.05%	
862-1600	$8.295 \times 10^{-3}$	1.538			0.25%	

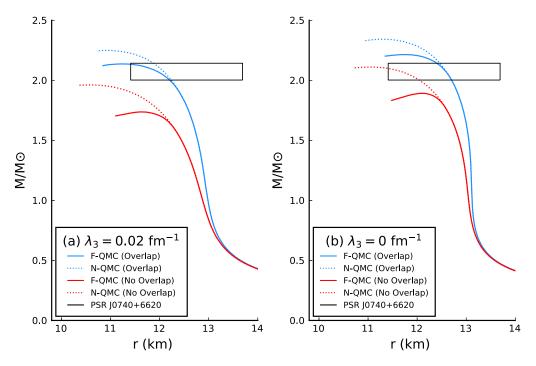


FIG. 3. The mass-radius sequence of stars computed from QMC EoS shown in Fig. 2. The box constraint (black) is PSR J0740+6620, extracted from Ref. [21]. In (a)  $\lambda_3 = 0.02 \text{ fm}^{-1}$  and has a reduced maximum mass compared to (b) when  $\lambda_3 = 0 \text{ fm}^{-1}$ .

The central pressure is chosen at r = 0 and integrated outwards until p(R) = 0, where *R* is the final radius of the star. This process is repeated for different central pressures to form the sequence of stars plotted in Fig. 3. The box denotes the constraint corresponding to pulsar PSR J0740+6620,  $M = 2.072^{+0.067}_{-0.066} M_{\odot}$  and  $R = 12.39^{+1.30}_{-0.98}$  km. The total baryon number is given by

$$A = \int_0^R \frac{4\pi r^2 n_B(r)}{\left(1 - \frac{2GM(r)}{r}\right)^{\frac{1}{2}}} dr.$$
 (27)

GW 170817 is a binary system with a total mass of  $2.73^{+0.04}_{-0.01} M_{\odot}$  [24]. The component masses in the low spin case are  $m_1 \in [1.18, 1.36] M_{\odot}$  and  $m_2 \in [1.36, 1.58] M_{\odot}$ . The tidal deformability may be computed as

$$\Lambda_M = \frac{2}{3} k_2 \left(\frac{R}{M}\right)^5. \tag{28}$$

The dimensionless constant,  $k_2$ , is the tidal Love number and the full equation is given in Refs. [27,75]. Equation (28) gives the one-sided tidal deformation, which is constrained by GW170817 for a  $1.4M_{\odot}$  star to lie in the range  $\Lambda_{1.4} = 190^{+390}_{-120}$ [24]. This is shown as a black line in Fig. 4. However, we note that other work has reported that the upper limit on  $\Lambda_{1.4}$  could be as large as 800 [76].

### 1. Mass-radius relation

Table III summarizes the properties of a NS with maximum mass,  $M_{\text{max}}$ , predicted by QMC. All entries are for the case where hyperons are included, unless otherwise indicated. The overlap parameters alter the NS properties in predictable ways. Increasing  $E_0$  has mild effects on the maximum mass of the star, with little change to its radius. The range parameter

TABLE III. Macroscopic properties of the NS (including hyperons unless otherwise indicated) are computed with variations of the overlap parameters. The range parameter *b* and overlap energy  $E_0$ have units of fm and MeV, respectively. The results summarize the maximum mass ( $M_{\text{max}}$ ,  $M_{\odot}$ ), total baryon number (A, 10<sup>57</sup>) and central number density ( $n_c$ , fm<sup>-3</sup>), central pressure ( $P_c$ , MeV fm<sup>-3</sup>), and central energy density ( $\epsilon_c$ , MeV fm<sup>-3</sup>) for each parameter set used.

b	$E_0$	M <sub>max</sub>	Α	$n_c$	$P_{c}$	$\epsilon_{c}$
		λ3	= 0.02  fr	$n^{-1}$		
0.4	3500	1.77	2.41	1.01	256	1191
0.4	4500	1.78	2.43	1.02	268	1205
0.4	5500	1.79	2.45	1.03	280	1219
0.5	3500	2.02	2.82	1.02	408	1260
0.5	4500	2.08	2.92	1.01	448	1258
0.5	5500	2.14	3.02	1.00	492	1267
0.5 <sup>a</sup>	5500	2.25	3.22	1.00	680	1314
b	0	1.74	2.36	0.961	213	1111
с	0	1.96	2.74	1.15	559	1461
		;	$\lambda_3 = 0 \text{ fm}^3$	-1		
0.4	3500	1.91	2.63	0.883	221	1030
0.4	4500	1.92	2.64	0.900	231	1055
0.4	5500	1.92	2.65	0.915	242	1078
0.5	3500	2.11	2.97	0.906	336	1102
0.5	4500	2.17	3.05	0.923	387	1144
0.5	5500	2.21	3.14	0.903	406	1123
0.5	5500	2.34	3.37	0.934	643	1222
	0	1.89	2.60	0.867	201	1005
	0	2.11	2.97	1.03	529	1305

<sup>a</sup>N-QMC (overlap).

<sup>b</sup>F-QMC (no overlap).

<sup>c</sup>N-QMC (no overlap).

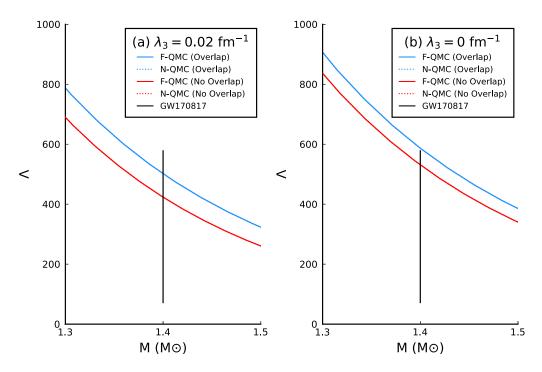


FIG. 4. Tidal deformability for the QMC EoS when (a)  $\lambda_3 = 0.02 \text{ fm}^{-1}$  and (b)  $\lambda_3 = 0 \text{ fm}^{-1}$ . Given that no hyperons appear in a star of mass 1.4 $M_{\odot}$  the solid (F-QMC) and dashed (N-QMC) lines are superimposed. GW170817 is shown by the solid black line and extracted from [24].

*b*, however, raises the mass substantially as it is increased. For b = 0.4 fm and  $\lambda_3 = 0.02$  fm<sup>-1</sup>, the maximum mass is predicted to be  $M_{\text{max}} < 2M_{\odot}$ , which is unsatisfactory.

Table IV reflects the central properties of different mass NSs predicted for F-QMC with overlap. The central density, pressure, and energy densities are all greater when  $\lambda_3 = 0.02 \text{ fm}^{-1}$ , for all masses. For a star of mass  $M = 1.4M_{\odot}$ , the number density is lower than the threshold density for hyperons and hence there are no hyperons in these stars. However, as the density increases the star's core is then populated by hyperonic matter.

TABLE IV. The central number density  $(n_c, \text{ fm}^{-3})$ , pressure  $(P_c, \text{ MeV fm}^{-3})$  and energy density  $(\epsilon_c, \text{ MeV fm}^{-3})$  for different mass stars  $(M_{\odot})$  predicted by F-QMC with overlap  $(E_0 = 5500 \text{ MeV}, b = 0.5 \text{ fm})$ .

Mass	$n_c$	$P_c$	$\epsilon_c$		
	$\lambda_3=0.02~\text{fm}^{-1}$				
1.0	0.341	30	336		
1.4	0.427	59	432		
1.6	0.482	86	496		
1.8	0.547	124	576		
2.0	0.663	196	733		
		$\lambda_3=0~\text{fm}^{-1}$			
1.0	0.317	27	311		
1.4	0.395	53	397		
1.6	0.438	74	447		
1.8	0.487	102	507		
2.0	0.564	154	607		

In Fig. 3, the mass-radius curve shows that the overlap term is essential in predicting a heavy NS,  $M > 2M_{\odot}$ , once the incompressibility is reduced to the preferred range [i.e., with  $\lambda_3 = 0.02 \text{ fm}^{-1}$  in Fig. 3(a)]. Without the overlap term, the mass of the star is significantly lower. The inclusion of the  $\sigma^3$  term acts to reduce the radius and lower the star's mass. The mass reduction is caused by the additional scalar meson attraction, with a consequent softening of the EoS. For  $\lambda_3 = 0.02 \text{ fm}^{-1}$ , the radius of the star slightly increases as the mass decreases from  $1.5M_{\odot}$  to  $1.0M_{\odot}$ , in contrast to the case  $\lambda_3 = 0 \text{ fm}^{-1}$ , where there are no significant changes to the radius. In the phenomenologically interesting region,  $M \approx 1.4M_{\odot}$ , the radius of the star is significantly lower when  $\lambda_3 = 0.02 \text{ fm}^{-1}$ .

The presence of hyperons reduces the maximum mass, as well as increasing the radius at maximum mass. From Fig. 3 we see that the overlap term decreases the radius at maximum mass for F-QMC, whereas for N-QMC the radius is increased at maximum mass when the overlap term is present.

Figure 4 illustrates the tidal deformability for the QMC EoS. The dashed line for the nucleon only case cannot be distinguished from F-QMC because the QMC model predicts no hyperons in a  $1.4M_{\odot}$  star. For  $\lambda_3 = 0.02$  fm<sup>-1</sup> the tidal deformation is acceptable with and without overlap. The introduction of the overlap term does causes an increase in the radius around the canonical mass, and this in turn increases the tidal deformability.

### E. Speed of sound

In the absence of direct observations of the composition of the core of a NS, theoretical calculations of the speed of sound

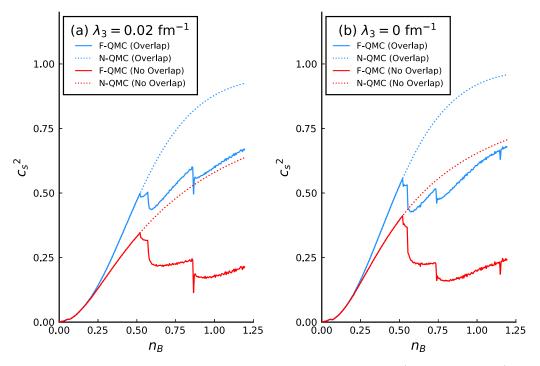


FIG. 5. The speed of sound  $(c_s^2)$  is given as a fraction of the speed of light for (a)  $\lambda_3 = 0.02$  fm<sup>-1</sup> and (b)  $\lambda_3 = 0$  fm<sup>-1</sup>. The number density  $n_B$  is given in fm<sup>-3</sup>. Nontrivial drops in  $c_s^2$  correspond to the initial appearance of hyperons (see Fig. 1).

(equivalently, polytropic index  $\gamma$ ) do offer valuable insights. It has previously been suggested that nontrivial changes in the EoS correspond to phase changes from hadronic matter to either quark matter [6] or the threshold of creation of hyperons [37].

The EoSs at low and extremely high densities have been extensively studied in effective field theory  $(n_B < 1.1n_0)$  and perturbative QCD ( $n_B > 40n_0$ ), respectively. Annala *et al.* [6] suggest that, for stars with  $M = 1.4M_{\odot}$ , hadronic nuclear theories are suitable in predicting the EoS giving rise to canonical mass stars. The QMC model is consistent with their EoS in that region. However, for  $M > 2M_{\odot}$ , Annala *et al.* suggest that the central density becomes so large that the cores of the stars may be populated by deconfined quark matter and gluons [6]. Quark matter, being conformal and scale invariant, would then have a speed of sound,  $c_s^2 = \frac{1}{3}$ , approaching logarithmically from below as the density increases. On the other hand, it has been shown that this is not a distinct feature of conformal matter [37,38]. Since the QMC model does not contain any elements of deconfined quarks,  $c_s^2 < \frac{1}{3}$  approaching from below, may also be interpreted as the creation of hyperons. Recall that the speed of sound is defined as

$$c_s^2 = \frac{dP}{d\epsilon}.$$
 (29)

Figure 5 shows the three sudden changes in  $c_s^2$  which occur at those number densities where the different species of hyperons first appear (see Fig. 1). In order of appearance, these are the  $\Lambda$ ,  $\Xi^-$ , and  $\Xi^0$ . Without the overlap term, the results reflect those reported by earlier QMC models [37,38]. However, when the overlap terms are included,  $c_s^2$  can be as

large as 0.5 or more. This is consistent with pure hadronic matter, as cited in [6].

#### **IV. CONCLUSION**

The excitement of studying neutron stars is that they contain the most dense matter in the Universe. Thus they may be expected to yield insights into the EoS of strongly interacting matter at densities inaccessible in any other way.

Relativistic descriptions of nuclear matter typically lead to higher values of the incompressibility of nuclear matter,  $K_{\infty}$ , than nonrelativistic Skyrme forces. While the connection between the incompressibility and the energies of GMR is complex, there is a tendency for lower values of  $K_{\infty}$  to be preferred. The energy density functional derived within the QMC model requires a small cubic term involving the scalar field in order to reproduce the observed GMR energies. This in turn lowers the maximum masses of the neutron stars generated by the model.

In order to solve that problem, we have explored the impact on the EoS of dense matter and the properties of neutron stars by introducing a phenomenological repulsive contribution to the energy density as the degree to which the baryonic overlap increases. This new term is designed such that it does not alter the properties of nuclear matter at saturation density.

The effects of this new overlap term are indeed to generate neutron stars with a maximum mass above  $2.1M_{\odot}$ , even when hyperons are included. The radii of stars around  $1.4M_{\odot}$  lie just below 13 km, which is the upper end of the range preferred by the analysis of the gravitational wave data from GW170817.

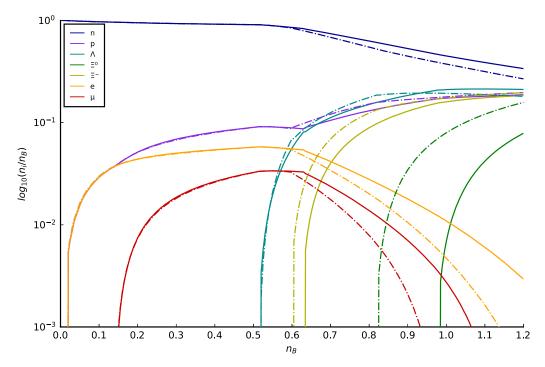


FIG. 6.  $\beta$ -equilibrium was calculated for  $0 < n_B < 1.2 \text{ fm}^{-3}$  with  $G_{\delta} = 0 \text{ fm}^2$ . Only the F-QMC with overlap species fraction is shown with the solid line corresponding the  $\lambda_3 = 0.02 \text{ fm}^{-1}$  and the dashed line showing the case for  $\lambda_3 = 0 \text{ fm}^{-1}$ . The no overlap case is not shown as it produces identical species fractions.

The calculated values of the tidal deformability also lie within the bounds determined from that data. The one qualitative change from earlier work is that in contrast with the values of  $c_s^2$  found with hyperons, which lie below  $\frac{1}{3}$ , with the overlap term they become as large as 0.6. Such large values are consistent with those expected in hadronic models, in contrast with the lower range anticipated for quark matter.

## ACKNOWLEDGMENTS

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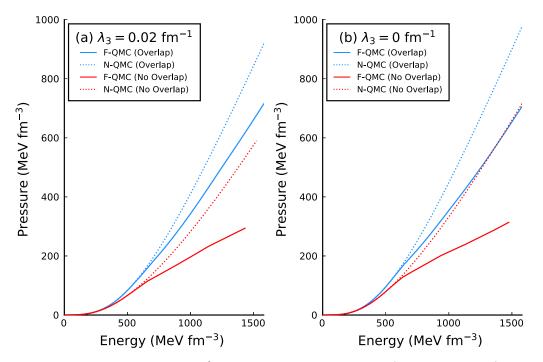


FIG. 7. QMC EoS with  $G_{\delta} = 0 \text{ fm}^2$  for values of  $\lambda_3$  as (a)  $\lambda_3 = 0.02 \text{ fm}^{-1}$  and (b)  $\lambda_3 = 0 \text{ fm}^{-1}$ .

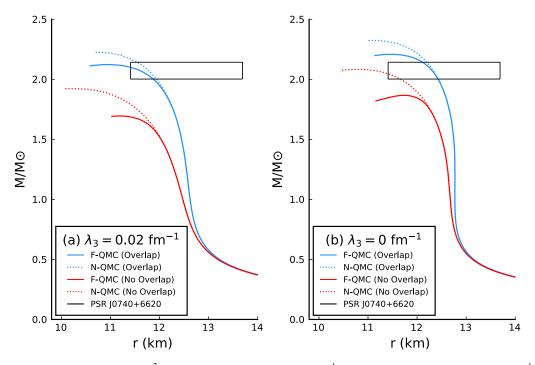


FIG. 8. MR curve for  $G_{\delta} = 0$  fm<sup>2</sup>. The left panel is (a)  $\lambda_3 = 0.02$  fm<sup>-1</sup>, while the right panel is (b)  $\lambda_3 = 0$  fm<sup>-1</sup>.

# APPENDIX: QMC OVERLAP WITH $G_{\delta} = 0$

The results (Sec. III) are replicated with no isovector scalar meson, i.e.,  $G_{\delta} = 0$  fm<sup>2</sup>. As shown by Motta *et al.*, the isovector  $\delta$  increases the maximum mass only marginally but changes the radius significantly [39]. For brevity, not all the results are included; only the preferred values of the overlap parameters are shown ( $E_0 = 5500$  MeV and b = 0.5 fm). One

finds that the largest effect of the change in  $G_{\delta}$  is that the slope of the symmetry energy, *L*, decreases to 53 MeV for  $\lambda_3 = 0.02 \text{ fm}^{-1}$  (52 MeV for  $\lambda_3 = 0 \text{ fm}^{-1}$ ).

The species fractions are given in Fig. 6. The  $\Lambda$  appears sooner without the  $\delta$  meson, while the  $\Xi^{0,-}$  both appear later.

Figure 7 shows the EoS, with the corresponding MR curve in Fig. 8. As expected, there is a slight decrease in the mass

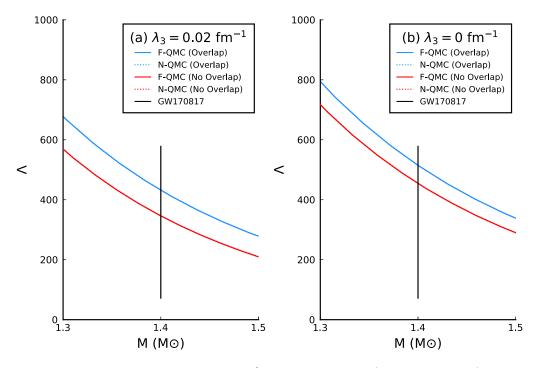


FIG. 9. Tidal deformation with  $G_{\delta} = 0 \text{ fm}^2$  with (a)  $\lambda_3 = 0.02 \text{ fm}^{-1}$  and (b)  $\lambda_3 = 0 \text{ fm}^{-1}$ .

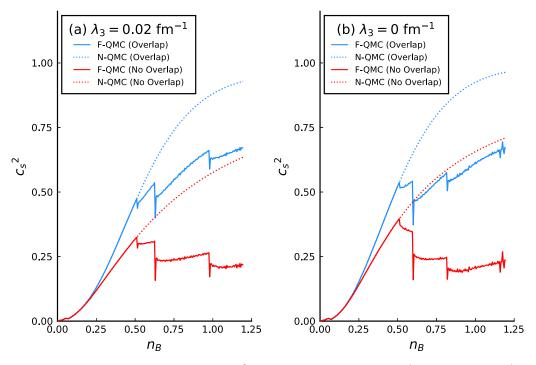


FIG. 10. Speed of sound  $(c_s^2)$  when  $G_{\delta} = 0$  fm<sup>2</sup> with values of (a)  $\lambda_3 = 0.02$  fm<sup>-1</sup> and (b)  $\lambda_3 = 0$  fm<sup>-1</sup>.

and a substantial reduction in the radius. Without overlap and  $\lambda_3$ , the results show the same pattern reported in Ref. [39]. The reduction in the radius has an impact in the tidal deformability, which is presented in Fig. 9. In Fig. 4(b) we saw that the  $\lambda_3 = 0$  fm<sup>-1</sup> tidal deformation lay at the upper end of the experimental bound reported in Ref. [24]. However, with the

reduction of the radii resulting when the  $\delta$  meson is excluded, QMC with overlap now falls within GW170817 constraints. Again the inclusion of the  $\sigma^3$  term with  $\lambda_3 = 0.02$  fm<sup>-1</sup> reduced the radius and fit the GW170817 constraint. Finally the speed of sound is shown in Fig. 10. The interpretation of these results may be found in Sec. III E.

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